Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.1-Binomial-products/1.1.4-Improper/31-1.1.4.3-e-x-^m-a-x^j+b-x^k-^p-c+d-x^n-^q

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [298]. This is test number [31].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (298)	0.00 (0)
Mathematica	99.33 (296)	0.67 (2)
Fricas	92.95 (277)	7.05 (21)
Maple	92.28 (275)	7.72 (23)
Giac	76.17 (227)	23.83 (71)
Maxima	71.14 (212)	28.86 (86)
Mupad	66.11 (197)	33.89 (101)
Sympy	46.31 (138)	53.69 (160)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

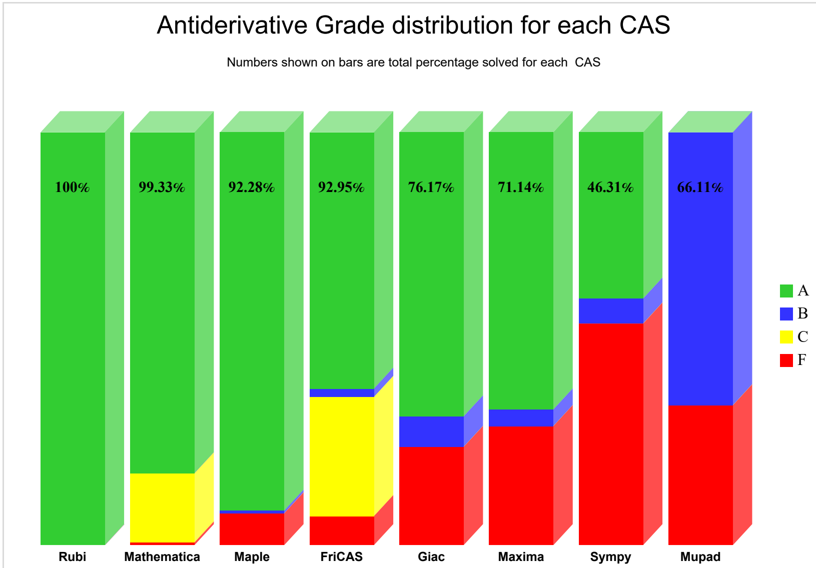
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

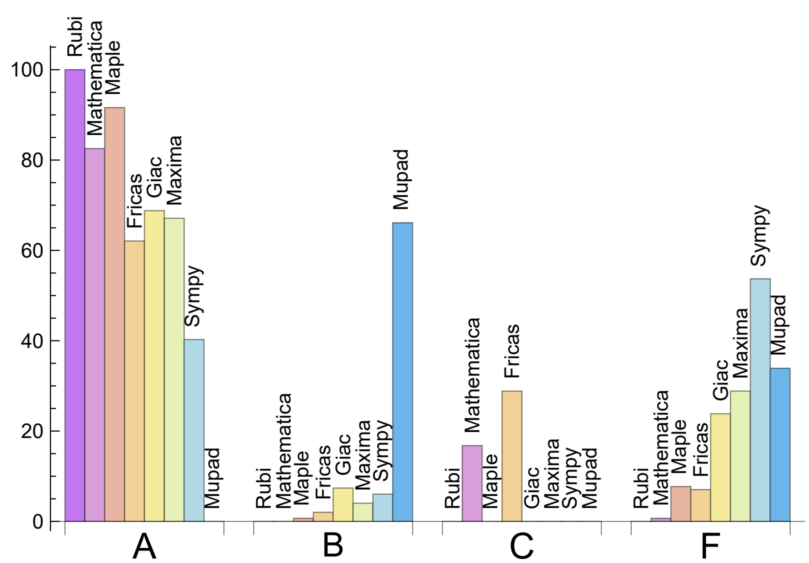
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Maple	91.611	0.671	0.000	7.718
Mathematica	82.550	0.000	16.779	0.671
Giac	68.792	7.383	0.000	23.826
Maxima	67.114	4.027	0.000	28.859
Fricas	62.081	2.013	28.859	7.047
Sympy	40.268	6.040	0.000	53.691
Mupad	0.000	66.107	0.000	33.893

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	2	100.00	0.00	0.00
Fricas	21	100.00	0.00	0.00
Maple	23	100.00	0.00	0.00
Giac	71	100.00	0.00	0.00
Maxima	86	100.00	0.00	0.00
Mupad	101	0.00	100.00	0.00
Sympy	160	69.38	30.62	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.24
Fricas	0.28
Rubi	0.30
Giac	0.36
Maple	1.80
Mathematica	1.88
Sympy	2.81
Mupad	5.66

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	97.26	0.83	87.50	0.90
Maple	122.09	0.90	95.00	0.89
Maxima	124.24	1.08	84.00	0.99
Rubi	142.80	1.00	102.00	1.00
Sympy	145.19	1.66	80.00	1.18
Giac	145.42	1.27	103.00	1.04
Mupad	151.83	1.14	76.00	0.96
Fricas	211.97	1.44	106.00	1.10

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

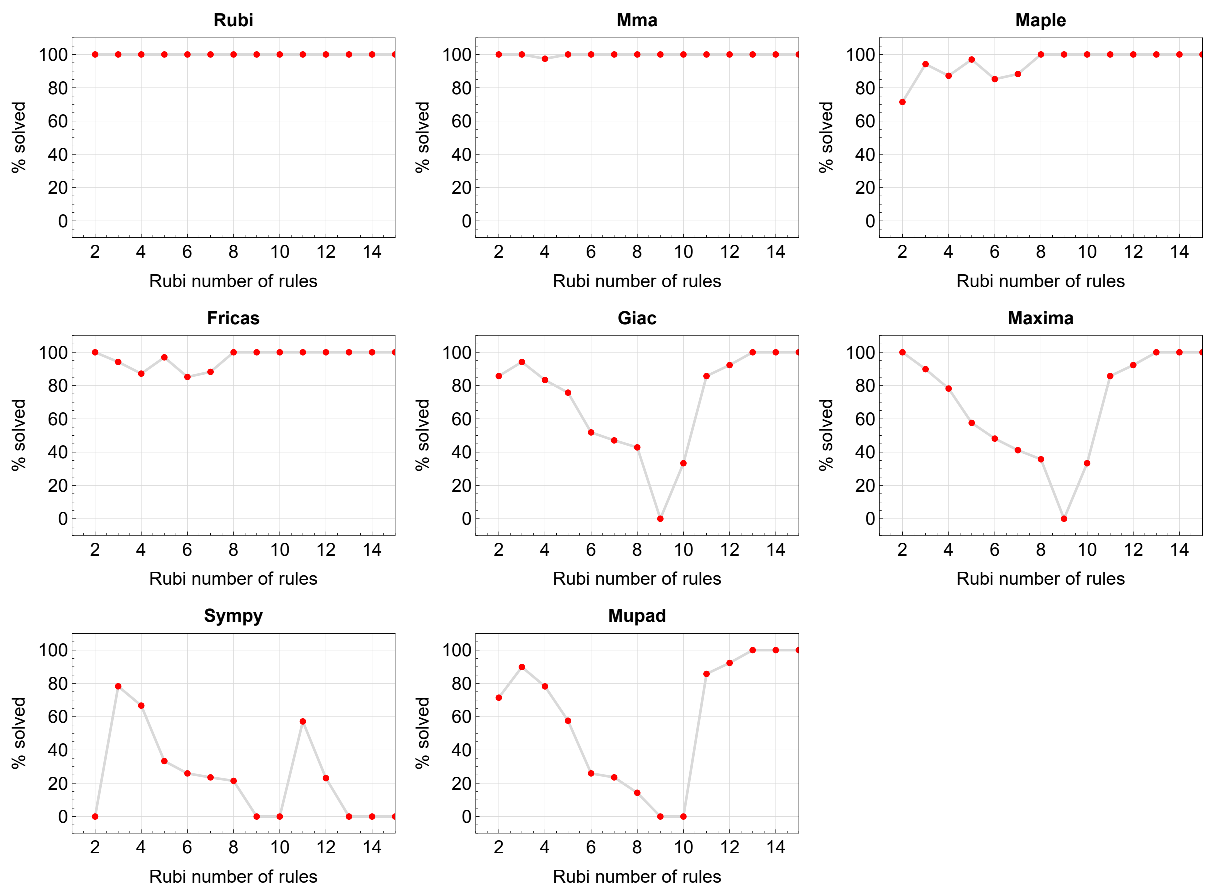


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

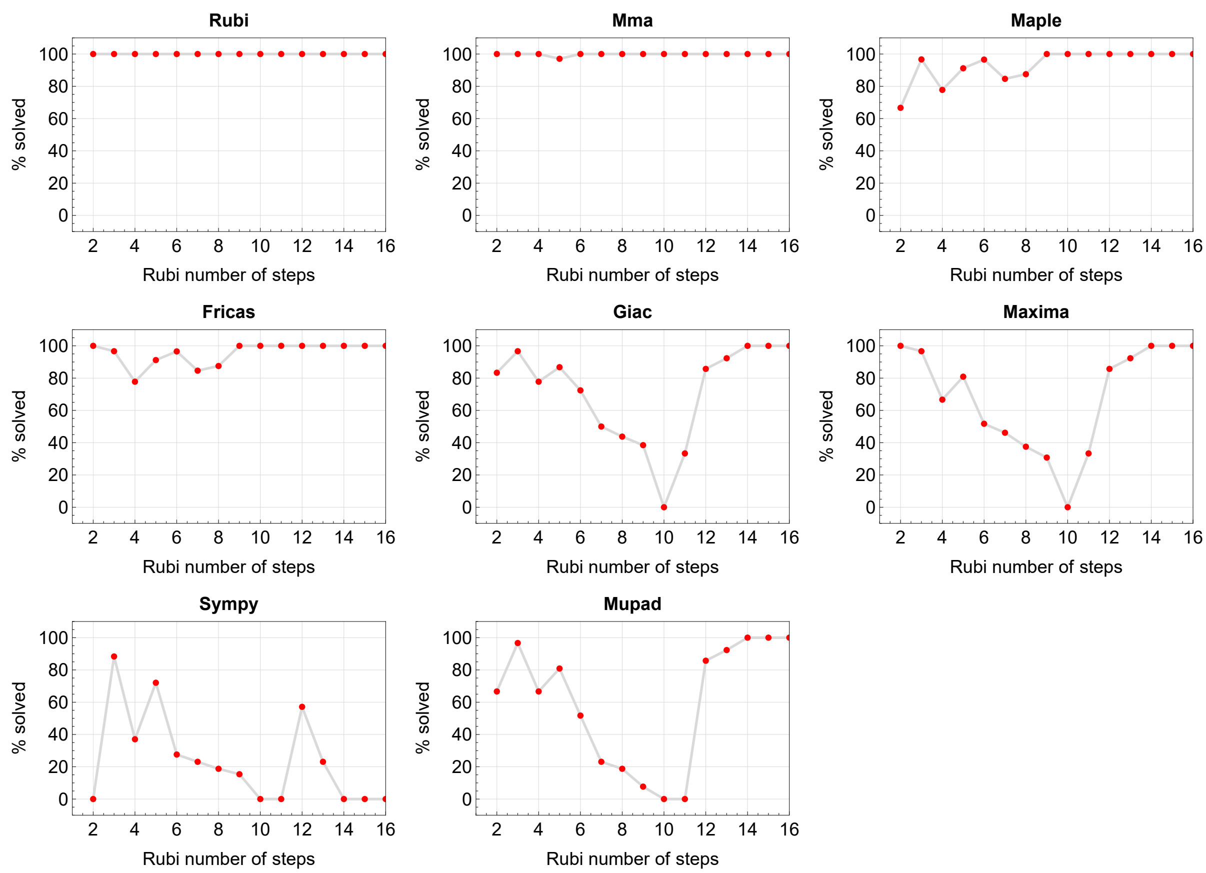


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

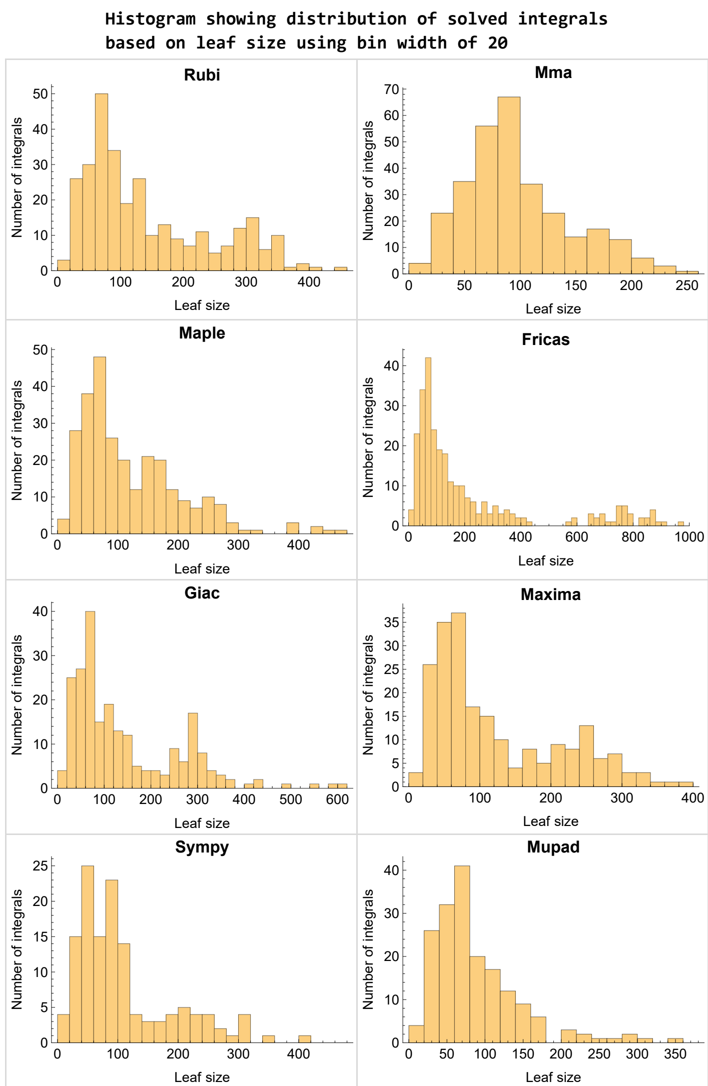


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

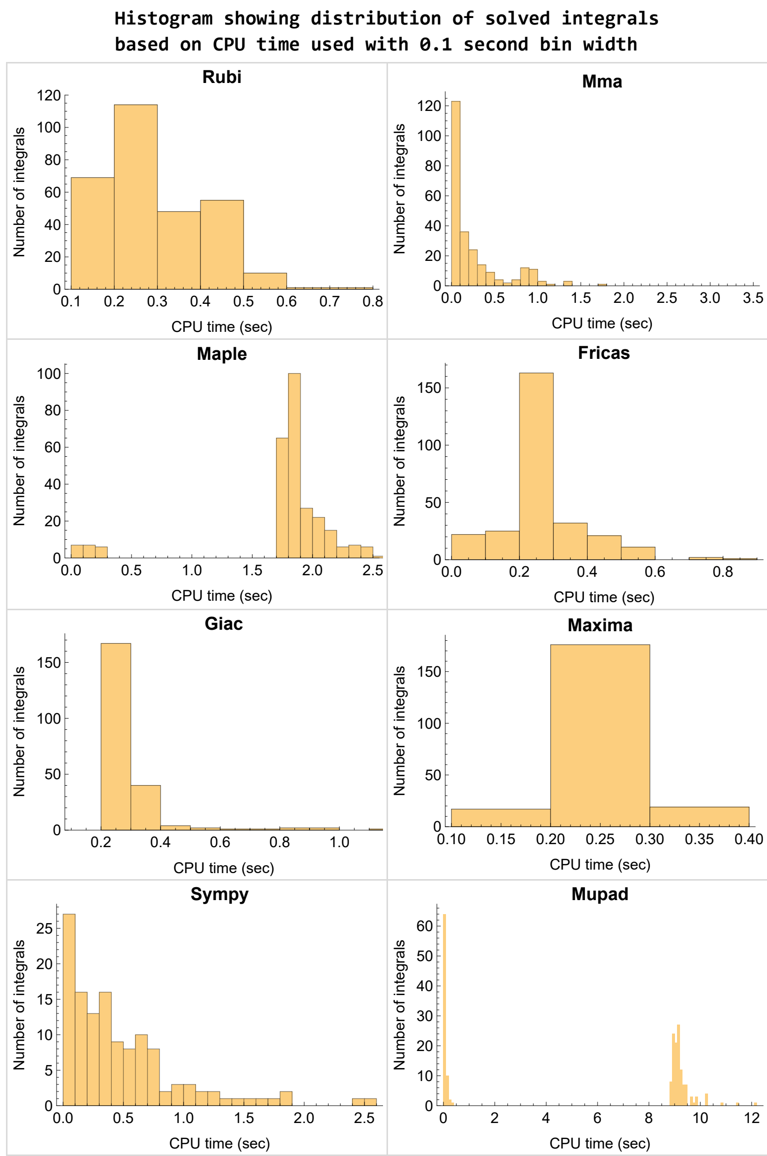


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

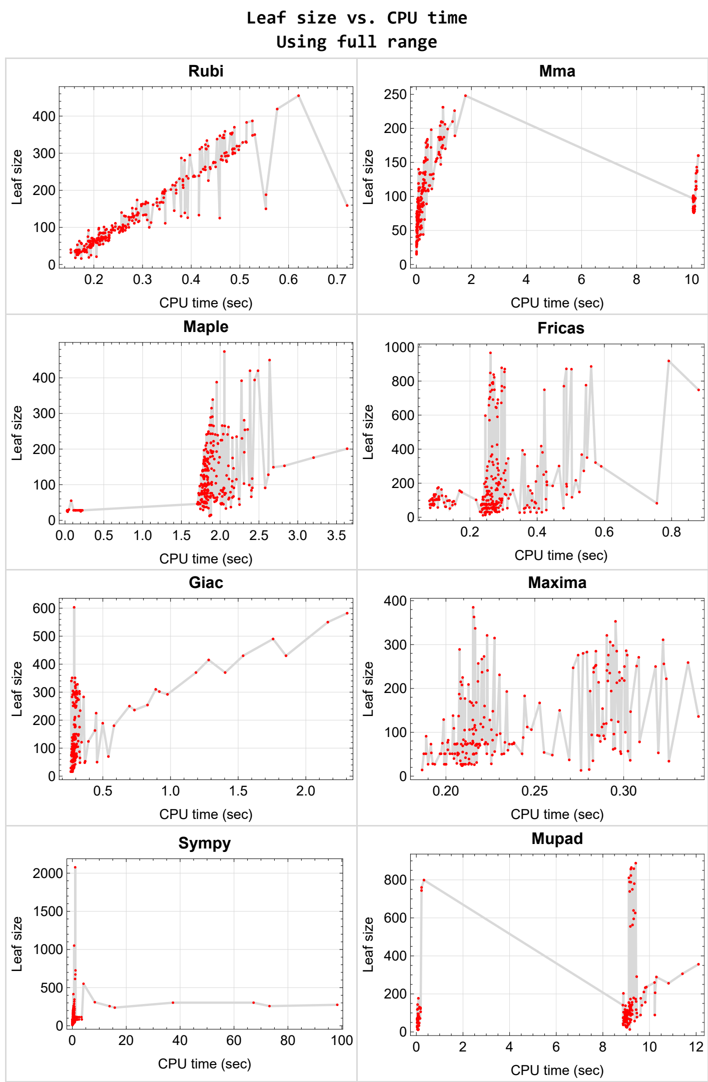


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

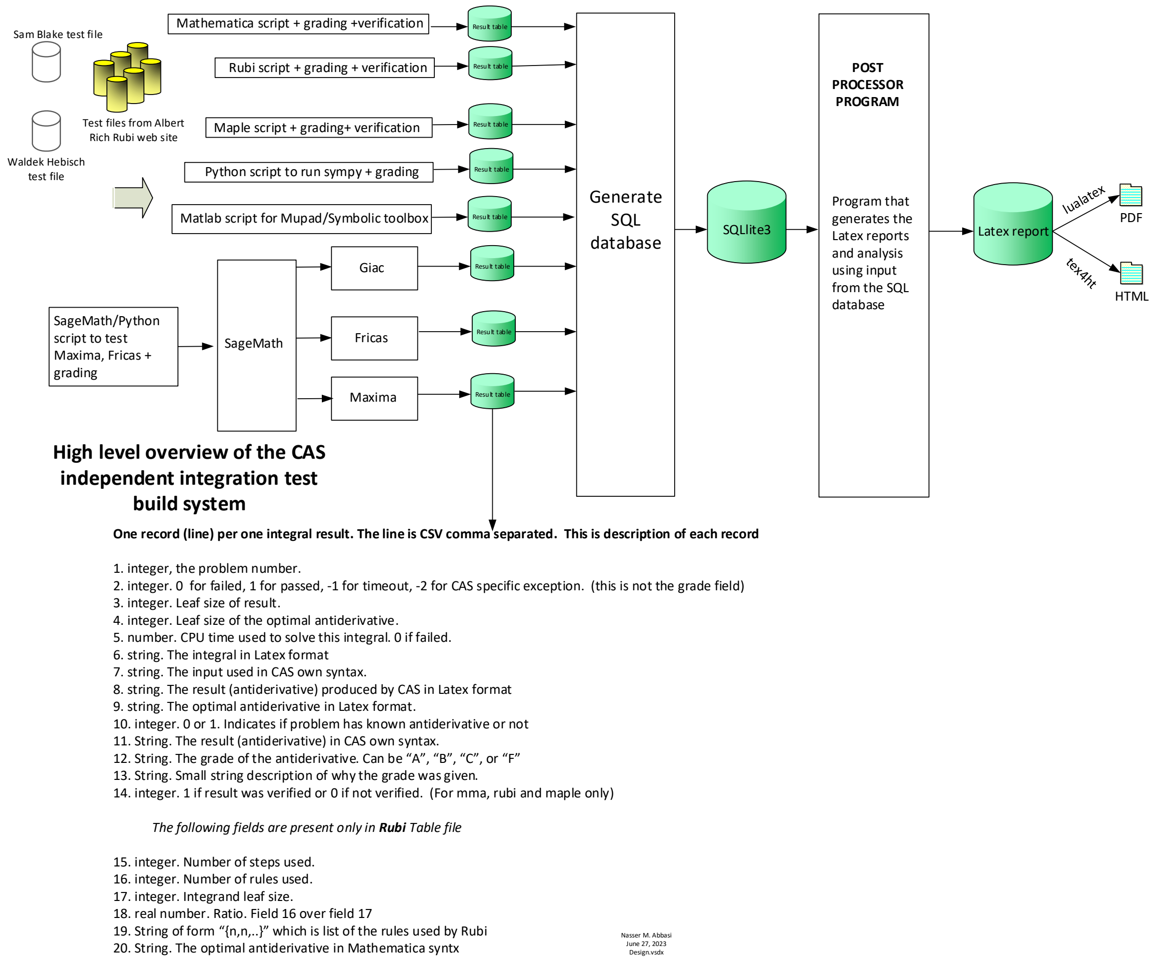
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298 }

B grade { }

C grade { 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 283 }

F normal fail { 284, 292 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 271, 278, 279, 280, 281 }

B grade { 269, 270 }

C grade { }

F normal fail { 272, 273, 274, 275, 276, 277, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 275, 278, 279, 280, 281 }

B grade { 84, 86, 269, 270, 271, 283 }

C grade { 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268 }

F normal fail { 272, 273, 274, 276, 277, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 95, 97, 98, 99, 100, 101, 102, 106, 109, 110, 111, 112, 119, 120, 121, 122, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 269, 270, 271, 275, 278, 280, 281 }

B grade { 91, 96, 107, 108, 113, 114, 115, 116, 117, 118, 279, 283 }

C grade { }

F normal fail { 103, 104, 105, 123, 124, 125, 126, 127, 128, 129, 142, 143, 144, 156, 157, 158, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 276, 277, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 278, 279, 280, 281 }

B grade { 28, 95, 96, 97, 98, 99, 113, 114, 115, 116, 117, 118, 135, 136, 137, 150, 151, 152, 269, 270, 271, 283 }

C grade { }

F normal fail { 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 275, 276, 277, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 96, 97, 98, 99, 100, 101, 102, 103, 108, 114, 115, 116, 117, 118, 119, 120, 121, 122, 133, 134, 135, 136, 137, 138, 139, 140, 141, 148, 149, 150, 151, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 269, 270, 271, 278, 279, 280, 281 }

C grade { }

F normal fail { }

F(-1) timedout fail { 94, 95, 104, 105, 106, 107, 109, 110, 111, 112, 113, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 142, 143, 144, 145, 146, 147, 156, 157, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 275, 276, 277, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 46, 47, 48, 50, 53, 55, 57, 58, 59, 61, 62, 63, 64, 65, 67, 68, 69, 71, 73, 74, 75, 76, 78, 79, 80, 81, 82, 84, 86, 87, 88, 89, 90, 91, 92, 106, 107, 108, 109, 130, 131, 132, 133, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 186, 187, 188, 189, 190, 191, 192, 278, 279, 280, 281 }

B grade { 28, 43, 45, 49, 51, 52, 54, 56, 60, 66, 70, 72, 77, 83, 85, 269, 270, 271 }

C grade { }

F normal fail { 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 234, 235, 236, 237, 238, 239, 240, 241, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 262, 263, 264, 265, 266, 267, 272, 273, 274, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

F(-1) timedout fail { 183, 184, 185, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 230, 231, 232, 233, 242, 243, 244, 245, 246, 257, 258, 259, 260, 261, 268, 275, 276, 277, 283 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.159	0.009	0.166	0.211	0.398	0.015	0.276	0.045

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	37	33	28	27	27	29	29	28
N.S.	1	1.12	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.178	0.009	0.145	0.199	0.362	0.016	0.276	0.040

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.185	0.006	0.133	0.214	0.348	0.017	0.269	9.028

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	34	33	28	27	27	29	29	28
N.S.	1	1.03	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.178	0.009	0.129	0.210	0.244	0.017	0.276	0.039

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.89
time (sec)	N/A	0.168	0.006	0.203	0.209	0.246	0.017	0.272	0.038

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	32	29	28	28	25	27	30	26
N.S.	1	1.10	1.00	0.97	0.97	0.86	0.93	1.03	0.90
time (sec)	N/A	0.171	0.011	0.111	0.193	0.246	0.046	0.283	9.003

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	24	28	20	23	24
N.S.	1	1.00	1.00	0.92	0.92	1.08	0.77	0.88	0.92
time (sec)	N/A	0.164	0.010	0.034	0.209	0.245	0.044	0.279	9.031

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	30	29	26	28	30	26	42	25
N.S.	1	1.03	1.00	0.90	0.97	1.03	0.90	1.45	0.86
time (sec)	N/A	0.168	0.014	0.036	0.226	0.252	0.093	0.295	0.044

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	27	25	26	29	27	28	26
N.S.	1	1.00	1.04	0.96	1.00	1.12	1.04	1.08	1.00
time (sec)	N/A	0.162	0.013	0.033	0.216	0.246	0.099	0.274	0.036

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	33	31	26	30	31	29	39	29
N.S.	1	1.14	1.07	0.90	1.03	1.07	1.00	1.34	1.00
time (sec)	N/A	0.174	0.019	0.027	0.213	0.380	0.196	0.286	0.053

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	33	28	29	29	32	31	29
N.S.	1	1.00	1.06	0.90	0.94	0.94	1.03	1.00	0.94
time (sec)	N/A	0.166	0.013	0.028	0.206	0.414	0.188	0.286	0.039

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	56	53	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	1.02	0.96	0.93
time (sec)	N/A	0.216	0.012	1.787	0.207	0.405	0.024	0.278	0.061

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	59	55	52	51	51	53	53	51
N.S.	1	1.07	1.00	0.95	0.93	0.93	0.96	0.96	0.93
time (sec)	N/A	0.213	0.009	1.772	0.216	0.361	0.022	0.279	0.049

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	56	53	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	1.02	0.96	0.93
time (sec)	N/A	0.198	0.012	1.797	0.204	0.258	0.025	0.272	0.051

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	46	51	52	51	51	53	53	51
N.S.	1	1.10	1.21	1.24	1.21	1.21	1.26	1.26	1.21
time (sec)	N/A	0.202	0.014	1.764	0.233	0.245	0.022	0.265	0.048

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96
time (sec)	N/A	0.194	0.010	1.793	0.260	0.251	0.022	0.457	0.047

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	50	51	51	52	49	49	53	48
N.S.	1	1.16	1.19	1.19	1.21	1.14	1.14	1.23	1.12
time (sec)	N/A	0.187	0.019	1.714	0.212	0.237	0.061	0.370	0.042

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	49	48	53	48	48	48
N.S.	1	1.00	1.00	1.02	1.00	1.10	1.00	1.00	1.00
time (sec)	N/A	0.191	0.021	1.724	0.207	0.242	0.059	0.366	0.051

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	52	49	48	52	54	48	70	48
N.S.	1	1.02	0.96	0.94	1.02	1.06	0.94	1.37	0.94
time (sec)	N/A	0.199	0.028	1.807	0.217	0.245	0.114	0.541	0.048

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	50	46	50	52	51	50	50
N.S.	1	1.00	1.04	0.96	1.04	1.08	1.06	1.04	1.04
time (sec)	N/A	0.191	0.022	1.726	0.231	0.254	0.123	0.277	0.050

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	52	50	46	54	55	51	72	51
N.S.	1	1.02	0.98	0.90	1.06	1.08	1.00	1.41	1.00
time (sec)	N/A	0.199	0.032	1.743	0.255	0.391	0.281	0.284	9.035

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	51	53	54	53	50
N.S.	1	1.00	1.00	0.94	1.06	1.10	1.12	1.10	1.04
time (sec)	N/A	0.189	0.023	1.713	0.202	0.478	0.313	0.289	9.035

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	55	53	46	55	55	56	66	51
N.S.	1	1.08	1.04	0.90	1.08	1.08	1.10	1.29	1.00
time (sec)	N/A	0.201	0.030	1.705	0.209	0.365	0.529	0.289	9.174

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	59	48	53	53	58	55	52
N.S.	1	1.00	1.11	0.91	1.00	1.00	1.09	1.04	0.98
time (sec)	N/A	0.197	0.020	1.729	0.207	0.311	0.570	0.291	9.001

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	76	73	73	80	77	69
N.S.	1	1.00	1.00	1.01	0.97	0.97	1.07	1.03	0.92
time (sec)	N/A	0.221	0.016	1.781	0.219	0.262	0.026	0.291	0.039

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	72	69	76	73	73	82	77	69
N.S.	1	1.06	1.01	1.12	1.07	1.07	1.21	1.13	1.01
time (sec)	N/A	0.231	0.021	1.767	0.224	0.251	0.028	0.286	0.032

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	76	73	73	82	77	69
N.S.	1	1.00	1.00	1.01	0.97	0.97	1.09	1.03	0.92
time (sec)	N/A	0.211	0.013	1.792	0.215	0.260	0.028	0.282	0.032

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	46	69	76	73	73	80	77	69
N.S.	1	1.10	1.64	1.81	1.74	1.74	1.90	1.83	1.64
time (sec)	N/A	0.193	0.021	1.773	0.219	0.270	0.028	0.288	0.032

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	70	70	76	73	65
N.S.	1	1.00	1.00	1.04	1.00	1.00	1.09	1.04	0.93
time (sec)	N/A	0.210	0.013	1.794	0.217	0.233	0.024	0.281	0.031

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	63	71	76	74	71	80	78	67
N.S.	1	1.05	1.18	1.27	1.23	1.18	1.33	1.30	1.12
time (sec)	N/A	0.195	0.023	1.722	0.206	0.248	0.079	0.292	0.037

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	71	69	75	68	70	65
N.S.	1	1.00	1.00	1.09	1.06	1.15	1.05	1.08	1.00
time (sec)	N/A	0.205	0.026	1.730	0.209	0.245	0.077	0.286	0.034

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	73	73	73	74	77	78	97	67
N.S.	1	1.03	1.03	1.03	1.04	1.08	1.10	1.37	0.94
time (sec)	N/A	0.223	0.033	1.764	0.212	0.238	0.128	0.269	0.041

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	71	70	73	75	75	74	68
N.S.	1	1.00	1.03	1.01	1.06	1.09	1.09	1.07	0.99
time (sec)	N/A	0.219	0.026	1.909	0.206	0.256	0.138	0.267	0.058

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	73	73	68	76	76	75	98	76
N.S.	1	1.01	1.01	0.94	1.06	1.06	1.04	1.36	1.06
time (sec)	N/A	0.221	0.034	1.879	0.238	0.243	0.306	0.276	8.875

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	64	73	75	78	75	73
N.S.	1	1.00	1.00	0.94	1.07	1.10	1.15	1.10	1.07
time (sec)	N/A	0.208	0.028	1.916	0.206	0.238	0.339	0.279	0.057

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	72	71	64	77	77	78	99	75
N.S.	1	1.01	1.00	0.90	1.08	1.08	1.10	1.39	1.06
time (sec)	N/A	0.221	0.040	2.157	0.233	0.236	0.667	0.284	0.067

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	66	63	73	75	80	77	71
N.S.	1	1.00	1.00	0.95	1.11	1.14	1.21	1.17	1.08
time (sec)	N/A	0.213	0.031	2.116	0.237	0.234	0.769	0.283	8.912

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	66	77	64	77	77	82	90	75
N.S.	1	1.05	1.22	1.02	1.22	1.22	1.30	1.43	1.19
time (sec)	N/A	0.201	0.036	1.873	0.222	0.236	1.272	0.276	0.082

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	66	75	75	83	79	74
N.S.	1	1.00	1.00	0.90	1.03	1.03	1.14	1.08	1.01
time (sec)	N/A	0.211	0.031	1.862	0.214	0.243	2.469	0.276	8.867

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	53	78	66	75	75	83	79	76
N.S.	1	1.08	1.59	1.35	1.53	1.53	1.69	1.61	1.55
time (sec)	N/A	0.173	0.021	1.856	0.198	0.235	3.521	0.291	8.907

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	93	119	123	124	274	204	133	144
N.S.	1	0.78	1.00	1.03	1.04	2.30	1.71	1.12	1.21
time (sec)	N/A	0.228	0.081	1.781	0.297	0.251	0.237	0.297	0.055

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	95	92	92	97	98	94	101	100
N.S.	1	0.99	0.96	0.96	1.01	1.02	0.98	1.05	1.04
time (sec)	N/A	0.266	0.045	1.772	0.231	0.250	0.190	0.292	8.933

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	80	98	99	100	228	180	108	118
N.S.	1	0.82	1.00	1.01	1.02	2.33	1.84	1.10	1.20
time (sec)	N/A	0.216	0.068	1.767	0.301	0.251	0.218	0.301	0.042

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	74	71	73	74	75	70	77	76
N.S.	1	0.99	0.95	0.97	0.99	1.00	0.93	1.03	1.01
time (sec)	N/A	0.233	0.032	1.754	0.238	0.244	0.179	0.276	0.060

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	67	77	75	78	178	153	85	96
N.S.	1	0.87	1.00	0.97	1.01	2.31	1.99	1.10	1.25
time (sec)	N/A	0.207	0.055	1.821	0.309	0.246	0.196	0.271	0.067

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	53	47	50	50	51	46	52	52
N.S.	1	0.98	0.87	0.93	0.93	0.94	0.85	0.96	0.96
time (sec)	N/A	0.206	0.023	1.821	0.216	0.240	0.154	0.271	0.071

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	56	57	51	53	129	90	57	70
N.S.	1	0.97	0.98	0.88	0.91	2.22	1.55	0.98	1.21
time (sec)	N/A	0.181	0.041	1.781	0.319	0.261	0.178	0.289	8.986

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	34	31	32	31	30	27	32	31
N.S.	1	0.97	0.89	0.91	0.89	0.86	0.77	0.91	0.89
time (sec)	N/A	0.181	0.014	1.745	0.217	0.234	0.130	0.273	0.057

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	34	34	99	82	34	31
N.S.	1	1.00	1.00	0.85	0.85	2.48	2.05	0.85	0.78
time (sec)	N/A	0.158	0.023	1.849	0.325	0.255	0.144	0.274	0.054

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	37	34	33	35	32	26	34	32
N.S.	1	1.09	1.00	0.97	1.03	0.94	0.76	1.00	0.94
time (sec)	N/A	0.186	0.016	2.074	0.208	0.230	0.392	0.288	8.984

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	36	105	82	36	35
N.S.	1	1.00	1.00	0.88	0.86	2.50	1.95	0.86	0.83
time (sec)	N/A	0.186	0.029	1.800	0.304	0.238	0.169	0.280	8.966

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	39	51	103	75	38	33
N.S.	1	1.00	1.00	0.95	1.24	2.51	1.83	0.93	0.80
time (sec)	N/A	0.188	0.028	1.789	0.298	0.240	0.183	0.286	0.112

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	51	49	46	48	47	41	71	46
N.S.	1	1.04	1.00	0.94	0.98	0.96	0.84	1.45	0.94
time (sec)	N/A	0.203	0.025	1.729	0.226	0.268	0.401	0.272	0.108

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	58	60	54	56	135	129	57	53
N.S.	1	0.95	0.98	0.89	0.92	2.21	2.11	0.93	0.87
time (sec)	N/A	0.175	0.058	1.946	0.298	0.254	0.212	0.273	8.991

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	71	70	64	70	73	61	100	70
N.S.	1	1.01	1.00	0.91	1.00	1.04	0.87	1.43	1.00
time (sec)	N/A	0.221	0.033	1.962	0.217	0.237	0.454	0.293	8.940

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	75	78	74	79	184	163	81	70
N.S.	1	0.96	1.00	0.95	1.01	2.36	2.09	1.04	0.90
time (sec)	N/A	0.191	0.055	1.991	0.287	0.446	0.250	0.279	8.942

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	93	96	86	96	98	88	126	92
N.S.	1	1.01	1.04	0.93	1.04	1.07	0.96	1.37	1.00
time (sec)	N/A	0.240	0.042	1.840	0.216	0.384	0.496	0.275	8.944

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	134	123	136	350	238	139	203
N.S.	1	1.00	1.01	0.92	1.02	2.63	1.79	1.05	1.53
time (sec)	N/A	0.416	0.113	1.802	0.342	0.548	0.419	0.280	8.881

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	102	93	103	107	148	104	135	121
N.S.	1	0.97	0.89	0.98	1.02	1.41	0.99	1.29	1.15
time (sec)	N/A	0.270	0.074	1.804	0.215	0.526	0.392	0.278	0.066

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	111	111	100	112	298	211	115	141
N.S.	1	1.01	1.01	0.91	1.02	2.71	1.92	1.05	1.28
time (sec)	N/A	0.357	0.092	1.827	0.288	0.252	0.387	0.273	8.857

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	79	72	76	82	121	78	106	86
N.S.	1	0.95	0.87	0.92	0.99	1.46	0.94	1.28	1.04
time (sec)	N/A	0.238	0.061	1.881	0.201	0.245	0.366	0.282	0.074

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	75	85	240	129	88	104
N.S.	1	1.00	1.00	0.84	0.96	2.70	1.45	0.99	1.17
time (sec)	N/A	0.259	0.078	1.779	0.289	0.264	0.350	0.286	8.888

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	58	50	59	60	81	56	70	62
N.S.	1	0.95	0.82	0.97	0.98	1.33	0.92	1.15	1.02
time (sec)	N/A	0.212	0.043	1.763	0.201	0.279	0.311	0.288	0.075

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	71	68	57	61	208	114	59	59
N.S.	1	1.04	1.00	0.84	0.90	3.06	1.68	0.87	0.87
time (sec)	N/A	0.195	0.056	1.843	0.294	0.247	0.274	0.277	8.893

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	39	41	38	40	44	36	37	37
N.S.	1	0.95	1.00	0.93	0.98	1.07	0.88	0.90	0.90
time (sec)	N/A	0.188	0.015	1.789	0.205	0.237	0.189	0.276	8.872

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	57	182	112	57	51
N.S.	1	1.00	1.00	0.90	0.90	2.89	1.78	0.90	0.81
time (sec)	N/A	0.177	0.049	1.956	0.302	0.374	0.214	0.295	8.921

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	53	46	48	51	70	46	52	47
N.S.	1	1.04	0.90	0.94	1.00	1.37	0.90	1.02	0.92
time (sec)	N/A	0.202	0.035	2.024	0.200	0.371	0.227	0.271	0.126

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	72	70	62	63	210	114	62	63
N.S.	1	1.03	1.00	0.89	0.90	3.00	1.63	0.89	0.90
time (sec)	N/A	0.209	0.036	1.822	0.295	0.396	0.262	0.287	9.029

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	72	64	76	76	117	70	80	78
N.S.	1	0.99	0.88	1.04	1.04	1.60	0.96	1.10	1.07
time (sec)	N/A	0.233	0.058	1.754	0.215	0.504	0.532	0.303	8.937

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	100	90	78	93	250	184	85	83
N.S.	1	1.11	1.00	0.87	1.03	2.78	2.04	0.94	0.92
time (sec)	N/A	0.314	0.080	1.802	0.285	0.284	0.311	0.271	8.946

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	96	85	96	106	154	100	150	100
N.S.	1	0.99	0.88	0.99	1.09	1.59	1.03	1.55	1.03
time (sec)	N/A	0.260	0.107	1.737	0.213	0.256	0.587	0.293	8.952

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	125	112	99	119	308	218	112	104
N.S.	1	1.13	1.01	0.89	1.07	2.77	1.96	1.01	0.94
time (sec)	N/A	0.462	0.090	1.833	0.289	0.270	0.357	0.280	8.992

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	150	133	119	147	416	252	138	177
N.S.	1	1.07	0.95	0.85	1.05	2.97	1.80	0.99	1.26
time (sec)	N/A	0.562	0.119	1.788	0.304	0.266	0.758	0.273	0.087

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	108	94	102	116	179	119	132	118
N.S.	1	0.97	0.85	0.92	1.05	1.61	1.07	1.19	1.06
time (sec)	N/A	0.279	0.070	1.756	0.216	0.251	0.807	0.285	8.946

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	126	113	95	120	358	214	111	138
N.S.	1	1.07	0.96	0.81	1.02	3.03	1.81	0.94	1.17
time (sec)	N/A	0.404	0.096	1.817	0.290	0.263	0.683	0.281	8.973

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	86	92	82	94	142	94	93	95
N.S.	1	0.97	1.03	0.92	1.06	1.60	1.06	1.04	1.07
time (sec)	N/A	0.242	0.039	1.765	0.229	0.252	0.694	0.288	0.092

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	101	92	77	94	328	194	80	92
N.S.	1	1.06	0.97	0.81	0.99	3.45	2.04	0.84	0.97
time (sec)	N/A	0.263	0.076	1.767	0.282	0.257	0.584	0.304	9.016

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	65	64	57	72	89	70	55	70
N.S.	1	0.97	0.96	0.85	1.07	1.33	1.04	0.82	1.04
time (sec)	N/A	0.225	0.031	1.752	0.212	0.393	0.494	0.292	8.921

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	95	83	76	92	301	155	78	82
N.S.	1	1.06	0.92	0.84	1.02	3.34	1.72	0.87	0.91
time (sec)	N/A	0.216	0.088	1.773	0.289	0.465	0.408	0.290	0.112

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	30	29	42	42	42	28	44
N.S.	1	1.00	0.94	0.91	1.31	1.31	1.31	0.88	1.38
time (sec)	N/A	0.158	0.017	1.780	0.211	0.427	0.278	0.279	8.866

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	84	77	92	300	150	78	82
N.S.	1	1.00	0.91	0.84	1.00	3.26	1.63	0.85	0.89
time (sec)	N/A	0.196	0.064	1.836	0.286	0.401	0.302	0.286	8.967

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	69	59	61	77	119	75	76	71
N.S.	1	1.01	0.87	0.90	1.13	1.75	1.10	1.12	1.04
time (sec)	N/A	0.222	0.053	1.822	0.201	0.239	0.294	0.288	0.145

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	107	96	82	96	324	194	82	113
N.S.	1	1.11	1.00	0.85	1.00	3.38	2.02	0.85	1.18
time (sec)	N/A	0.268	0.066	1.836	0.294	0.267	0.358	0.282	8.984

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	96	86	100	109	197	107	105	107
N.S.	1	0.99	0.89	1.03	1.12	2.03	1.10	1.08	1.10
time (sec)	N/A	0.265	0.064	1.777	0.204	0.243	0.594	0.274	8.976

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	130	119	98	128	368	226	108	114
N.S.	1	1.11	1.02	0.84	1.09	3.15	1.93	0.92	0.97
time (sec)	N/A	0.384	0.075	1.789	0.293	0.266	0.401	0.281	9.045

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	108	123	137	229	136	132	131
N.S.	1	1.00	0.89	1.02	1.13	1.89	1.12	1.09	1.08
time (sec)	N/A	0.292	0.084	1.789	0.207	0.249	0.668	0.280	0.132

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	159	140	119	154	426	260	135	135
N.S.	1	1.14	1.00	0.85	1.10	3.04	1.86	0.96	0.96
time (sec)	N/A	0.719	0.082	1.839	0.288	0.281	0.447	0.299	9.128

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	149	135	143	170	267	165	201	155
N.S.	1	1.01	0.91	0.97	1.15	1.80	1.11	1.36	1.05
time (sec)	N/A	0.339	0.129	1.809	0.214	0.427	0.697	0.290	9.113

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	214	189	178	321	368	345	245	289
N.S.	1	0.98	0.87	0.82	1.47	1.69	1.58	1.12	1.33
time (sec)	N/A	0.391	1.399	1.873	0.223	0.533	0.768	0.297	10.298

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	180	170	159	273	321	318	211	233
N.S.	1	0.99	0.94	0.88	1.51	1.77	1.76	1.17	1.29
time (sec)	N/A	0.359	1.052	1.804	0.221	0.574	0.714	0.294	9.819

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	128	147	140	225	272	287	177	177
N.S.	1	1.02	1.18	1.12	1.80	2.18	2.30	1.42	1.42
time (sec)	N/A	0.271	0.886	1.809	0.209	0.539	0.680	0.296	9.624

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	111	126	116	177	223	255	140	140
N.S.	1	1.04	1.18	1.08	1.65	2.08	2.38	1.31	1.31
time (sec)	N/A	0.247	0.615	2.114	0.211	0.269	0.626	0.297	9.673

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	94	97	91	128	172	0	103	117
N.S.	1	0.94	0.97	0.91	1.28	1.72	0.00	1.03	1.17
time (sec)	N/A	0.261	0.189	2.046	0.215	0.264	0.000	0.284	9.428

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	89	87	83	105	161	0	87	0
N.S.	1	0.92	0.90	0.86	1.08	1.66	0.00	0.90	0.00
time (sec)	N/A	0.265	0.263	1.845	0.211	0.282	0.000	0.327	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	86	99	86	96	160	0	163	0
N.S.	1	1.08	1.24	1.08	1.20	2.00	0.00	2.04	0.00
time (sec)	N/A	0.260	0.164	2.038	0.215	0.277	0.000	0.442	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	65	44	47	111	59	0	250	113
N.S.	1	1.07	0.72	0.77	1.82	0.97	0.00	4.10	1.85
time (sec)	N/A	0.228	0.164	2.007	0.216	0.268	0.000	0.696	9.256

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	99	66	66	161	85	0	310	160
N.S.	1	1.03	0.69	0.69	1.68	0.89	0.00	3.23	1.67
time (sec)	N/A	0.262	0.191	1.807	0.215	0.294	0.000	0.891	9.428

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	88	84	209	109	0	370	210
N.S.	1	1.00	0.66	0.63	1.57	0.82	0.00	2.78	1.58
time (sec)	N/A	0.293	0.245	1.795	0.213	0.289	0.000	1.404	9.760

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	167	110	104	257	133	0	430	260
N.S.	1	0.98	0.65	0.61	1.51	0.78	0.00	2.53	1.53
time (sec)	N/A	0.339	0.260	1.944	0.218	0.488	0.000	1.854	10.218

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	127	82	91	106	106	0	140	103
N.S.	1	0.97	0.63	0.69	0.81	0.81	0.00	1.07	0.79
time (sec)	N/A	0.278	0.088	2.415	0.226	0.426	0.000	0.294	9.127

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	93	64	67	83	82	0	105	83
N.S.	1	0.99	0.68	0.71	0.88	0.87	0.00	1.12	0.88
time (sec)	N/A	0.244	0.070	2.172	0.220	0.396	0.000	0.315	9.026

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	41	45	51	57	0	72	60
N.S.	1	1.00	0.67	0.74	0.84	0.93	0.00	1.18	0.98
time (sec)	N/A	0.196	0.047	2.015	0.223	0.377	0.000	0.281	8.967

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	79	84	84	0	159	0	116	99
N.S.	1	1.01	1.08	1.08	0.00	2.04	0.00	1.49	1.27
time (sec)	N/A	0.234	0.132	2.010	0.000	0.328	0.000	0.286	9.457

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	92	94	95	0	169	0	76	0
N.S.	1	0.92	0.94	0.95	0.00	1.69	0.00	0.76	0.00
time (sec)	N/A	0.247	0.158	2.152	0.000	0.281	0.000	0.303	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	99	106	103	0	198	0	132	0
N.S.	1	0.96	1.03	1.00	0.00	1.92	0.00	1.28	0.00
time (sec)	N/A	0.247	0.211	2.307	0.000	0.289	0.000	0.298	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	221	248	197	363	418	726	280	0
N.S.	1	0.99	1.11	0.88	1.63	1.87	3.26	1.26	0.00
time (sec)	N/A	0.389	1.784	1.792	0.216	0.412	1.175	0.302	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	169	226	178	315	369	672	246	0
N.S.	1	1.01	1.35	1.07	1.89	2.21	4.02	1.47	0.00
time (sec)	N/A	0.317	1.388	1.797	0.227	0.363	1.126	0.300	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	152	199	159	267	316	614	207	236
N.S.	1	1.03	1.34	1.07	1.80	2.14	4.15	1.40	1.59
time (sec)	N/A	0.286	1.137	1.859	0.220	0.308	1.013	0.293	9.865

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	140	157	140	216	275	551	178	0
N.S.	1	0.97	1.09	0.97	1.50	1.91	3.83	1.24	0.00
time (sec)	N/A	0.304	0.887	1.830	0.218	0.285	4.144	0.297	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	134	124	116	168	224	0	142	0
N.S.	1	0.98	0.91	0.85	1.23	1.64	0.00	1.04	0.00
time (sec)	N/A	0.316	0.292	1.792	0.217	0.293	0.000	0.289	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	117	116	101	148	209	0	121	0
N.S.	1	0.91	0.91	0.79	1.16	1.63	0.00	0.95	0.00
time (sec)	N/A	0.298	0.490	1.834	0.226	0.429	0.000	0.343	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	120	120	101	167	189	0	225	0
N.S.	1	0.88	0.88	0.74	1.23	1.39	0.00	1.65	0.00
time (sec)	N/A	0.314	0.339	1.854	0.215	0.429	0.000	0.450	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	111	110	107	177	207	0	254	0
N.S.	1	1.07	1.06	1.03	1.70	1.99	0.00	2.44	0.00
time (sec)	N/A	0.286	0.278	1.892	0.208	0.429	0.000	0.830	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	65	44	48	193	82	0	370	156
N.S.	1	1.07	0.72	0.79	3.16	1.34	0.00	6.07	2.56
time (sec)	N/A	0.235	0.251	1.827	0.234	0.756	0.000	1.188	9.820

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	99	66	68	241	109	0	430	206
N.S.	1	1.03	0.69	0.71	2.51	1.14	0.00	4.48	2.15
time (sec)	N/A	0.264	0.299	1.832	0.223	0.296	0.000	1.537	10.241

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	89	87	289	134	0	490	256
N.S.	1	1.00	0.67	0.65	2.17	1.01	0.00	3.68	1.92
time (sec)	N/A	0.307	0.370	1.804	0.208	0.320	0.000	1.758	10.817

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	167	110	106	337	157	0	550	306
N.S.	1	0.98	0.65	0.62	1.98	0.92	0.00	3.24	1.80
time (sec)	N/A	0.355	0.422	1.845	0.216	0.380	0.000	2.163	11.415

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	201	132	124	385	181	0	582	356
N.S.	1	0.97	0.64	0.60	1.86	0.87	0.00	2.81	1.72
time (sec)	N/A	0.381	0.497	1.825	0.215	0.488	0.000	2.305	12.101

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	161	113	115	150	154	0	175	143
N.S.	1	0.96	0.67	0.68	0.89	0.92	0.00	1.04	0.85
time (sec)	N/A	0.328	0.129	2.209	0.264	0.251	0.000	0.294	9.254

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	127	94	91	128	131	0	140	124
N.S.	1	0.97	0.72	0.69	0.98	1.00	0.00	1.07	0.95
time (sec)	N/A	0.286	0.102	2.177	0.225	0.250	0.000	0.286	9.169

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	95	70	67	105	106	0	105	103
N.S.	1	0.99	0.73	0.70	1.09	1.10	0.00	1.09	1.07
time (sec)	N/A	0.234	0.079	2.027	0.217	0.263	0.000	0.296	9.059

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	41	45	80	80	0	72	83
N.S.	1	1.00	0.67	0.74	1.31	1.31	0.00	1.18	1.36
time (sec)	N/A	0.213	0.053	2.042	0.215	0.269	0.000	0.287	9.014

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	104	108	99	0	206	0	140	0
N.S.	1	1.02	1.06	0.97	0.00	2.02	0.00	1.37	0.00
time (sec)	N/A	0.276	0.190	2.375	0.000	0.286	0.000	0.288	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	118	109	149	0	195	0	115	0
N.S.	1	0.89	0.82	1.12	0.00	1.47	0.00	0.86	0.00
time (sec)	N/A	0.288	0.210	2.686	0.000	0.487	0.000	0.307	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	120	114	117	0	217	0	145	0
N.S.	1	0.89	0.84	0.87	0.00	1.61	0.00	1.07	0.00
time (sec)	N/A	0.279	0.240	2.413	0.000	0.515	0.000	0.318	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	127	132	128	0	250	0	175	0
N.S.	1	0.91	0.94	0.91	0.00	1.79	0.00	1.25	0.00
time (sec)	N/A	0.290	0.316	2.621	0.000	0.419	0.000	0.328	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	158	154	153	0	299	0	214	0
N.S.	1	0.89	0.87	0.86	0.00	1.69	0.00	1.21	0.00
time (sec)	N/A	0.342	0.403	2.829	0.000	0.591	0.000	0.322	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	192	172	176	0	345	0	234	0
N.S.	1	0.90	0.80	0.82	0.00	1.61	0.00	1.09	0.00
time (sec)	N/A	0.386	0.528	3.204	0.000	0.314	0.000	0.332	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	226	198	201	0	393	0	294	0
N.S.	1	0.90	0.79	0.80	0.00	1.57	0.00	1.17	0.00
time (sec)	N/A	0.441	0.541	3.637	0.000	0.357	0.000	0.328	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	168	155	140	231	275	224	184	0
N.S.	1	0.95	0.88	0.80	1.31	1.56	1.27	1.05	0.00
time (sec)	N/A	0.345	0.734	1.849	0.230	0.305	0.848	0.287	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	134	133	121	183	226	201	150	0
N.S.	1	0.96	0.96	0.87	1.32	1.63	1.45	1.08	0.00
time (sec)	N/A	0.310	0.618	1.839	0.244	0.293	0.795	0.304	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	87	108	100	134	177	170	112	0
N.S.	1	1.05	1.30	1.20	1.61	2.13	2.05	1.35	0.00
time (sec)	N/A	0.236	0.435	1.891	0.222	0.300	0.751	0.292	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	65	91	84	88	131	139	77	89
N.S.	1	0.98	1.38	1.27	1.33	1.98	2.11	1.17	1.35
time (sec)	N/A	0.220	0.254	1.833	0.243	0.308	0.705	0.291	9.618

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	62	77	67	56	136	0	66	57
N.S.	1	1.09	1.35	1.18	0.98	2.39	0.00	1.16	1.00
time (sec)	N/A	0.220	0.112	1.807	0.227	0.304	0.000	0.302	9.362

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	65	43	40	70	38	0	124	39
N.S.	1	1.07	0.70	0.66	1.15	0.62	0.00	2.03	0.64
time (sec)	N/A	0.231	0.127	1.777	0.219	0.273	0.000	0.393	9.091

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	99	64	63	119	62	0	180	62
N.S.	1	1.03	0.67	0.66	1.24	0.65	0.00	1.88	0.65
time (sec)	N/A	0.267	0.155	1.933	0.220	0.271	0.000	0.582	9.175

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	89	85	167	86	0	236	121
N.S.	1	1.00	0.67	0.64	1.26	0.65	0.00	1.77	0.91
time (sec)	N/A	0.301	0.195	1.808	0.253	0.286	0.000	0.734	9.206

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	167	110	104	215	110	0	292	156
N.S.	1	0.98	0.65	0.61	1.26	0.65	0.00	1.72	0.92
time (sec)	N/A	0.342	0.228	1.882	0.208	0.315	0.000	0.978	9.253

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	127	85	84	106	83	0	130	87
N.S.	1	0.97	0.65	0.64	0.81	0.63	0.00	0.99	0.66
time (sec)	N/A	0.279	0.096	2.403	0.248	0.265	0.000	0.289	9.192

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	93	63	60	83	59	0	98	64
N.S.	1	0.99	0.67	0.64	0.88	0.63	0.00	1.04	0.68
time (sec)	N/A	0.251	0.073	2.243	0.210	0.270	0.000	0.281	9.210

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	40	37	50	36	0	67	41
N.S.	1	1.00	0.68	0.63	0.85	0.61	0.00	1.14	0.69
time (sec)	N/A	0.202	0.050	2.138	0.215	0.277	0.000	0.287	9.115

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	73	72	0	138	0	80	0
N.S.	1	1.00	1.33	1.31	0.00	2.51	0.00	1.45	0.00
time (sec)	N/A	0.188	0.077	2.057	0.000	0.265	0.000	0.296	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	86	90	0	152	0	66	0
N.S.	1	1.00	1.26	1.32	0.00	2.24	0.00	0.97	0.00
time (sec)	N/A	0.209	0.132	2.051	0.000	0.258	0.000	0.299	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	102	104	110	0	199	0	125	0
N.S.	1	0.99	1.01	1.07	0.00	1.93	0.00	1.21	0.00
time (sec)	N/A	0.253	0.192	2.156	0.000	0.279	0.000	0.318	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	188	156	145	237	340	0	183	0
N.S.	1	1.02	0.85	0.79	1.29	1.85	0.00	0.99	0.00
time (sec)	N/A	0.582	0.761	1.889	0.220	0.280	0.000	0.295	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	139	132	140	187	289	0	145	0
N.S.	1	0.95	0.90	0.95	1.27	1.97	0.00	0.99	0.00
time (sec)	N/A	0.395	0.551	1.921	0.207	0.282	0.000	0.287	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	100	108	114	138	230	0	104	0
N.S.	1	0.89	0.96	1.02	1.23	2.05	0.00	0.93	0.00
time (sec)	N/A	0.285	0.344	1.837	0.204	0.276	0.000	0.305	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	72	77	75	79	188	0	70	78
N.S.	1	1.07	1.15	1.12	1.18	2.81	0.00	1.04	1.16
time (sec)	N/A	0.229	0.138	1.819	0.211	0.284	0.000	0.308	9.460

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	37	65	49	0	65	53
N.S.	1	1.00	1.00	1.00	1.76	1.32	0.00	1.76	1.43
time (sec)	N/A	0.187	0.112	1.825	0.218	0.274	0.000	0.331	8.975

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	70	64	59	112	72	0	189	70
N.S.	1	1.06	0.97	0.89	1.70	1.09	0.00	2.86	1.06
time (sec)	N/A	0.227	0.161	2.108	0.221	0.277	0.000	0.499	9.083

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	104	85	80	160	98	0	302	95
N.S.	1	1.03	0.84	0.79	1.58	0.97	0.00	2.99	0.94
time (sec)	N/A	0.265	0.217	2.032	0.222	0.295	0.000	0.917	9.246

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	108	97	208	121	0	415	173
N.S.	1	1.00	0.78	0.70	1.51	0.88	0.00	3.01	1.25
time (sec)	N/A	0.298	0.252	1.796	0.209	0.266	0.000	1.282	9.406

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	135	82	91	82	93	0	145	92
N.S.	1	0.97	0.59	0.65	0.59	0.67	0.00	1.04	0.66
time (sec)	N/A	0.300	0.095	2.580	0.226	0.279	0.000	0.285	9.263

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	101	60	66	59	68	0	107	67
N.S.	1	0.97	0.58	0.63	0.57	0.65	0.00	1.03	0.64
time (sec)	N/A	0.255	0.069	2.401	0.235	0.291	0.000	0.279	9.159

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	35	44	39	45	0	63	44
N.S.	1	1.00	0.51	0.64	0.57	0.65	0.00	0.91	0.64
time (sec)	N/A	0.214	0.054	2.250	0.213	0.298	0.000	0.289	9.097

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	73	79	0	199	0	109	0
N.S.	1	1.00	1.14	1.23	0.00	3.11	0.00	1.70	0.00
time (sec)	N/A	0.211	0.109	2.133	0.000	0.285	0.000	0.277	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	130	96	126	0	260	0	107	0
N.S.	1	0.92	0.68	0.89	0.00	1.83	0.00	0.75	0.00
time (sec)	N/A	0.271	0.182	2.083	0.000	0.301	0.000	0.297	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	130	117	153	0	315	0	149	89
N.S.	1	0.95	0.85	1.12	0.00	2.30	0.00	1.09	0.65
time (sec)	N/A	0.287	0.257	2.106	0.000	0.283	0.000	0.320	10.229

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.170	0.039	0.223	0.200	0.253	0.905	0.282	0.056

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.174	0.038	0.200	0.210	0.249	0.593	0.276	9.046

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	41	28	27	32	46	29	31
N.S.	1	1.00	1.05	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.166	0.035	0.200	0.213	0.247	0.396	0.280	0.044

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	37	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	0.95	0.74	0.79
time (sec)	N/A	0.168	0.035	0.203	0.190	0.276	0.579	0.265	0.045

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.162	0.032	0.205	0.209	0.287	0.247	0.264	9.004

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	30	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.77	1.18	0.74	0.79
time (sec)	N/A	0.169	0.030	0.185	0.194	0.281	0.270	0.286	0.045

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	28	27	29	44	29	31
N.S.	1	1.00	0.95	0.76	0.73	0.78	1.19	0.78	0.84
time (sec)	N/A	0.167	0.034	0.191	0.208	0.259	0.307	0.281	0.045

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	30	27	29	44	29	31
N.S.	1	1.00	0.95	0.81	0.73	0.78	1.19	0.78	0.84
time (sec)	N/A	0.165	0.038	0.050	0.200	0.264	0.444	0.276	9.014

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	80	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.191	0.057	1.790	0.188	0.267	1.874	0.279	8.998

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	80	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.190	0.058	1.813	0.205	0.250	1.294	0.285	0.050

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	80	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.190	0.053	1.789	0.199	0.278	0.962	0.276	0.051

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	66	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.05	0.84	0.81
time (sec)	N/A	0.190	0.051	1.801	0.197	0.264	1.056	0.272	0.050

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	80	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.193	0.051	1.802	0.243	0.255	0.626	0.277	0.050

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	80	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.188	0.054	1.793	0.197	0.248	0.664	0.286	0.050

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	80	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.185	0.052	1.766	0.207	0.285	0.718	0.271	0.051

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	54	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.86	1.27	0.84	0.81
time (sec)	N/A	0.180	0.050	1.928	0.189	0.254	0.972	0.276	0.056

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	76	73	78	114	77	69
N.S.	1	1.00	1.00	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.213	0.121	1.838	0.218	0.269	3.359	0.274	0.041

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	76	73	78	114	77	69
N.S.	1	1.00	1.00	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.206	0.113	2.058	0.211	0.257	2.512	0.268	0.034

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	97	76	73	78	114	77	69
N.S.	1	1.00	1.14	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.208	0.076	2.070	0.236	0.276	1.789	0.278	0.034

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	83	76	73	78	95	77	69
N.S.	1	1.00	0.98	0.89	0.86	0.92	1.12	0.91	0.81
time (sec)	N/A	0.208	0.076	1.893	0.222	0.280	1.607	0.282	0.034

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	83	76	73	78	114	77	69
N.S.	1	1.00	0.98	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.211	0.077	1.813	0.192	0.249	1.398	0.279	0.034

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	83	76	73	78	114	77	69
N.S.	1	1.00	0.98	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.215	0.069	1.875	0.209	0.242	1.413	0.273	0.034

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	83	76	73	78	114	77	69
N.S.	1	1.00	0.98	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.220	0.068	1.807	0.205	0.247	1.563	0.269	0.035

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	83	76	73	78	114	77	69
N.S.	1	1.00	0.98	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.215	0.065	1.773	0.217	0.266	1.801	0.279	0.037

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	277	172	164	237	770	0	298	115
N.S.	1	1.00	0.62	0.59	0.85	2.77	0.00	1.07	0.41
time (sec)	N/A	0.482	0.365	1.839	0.283	0.295	0.000	0.311	9.150

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	275	173	162	259	647	0	298	788
N.S.	1	1.00	0.63	0.59	0.94	2.34	0.00	1.08	2.86
time (sec)	N/A	0.462	0.391	1.980	0.336	0.277	0.000	0.297	9.199

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	258	152	140	214	749	0	264	92
N.S.	1	1.00	0.59	0.54	0.83	2.91	0.00	1.03	0.36
time (sec)	N/A	0.456	0.282	1.773	0.286	0.273	0.000	0.302	9.111

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	256	150	138	235	597	275	263	789
N.S.	1	1.00	0.59	0.54	0.92	2.34	1.08	1.03	3.09
time (sec)	N/A	0.439	0.276	1.773	0.292	0.246	98.313	0.279	9.143

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	239	135	124	194	691	303	251	71
N.S.	1	1.01	0.57	0.52	0.82	2.92	1.28	1.06	0.30
time (sec)	N/A	0.418	0.254	1.952	0.281	0.269	37.344	0.287	0.162

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	237	134	127	218	570	238	251	739
N.S.	1	1.01	0.57	0.54	0.93	2.43	1.01	1.07	3.14
time (sec)	N/A	0.420	0.251	1.790	0.298	0.258	15.751	0.298	9.150

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	237	135	127	194	705	309	251	71
N.S.	1	1.01	0.57	0.54	0.83	3.00	1.31	1.07	0.30
time (sec)	N/A	0.411	0.315	1.754	0.293	0.260	8.341	0.303	9.058

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	239	136	124	218	590	257	251	811
N.S.	1	1.01	0.57	0.52	0.92	2.49	1.08	1.06	3.42
time (sec)	N/A	0.424	0.327	1.776	0.300	0.282	13.820	0.280	9.117

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	152	140	213	738	258	268	90
N.S.	1	1.00	0.60	0.55	0.84	2.89	1.01	1.05	0.35
time (sec)	N/A	0.438	0.330	1.789	0.296	0.261	73.160	0.305	9.050

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	154	141	247	643	303	257	555
N.S.	1	1.00	0.60	0.55	0.96	2.50	1.18	1.00	2.16
time (sec)	N/A	0.436	0.348	1.777	0.283	0.279	67.256	0.290	9.180

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	274	174	158	237	784	0	291	107
N.S.	1	0.99	0.63	0.57	0.86	2.84	0.00	1.05	0.39
time (sec)	N/A	0.468	0.404	1.789	0.298	0.269	0.000	0.290	9.056

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	276	176	160	276	672	0	291	563
N.S.	1	0.99	0.63	0.58	0.99	2.42	0.00	1.05	2.03
time (sec)	N/A	0.452	0.411	1.785	0.274	0.293	0.000	0.277	9.273

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	319	206	193	298	748	0	335	857
N.S.	1	0.96	0.62	0.58	0.90	2.25	0.00	1.01	2.58
time (sec)	N/A	0.511	1.039	1.833	0.294	0.266	0.000	0.292	9.198

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	302	184	170	247	848	0	299	127
N.S.	1	0.97	0.59	0.55	0.80	2.74	0.00	0.96	0.41
time (sec)	N/A	0.493	0.852	1.821	0.272	0.261	0.000	0.287	0.180

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	300	183	169	271	696	0	298	823
N.S.	1	0.97	0.59	0.55	0.87	2.25	0.00	0.96	2.65
time (sec)	N/A	0.487	0.846	1.808	0.308	0.257	0.000	0.297	9.200

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	283	162	153	223	793	0	283	106
N.S.	1	0.98	0.56	0.53	0.77	2.74	0.00	0.98	0.37
time (sec)	N/A	0.479	0.831	1.823	0.299	0.268	0.000	0.298	0.185

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	281	161	152	250	669	0	283	744
N.S.	1	0.97	0.56	0.53	0.87	2.31	0.00	0.98	2.57
time (sec)	N/A	0.478	0.851	1.826	0.318	0.282	0.000	0.272	0.222

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	263	152	146	217	776	0	273	91
N.S.	1	1.01	0.58	0.56	0.83	2.97	0.00	1.05	0.35
time (sec)	N/A	0.452	0.719	1.839	0.291	0.259	0.000	0.315	9.187

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	263	151	146	241	658	0	273	750
N.S.	1	1.01	0.58	0.56	0.92	2.52	0.00	1.05	2.87
time (sec)	N/A	0.442	0.700	2.172	0.291	0.257	0.000	0.321	9.265

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	280	162	153	222	789	0	278	104
N.S.	1	0.99	0.57	0.54	0.78	2.78	0.00	0.98	0.37
time (sec)	N/A	0.476	0.814	1.805	0.324	0.266	0.000	0.311	9.155

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	283	165	153	251	691	0	283	859
N.S.	1	0.98	0.57	0.53	0.87	2.39	0.00	0.98	2.97
time (sec)	N/A	0.479	0.819	1.807	0.307	0.283	0.000	0.358	9.329

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	300	187	170	250	838	0	303	121
N.S.	1	0.97	0.60	0.55	0.81	2.70	0.00	0.98	0.39
time (sec)	N/A	0.479	0.804	1.790	0.300	0.271	0.000	0.327	9.196

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	302	187	170	286	742	0	292	595
N.S.	1	0.97	0.60	0.55	0.92	2.39	0.00	0.94	1.92
time (sec)	N/A	0.490	0.861	1.804	0.301	0.295	0.000	0.308	9.322

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	319	209	190	276	886	0	328	142
N.S.	1	0.96	0.63	0.57	0.83	2.67	0.00	0.99	0.43
time (sec)	N/A	0.519	0.954	1.846	0.302	0.561	0.000	0.313	9.171

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	329	204	189	306	776	0	321	865
N.S.	1	0.96	0.59	0.55	0.89	2.26	0.00	0.94	2.52
time (sec)	N/A	0.512	0.865	1.898	0.292	0.545	0.000	0.296	9.223

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	312	184	173	256	872	0	304	138
N.S.	1	0.97	0.57	0.54	0.80	2.71	0.00	0.94	0.43
time (sec)	N/A	0.511	0.937	1.839	0.323	0.487	0.000	0.305	9.175

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	310	184	172	283	748	0	304	760
N.S.	1	0.96	0.57	0.53	0.88	2.32	0.00	0.94	2.36
time (sec)	N/A	0.503	0.917	1.924	0.279	0.880	0.000	0.279	0.225

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	292	171	166	251	871	0	293	122
N.S.	1	1.00	0.58	0.57	0.86	2.97	0.00	1.00	0.42
time (sec)	N/A	0.477	0.946	1.837	0.284	0.304	0.000	0.299	0.179

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	293	173	167	280	763	0	298	799
N.S.	1	0.98	0.58	0.56	0.94	2.56	0.00	1.00	2.68
time (sec)	N/A	0.473	0.938	1.806	0.277	0.303	0.000	0.289	0.327

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	293	175	168	253	878	0	298	124
N.S.	1	0.98	0.59	0.56	0.85	2.95	0.00	1.00	0.42
time (sec)	N/A	0.490	0.962	1.805	0.284	0.295	0.000	0.286	9.185

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	292	172	166	276	749	0	293	780
N.S.	1	1.00	0.59	0.57	0.94	2.56	0.00	1.00	2.66
time (sec)	N/A	0.490	0.917	1.789	0.291	0.422	0.000	0.285	9.348

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	309	187	173	255	870	0	300	133
N.S.	1	0.98	0.59	0.55	0.81	2.75	0.00	0.95	0.42
time (sec)	N/A	0.511	1.001	1.844	0.291	0.502	0.000	0.301	9.208

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	312	187	173	285	770	0	304	888
N.S.	1	0.97	0.58	0.54	0.89	2.39	0.00	0.94	2.76
time (sec)	N/A	0.507	0.889	1.810	0.284	0.480	0.000	0.279	9.411

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	329	209	190	285	918	0	326	152
N.S.	1	0.96	0.61	0.55	0.83	2.68	0.00	0.95	0.44
time (sec)	N/A	0.519	0.904	1.888	0.296	0.792	0.000	0.302	9.131

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	331	210	190	321	822	0	315	626
N.S.	1	0.97	0.61	0.55	0.94	2.40	0.00	0.92	1.83
time (sec)	N/A	0.524	0.906	1.841	0.291	0.272	0.000	0.297	9.387

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	348	231	210	311	966	0	351	173
N.S.	1	0.95	0.63	0.58	0.85	2.65	0.00	0.96	0.47
time (sec)	N/A	0.528	0.962	1.814	0.322	0.261	0.000	0.273	9.182

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	350	231	210	353	854	0	351	639
N.S.	1	0.96	0.63	0.58	0.97	2.34	0.00	0.96	1.75
time (sec)	N/A	0.537	0.970	1.913	0.295	0.303	0.000	0.294	9.315

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	232	136	241	0	123	0	0	0
N.S.	1	0.95	0.56	0.99	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.410	10.193	1.936	0.000	0.129	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	353	111	267	0	103	0	0	0
N.S.	1	0.96	0.30	0.72	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.483	10.160	1.868	0.000	0.109	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	196	111	217	0	98	0	0	0
N.S.	1	0.96	0.54	1.06	0.00	0.48	0.00	0.00	0.00
time (sec)	N/A	0.349	10.167	1.828	0.000	0.137	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	316	94	243	0	77	0	0	0
N.S.	1	0.97	0.29	0.75	0.00	0.24	0.00	0.00	0.00
time (sec)	N/A	0.430	10.094	1.875	0.000	0.156	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	162	94	193	0	74	0	0	0
N.S.	1	0.98	0.57	1.17	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.307	10.057	1.909	0.000	0.081	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	311	97	232	0	71	0	0	0
N.S.	1	0.96	0.30	0.72	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.431	10.047	2.157	0.000	0.087	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	158	97	180	0	61	0	0	0
N.S.	1	0.97	0.60	1.10	0.00	0.37	0.00	0.00	0.00
time (sec)	N/A	0.309	10.048	1.903	0.000	0.128	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	312	96	243	0	76	0	0	0
N.S.	1	0.95	0.29	0.74	0.00	0.23	0.00	0.00	0.00
time (sec)	N/A	0.435	10.048	1.901	0.000	0.087	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	163	98	194	0	71	0	0	0
N.S.	1	0.98	0.59	1.16	0.00	0.43	0.00	0.00	0.00
time (sec)	N/A	0.302	10.049	1.889	0.000	0.148	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	349	99	267	0	103	0	0	0
N.S.	1	0.95	0.27	0.72	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.470	10.061	1.890	0.000	0.104	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	196	98	218	0	96	0	0	0
N.S.	1	0.96	0.48	1.07	0.00	0.47	0.00	0.00	0.00
time (sec)	N/A	0.350	10.060	1.881	0.000	0.080	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	486	455	160	339	0	175	0	0	0
N.S.	1	0.94	0.33	0.70	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.626	10.245	1.904	0.000	0.106	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	298	160	289	0	170	0	0	0
N.S.	1	0.93	0.50	0.90	0.00	0.53	0.00	0.00	0.00
time (sec)	N/A	0.505	10.232	1.877	0.000	0.104	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	447	419	138	315	0	150	0	0	0
N.S.	1	0.94	0.31	0.70	0.00	0.34	0.00	0.00	0.00
time (sec)	N/A	0.583	10.194	1.889	0.000	0.174	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	262	138	265	0	147	0	0	0
N.S.	1	0.93	0.49	0.94	0.00	0.52	0.00	0.00	0.00
time (sec)	N/A	0.458	10.190	1.915	0.000	0.176	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	383	115	291	0	127	0	0	0
N.S.	1	0.94	0.28	0.71	0.00	0.31	0.00	0.00	0.00
time (sec)	N/A	0.523	10.161	1.899	0.000	0.117	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	225	115	241	0	121	0	0	0
N.S.	1	0.94	0.48	1.01	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.384	10.162	1.824	0.000	0.094	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	347	98	267	0	102	0	0	0
N.S.	1	0.94	0.27	0.72	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.473	10.124	1.878	0.000	0.099	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	192	97	217	0	97	0	0	0
N.S.	1	0.96	0.48	1.08	0.00	0.48	0.00	0.00	0.00
time (sec)	N/A	0.345	10.079	1.880	0.000	0.091	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	341	85	249	0	94	0	0	0
N.S.	1	0.96	0.24	0.70	0.00	0.26	0.00	0.00	0.00
time (sec)	N/A	0.476	10.078	1.849	0.000	0.152	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	193	101	200	0	86	0	0	0
N.S.	1	0.96	0.50	1.00	0.00	0.43	0.00	0.00	0.00
time (sec)	N/A	0.346	10.064	1.815	0.000	0.123	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	338	99	243	0	81	0	0	0
N.S.	1	0.95	0.28	0.69	0.00	0.23	0.00	0.00	0.00
time (sec)	N/A	0.473	10.051	1.842	0.000	0.090	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	191	101	196	0	75	0	0	0
N.S.	1	0.94	0.50	0.96	0.00	0.37	0.00	0.00	0.00
time (sec)	N/A	0.354	10.053	1.878	0.000	0.094	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	342	100	267	0	102	0	0	0
N.S.	1	0.94	0.27	0.73	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.480	10.063	1.864	0.000	0.095	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	238	143	239	0	122	0	0	0
N.S.	1	0.98	0.59	0.98	0.00	0.50	0.00	0.00	0.00
time (sec)	N/A	0.402	10.179	1.968	0.000	0.108	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	359	122	265	0	102	0	0	0
N.S.	1	0.97	0.33	0.72	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.499	10.155	2.059	0.000	0.219	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	202	122	215	0	99	0	0	0
N.S.	1	0.99	0.60	1.05	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.366	10.147	1.972	0.000	0.292	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	323	97	241	0	79	0	0	0
N.S.	1	0.98	0.29	0.73	0.00	0.24	0.00	0.00	0.00
time (sec)	N/A	0.453	10.130	2.096	0.000	0.093	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	166	97	191	0	73	0	0	0
N.S.	1	0.99	0.58	1.14	0.00	0.44	0.00	0.00	0.00
time (sec)	N/A	0.320	10.131	2.311	0.000	0.086	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	287	81	225	0	55	0	0	0
N.S.	1	0.98	0.28	0.77	0.00	0.19	0.00	0.00	0.00
time (sec)	N/A	0.399	10.108	2.023	0.000	0.082	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	80	175	0	51	0	0	0
N.S.	1	1.00	0.62	1.35	0.00	0.39	0.00	0.00	0.00
time (sec)	N/A	0.273	10.067	2.160	0.000	0.140	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	82	225	0	62	0	0	0
N.S.	1	1.00	0.29	0.80	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.404	10.054	2.008	0.000	0.127	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	82	177	0	57	0	0	0
N.S.	1	1.00	0.63	1.35	0.00	0.44	0.00	0.00	0.00
time (sec)	N/A	0.268	10.048	1.993	0.000	0.104	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	319	83	242	0	76	0	0	0
N.S.	1	0.96	0.25	0.73	0.00	0.23	0.00	0.00	0.00
time (sec)	N/A	0.446	10.060	2.040	0.000	0.083	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	166	85	193	0	69	0	0	0
N.S.	1	0.99	0.51	1.16	0.00	0.41	0.00	0.00	0.00
time (sec)	N/A	0.326	10.056	1.954	0.000	0.092	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	355	84	266	0	104	0	0	0
N.S.	1	0.96	0.23	0.72	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.480	10.058	2.026	0.000	0.084	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	202	84	216	0	96	0	0	0
N.S.	1	0.99	0.41	1.06	0.00	0.47	0.00	0.00	0.00
time (sec)	N/A	0.355	10.057	1.970	0.000	0.133	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	249	134	281	0	156	0	0	0
N.S.	1	0.99	0.53	1.12	0.00	0.62	0.00	0.00	0.00
time (sec)	N/A	0.421	10.155	2.309	0.000	0.169	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	370	110	420	0	134	0	0	0
N.S.	1	0.98	0.29	1.11	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.510	10.136	2.488	0.000	0.098	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	213	110	255	0	129	0	0	0
N.S.	1	1.00	0.51	1.19	0.00	0.60	0.00	0.00	0.00
time (sec)	N/A	0.368	10.136	2.355	0.000	0.093	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	334	85	394	0	106	0	0	0
N.S.	1	0.98	0.25	1.16	0.00	0.31	0.00	0.00	0.00
time (sec)	N/A	0.440	10.115	2.444	0.000	0.100	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	177	86	230	0	102	0	0	0
N.S.	1	0.99	0.48	1.29	0.00	0.57	0.00	0.00	0.00
time (sec)	N/A	0.335	10.111	2.281	0.000	0.088	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	295	78	388	0	98	0	0	0
N.S.	1	0.99	0.26	1.30	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.399	10.101	1.955	0.000	0.140	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	76	222	0	90	0	0	0
N.S.	1	1.00	0.55	1.62	0.00	0.66	0.00	0.00	0.00
time (sec)	N/A	0.280	10.077	1.915	0.000	0.148	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	311	77	392	0	115	0	0	0
N.S.	1	0.98	0.24	1.23	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.446	10.057	2.276	0.000	0.107	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	163	92	235	0	111	0	0	0
N.S.	1	0.98	0.55	1.41	0.00	0.66	0.00	0.00	0.00
time (sec)	N/A	0.318	10.059	2.212	0.000	0.088	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	351	79	420	0	134	0	0	0
N.S.	1	0.95	0.21	1.14	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.489	10.053	2.385	0.000	0.093	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	198	79	254	0	126	0	0	0
N.S.	1	0.98	0.39	1.25	0.00	0.62	0.00	0.00	0.00
time (sec)	N/A	0.356	10.052	2.319	0.000	0.094	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	387	79	450	0	163	0	0	0
N.S.	1	0.96	0.20	1.11	0.00	0.40	0.00	0.00	0.00
time (sec)	N/A	0.530	10.061	2.637	0.000	0.115	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	89	474	129	381	2077	603	291
N.S.	1	1.00	0.93	4.94	1.34	3.97	21.64	6.28	3.03
time (sec)	N/A	0.250	0.180	2.054	0.199	0.416	1.056	0.288	9.450

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	66	262	91	217	1051	340	179
N.S.	1	1.00	0.93	3.69	1.28	3.06	14.80	4.79	2.52
time (sec)	N/A	0.225	0.113	2.107	0.189	0.305	0.667	0.267	9.321

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	55	53	94	415	149	97
N.S.	1	1.00	0.93	1.22	1.18	2.09	9.22	3.31	2.16
time (sec)	N/A	0.197	0.079	0.079	0.192	0.268	0.395	0.283	9.197

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	55	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.208	0.118	0.000	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	80	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.217	0.151	0.000	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	131	135	0	0	0	0	0	0
N.S.	1	0.94	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.287	0.156	0.000	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	92	63	0	112	140	0	0	0
N.S.	1	0.97	0.66	0.00	1.18	1.47	0.00	0.00	0.00
time (sec)	N/A	0.267	0.143	0.000	0.246	0.385	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	113	113	111	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	0.561	0.000	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	145	210	0	0	0	0	0	0
N.S.	1	1.12	1.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.367	1.313	0.000	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	25	19	14	15	15	17	17	13
N.S.	1	1.32	1.00	0.74	0.79	0.79	0.89	0.89	0.68
time (sec)	N/A	0.197	0.008	1.876	0.281	0.246	0.043	0.266	0.090

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	19	15	12	35	11	10	16	11
N.S.	1	1.27	1.00	0.80	2.33	0.73	0.67	1.07	0.73
time (sec)	N/A	0.188	0.006	1.863	0.283	0.239	0.051	0.276	0.071

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	21	17	14	13	13	14	16	13
N.S.	1	1.24	1.00	0.82	0.76	0.76	0.82	0.94	0.76
time (sec)	N/A	0.202	0.008	1.871	0.276	0.246	0.058	0.263	9.156

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	18	10	16	14
N.S.	1	1.00	1.00	0.94	0.88	1.12	0.62	1.00	0.88
time (sec)	N/A	0.174	0.005	1.882	0.187	0.259	0.045	0.269	0.047

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.218	0.144	0.000	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	116	0	37	43	0	48	0
N.S.	1	1.00	6.44	0.00	2.06	2.39	0.00	2.67	0.00
time (sec)	N/A	0.163	0.465	0.000	0.269	0.277	0.000	0.331	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	214	157	0	0	0	0	0	0
N.S.	1	1.10	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.375	0.331	0.000	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	140	119	0	0	0	0	0	0
N.S.	1	1.07	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.265	0.239	0.000	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	0.193	0.000	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	0.193	0.000	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	0.213	0.000	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	115	112	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.295	0.301	0.000	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	207	184	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.400	0.401	0.000	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.240	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	231	179	0	0	0	0	0	0
N.S.	1	1.14	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.384	0.345	0.000	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	174	120	0	0	0	0	0	0
N.S.	1	1.16	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	0.286	0.000	0.000	0.000	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.199	0.177	0.000	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	88	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.242	0.220	0.000	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	150	125	0	0	0	0	0	0
N.S.	1	1.13	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.300	0.258	0.000	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	229	182	0	0	0	0	0	0
N.S.	1	1.06	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.396	0.382	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [218] had the largest ratio of [.5769229999999999964]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	22	0.136
2	A	5	4	1.12	20	0.200
3	A	3	3	1.00	19	0.158
4	A	5	4	1.03	22	0.182
5	A	3	3	1.00	22	0.136
6	A	5	4	1.10	22	0.182
7	A	3	3	1.00	22	0.136
8	A	5	4	1.03	22	0.182
9	A	3	3	1.00	22	0.136
10	A	5	4	1.14	22	0.182
11	A	3	3	1.00	22	0.136
12	A	3	3	1.00	21	0.143
13	A	5	4	1.07	24	0.167
14	A	3	3	1.00	24	0.125
15	A	5	4	1.10	24	0.167
16	A	3	3	1.00	24	0.125
17	A	6	5	1.16	24	0.208
18	A	3	3	1.00	24	0.125
19	A	5	4	1.02	24	0.167
20	A	3	3	1.00	24	0.125
21	A	5	4	1.02	24	0.167
22	A	3	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	5	4	1.08	24	0.167
24	A	3	3	1.00	24	0.125
25	A	3	3	1.00	24	0.125
26	A	5	4	1.06	24	0.167
27	A	3	3	1.00	24	0.125
28	A	5	4	1.10	24	0.167
29	A	3	3	1.00	24	0.125
30	A	6	5	1.05	24	0.208
31	A	3	3	1.00	24	0.125
32	A	5	4	1.03	24	0.167
33	A	3	3	1.00	24	0.125
34	A	5	4	1.01	24	0.167
35	A	3	3	1.00	24	0.125
36	A	5	4	1.01	24	0.167
37	A	3	3	1.00	24	0.125
38	A	6	5	1.05	24	0.208
39	A	3	3	1.00	24	0.125
40	A	5	4	1.08	24	0.167
41	A	4	4	0.78	24	0.167
42	A	5	4	0.99	24	0.167
43	A	4	4	0.82	24	0.167
44	A	5	4	0.99	24	0.167
45	A	4	4	0.87	24	0.167
46	A	5	4	0.98	24	0.167
47	A	4	4	0.97	24	0.167
48	A	5	4	0.97	24	0.167
49	A	3	3	1.00	24	0.125
50	A	5	4	1.09	22	0.182
51	A	3	3	1.00	21	0.143
52	A	3	3	1.00	22	0.136
53	A	5	4	1.04	24	0.167
54	A	4	4	0.95	24	0.167
55	A	5	4	1.01	24	0.167
56	A	5	5	0.96	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	5	4	1.01	24	0.167
58	A	5	5	1.00	24	0.208
59	A	5	4	0.97	24	0.167
60	A	4	4	1.01	24	0.167
61	A	5	4	0.95	24	0.167
62	A	5	5	1.00	24	0.208
63	A	5	4	0.95	24	0.167
64	A	4	4	1.04	24	0.167
65	A	5	4	0.95	24	0.167
66	A	3	3	1.00	24	0.125
67	A	5	4	1.04	24	0.167
68	A	6	6	1.03	24	0.250
69	A	5	4	0.99	22	0.182
70	A	5	5	1.11	21	0.238
71	A	5	4	0.99	24	0.167
72	A	5	5	1.13	24	0.208
73	A	6	6	1.07	24	0.250
74	A	5	4	0.97	24	0.167
75	A	5	5	1.07	24	0.208
76	A	5	4	0.97	24	0.167
77	A	7	7	1.06	24	0.292
78	A	5	4	0.97	24	0.167
79	A	4	4	1.06	24	0.167
80	A	4	3	1.00	24	0.125
81	A	4	4	1.00	24	0.167
82	A	5	4	1.01	24	0.167
83	A	8	8	1.11	24	0.333
84	A	5	4	0.99	24	0.167
85	A	6	6	1.11	24	0.250
86	A	5	4	1.00	22	0.182
87	A	7	7	1.14	21	0.333
88	A	5	4	1.01	24	0.167
89	A	9	8	0.98	26	0.308
90	A	8	7	0.99	26	0.269

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	6	5	1.02	26	0.192
92	A	6	5	1.04	24	0.208
93	A	6	5	0.94	26	0.192
94	A	6	5	0.92	26	0.192
95	A	8	7	1.08	26	0.269
96	A	4	3	1.07	26	0.115
97	A	5	4	1.03	26	0.154
98	A	6	5	1.00	26	0.192
99	A	7	6	0.98	26	0.231
100	A	4	4	0.97	26	0.154
101	A	3	3	0.99	26	0.115
102	A	2	2	1.00	23	0.087
103	A	5	4	1.01	26	0.154
104	A	5	4	0.92	26	0.154
105	A	5	4	0.96	26	0.154
106	A	9	8	0.99	26	0.308
107	A	7	6	1.01	26	0.231
108	A	7	6	1.03	24	0.250
109	A	7	6	0.97	26	0.231
110	A	7	6	0.98	26	0.231
111	A	7	6	0.91	26	0.231
112	A	9	8	0.88	26	0.308
113	A	9	8	1.07	26	0.308
114	A	4	3	1.07	26	0.115
115	A	5	4	1.03	26	0.154
116	A	6	5	1.00	26	0.192
117	A	7	6	0.98	26	0.231
118	A	8	7	0.97	26	0.269
119	A	5	5	0.96	26	0.192
120	A	4	4	0.97	26	0.154
121	A	3	3	0.99	23	0.130
122	A	2	2	1.00	26	0.077
123	A	6	5	1.02	26	0.192
124	A	6	5	0.89	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	6	5	0.89	26	0.192
126	A	6	5	0.91	26	0.192
127	A	7	6	0.89	26	0.231
128	A	8	7	0.90	26	0.269
129	A	9	8	0.90	26	0.308
130	A	8	7	0.95	26	0.269
131	A	7	6	0.96	26	0.231
132	A	5	4	1.05	26	0.154
133	A	5	4	0.98	24	0.167
134	A	5	4	1.09	26	0.154
135	A	4	3	1.07	26	0.115
136	A	5	4	1.03	26	0.154
137	A	6	5	1.00	26	0.192
138	A	7	6	0.98	26	0.231
139	A	4	4	0.97	26	0.154
140	A	3	3	0.99	26	0.115
141	A	2	2	1.00	26	0.077
142	A	4	3	1.00	23	0.130
143	A	4	3	1.00	26	0.115
144	A	5	4	0.99	26	0.154
145	A	11	10	1.02	26	0.385
146	A	8	7	0.95	26	0.269
147	A	7	6	0.89	26	0.231
148	A	6	5	1.07	26	0.192
149	A	3	2	1.00	24	0.083
150	A	4	3	1.06	26	0.115
151	A	5	4	1.03	26	0.154
152	A	6	5	1.00	26	0.192
153	A	4	4	0.97	26	0.154
154	A	3	3	0.97	26	0.115
155	A	2	2	1.00	26	0.077
156	A	4	3	1.00	26	0.115
157	A	6	5	0.92	23	0.217
158	A	6	5	0.95	26	0.192

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
159	A	3	3	1.00	24	0.125
160	A	3	3	1.00	24	0.125
161	A	3	3	1.00	24	0.125
162	A	3	3	1.00	24	0.125
163	A	3	3	1.00	24	0.125
164	A	3	3	1.00	24	0.125
165	A	3	3	1.00	24	0.125
166	A	3	3	1.00	24	0.125
167	A	3	3	1.00	26	0.115
168	A	3	3	1.00	26	0.115
169	A	3	3	1.00	26	0.115
170	A	3	3	1.00	26	0.115
171	A	3	3	1.00	26	0.115
172	A	3	3	1.00	26	0.115
173	A	3	3	1.00	26	0.115
174	A	3	3	1.00	26	0.115
175	A	3	3	1.00	26	0.115
176	A	3	3	1.00	26	0.115
177	A	3	3	1.00	26	0.115
178	A	3	3	1.00	26	0.115
179	A	3	3	1.00	26	0.115
180	A	3	3	1.00	26	0.115
181	A	3	3	1.00	26	0.115
182	A	3	3	1.00	26	0.115
183	A	14	13	1.00	26	0.500
184	A	14	13	1.00	26	0.500
185	A	13	12	1.00	26	0.462
186	A	13	12	1.00	26	0.462
187	A	12	11	1.01	26	0.423
188	A	12	11	1.01	26	0.423
189	A	12	11	1.01	26	0.423
190	A	12	11	1.01	26	0.423
191	A	13	12	1.00	26	0.462
192	A	13	12	1.00	26	0.462

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	14	13	0.99	26	0.500
194	A	14	13	0.99	26	0.500
195	A	15	14	0.96	26	0.538
196	A	14	13	0.97	26	0.500
197	A	14	13	0.97	26	0.500
198	A	13	12	0.98	26	0.462
199	A	13	12	0.97	26	0.462
200	A	12	11	1.01	26	0.423
201	A	12	11	1.01	26	0.423
202	A	13	12	0.99	26	0.462
203	A	13	12	0.98	26	0.462
204	A	14	13	0.97	26	0.500
205	A	14	13	0.97	26	0.500
206	A	15	14	0.96	26	0.538
207	A	15	14	0.96	26	0.538
208	A	14	13	0.97	26	0.500
209	A	14	13	0.96	26	0.500
210	A	13	12	1.00	26	0.462
211	A	13	12	0.98	26	0.462
212	A	13	12	0.98	26	0.462
213	A	13	12	1.00	26	0.462
214	A	14	13	0.98	26	0.500
215	A	14	13	0.97	26	0.500
216	A	15	14	0.96	26	0.538
217	A	15	14	0.97	26	0.538
218	A	16	15	0.95	26	0.577
219	A	16	15	0.96	26	0.577
220	A	8	7	0.95	28	0.250
221	A	10	9	0.96	28	0.321
222	A	7	6	0.96	28	0.214
223	A	9	8	0.97	28	0.286
224	A	6	5	0.98	28	0.179
225	A	9	8	0.96	28	0.286
226	A	6	5	0.97	28	0.179

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
227	A	9	8	0.95	28	0.286
228	A	6	5	0.98	28	0.179
229	A	10	9	0.95	28	0.321
230	A	7	6	0.96	28	0.214
231	A	13	12	0.94	28	0.429
232	A	10	9	0.93	28	0.321
233	A	12	11	0.94	28	0.393
234	A	9	8	0.93	28	0.286
235	A	11	10	0.94	28	0.357
236	A	8	7	0.94	28	0.250
237	A	10	9	0.94	28	0.321
238	A	7	6	0.96	28	0.214
239	A	10	9	0.96	28	0.321
240	A	7	6	0.96	28	0.214
241	A	10	9	0.95	28	0.321
242	A	7	6	0.94	28	0.214
243	A	10	9	0.94	28	0.321
244	A	8	7	0.98	28	0.250
245	A	10	9	0.97	28	0.321
246	A	7	6	0.99	28	0.214
247	A	9	8	0.98	28	0.286
248	A	6	5	0.99	28	0.179
249	A	8	7	0.98	28	0.250
250	A	5	4	1.00	28	0.143
251	A	8	7	1.00	28	0.250
252	A	5	4	1.00	28	0.143
253	A	9	8	0.96	28	0.286
254	A	6	5	0.99	28	0.179
255	A	10	9	0.96	28	0.321
256	A	7	6	0.99	28	0.214
257	A	8	7	0.99	28	0.250
258	A	10	9	0.98	28	0.321
259	A	7	6	1.00	28	0.214
260	A	9	8	0.98	28	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
261	A	6	5	0.99	28	0.179
262	A	8	7	0.99	28	0.250
263	A	5	4	1.00	28	0.143
264	A	9	8	0.98	28	0.286
265	A	6	5	0.98	28	0.179
266	A	10	9	0.95	28	0.321
267	A	7	6	0.98	28	0.214
268	A	11	10	0.96	28	0.357
269	A	3	3	1.00	24	0.125
270	A	3	3	1.00	24	0.125
271	A	3	3	1.00	22	0.136
272	A	3	3	1.00	24	0.125
273	A	3	3	1.00	24	0.125
274	A	4	4	0.94	24	0.167
275	A	2	2	0.97	32	0.062
276	A	4	4	1.00	30	0.133
277	A	4	4	1.12	34	0.118
278	A	5	4	1.32	19	0.211
279	A	5	4	1.27	15	0.267
280	A	5	4	1.24	19	0.211
281	A	3	3	1.00	17	0.176
282	A	4	4	1.00	25	0.160
283	A	2	2	1.00	39	0.051
284	A	5	4	1.00	20	0.200
285	A	8	7	1.10	20	0.350
286	A	7	6	1.07	18	0.333
287	A	6	5	1.00	17	0.294
288	A	4	3	1.00	20	0.150
289	A	5	4	1.00	20	0.200
290	A	5	4	1.00	20	0.200
291	A	5	4	1.00	20	0.200
292	A	5	4	1.00	20	0.200
293	A	8	7	1.14	20	0.350
294	A	7	6	1.16	18	0.333

Continued on next page

2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
295	A	4	3	1.00	17	0.176
296	A	5	4	1.00	20	0.200
297	A	7	6	1.13	20	0.300
298	A	7	6	1.06	20	0.300

CHAPTER 3

Listing of integrals

3.1 $\int x^2(A+Bx^2)(bx^2+cx^4)\,dx$

3.1.1 Optimal result

Integrand size = 22, antiderivative size = 33

$$\int x^2(A+Bx^2)(bx^2+cx^4)\,dx=\frac{1}{5}Abx^5+\frac{1}{7}(bB+Ac)x^7+\frac{1}{9}Bcx^9$$

output
```
1/5*A*b*x^5+1/7*(A*c+B*b)*x^7+1/9*B*c*x^9
```

3.1.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x^2(A+Bx^2)(bx^2+cx^4)\,dx=\frac{1}{5}Abx^5+\frac{1}{7}(bB+Ac)x^7+\frac{1}{9}Bcx^9$$

input
```
Integrate[x^2*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

output
```
(A*b*x^5)/5 + ((b*B + A*c)*x^7)/7 + (B*c*x^9)/9
```

3.1.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (A+Bx^2) \left(bx^2+cx^4\right) \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^4 (A+Bx^2) \left(b+cx^2\right) dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(x^6(Ac+bB)+Abx^4+Bcx^8\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{7}x^7(Ac+bB)+\frac{1}{5}Abx^5+\frac{1}{9}Bcx^9$$

input
```
Int[x^2*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

output
```
(A*b*x^5)/5 + ((b*B + A*c)*x^7)/7 + (B*c*x^9)/9
```

3.1.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{Ab\,x^{5}}{5}+\frac{(Ac+Bb)x^{7}}{7}+\frac{Bc\,x^{9}}{9}$	28
norman	$\frac{Bc\,x^{9}}{9}+\left(\frac{Ac}{7}+\frac{Bb}{7}\right)x^{7}+\frac{Ab\,x^{5}}{5}$	29
risch	$\frac{1}{5}Ab\,x^{5}+\frac{1}{7}x^{7}Ac+\frac{1}{7}bB\,x^{7}+\frac{1}{9}Bc\,x^{9}$	30
parallelrisch	$\frac{1}{5}Ab\,x^{5}+\frac{1}{7}x^{7}Ac+\frac{1}{7}bB\,x^{7}+\frac{1}{9}Bc\,x^{9}$	30
gosper	$\frac{x^{5}(35Bc\,x^{4}+45Ac\,x^{2}+45bB\,x^{2}+63Ab)}{315}$	32

input
```
int(x^2*(B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/5*A*b*x^5+1/7*(A*c+B*b)*x^7+1/9*B*c*x^9
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^2(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{1}{9}\,Bcx^9+\frac{1}{7}\,(Bb+Ac)x^7+\frac{1}{5}\,Abx^5$$

input
```
integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/9*B*c*x^9 + 1/7*(B*b + A*c)*x^7 + 1/5*A*b*x^5
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^2(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{Abx^5}{5}+\frac{Bcx^9}{9}+x^7\left(\frac{Ac}{7}+\frac{Bb}{7}\right)$$

input
```
integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2),x)
```

output
```
A*b*x**5/5 + B*c*x**9/9 + x**7*(A*c/7 + B*b/7)
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^2(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{1}{9}\,Bcx^9+\frac{1}{7}\,(Bb+Ac)x^7+\frac{1}{5}\,Abx^5$$

input
```
integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/9*B*c*x^9 + 1/7*(B*b + A*c)*x^7 + 1/5*A*b*x^5
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^2(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{1}{9}\,Bcx^9+\frac{1}{7}\,Bbx^7+\frac{1}{7}\,Acx^7+\frac{1}{5}\,Abx^5$$

input
```
integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/9*B*c*x^9 + 1/7*B*b*x^7 + 1/7*A*c*x^7 + 1/5*A*b*x^5
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x^2(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{B\,c\,x^9}{9}+\left(\frac{A\,c}{7}+\frac{B\,b}{7}\right)\,x^7+\frac{A\,b\,x^5}{5}$$

input
```
int(x^2*(A + B*x^2)*(b*x^2 + c*x^4),x)
```

output
```
x^7*((A*c)/7 + (B*b)/7) + (A*b*x^5)/5 + (B*c*x^9)/9
```

3.2 $\int x(A+Bx^2)\left(bx^2+cx^4\right)\,dx$

3.2.1 Optimal result

Integrand size = 20, antiderivative size = 33

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{1}{4}Abx^4+\frac{1}{6}(bB+Ac)x^6+\frac{1}{8}Bcx^8$$

output
```
1/4*A*b*x^4+1/6*(A*c+B*b)*x^6+1/8*B*c*x^8
```

3.2.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{1}{4}Abx^4+\frac{1}{6}(bB+Ac)x^6+\frac{1}{8}Bcx^8$$

input
```
Integrate[x*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

output
```
(A*b*x^4)/4 + ((b*B + A*c)*x^6)/6 + (B*c*x^8)/8
```

3.2.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx$$

$$\Big\downarrow \; 9$$

$$\int x^3\left(A+Bx^2\right)\left(b+cx^2\right)dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int x^2\left(Bx^2+A\right)\left(cx^2+b\right)dx^2$$

$$\Big\downarrow \; 85$$

$$\frac{1}{2}\int \left(Bcx^6+(bB+Ac)x^4+Abx^2\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{1}{3}x^6(Ac+bB)+\frac{1}{2}Abx^4+\frac{1}{4}Bcx^8\right)$$

input
```
Int[x*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

output
```
((A*b*x^4)/2 + ((b*B + A*c)*x^6)/3 + (B*c*x^8)/4)/2
```

3.2.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.2.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{Ab\,x^4}{4}+\frac{(Ac+Bb)x^6}{6}+\frac{Bc\,x^8}{8}$	28
norman	$\frac{Bc\,x^8}{8}+\left(\frac{Ac}{6}+\frac{Bb}{6}\right)x^6+\frac{Ab\,x^4}{4}$	29
risch	$\frac{1}{4}Ab\,x^4+\frac{1}{6}x^6Ac+\frac{1}{6}bB\,x^6+\frac{1}{8}Bc\,x^8$	30
parallelrisch	$\frac{1}{4}Ab\,x^4+\frac{1}{6}x^6Ac+\frac{1}{6}bB\,x^6+\frac{1}{8}Bc\,x^8$	30
gosper	$\frac{x^4(3Bc\,x^4+4Ac\,x^2+4bB\,x^2+6Ab)}{24}$	32

input
```
int(x*(B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*A*b*x^4+1/6*(A*c+B*b)*x^6+1/8*B*c*x^8
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{1}{8}\,Bcx^8+\frac{1}{6}\,(Bb+Ac)x^6+\frac{1}{4}\,Abx^4$$

input
```
integrate(x*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/8*B*c*x^8 + 1/6*(B*b + A*c)*x^6 + 1/4*A*b*x^4
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{Abx^4}{4}+\frac{Bcx^8}{8}+x^6\left(\frac{Ac}{6}+\frac{Bb}{6}\right)$$

input
```
integrate(x*(B*x**2+A)*(c*x**4+b*x**2),x)
```

output
```
A*b*x**4/4 + B*c*x**8/8 + x**6*(A*c/6 + B*b/6)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{1}{8}\,Bcx^8+\frac{1}{6}\,(Bb+Ac)x^6+\frac{1}{4}\,Abx^4$$

input
```
integrate(x*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/8*B*c*x^8 + 1/6*(B*b + A*c)*x^6 + 1/4*A*b*x^4
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{1}{8}\,Bcx^8+\frac{1}{6}\,Bbx^6+\frac{1}{6}\,Acx^6+\frac{1}{4}\,Abx^4$$

input
```
integrate(x*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/8*B*c*x^8 + 1/6*B*b*x^6 + 1/6*A*c*x^6 + 1/4*A*b*x^4
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{B\,c\,x^8}{8}+\left(\frac{A\,c}{6}+\frac{B\,b}{6}\right)\,x^6+\frac{A\,b\,x^4}{4}$$

input
```
int(x*(A + B*x^2)*(b*x^2 + c*x^4),x)
```

output
```
x^6*((A*c)/6 + (B*b)/6) + (A*b*x^4)/4 + (B*c*x^8)/8
```

3.3 $\int (A+Bx^2)(bx^2+cx^4)\,dx$

3.3.1 Optimal result

Integrand size = 19, antiderivative size = 33

$$\int (A+Bx^2)(bx^2+cx^4)\,dx=\frac{1}{3}Abx^3+\frac{1}{5}(bB+Ac)x^5+\frac{1}{7}Bcx^7$$

output
```
1/3*A*b*x^3+1/5*(A*c+B*b)*x^5+1/7*B*c*x^7
```

3.3.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int (A+Bx^2)(bx^2+cx^4)\,dx=\frac{1}{3}Abx^3+\frac{1}{5}(bB+Ac)x^5+\frac{1}{7}Bcx^7$$

input
```
Integrate[(A + B*x^2)*(b*x^2 + c*x^4),x]
```

output
```
(A*b*x^3)/3 + ((b*B + A*c)*x^5)/5 + (B*c*x^7)/7
```

3.3.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2027, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx^2)\,(bx^2+cx^4)\,dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int x^2(A+Bx^2)\,(b+cx^2)\,dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(x^4(Ac+bB)+Abx^2+Bcx^6\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}x^5(Ac+bB)+\frac{1}{3}Abx^3+\frac{1}{7}Bcx^7$$

input
```
Int[(A + B*x^2)*(b*x^2 + c*x^4),x]
```

output
```
(A*b*x^3)/3 + ((b*B + A*c)*x^5)/5 + (B*c*x^7)/7
```

3.3.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.3.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{Ab\,x^{3}}{3}+\frac{(Ac+Bb)x^{5}}{5}+\frac{Bc\,x^{7}}{7}$	28
norman	$\frac{Bc\,x^{7}}{7}+\left(\frac{Ac}{5}+\frac{Bb}{5}\right)x^{5}+\frac{Ab\,x^{3}}{3}$	29
risch	$\frac{1}{3}Ab\,x^{3}+\frac{1}{5}x^{5}Ac+\frac{1}{5}bB\,x^{5}+\frac{1}{7}Bc\,x^{7}$	30
parallelrisch	$\frac{1}{3}Ab\,x^{3}+\frac{1}{5}x^{5}Ac+\frac{1}{5}bB\,x^{5}+\frac{1}{7}Bc\,x^{7}$	30
gosper	$\frac{x^{3}\left(15Bc\,x^{4}+21Ac\,x^{2}+21bB\,x^{2}+35Ab\right)}{105}$	32

input
```
int((B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/3*A*b*x^3+1/5*(A*c+B*b)*x^5+1/7*B*c*x^7
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{1}{7}\,Bcx^7+\frac{1}{5}\,(Bb+Ac)x^5+\frac{1}{3}\,Abx^3$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/7*B*c*x^7 + 1/5*(B*b + A*c)*x^5 + 1/3*A*b*x^3
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{Abx^3}{3}+\frac{Bcx^7}{7}+x^5\left(\frac{Ac}{5}+\frac{Bb}{5}\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2),x)
```

output
```
A*b*x**3/3 + B*c*x**7/7 + x**5*(A*c/5 + B*b/5)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int (A + Bx^2) (bx^2 + cx^4) \, dx = \frac{1}{7} Bcx^7 + \frac{1}{5} (Bb + Ac)x^5 + \frac{1}{3} Abx^3$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/7*B*c*x^7 + 1/5*(B*b + A*c)*x^5 + 1/3*A*b*x^3
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int (A + Bx^2) (bx^2 + cx^4) \, dx = \frac{1}{7} Bcx^7 + \frac{1}{5} Bbx^5 + \frac{1}{5} Acx^5 + \frac{1}{3} Abx^3$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/7*B*c*x^7 + 1/5*B*b*x^5 + 1/5*A*c*x^5 + 1/3*A*b*x^3
```

3.3.9 Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int (A + Bx^2) (bx^2 + cx^4) \, dx = \frac{B\,c\,x^7}{7} + \left(\frac{A\,c}{5} + \frac{B\,b}{5}\right) x^5 + \frac{A\,b\,x^3}{3}$$

input
```
int((A + B*x^2)*(b*x^2 + c*x^4),x)
```

output
```
x^5*((A*c)/5 + (B*b)/5) + (A*b*x^3)/3 + (B*c*x^7)/7
```

3.4 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x}\,dx$

3.4.1 Optimal result

Integrand size = 22, antiderivative size = 33

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x}\,dx = \frac{1}{2}Abx^2 + \frac{1}{4}(bB+Ac)x^4 + \frac{1}{6}Bcx^6$$

output
```
1/2*A*b*x^2+1/4*(A*c+B*b)*x^4+1/6*B*c*x^6
```

3.4.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x}\,dx = \frac{1}{2}Abx^2 + \frac{1}{4}(bB+Ac)x^4 + \frac{1}{6}Bcx^6$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x,x]
```

output
```
(A*b*x^2)/2 + ((b*B + A*c)*x^4)/4 + (B*c*x^6)/6
```

3.4.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {9, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x}dx$$

$$\Big\downarrow \text{9}$$

$$\int x(A+Bx^2)(b+cx^2)dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int (Bx^2+A)(cx^2+b)dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int (Bcx^4+(bB+Ac)x^2+Ab)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4(Ac+bB)+Abx^2+\frac{1}{3}Bcx^6\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4))/x,x]
```

output
```
(A*b*x^2 + ((b*B + A*c)*x^4)/2 + (B*c*x^6)/3)/2
```

3.4.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.4.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{Ab\,x^2}{2}+\frac{(Ac+Bb)x^4}{4}+\frac{Bc\,x^6}{6}$	28
norman	$\frac{Bc\,x^6}{6}+\left(\frac{Ac}{4}+\frac{Bb}{4}\right)x^4+\frac{Ab\,x^2}{2}$	29
risch	$\frac{1}{2}Ab\,x^2+\frac{1}{4}x^4Ac+\frac{1}{4}bB\,x^4+\frac{1}{6}Bc\,x^6$	30
parallelrisch	$\frac{1}{2}Ab\,x^2+\frac{1}{4}x^4Ac+\frac{1}{4}bB\,x^4+\frac{1}{6}Bc\,x^6$	30
gosper	$\frac{x^2\left(2Bc\,x^4+3Ac\,x^2+3bB\,x^2+6Ab\right)}{12}$	32

input
```
int((B*x^2+A)*(c*x^4+b*x^2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*A*b*x^2+1/4*(A*c+B*b)*x^4+1/6*B*c*x^6
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{x}\,dx = \frac{1}{6}\,Bcx^6+\frac{1}{4}\,(Bb+Ac)x^4+\frac{1}{2}\,Abx^2$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x,x, algorithm="fricas")
```

output
```
1/6*B*c*x^6 + 1/4*(B*b + A*c)*x^4 + 1/2*A*b*x^2
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x}\,dx = \frac{Abx^2}{2} + \frac{Bcx^6}{6} + x^4\left(\frac{Ac}{4} + \frac{Bb}{4}\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)/x,x)
```

output
```
A*b*x**2/2 + B*c*x**6/6 + x**4*(A*c/4 + B*b/4)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x}\,dx = \frac{1}{6}\,Bcx^6 + \frac{1}{4}\,(Bb+Ac)x^4 + \frac{1}{2}\,Abx^2$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x,x, algorithm="maxima")
```

output
```
1/6*B*c*x^6 + 1/4*(B*b + A*c)*x^4 + 1/2*A*b*x^2
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x}\,dx = \frac{1}{6}\,Bcx^6 + \frac{1}{4}\,Bbx^4 + \frac{1}{4}\,Acx^4 + \frac{1}{2}\,Abx^2$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x,x, algorithm="giac")
```

output
```
1/6*B*c*x^6 + 1/4*B*b*x^4 + 1/4*A*c*x^4 + 1/2*A*b*x^2
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x}\,dx = \frac{B\,c\,x^6}{6} + \left(\frac{A\,c}{4} + \frac{B\,b}{4}\right)x^4 + \frac{A\,b\,x^2}{2}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4))/x,x)
```

output
```
x^4*((A*c)/4 + (B*b)/4) + (A*b*x^2)/2 + (B*c*x^6)/6
```

3.5 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2} dx$

3.5.1 Optimal result

Integrand size = 22, antiderivative size = 28

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2} dx = Abx + \frac{1}{3}(bB+Ac)x^3 + \frac{1}{5}Bcx^5$$

output
```
A*b*x+1/3*(A*c+B*b)*x^3+1/5*B*c*x^5
```

3.5.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2} dx = Abx + \frac{1}{3}(bB+Ac)x^3 + \frac{1}{5}Bcx^5$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^2,x]
```

output
```
A*b*x + ((b*B + A*c)*x^3)/3 + (B*c*x^5)/5
```

3.5.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}dx$$

$$\Big\downarrow \; 9$$

$$\int (A+Bx^2)(b+cx^2)dx$$

$$\Big\downarrow \; 290$$

$$\int (x^2(Ac+bB)+Ab+Bcx^4)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}x^3(Ac+bB)+Abx+\frac{1}{5}Bcx^5$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^2,x]
```

output
```
A*b*x + ((b*B + A*c)*x^3)/3 + (B*c*x^5)/5
```

3.5.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.5.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$Abx+\frac{(Ac+Bb)x^{3}}{3}+\frac{Bc\,x^{5}}{5}$	25
risch	$Abx+\frac{1}{3}x^{3}Ac+\frac{1}{3}bB\,x^{3}+\frac{1}{5}Bc\,x^{5}$	27
parallelrisch	$Abx+\frac{1}{3}x^{3}Ac+\frac{1}{3}bB\,x^{3}+\frac{1}{5}Bc\,x^{5}$	27
gosper	$\frac{x(3Bc\,x^{4}+5Ac\,x^{2}+5bB\,x^{2}+15Ab)}{15}$	30
norman	$\frac{\left(\frac{Ac}{3}+\frac{Bb}{3}\right)x^{4}+Ab\,x^{2}+\frac{Bc\,x^{6}}{5}}{x}$	32

input
```
int((B*x^2+A)*(c*x^4+b*x^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
A*b*x+1/3*(A*c+B*b)*x^3+1/5*B*c*x^5
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}\,dx=\frac{1}{5}\,Bcx^5+\frac{1}{3}\,(Bb+Ac)x^3+Abx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^2,x, algorithm="fricas")
```

output
```
1/5*B*c*x^5 + 1/3*(B*b + A*c)*x^3 + A*b*x
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}\,dx=Abx+\frac{Bcx^5}{5}+x^3\left(\frac{Ac}{3}+\frac{Bb}{3}\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)/x**2,x)
```

output
```
A*b*x + B*c*x**5/5 + x**3*(A*c/3 + B*b/3)
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}\,dx = \frac{1}{5}\,Bcx^5 + \frac{1}{3}\,(Bb+Ac)x^3 + Abx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^2,x, algorithm="maxima")
```

output
```
1/5*B*c*x^5 + 1/3*(B*b + A*c)*x^3 + A*b*x
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}\,dx = \frac{1}{5}\,Bcx^5 + \frac{1}{3}\,Bbx^3 + \frac{1}{3}\,Acx^3 + Abx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^2,x, algorithm="giac")
```

output
```
1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/3*A*c*x^3 + A*b*x
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}\,dx = \frac{B\,c\,x^5}{5} + \left(\frac{A\,c}{3} + \frac{B\,b}{3}\right)\,x^3 + A\,b\,x$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4))/x^2,x)
```

output
```
x^3*((A*c)/3 + (B*b)/3) + A*b*x + (B*c*x^5)/5
```

3.6 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3}\,dx$

3.6.1 Optimal result

Integrand size = 22, antiderivative size = 29

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3}\,dx = \frac{1}{2}(bB+Ac)x^2+\frac{1}{4}Bcx^4+Ab\log(x)$$

output
```
1/2*(A*c+B*b)*x^2+1/4*B*c*x^4+A*b*ln(x)
```

3.6.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3}\,dx = \frac{1}{2}(bB+Ac)x^2+\frac{1}{4}Bcx^4+Ab\log(x)$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^3,x]
```

output
```
((b*B + A*c)*x^2)/2 + (B*c*x^4)/4 + A*b*Log[x]
```

3.6.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {9, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3}dx$$
$$\Big\downarrow 9$$
$$\int \frac{(A+Bx^2)(b+cx^2)}{x}dx$$
$$\Big\downarrow 354$$
$$\frac{1}{2}\int \frac{(Bx^2+A)(cx^2+b)}{x^2}dx^2$$
$$\Big\downarrow 85$$
$$\frac{1}{2}\int \left(Bcx^2+bB+Ac+\frac{Ab}{x^2}\right)dx^2$$
$$\Big\downarrow 2009$$
$$\frac{1}{2}\left(x^2(Ac+bB)+Ab\log\left(x^2\right)+\frac{1}{2}Bcx^4\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^3,x]
```

output
```
((b*B + A*c)*x^2 + (B*c*x^4)/2 + A*b*Log[x^2])/2
```

3.6.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.6.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{Bc\,x^{4}}{4}+\frac{Ac\,x^{2}}{2}+\frac{bB\,x^{2}}{2}+Ab\ln\left(x\right)$	28
parallelrisch	$\frac{Bc\,x^{4}}{4}+\frac{Ac\,x^{2}}{2}+\frac{bB\,x^{2}}{2}+Ab\ln\left(x\right)$	28
norman	$\frac{\left(\frac{Ac}{2}+\frac{Bb}{2}\right)x^{4}+\frac{Bc\,x^{6}}{4}}{x^{2}}+Ab\ln\left(x\right)$	32
risch	$\frac{Bc\,x^{4}}{4}+\frac{Ac\,x^{2}}{2}+\frac{bB\,x^{2}}{2}+\frac{c\,A^{2}}{4B}+\frac{Ab}{2}+\frac{B\,b^{2}}{4c}+Ab\ln\left(x\right)$	50

input
```
int((B*x^2+A)*(c*x^4+b*x^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*B*c*x^4+1/2*A*c*x^2+1/2*b*B*x^2+A*b*ln(x)
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3}\,dx = \frac{1}{4}\,Bcx^4 + \frac{1}{2}\,(Bb+Ac)x^2 + Ab\log{(x)}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^3,x, algorithm="fricas")
```

output
```
1/4*B*c*x^4 + 1/2*(B*b + A*c)*x^2 + A*b*log(x)
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3}\,dx = Ab\log{(x)} + \frac{Bcx^4}{4} + x^2\left(\frac{Ac}{2} + \frac{Bb}{2}\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)/x**3,x)
```

output
```
A*b*log(x) + B*c*x**4/4 + x**2*(A*c/2 + B*b/2)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3}\,dx = \frac{1}{4}\,Bcx^4 + \frac{1}{2}\,(Bb+Ac)x^2 + \frac{1}{2}\,Ab\log{(x^2)}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^3,x, algorithm="maxima")
```

output
```
1/4*B*c*x^4 + 1/2*(B*b + A*c)*x^2 + 1/2*A*b*log(x^2)
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3}\,dx = \frac{1}{4}Bcx^4 + \frac{1}{2}Bbx^2 + \frac{1}{2}Acx^2 + \frac{1}{2}Ab\log\left(x^2\right)$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^3,x, algorithm="giac")
```

output
```
1/4*B*c*x^4 + 1/2*B*b*x^2 + 1/2*A*c*x^2 + 1/2*A*b*log(x^2)
```

3.6.9 Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3}\,dx = x^2\left(\frac{A\,c}{2}+\frac{B\,b}{2}\right)+\frac{B\,c\,x^4}{4}+A\,b\,\ln(x)$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4))/x^3,x)
```

output
```
x^2*((A*c)/2 + (B*b)/2) + (B*c*x^4)/4 + A*b*log(x)
```

3.7 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4} \, dx$

3.7.1 Optimal result

Integrand size = 22, antiderivative size = 26

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4} \, dx = -\frac{Ab}{x} + (bB+Ac)x + \frac{1}{3}Bcx^3$$

output
```
-A*b/x+(A*c+B*b)*x+1/3*B*c*x^3
```

3.7.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4} \, dx = -\frac{Ab}{x} + (bB+Ac)x + \frac{1}{3}Bcx^3$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^4,x]
```

output
```
-((A*b)/x) + (b*B + A*c)*x + (B*c*x^3)/3
```

3.7.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(A+Bx^2)(b+cx^2)}{x^2}dx$$

$$\Big\downarrow \; 355$$

$$\int \left(bB\left(\frac{Ac}{bB}+1\right)+\frac{Ab}{x^2}+Bcx^2\right)dx$$

$$\Big\downarrow \; 2009$$

$$x(Ac+bB)-\frac{Ab}{x}+\frac{1}{3}Bcx^3$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^4,x]
```

output
```
-((A*b)/x) + (b*B + A*c)*x + (B*c*x^3)/3
```

3.7.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.7.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{B c \,x^{3}}{3}+A c x +b B x -\frac{A b}{x}$	24
risch	$\frac{B c \,x^{3}}{3}+A c x +b B x -\frac{A b}{x}$	24
norman	$\frac{\left(A c +B b \right) x^{4}-A b \,x^{2}+\frac{B c \,x^{6}}{3}}{x^{3}}$	31
parallelrisch	$\frac{B c \,x^{4}+3 A c \,x^{2}+3 b B \,x^{2}-3 A b}{3 x}$	31
gosper	$-\frac{-B c \,x^{4}-3 A c \,x^{2}-3 b B \,x^{2}+3 A b}{3 x}$	32

input
```
int((B*x^2+A)*(c*x^4+b*x^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*B*c*x^3+A*c*x+b*B*x-A*b/x
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4}\,dx=\frac{Bcx^4+3\,(Bb+Ac)x^2-3\,Ab}{3\,x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^4,x, algorithm="fricas")
```

output
```
1/3*(B*c*x^4 + 3*(B*b + A*c)*x^2 - 3*A*b)/x
```

3.7.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4}\,dx = -\frac{Ab}{x} + \frac{Bcx^3}{3} + x(Ac+Bb)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)/x**4,x)
```

output
```
-A*b/x + B*c*x**3/3 + x*(A*c + B*b)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4}\,dx = \frac{1}{3}Bcx^3 + (Bb+Ac)x - \frac{Ab}{x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^4,x, algorithm="maxima")
```

output
```
1/3*B*c*x^3 + (B*b + A*c)*x - A*b/x
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4}\,dx = \frac{1}{3}Bcx^3 + Bbx + Acx - \frac{Ab}{x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^4,x, algorithm="giac")
```

output
```
1/3*B*c*x^3 + B*b*x + A*c*x - A*b/x
```

3.7.9 Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4}\,dx = x\,(A\,c+B\,b)-\frac{A\,b}{x}+\frac{B\,c\,x^3}{3}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4))/x^4,x)
```

output
```
x*(A*c + B*b) - (A*b)/x + (B*c*x^3)/3
```

3.8 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5}\,dx$

3.8.1 Optimal result

Integrand size = 22, antiderivative size = 29

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5}\,dx = -\frac{Ab}{2x^2}+\frac{1}{2}Bcx^2+(bB+Ac)\log(x)$$

output
```
-1/2*A*b/x^2+1/2*B*c*x^2+(A*c+B*b)*ln(x)
```

3.8.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5}\,dx = -\frac{Ab}{2x^2}+\frac{1}{2}Bcx^2+(bB+Ac)\log(x)$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^5,x]
```

output
```
-1/2*(A*b)/x^2 + (B*c*x^2)/2 + (b*B + A*c)*Log[x]
```

3.8.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {9, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(A+Bx^2)(b+cx^2)}{x^3} dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{(Bx^2+A)(cx^2+b)}{x^4} dx^2$$

$$\Big\downarrow \; 85$$

$$\frac{1}{2}\int \left(\frac{Ab}{x^4}+Bc+\frac{bB+Ac}{x^2}\right) dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\log\left(x^2\right)(Ac+bB)-\frac{Ab}{x^2}+Bcx^2\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^5,x]
```

output
```
(-((A*b)/x^2) + B*c*x^2 + (b*B + A*c)*Log[x^2])/2
```

3.8.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.8.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{Ab}{2x^2}+\frac{Bc\,x^2}{2}+(Ac+Bb)\ln(x)$	26
risch	$-\frac{Ab}{2x^2}+\frac{Bc\,x^2}{2}+A\ln(x)\,c+bB\ln(x)$	26
norman	$\frac{-\frac{1}{2}Ab\,x^2+\frac{1}{2}Bc\,x^6}{x^4}+(Ac+Bb)\ln(x)$	31
parallelrisch	$\frac{Bc\,x^4+2A\ln(x)x^2c+2B\ln(x)x^2b-Ab}{2x^2}$	35

input
```
int((B*x^2+A)*(c*x^4+b*x^2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A*b/x^2+1/2*B*c*x^2+(A*c+B*b)*ln(x)
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5}\,dx = \frac{Bcx^4+2\,(Bb+Ac)x^2\log{(x)}-Ab}{2\,x^2}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^5,x, algorithm="fricas")
```

output
```
1/2*(B*c*x^4 + 2*(B*b + A*c)*x^2*log(x) - A*b)/x^2
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5}\,dx = -\frac{Ab}{2x^2}+\frac{Bcx^2}{2}+(Ac+Bb)\log{(x)}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)/x**5,x)
```

output
```
-A*b/(2*x**2) + B*c*x**2/2 + (A*c + B*b)*log(x)
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5}\,dx = \frac{1}{2}\,Bcx^2+\frac{1}{2}\,(Bb+Ac)\log{(x^2)}-\frac{Ab}{2\,x^2}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^5,x, algorithm="maxima")
```

output
```
1/2*B*c*x^2 + 1/2*(B*b + A*c)*log(x^2) - 1/2*A*b/x^2
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5}\,dx = \frac{1}{2}\,Bcx^2 + \frac{1}{2}\,(Bb+Ac)\log\left(x^2\right) - \frac{Bbx^2+Acx^2+Ab}{2\,x^2}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^5,x, algorithm="giac")
```

output
```
1/2*B*c*x^2 + 1/2*(B*b + A*c)*log(x^2) - 1/2*(B*b*x^2 + A*c*x^2 + A*b)/x^2
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5}\,dx = \ln(x)\,\left(A\,c+B\,b\right) - \frac{A\,b}{2\,x^2} + \frac{B\,c\,x^2}{2}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4))/x^5,x)
```

output
```
log(x)*(A*c + B*b) - (A*b)/(2*x^2) + (B*c*x^2)/2
```

3.9 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^6} dx$

3.9.1 Optimal result

Integrand size = 22, antiderivative size = 26

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^6} dx = -\frac{Ab}{3x^3} - \frac{bB+Ac}{x} + Bcx$$

output
```
-1/3*A*b/x^3+(-A*c-B*b)/x+B*c*x
```

3.9.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^6} dx = -\frac{Ab}{3x^3} + \frac{-bB-Ac}{x} + Bcx$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^6,x]
```

output
```
-1/3*(A*b)/x^3 + (-(b*B) - A*c)/x + B*c*x
```

3.9.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^6}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx^2)(b+cx^2)}{x^4}dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(\frac{Ac+bB}{x^2}+\frac{Ab}{x^4}+Bc\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{Ac+bB}{x}-\frac{Ab}{3x^3}+Bcx$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^6,x]
```

output
```
-1/3*(A*b)/x^3 - (b*B + A*c)/x + B*c*x
```

3.9.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.9.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result	size
default	$Bcx-\frac{Ab}{3x^{3}}-\frac{Ac+Bb}{x}$	25
risch	$Bcx+\frac{\left(-Ac-Bb\right)x^{2}-\frac{Ab}{3}}{x^{3}}$	28
gosper	$-\frac{-3Bc\,x^{4}+3Ac\,x^{2}+3bB\,x^{2}+Ab}{3x^{3}}$	31
parallelrisch	$-\frac{-3Bc\,x^{4}+3Ac\,x^{2}+3bB\,x^{2}+Ab}{3x^{3}}$	31
norman	$\frac{\left(-Ac-Bb\right)x^{4}+Bc\,x^{6}-\frac{Ab\,x^{2}}{3}}{x^{5}}$	32

input
```
int((B*x^2+A)*(c*x^4+b*x^2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
B*c*x-1/3*A*b/x^3-(A*c+B*b)/x
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{x^6}\,dx=\frac{3\,Bcx^4-3\,(Bb+Ac)x^2-Ab}{3\,x^3}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^6,x, algorithm="fricas")
```

output
```
1/3*(3*B*c*x^4 - 3*(B*b + A*c)*x^2 - A*b)/x^3
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^6} \, dx = Bcx + \frac{-Ab + x^2(-3Ac - 3Bb)}{3x^3}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)/x**6,x)
```

output
```
B*c*x + (-A*b + x**2*(-3*A*c - 3*B*b))/(3*x**3)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^6} \, dx = Bcx - \frac{3(Bb+Ac)x^2 + Ab}{3x^3}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^6,x, algorithm="maxima")
```

output
```
B*c*x - 1/3*(3*(B*b + A*c)*x^2 + A*b)/x^3
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^6} \, dx = Bcx - \frac{3Bbx^2 + 3Acx^2 + Ab}{3x^3}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^6,x, algorithm="giac")
```

output
```
B*c*x - 1/3*(3*B*b*x^2 + 3*A*c*x^2 + A*b)/x^3
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^6}\,dx = B\,c\,x - \frac{(A\,c+B\,b)\;x^2+\frac{A\,b}{3}}{x^3}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4))/x^6,x)
```

output
```
B*c*x - ((A*b)/3 + x^2*(A*c + B*b))/x^3
```

3.10 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^7} dx$

3.10.1 Optimal result

Integrand size = 22, antiderivative size = 29

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^7} dx = -\frac{Ab}{4x^4} - \frac{bB+Ac}{2x^2} + Bc\log(x)$$

output
```
-1/4*A*b/x^4+1/2*(-A*c-B*b)/x^2+B*c*ln(x)
```

3.10.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^7} dx = -\frac{Ab}{4x^4} + \frac{-bB-Ac}{2x^2} + Bc\log(x)$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^7,x]
```

output
```
-1/4*(A*b)/x^4 + (-(b*B) - A*c)/(2*x^2) + B*c*Log[x]
```

3.10.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {9, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^7}dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{(A+Bx^2)(b+cx^2)}{x^5}dx$$
$$\Big\downarrow \; 354$$
$$\frac{1}{2}\int \frac{(Bx^2+A)(cx^2+b)}{x^6}dx^2$$
$$\Big\downarrow \; 85$$
$$\frac{1}{2}\int \left(\frac{Ab}{x^6}+\frac{Bc}{x^2}+\frac{bB+Ac}{x^4}\right)dx^2$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{2}\left(-\frac{Ac+bB}{x^2}-\frac{Ab}{2x^4}+Bc\log\left(x^2\right)\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^7,x]
```

output
```
(-1/2*(A*b)/x^4 - (b*B + A*c)/x^2 + B*c*Log[x^2])/2
```

3.10.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.10.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$Bc\ln\left(x\right)-\frac{Ac+Bb}{2x^{2}}-\frac{Ab}{4x^{4}}$	26
risch	$\frac{\left(-\frac{Ac}{2}-\frac{Bb}{2}\right)x^{2}-\frac{Ab}{4}}{x^{4}}+Bc\ln\left(x\right)$	29
norman	$\frac{\left(-\frac{Ac}{2}-\frac{Bb}{2}\right)x^{4}-\frac{Ab\,x^{2}}{4}}{x^{6}}+Bc\ln\left(x\right)$	32
parallelrisch	$-\frac{-4Bc\ln(x)x^{4}+2Ac\,x^{2}+2bB\,x^{2}+Ab}{4x^{4}}$	33

input
```
int((B*x^2+A)*(c*x^4+b*x^2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
B*c*ln(x)-1/2*(A*c+B*b)/x^2-1/4*A*b/x^4
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{x^7}\,dx = \frac{4\,Bcx^4\log{(x)}-2\,(Bb+Ac)x^2-Ab}{4\,x^4}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^7,x, algorithm="fricas")
```

output
```
1/4*(4*B*c*x^4*log(x) - 2*(B*b + A*c)*x^2 - A*b)/x^4
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{x^7}\,dx = Bc\log{(x)} + \frac{-Ab+x^2(-2Ac-2Bb)}{4x^4}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)/x**7,x)
```

output
```
B*c*log(x) + (-A*b + x**2*(-2*A*c - 2*B*b))/(4*x**4)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{x^7}\,dx = \frac{1}{2}\,Bc\log{(x^2)} - \frac{2\,(Bb+Ac)x^2+Ab}{4\,x^4}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^7,x, algorithm="maxima")
```

output
```
1/2*B*c*log(x^2) - 1/4*(2*(B*b + A*c)*x^2 + A*b)/x^4
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^7}\,dx = \frac{1}{2}Bc\log(x^2) - \frac{3\,Bcx^4+2\,Bbx^2+2\,Acx^2+Ab}{4\,x^4}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^7,x, algorithm="giac")
```

output
```
1/2*B*c*log(x^2) - 1/4*(3*B*c*x^4 + 2*B*b*x^2 + 2*A*c*x^2 + A*b)/x^4
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^7}\,dx = B\,c\,\ln(x) - \frac{\left(\frac{A\,c}{2}+\frac{B\,b}{2}\right)\,x^2+\frac{A\,b}{4}}{x^4}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4))/x^7,x)
```

output
```
B*c*log(x) - ((A*b)/4 + x^2*((A*c)/2 + (B*b)/2))/x^4
```

3.11 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8} \, dx$

3.11.1 Optimal result

Integrand size = 22, antiderivative size = 31

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8} \, dx = -\frac{Ab}{5x^5} - \frac{bB+Ac}{3x^3} - \frac{Bc}{x}$$

output
```
-1/5*A*b/x^5+1/3*(-A*c-B*b)/x^3-B*c/x
```

3.11.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8} \, dx = -\frac{Ab}{5x^5} + \frac{-bB-Ac}{3x^3} - \frac{Bc}{x}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^8,x]
```

output
```
-1/5*(A*b)/x^5 + (-(b*B) - A*c)/(3*x^3) - (B*c)/x
```

3.11.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8}\,dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{(A+Bx^2)(b+cx^2)}{x^6}\,dx$$
$$\Big\downarrow \; 355$$
$$\int \left(\frac{Ac+bB}{x^4}+\frac{Ab}{x^6}+\frac{Bc}{x^2}\right)dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{Ac+bB}{3x^3}-\frac{Ab}{5x^5}-\frac{Bc}{x}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^8,x]
```

output
```
-1/5*(A*b)/x^5 - (b*B + A*c)/(3*x^3) - (B*c)/x
```

3.11.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.11.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{Ac+Bb}{3x^3}-\frac{Bc}{x}-\frac{Ab}{5x^5}$	28
risch	$\frac{-Bc\,x^4+\left(-\frac{Ac}{3}-\frac{Bb}{3}\right)x^2-\frac{Ab}{5}}{x^5}$	30
gosper	$-\frac{15Bc\,x^4+5Ac\,x^2+5bB\,x^2+3Ab}{15x^5}$	32
parallelrisch	$-\frac{15Bc\,x^4+5Ac\,x^2+5bB\,x^2+3Ab}{15x^5}$	32
norman	$\frac{\left(-\frac{Ac}{3}-\frac{Bb}{3}\right)x^4-\frac{Ab\,x^2}{5}-Bc\,x^6}{x^7}$	33

input
```
int((B*x^2+A)*(c*x^4+b*x^2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(A*c+B*b)/x^3-B*c/x-1/5*A*b/x^5
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{x^8}\,dx=-\frac{15\,Bcx^4+5\,(Bb+Ac)x^2+3\,Ab}{15\,x^5}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^8,x, algorithm="fricas")
```

output
```
-1/15*(15*B*c*x^4 + 5*(B*b + A*c)*x^2 + 3*A*b)/x^5
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8}\,dx = \frac{-3Ab-15Bcx^4+x^2(-5Ac-5Bb)}{15x^5}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)/x**8,x)
```

output
```
(-3*A*b - 15*B*c*x**4 + x**2*(-5*A*c - 5*B*b))/(15*x**5)
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8}\,dx = -\frac{15\,Bcx^4+5\,(Bb+Ac)x^2+3\,Ab}{15\,x^5}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^8,x, algorithm="maxima")
```

output
```
-1/15*(15*B*c*x^4 + 5*(B*b + A*c)*x^2 + 3*A*b)/x^5
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8}\,dx = -\frac{15\,Bcx^4+5\,Bbx^2+5\,Acx^2+3\,Ab}{15\,x^5}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^8,x, algorithm="giac")
```

output
```
-1/15*(15*B*c*x^4 + 5*B*b*x^2 + 5*A*c*x^2 + 3*A*b)/x^5
```

3.11.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8}\,dx = -\frac{B\,c\,x^4+\left(\frac{A\,c}{3}+\frac{B\,b}{3}\right)\,x^2+\frac{A\,b}{5}}{x^5}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4))/x^8,x)
```

output
```
-((A*b)/5 + x^2*((A*c)/3 + (B*b)/3) + B*c*x^4)/x^5
```

3.12 $\int (A+Bx^2)(bx^2+cx^4)^2\,dx$

3.12.1 Optimal result

Integrand size = 21, antiderivative size = 55

$$\int (A+Bx^2)(bx^2+cx^4)^2\,dx = \frac{1}{5}Ab^2x^5+\frac{1}{7}b(bB+2Ac)x^7+\frac{1}{9}c(2bB+Ac)x^9+\frac{1}{11}Bc^2x^{11}$$

output
```
1/5*A*b^2*x^5+1/7*b*(2*A*c+B*b)*x^7+1/9*c*(A*c+2*B*b)*x^9+1/11*B*c^2*x^11
```

3.12.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int (A+Bx^2)(bx^2+cx^4)^2\,dx = \frac{1}{5}Ab^2x^5+\frac{1}{7}b(bB+2Ac)x^7+\frac{1}{9}c(2bB+Ac)x^9+\frac{1}{11}Bc^2x^{11}$$

input
```
Integrate[(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

output
```
(A*b^2*x^5)/5 + (b*(b*B + 2*A*c)*x^7)/7 + (c*(2*b*B + A*c)*x^9)/9 + (B*c^2
*x^11)/11
```

3.12.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2027, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$$

$$\Big\downarrow \; 2027$$

$$\int x^4(A+Bx^2)\left(b+cx^2\right)^2\,dx$$

$$\Big\downarrow \; 355$$

$$\int \left(Ab^2x^4+cx^8(Ac+2bB)+bx^6(2Ac+bB)+Bc^2x^{10}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{5}Ab^2x^5+\frac{1}{9}cx^9(Ac+2bB)+\frac{1}{7}bx^7(2Ac+bB)+\frac{1}{11}Bc^2x^{11}$$

input
```
Int[(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

output
```
(A*b^2*x^5)/5 + (b*(b*B + 2*A*c)*x^7)/7 + (c*(2*b*B + A*c)*x^9)/9 + (B*c^2
*x^11)/11
```

3.12.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.12.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{B\,c^{2} x^{11}}{11}+\frac{\left(A\,c^{2}+2 B b c \right) x^{9}}{9}+\frac{\left(2 A b c +B\,b^{2}\right) x^{7}}{7}+\frac{A\,b^{2} x^{5}}{5}$	52
norman	$\frac{B\,c^{2} x^{11}}{11}+\left(\frac{1}{9} A\,c^{2}+\frac{2}{9} B b c \right) x^{9}+\left(\frac{2}{7} A b c +\frac{1}{7} B\,b^{2}\right) x^{7}+\frac{A\,b^{2} x^{5}}{5}$	52
risch	$\frac{1}{11} B\,c^{2} x^{11}+\frac{1}{9} x^{9} A\,c^{2}+\frac{2}{9} x^{9} B b c +\frac{2}{7} x^{7} A b c +\frac{1}{7} b^{2} B\,x^{7}+\frac{1}{5} A\,b^{2} x^{5}$	54
parallelrisch	$\frac{1}{11} B\,c^{2} x^{11}+\frac{1}{9} x^{9} A\,c^{2}+\frac{2}{9} x^{9} B b c +\frac{2}{7} x^{7} A b c +\frac{1}{7} b^{2} B\,x^{7}+\frac{1}{5} A\,b^{2} x^{5}$	54
gosper	$\frac{x^{5} \left(315 B\,c^{2} x^{6}+385 A\,c^{2} x^{4}+770 x^{4} B b c +990 A b c\,x^{2}+495 b^{2} B\,x^{2}+693 b^{2} A \right)}{3465}$	56

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/11*B*c^2*x^11+1/9*(A*c^2+2*B*b*c)*x^9+1/7*(2*A*b*c+B*b^2)*x^7+1/5*A*b^2*
x^5
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx=\frac{1}{11}\,Bc^2x^{11}+\frac{1}{9}\,(2\,Bbc+Ac^2)x^9+\frac{1}{5}\,Ab^2x^5+\frac{1}{7}\,(Bb^2+2\,Abc)x^7$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/11*B*c^2*x^11 + 1/9*(2*B*b*c + A*c^2)*x^9 + 1/5*A*b^2*x^5 + 1/7*(B*b^2 +
 2*A*b*c)*x^7
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int (A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{Ab^2x^5}{5}+\frac{Bc^2x^{11}}{11}+x^9\left(\frac{Ac^2}{9}+\frac{2Bbc}{9}\right)+x^7\cdot\left(\frac{2Abc}{7}+\frac{Bb^2}{7}\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

output
```
A*b**2*x**5/5 + B*c**2*x**11/11 + x**9*(A*c**2/9 + 2*B*b*c/9) + x**7*(2*A*
b*c/7 + B*b**2/7)
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int (A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{1}{11}\,Bc^2x^{11}+\frac{1}{9}\,(2\,Bbc+Ac^2)x^9+\frac{1}{5}\,Ab^2x^5+\frac{1}{7}\,(Bb^2+2\,Abc)x^7$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/11*B*c^2*x^11 + 1/9*(2*B*b*c + A*c^2)*x^9 + 1/5*A*b^2*x^5 + 1/7*(B*b^2 +
 2*A*b*c)*x^7
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int (A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{1}{11}\,Bc^2x^{11}+\frac{2}{9}\,Bbcx^9+\frac{1}{9}\,Ac^2x^9+\frac{1}{7}\,Bb^2x^7+\frac{2}{7}\,Abcx^7+\frac{1}{5}\,Ab^2x^5$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/11*B*c^2*x^11 + 2/9*B*b*c*x^9 + 1/9*A*c^2*x^9 + 1/7*B*b^2*x^7 + 2/7*A*b*
c*x^7 + 1/5*A*b^2*x^5
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int (A+Bx^2) (bx^2+cx^4)^2 \, dx = x^7 \left(\frac{B\,b^2}{7}+\frac{2\,A\,c\,b}{7}\right)+x^9 \left(\frac{A\,c^2}{9}+\frac{2\,B\,b\,c}{9}\right)+\frac{A\,b^2\,x^5}{5}+\frac{B\,c^2\,x^{11}}{11}$$

input
```
int((A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

output
```
x^7*((B*b^2)/7 + (2*A*b*c)/7) + x^9*((A*c^2)/9 + (2*B*b*c)/9) + (A*b^2*x^5
)/5 + (B*c^2*x^11)/11
```

3.13 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x} dx$

3.13.1 Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x} dx = \frac{1}{4}Ab^2x^4 + \frac{1}{6}b(bB+2Ac)x^6 + \frac{1}{8}c(2bB+Ac)x^8 + \frac{1}{10}Bc^2x^{10}$$

output
```
1/4*A*b^2*x^4+1/6*b*(2*A*c+B*b)*x^6+1/8*c*(A*c+2*B*b)*x^8+1/10*B*c^2*x^10
```

3.13.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x} dx = \frac{1}{4}Ab^2x^4 + \frac{1}{6}b(bB+2Ac)x^6 + \frac{1}{8}c(2bB+Ac)x^8 + \frac{1}{10}Bc^2x^{10}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x,x]
```

output
```
(A*b^2*x^4)/4 + (b*(b*B + 2*A*c)*x^6)/6 + (c*(2*b*B + A*c)*x^8)/8 + (B*c^2
*x^10)/10
```

3.13.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int x^3(A+Bx^2)(b+cx^2)^2dx$$
$$\Big\downarrow \text{ 354}$$
$$\frac{1}{2}\int x^2(Bx^2+A)(cx^2+b)^2dx^2$$
$$\Big\downarrow \text{ 85}$$
$$\frac{1}{2}\int (Bc^2x^8+c(2bB+Ac)x^6+b(bB+2Ac)x^4+Ab^2x^2)\,dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(\frac{1}{2}Ab^2x^4+\frac{1}{4}cx^8(Ac+2bB)+\frac{1}{3}bx^6(2Ac+bB)+\frac{1}{5}Bc^2x^{10}\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x,x]
```

output
```
((A*b^2*x^4)/2 + (b*(b*B + 2*A*c)*x^6)/3 + (c*(2*b*B + A*c)*x^8)/4 + (B*c^
2*x^10)/5)/2
```

3.13.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.13.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{B\,c^{2} x^{10}}{10}+\frac{\left(A\,c^{2}+2 B b c \right) x^{8}}{8}+\frac{\left(2 A b c +B\,b^{2}\right) x^{6}}{6}+\frac{A\,b^{2} x^{4}}{4}$	52
norman	$\frac{B\,c^{2} x^{10}}{10}+\left(\frac{1}{8} A\,c^{2}+\frac{1}{4} B b c \right) x^{8}+\left(\frac{1}{3} A b c +\frac{1}{6} B\,b^{2}\right) x^{6}+\frac{A\,b^{2} x^{4}}{4}$	52
risch	$\frac{1}{10} B\,c^{2} x^{10}+\frac{1}{8} x^{8} A\,c^{2}+\frac{1}{4} x^{8} B b c +\frac{1}{3} x^{6} A b c +\frac{1}{6} b^{2} B\,x^{6}+\frac{1}{4} A\,b^{2} x^{4}$	54
parallelrisch	$\frac{1}{10} B\,c^{2} x^{10}+\frac{1}{8} x^{8} A\,c^{2}+\frac{1}{4} x^{8} B b c +\frac{1}{3} x^{6} A b c +\frac{1}{6} b^{2} B\,x^{6}+\frac{1}{4} A\,b^{2} x^{4}$	54
gosper	$\frac{x^{4} \left(12 B\,c^{2} x^{6}+15 A\,c^{2} x^{4}+30 x^{4} B b c +40 A b c\,x^{2}+20 b^{2} B\,x^{2}+30 b^{2} A \right)}{120}$	56

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/10*B*c^2*x^10+1/8*(A*c^2+2*B*b*c)*x^8+1/6*(2*A*b*c+B*b^2)*x^6+1/4*A*b^2*
x^4
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x}\,dx = \frac{1}{10}\,Bc^2x^{10}+\frac{1}{8}\,(2\,Bbc+Ac^2)x^8+\frac{1}{4}\,Ab^2x^4+\frac{1}{6}\,(Bb^2+2\,Abc)x^6$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x,x, algorithm="fricas")
```

output
```
1/10*B*c^2*x^10 + 1/8*(2*B*b*c + A*c^2)*x^8 + 1/4*A*b^2*x^4 + 1/6*(B*b^2 +
 2*A*b*c)*x^6
```

3.13.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x}\,dx = \frac{Ab^2x^4}{4}+\frac{Bc^2x^{10}}{10}+x^8\left(\frac{Ac^2}{8}+\frac{Bbc}{4}\right)+x^6\left(\frac{Abc}{3}+\frac{Bb^2}{6}\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x,x)
```

output
```
A*b**2*x**4/4 + B*c**2*x**10/10 + x**8*(A*c**2/8 + B*b*c/4) + x**6*(A*b*c/
3 + B*b**2/6)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x}\,dx = \frac{1}{10}\,Bc^2x^{10}+\frac{1}{8}\,(2\,Bbc+Ac^2)x^8+\frac{1}{4}\,Ab^2x^4+\frac{1}{6}\,(Bb^2+2\,Abc)x^6$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x,x, algorithm="maxima")
```

output
```
1/10*B*c^2*x^10 + 1/8*(2*B*b*c + A*c^2)*x^8 + 1/4*A*b^2*x^4 + 1/6*(B*b^2 +
 2*A*b*c)*x^6
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x}\,dx = \frac{1}{10}Bc^2x^{10} + \frac{1}{4}Bbcx^8 + \frac{1}{8}Ac^2x^8 + \frac{1}{6}Bb^2x^6 + \frac{1}{3}Abcx^6 + \frac{1}{4}Ab^2x^4$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x,x, algorithm="giac")
```

output
```
1/10*B*c^2*x^10 + 1/4*B*b*c*x^8 + 1/8*A*c^2*x^8 + 1/6*B*b^2*x^6 + 1/3*A*b*
c*x^6 + 1/4*A*b^2*x^4
```

3.13.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x}\,dx = x^6\left(\frac{B\,b^2}{6}+\frac{A\,c\,b}{3}\right)+x^8\left(\frac{A\,c^2}{8}+\frac{B\,b\,c}{4}\right)+\frac{A\,b^2\,x^4}{4}+\frac{B\,c^2\,x^{10}}{10}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x,x)
```

output
```
x^6*((B*b^2)/6 + (A*b*c)/3) + x^8*((A*c^2)/8 + (B*b*c)/4) + (A*b^2*x^4)/4
+ (B*c^2*x^10)/10
```

3.14 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2} dx$

3.14.1 Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2} dx = \frac{1}{3}Ab^2x^3 + \frac{1}{5}b(bB+2Ac)x^5 + \frac{1}{7}c(2bB+Ac)x^7 + \frac{1}{9}Bc^2x^9$$

output
```
1/3*A*b^2*x^3+1/5*b*(2*A*c+B*b)*x^5+1/7*c*(A*c+2*B*b)*x^7+1/9*B*c^2*x^9
```

3.14.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2} dx = \frac{1}{3}Ab^2x^3 + \frac{1}{5}b(bB+2Ac)x^5 + \frac{1}{7}c(2bB+Ac)x^7 + \frac{1}{9}Bc^2x^9$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^2,x]
```

output
```
(A*b^2*x^3)/3 + (b*(b*B + 2*A*c)*x^5)/5 + (c*(2*b*B + A*c)*x^7)/7 + (B*c^2
*x^9)/9
```

3.14.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2}dx$$
$$\Big\downarrow 9$$
$$\int x^2(A+Bx^2)(b+cx^2)^2dx$$
$$\Big\downarrow 355$$
$$\int (Ab^2x^2+cx^6(Ac+2bB)+bx^4(2Ac+bB)+Bc^2x^8)\,dx$$
$$\Big\downarrow 2009$$
$$\frac{1}{3}Ab^2x^3+\frac{1}{7}cx^7(Ac+2bB)+\frac{1}{5}bx^5(2Ac+bB)+\frac{1}{9}Bc^2x^9$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^2,x]
```

output
```
(A*b^2*x^3)/3 + (b*(b*B + 2*A*c)*x^5)/5 + (c*(2*b*B + A*c)*x^7)/7 + (B*c^2
*x^9)/9
```

3.14.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.14.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{B\,c^{2}x^{9}}{9}+\frac{\left(A\,c^{2}+2Bbc\right)x^{7}}{7}+\frac{\left(2Abc+B\,b^{2}\right)x^{5}}{5}+\frac{A\,b^{2}x^{3}}{3}$	52
risch	$\frac{1}{9}B\,c^{2}x^{9}+\frac{1}{7}x^{7}A\,c^{2}+\frac{2}{7}x^{7}Bbc+\frac{2}{5}x^{5}Abc+\frac{1}{5}b^{2}B\,x^{5}+\frac{1}{3}A\,b^{2}x^{3}$	54
parallelrisch	$\frac{1}{9}B\,c^{2}x^{9}+\frac{1}{7}x^{7}A\,c^{2}+\frac{2}{7}x^{7}Bbc+\frac{2}{5}x^{5}Abc+\frac{1}{5}b^{2}B\,x^{5}+\frac{1}{3}A\,b^{2}x^{3}$	54
gosper	$\frac{x^{3}\left(35B\,c^{2}x^{6}+45A\,c^{2}x^{4}+90x^{4}Bbc+126Abc\,x^{2}+63b^{2}B\,x^{2}+105b^{2}A\right)}{315}$	56
norman	$\frac{\left(\frac{1}{7}A\,c^{2}+\frac{2}{7}Bbc\right)x^{8}+\left(\frac{2}{5}Abc+\frac{1}{5}B\,b^{2}\right)x^{6}+\frac{A\,b^{2}x^{4}}{3}+\frac{B\,c^{2}x^{10}}{9}}{x}$	56

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*B*c^2*x^9+1/7*(A*c^2+2*B*b*c)*x^7+1/5*(2*A*b*c+B*b^2)*x^5+1/3*A*b^2*x^
3
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^2}\,dx=\frac{1}{9}\,Bc^2x^9+\frac{1}{7}\left(2\,Bbc+Ac^2\right)x^7+\frac{1}{3}\,Ab^2x^3+\frac{1}{5}\left(Bb^2+2\,Abc\right)x^5$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^2,x, algorithm="fricas")
```

output
```
1/9*B*c^2*x^9 + 1/7*(2*B*b*c + A*c^2)*x^7 + 1/3*A*b^2*x^3 + 1/5*(B*b^2 + 2
*A*b*c)*x^5
```

3.14.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2}\,dx = \frac{Ab^2x^3}{3}+\frac{Bc^2x^9}{9}+x^7\left(\frac{Ac^2}{7}+\frac{2Bbc}{7}\right)+x^5\cdot\left(\frac{2Abc}{5}+\frac{Bb^2}{5}\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**2,x)
```

output
```
A*b**2*x**3/3 + B*c**2*x**9/9 + x**7*(A*c**2/7 + 2*B*b*c/7) + x**5*(2*A*b*
c/5 + B*b**2/5)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2}\,dx = \frac{1}{9}\,Bc^2x^9+\frac{1}{7}\,(2\,Bbc+Ac^2)x^7+\frac{1}{3}\,Ab^2x^3+\frac{1}{5}\,(Bb^2+2\,Abc)x^5$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^2,x, algorithm="maxima")
```

output
```
1/9*B*c^2*x^9 + 1/7*(2*B*b*c + A*c^2)*x^7 + 1/3*A*b^2*x^3 + 1/5*(B*b^2 + 2
*A*b*c)*x^5
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2}\,dx = \frac{1}{9}\,Bc^2x^9+\frac{2}{7}\,Bbcx^7+\frac{1}{7}\,Ac^2x^7+\frac{1}{5}\,Bb^2x^5+\frac{2}{5}\,Abcx^5+\frac{1}{3}\,Ab^2x^3$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^2,x, algorithm="giac")
```

output
```
1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/7*A*c^2*x^7 + 1/5*B*b^2*x^5 + 2/5*A*b*c*
x^5 + 1/3*A*b^2*x^3
```

3.14.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2}\,dx = x^5\left(\frac{B\,b^2}{5}+\frac{2\,A\,c\,b}{5}\right)+x^7\left(\frac{A\,c^2}{7}+\frac{2\,B\,b\,c}{7}\right)+\frac{A\,b^2\,x^3}{3}+\frac{B\,c^2\,x^9}{9}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^2,x)
```

output
```
x^5*((B*b^2)/5 + (2*A*b*c)/5) + x^7*((A*c^2)/7 + (2*B*b*c)/7) + (A*b^2*x^3
)/3 + (B*c^2*x^9)/9
```

3.15 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^3} dx$

3.15.1 Optimal result

Integrand size = 24, antiderivative size = 42

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^3} dx = -\frac{(bB-Ac)(b+cx^2)^3}{6c^2} + \frac{B(b+cx^2)^4}{8c^2}$$

output
```
-1/6*(-A*c+B*b)*(c*x^2+b)^3/c^2+1/8*B*(c*x^2+b)^4/c^2
```

3.15.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^3} dx = \frac{1}{24}x^2(12Ab^2+6b(bB+2Ac)x^2+4c(2bB+Ac)x^4+3Bc^2x^6)$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^3,x]
```

output
```
(x^2*(12*A*b^2 + 6*b*(b*B + 2*A*c)*x^2 + 4*c*(2*b*B + A*c)*x^4 + 3*B*c^2*x
^6))/24
```

3.15.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x(A+Bx^2)(b+cx^2)^2dx$$

$$\Big\downarrow \text{ 353}$$

$$\frac{1}{2}\int (Bx^2+A)(cx^2+b)^2dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{B(cx^2+b)^3}{c}+\frac{(Ac-bB)(cx^2+b)^2}{c}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{B(b+cx^2)^4}{4c^2}-\frac{(b+cx^2)^3(bB-Ac)}{3c^2}\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^3,x]
```

output
```
(-1/3*((b*B - A*c)*(b + c*x^2)^3)/c^2 + (B*(b + c*x^2)^4)/(4*c^2))/2
```

3.15.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.15.4 Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{B\,c^2 x^8}{8}+\frac{(A\,c^2+2Bbc)x^6}{6}+\frac{(2Abc+B\,b^2)x^4}{4}+\frac{A\,b^2 x^2}{2}$	52
risch	$\frac{1}{8}B\,c^2x^8+\frac{1}{6}x^6A\,c^2+\frac{1}{3}x^6Bbc+\frac{1}{2}x^4Abc+\frac{1}{4}b^2B\,x^4+\frac{1}{2}A\,b^2x^2$	54
parallelrisch	$\frac{1}{8}B\,c^2x^8+\frac{1}{6}x^6A\,c^2+\frac{1}{3}x^6Bbc+\frac{1}{2}x^4Abc+\frac{1}{4}b^2B\,x^4+\frac{1}{2}A\,b^2x^2$	54
gosper	$\frac{x^2(3B\,c^2x^6+4A\,c^2x^4+8x^4Bbc+12Abc\,x^2+6b^2B\,x^2+12b^2A)}{24}$	56
norman	$\frac{(\frac{1}{6}A\,c^2+\frac{1}{3}Bbc)x^8+(\frac{1}{2}Abc+\frac{1}{4}B\,b^2)x^6+\frac{A\,b^2x^4}{2}+\frac{B\,c^2x^{10}}{8}}{x^2}$	56

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*B*c^2*x^8+1/6*(A*c^2+2*B*b*c)*x^6+1/4*(2*A*b*c+B*b^2)*x^4+1/2*A*b^2*x^
2
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^3}\,dx = \frac{1}{8}\,Bc^2x^8 + \frac{1}{6}\,(2\,Bbc + Ac^2)x^6 + \frac{1}{2}\,Ab^2x^2 + \frac{1}{4}\,(Bb^2 + 2\,Abc)x^4$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^3,x, algorithm="fricas")
```

output
```
1/8*B*c^2*x^8 + 1/6*(2*B*b*c + A*c^2)*x^6 + 1/2*A*b^2*x^2 + 1/4*(B*b^2 + 2
*A*b*c)*x^4
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^3}\,dx = \frac{Ab^2x^2}{2} + \frac{Bc^2x^8}{8} + x^6\left(\frac{Ac^2}{6} + \frac{Bbc}{3}\right) + x^4\left(\frac{Abc}{2} + \frac{Bb^2}{4}\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**3,x)
```

output
```
A*b**2*x**2/2 + B*c**2*x**8/8 + x**6*(A*c**2/6 + B*b*c/3) + x**4*(A*b*c/2
+ B*b**2/4)
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^3}\,dx = \frac{1}{8}\,Bc^2x^8 + \frac{1}{6}\,(2\,Bbc + Ac^2)x^6 + \frac{1}{2}\,Ab^2x^2 + \frac{1}{4}\,(Bb^2 + 2\,Abc)x^4$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^3,x, algorithm="maxima")
```

output
```
1/8*B*c^2*x^8 + 1/6*(2*B*b*c + A*c^2)*x^6 + 1/2*A*b^2*x^2 + 1/4*(B*b^2 + 2
*A*b*c)*x^4
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^3}\,dx = \frac{1}{8}Bc^2x^8 + \frac{1}{3}Bbcx^6 + \frac{1}{6}Ac^2x^6 + \frac{1}{4}Bb^2x^4 + \frac{1}{2}Abcx^4 + \frac{1}{2}Ab^2x^2$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^3,x, algorithm="giac")
```

output
```
1/8*B*c^2*x^8 + 1/3*B*b*c*x^6 + 1/6*A*c^2*x^6 + 1/4*B*b^2*x^4 + 1/2*A*b*c*
x^4 + 1/2*A*b^2*x^2
```

3.15.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^3}\,dx = x^4\left(\frac{B\,b^2}{4} + \frac{A\,c\,b}{2}\right) + x^6\left(\frac{A\,c^2}{6} + \frac{B\,b\,c}{3}\right) + \frac{A\,b^2\,x^2}{2} + \frac{B\,c^2\,x^8}{8}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^3,x)
```

output
```
x^4*((B*b^2)/4 + (A*b*c)/2) + x^6*((A*c^2)/6 + (B*b*c)/3) + (A*b^2*x^2)/2
+ (B*c^2*x^8)/8
```

3.16 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^4} \, dx$

3.16.1 Optimal result

Integrand size = 24, antiderivative size = 50

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^4} \, dx = Ab^2x + \frac{1}{3}b(bB+2Ac)x^3 + \frac{1}{5}c(2bB+Ac)x^5 + \frac{1}{7}Bc^2x^7$$

output
```
A*b^2*x+1/3*b*(2*A*c+B*b)*x^3+1/5*c*(A*c+2*B*b)*x^5+1/7*B*c^2*x^7
```

3.16.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^4} \, dx = Ab^2x + \frac{1}{3}b(bB+2Ac)x^3 + \frac{1}{5}c(2bB+Ac)x^5 + \frac{1}{7}Bc^2x^7$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^4,x]
```

output
```
A*b^2*x + (b*(b*B + 2*A*c)*x^3)/3 + (c*(2*b*B + A*c)*x^5)/5 + (B*c^2*x^7)/
7
```

3.16.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {9, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^4}\,dx$$

$$\Big\downarrow \; 9$$

$$\int (A+Bx^2)(b+cx^2)^2\,dx$$

$$\Big\downarrow \; 290$$

$$\int (Ab^2+cx^4(Ac+2bB)+bx^2(2Ac+bB)+Bc^2x^6)\,dx$$

$$\Big\downarrow \; 2009$$

$$Ab^2x+\frac{1}{5}cx^5(Ac+2bB)+\frac{1}{3}bx^3(2Ac+bB)+\frac{1}{7}Bc^2x^7$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^4,x]
```

output
```
A*b^2*x + (b*(b*B + 2*A*c)*x^3)/3 + (c*(2*b*B + A*c)*x^5)/5 + (B*c^2*x^7)/
7
```

3.16.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.16.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{B\,c^{2} x^{7}}{7}+\frac{\left(A\,c^{2}+2 B b c \right) x^{5}}{5}+\frac{\left(2 A b c +B\,b^{2}\right) x^{3}}{3}+A\,b^{2} x$	49
risch	$\frac{1}{7} B\,c^{2} x^{7}+\frac{1}{5} x^{5} A\,c^{2}+\frac{2}{5} x^{5} B b c +\frac{2}{3} x^{3} A b c +\frac{1}{3} b^{2} B\,x^{3}+A\,b^{2} x$	51
parallelrisch	$\frac{1}{7} B\,c^{2} x^{7}+\frac{1}{5} x^{5} A\,c^{2}+\frac{2}{5} x^{5} B b c +\frac{2}{3} x^{3} A b c +\frac{1}{3} b^{2} B\,x^{3}+A\,b^{2} x$	51
gosper	$\frac{x \left(15 B\,c^{2} x^{6}+21 A\,c^{2} x^{4}+42 x^{4} B b c +70 A b c\,x^{2}+35 b^{2} B\,x^{2}+105 b^{2} A \right)}{105}$	54
norman	$\frac{\left(\frac{1}{5} A\,c^{2}+\frac{2}{5} B b c \right) x^{8}+\left(\frac{2}{3} A b c +\frac{1}{3} B\,b^{2}\right) x^{6}+A\,b^{2} x^{4}+\frac{B\,c^{2} x^{10}}{7}}{x^{3}}$	55

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/7*B*c^2*x^7+1/5*(A*c^2+2*B*b*c)*x^5+1/3*(2*A*b*c+B*b^2)*x^3+A*b^2*x
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^4}\,dx=\frac{1}{7}\,Bc^2x^7+\frac{1}{5}\left(2\,Bbc+Ac^2\right)x^5+Ab^2x+\frac{1}{3}\left(Bb^2+2\,Abc\right)x^3$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^4,x, algorithm="fricas")
```

output
```
1/7*B*c^2*x^7 + 1/5*(2*B*b*c + A*c^2)*x^5 + A*b^2*x + 1/3*(B*b^2 + 2*A*b*c
)*x^3
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^4}\,dx = Ab^2x + \frac{Bc^2x^7}{7} + x^5\left(\frac{Ac^2}{5} + \frac{2Bbc}{5}\right) + x^3\cdot\left(\frac{2Abc}{3} + \frac{Bb^2}{3}\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**4,x)
```

output
```
A*b**2*x + B*c**2*x**7/7 + x**5*(A*c**2/5 + 2*B*b*c/5) + x**3*(2*A*b*c/3 +
 B*b**2/3)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^4}\,dx = \frac{1}{7}Bc^2x^7 + \frac{1}{5}(2Bbc+Ac^2)x^5 + Ab^2x + \frac{1}{3}(Bb^2+2Abc)x^3$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^4,x, algorithm="maxima")
```

output
```
1/7*B*c^2*x^7 + 1/5*(2*B*b*c + A*c^2)*x^5 + A*b^2*x + 1/3*(B*b^2 + 2*A*b*c
)*x^3
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^4}\,dx = \frac{1}{7}Bc^2x^7 + \frac{2}{5}Bbcx^5 + \frac{1}{5}Ac^2x^5 + \frac{1}{3}Bb^2x^3 + \frac{2}{3}Abcx^3 + Ab^2x$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^4,x, algorithm="giac")
```

output
```
1/7*B*c^2*x^7 + 2/5*B*b*c*x^5 + 1/5*A*c^2*x^5 + 1/3*B*b^2*x^3 + 2/3*A*b*c*
x^3 + A*b^2*x
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^4}\,dx = x^3\left(\frac{B\,b^2}{3}+\frac{2\,A\,c\,b}{3}\right)+x^5\left(\frac{A\,c^2}{5}+\frac{2\,B\,b\,c}{5}\right)+\frac{B\,c^2\,x^7}{7}+A\,b^2\,x$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^4,x)
```

output
```
x^3*((B*b^2)/3 + (2*A*b*c)/3) + x^5*((A*c^2)/5 + (2*B*b*c)/5) + (B*c^2*x^7
)/7 + A*b^2*x
```

3.17 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^5}\,dx$

3.17.1 Optimal result

Integrand size = 24, antiderivative size = 43

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^5}\,dx = Abcx^2 + \frac{1}{4}Ac^2x^4 + \frac{B(b+cx^2)^3}{6c} + Ab^2\log(x)$$

output
```
A*b*c*x^2+1/4*A*c^2*x^4+1/6*B*(c*x^2+b)^3/c+A*b^2*ln(x)
```

3.17.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^5}\,dx = \frac{1}{2}b(bB+2Ac)x^2 + \frac{1}{4}c(2bB+Ac)x^4 + \frac{1}{6}Bc^2x^6 + Ab^2\log(x)$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^5,x]
```

output
```
(b*(b*B + 2*A*c)*x^2)/2 + (c*(2*b*B + A*c)*x^4)/4 + (B*c^2*x^6)/6 + A*b^2*
Log[x]
```

3.17.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {9, 354, 90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^5}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{(A+Bx^2)(b+cx^2)^2}{x}dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2}\int \frac{(Bx^2+A)(cx^2+b)^2}{x^2}dx^2$$

$$\Big\downarrow \ 90$$

$$\frac{1}{2}\left(A\int \frac{(cx^2+b)^2}{x^2}dx^2+\frac{B(b+cx^2)^3}{3c}\right)$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2}\left(A\int \left(\frac{b^2}{x^2}+2cb+c^2x^2\right)dx^2+\frac{B(b+cx^2)^3}{3c}\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(A\left(b^2\log(x^2)+2bcx^2+\frac{c^2x^4}{2}\right)+\frac{B(b+cx^2)^3}{3c}\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^5,x]
```

output
```
((B*(b + c*x^2)^3)/(3*c) + A*(2*b*c*x^2 + (c^2*x^4)/2 + b^2*Log[x^2]))/2
```

3.17.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.17.4 Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{B\,c^{2} x^{6}}{6}+\frac{A\,c^{2} x^{4}}{4}+\frac{x^{4} B b c}{2}+A b c\,x^{2}+\frac{b^{2} B\,x^{2}}{2}+A\,b^{2} \ln \left(x \right)$	51
risch	$\frac{B\,c^{2} x^{6}}{6}+\frac{A\,c^{2} x^{4}}{4}+\frac{x^{4} B b c}{2}+A b c\,x^{2}+\frac{b^{2} B\,x^{2}}{2}+A\,b^{2} \ln \left(x \right)$	51
parallelrisch	$\frac{B\,c^{2} x^{6}}{6}+\frac{A\,c^{2} x^{4}}{4}+\frac{x^{4} B b c}{2}+A b c\,x^{2}+\frac{b^{2} B\,x^{2}}{2}+A\,b^{2} \ln \left(x \right)$	51
norman	$\frac{\left(\frac{1}{4} A\,c^{2}+\frac{1}{2} B b c \right) x^{8}+\left(A b c +\frac{1}{2} B\,b^{2}\right) x^{6}+\frac{B\,c^{2} x^{10}}{6}}{x^{4}}+A\,b^{2} \ln \left(x \right)$	54

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/6*B*c^2*x^6+1/4*A*c^2*x^4+1/2*x^4*B*b*c+A*b*c*x^2+1/2*b^2*B*x^2+A*b^2*ln
(x)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^5}\,dx = \frac{1}{6}Bc^2x^6 + \frac{1}{4}(2Bbc+Ac^2)x^4 + Ab^2\log(x) + \frac{1}{2}(Bb^2+2Abc)x^2$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^5,x, algorithm="fricas")
```

output
```
1/6*B*c^2*x^6 + 1/4*(2*B*b*c + A*c^2)*x^4 + A*b^2*log(x) + 1/2*(B*b^2 + 2*
A*b*c)*x^2
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^5}\,dx = Ab^2\log(x) + \frac{Bc^2x^6}{6} + x^4\left(\frac{Ac^2}{4} + \frac{Bbc}{2}\right) + x^2\left(Abc + \frac{Bb^2}{2}\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**5,x)
```

output
```
A*b**2*log(x) + B*c**2*x**6/6 + x**4*(A*c**2/4 + B*b*c/2) + x**2*(A*b*c +
B*b**2/2)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^5}\,dx = \frac{1}{6}Bc^2x^6 + \frac{1}{4}(2Bbc+Ac^2)x^4 + \frac{1}{2}Ab^2\log(x^2) + \frac{1}{2}(Bb^2+2Abc)x^2$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^5,x, algorithm="maxima")
```

output
```
1/6*B*c^2*x^6 + 1/4*(2*B*b*c + A*c^2)*x^4 + 1/2*A*b^2*log(x^2) + 1/2*(B*b^
2 + 2*A*b*c)*x^2
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^5}\,dx = \frac{1}{6}Bc^2x^6 + \frac{1}{2}Bbcx^4 + \frac{1}{4}Ac^2x^4 + \frac{1}{2}Bb^2x^2 + Abcx^2 + \frac{1}{2}Ab^2\log(x^2)$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^5,x, algorithm="giac")
```

output
```
1/6*B*c^2*x^6 + 1/2*B*b*c*x^4 + 1/4*A*c^2*x^4 + 1/2*B*b^2*x^2 + A*b*c*x^2
+ 1/2*A*b^2*log(x^2)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^5}\,dx = x^2\left(\frac{B\,b^2}{2}+A\,c\,b\right)+x^4\left(\frac{A\,c^2}{4}+\frac{B\,b\,c}{2}\right)+\frac{B\,c^2\,x^6}{6}+A\,b^2\,\ln(x)$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^5,x)
```

output
```
x^2*((B*b^2)/2 + A*b*c) + x^4*((A*c^2)/4 + (B*b*c)/2) + (B*c^2*x^6)/6 + A*
b^2*log(x)
```

3.18 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^6} dx$

3.18.1 Optimal result

Integrand size = 24, antiderivative size = 48

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^6} dx = -\frac{Ab^2}{x} + b(bB+2Ac)x + \frac{1}{3}c(2bB+Ac)x^3 + \frac{1}{5}Bc^2x^5$$

output
```
-A*b^2/x+b*(2*A*c+B*b)*x+1/3*c*(A*c+2*B*b)*x^3+1/5*B*c^2*x^5
```

3.18.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^6} dx = -\frac{Ab^2}{x} + b(bB+2Ac)x + \frac{1}{3}c(2bB+Ac)x^3 + \frac{1}{5}Bc^2x^5$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^6,x]
```

output
```
-((A*b^2)/x) + b*(b*B + 2*A*c)*x + (c*(2*b*B + A*c)*x^3)/3 + (B*c^2*x^5)/5
```

3.18.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^6}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx^2)(b+cx^2)^2}{x^2}dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(\frac{Ab^2}{x^2}+cx^2(Ac+2bB)+b(2Ac+bB)+Bc^2x^4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{Ab^2}{x}+\frac{1}{3}cx^3(Ac+2bB)+bx(2Ac+bB)+\frac{1}{5}Bc^2x^5$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^6,x]
```

output
```
-((A*b^2)/x) + b*(b*B + 2*A*c)*x + (c*(2*b*B + A*c)*x^3)/3 + (B*c^2*x^5)/5
```

3.18.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.18.4 Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{B\,c^{2} x^{5}}{5}+\frac{A\,c^{2} x^{3}}{3}+\frac{2 B b c\,x^{3}}{3}+2 A b c x+b^{2} B x-\frac{A\,b^{2}}{x}$	49
risch	$\frac{B\,c^{2} x^{5}}{5}+\frac{A\,c^{2} x^{3}}{3}+\frac{2 B b c\,x^{3}}{3}+2 A b c x+b^{2} B x-\frac{A\,b^{2}}{x}$	49
norman	$\frac{\left(\frac{1}{3} A\,c^{2}+\frac{2}{3} B b c\right) x^{8}+\left(2 A b c+B\,b^{2}\right) x^{6}-A\,b^{2} x^{4}+\frac{B\,c^{2} x^{10}}{5}}{x^{5}}$	55
gosper	$-\frac{-3 B\,c^{2} x^{6}-5 A\,c^{2} x^{4}-10 x^{4} B b c-30 A b c\,x^{2}-15 b^{2} B\,x^{2}+15 b^{2} A}{15 x}$	56
parallelrisch	$\frac{3 B\,c^{2} x^{6}+5 A\,c^{2} x^{4}+10 x^{4} B b c+30 A b c\,x^{2}+15 b^{2} B\,x^{2}-15 b^{2} A}{15 x}$	56

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/5*B*c^2*x^5+1/3*A*c^2*x^3+2/3*B*b*c*x^3+2*A*b*c*x+b^2*B*x-A*b^2/x
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^6}\,dx = \frac{3\,Bc^2x^6+5\,(2\,Bbc+Ac^2)x^4-15\,Ab^2+15\,(Bb^2+2\,Abc)x^2}{15\,x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^6,x, algorithm="fricas")
```

output
```
1/15*(3*B*c^2*x^6 + 5*(2*B*b*c + A*c^2)*x^4 - 15*A*b^2 + 15*(B*b^2 + 2*A*b
*c)*x^2)/x
```

3.18.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^6}\,dx = -\frac{Ab^2}{x} + \frac{Bc^2x^5}{5} + x^3\left(\frac{Ac^2}{3} + \frac{2Bbc}{3}\right) + x(2Abc + Bb^2)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**6,x)
```

output
```
-A*b**2/x + B*c**2*x**5/5 + x**3*(A*c**2/3 + 2*B*b*c/3) + x*(2*A*b*c + B*b
**2)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^6}\,dx = \frac{1}{5}\,Bc^2x^5 + \frac{1}{3}\,(2\,Bbc + Ac^2)x^3 - \frac{Ab^2}{x} + (Bb^2 + 2\,Abc)x$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^6,x, algorithm="maxima")
```

output
```
1/5*B*c^2*x^5 + 1/3*(2*B*b*c + A*c^2)*x^3 - A*b^2/x + (B*b^2 + 2*A*b*c)*x
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^6}\,dx = \frac{1}{5}\,Bc^2x^5 + \frac{2}{3}\,Bbcx^3 + \frac{1}{3}\,Ac^2x^3 + Bb^2x + 2\,Abcx - \frac{Ab^2}{x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^6,x, algorithm="giac")
```

output
```
1/5*B*c^2*x^5 + 2/3*B*b*c*x^3 + 1/3*A*c^2*x^3 + B*b^2*x + 2*A*b*c*x - A*b^
2/x
```

3.18.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^6}\,dx = x^3\left(\frac{A\,c^2}{3}+\frac{2\,B\,b\,c}{3}\right)+x\left(B\,b^2+2\,A\,c\,b\right)-\frac{A\,b^2}{x}+\frac{B\,c^2\,x^5}{5}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^6,x)
```

output
```
x^3*((A*c^2)/3 + (2*B*b*c)/3) + x*(B*b^2 + 2*A*b*c) - (A*b^2)/x + (B*c^2*x
^5)/5
```

3.19 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7}\,dx$

3.19.1 Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7}\,dx = -\frac{Ab^2}{2x^2}+\frac{1}{2}c(2bB+Ac)x^2+\frac{1}{4}Bc^2x^4+b(bB+2Ac)\log(x)$$

output
```
-1/2*A*b^2/x^2+1/2*c*(A*c+2*B*b)*x^2+1/4*B*c^2*x^4+b*(2*A*c+B*b)*ln(x)
```

3.19.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7}\,dx = \frac{1}{4}\left(-\frac{2Ab^2}{x^2}+2c(2bB+Ac)x^2+Bc^2x^4+4b(bB+2Ac)\log(x)\right)$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^7,x]
```

output
```
((-2*A*b^2)/x^2 + 2*c*(2*b*B + A*c)*x^2 + B*c^2*x^4 + 4*b*(b*B + 2*A*c)*Lo
g[x])/4
```

3.19.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{(A+Bx^2)(b+cx^2)^2}{x^3}dx$$
$$\Big\downarrow \text{ 354}$$
$$\frac{1}{2}\int \frac{(Bx^2+A)(cx^2+b)^2}{x^4}dx^2$$
$$\Big\downarrow \text{ 85}$$
$$\frac{1}{2}\int \left(\frac{Ab^2}{x^4}+\frac{(bB+2Ac)b}{x^2}+Bc^2x^2+c(2bB+Ac)\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(-\frac{Ab^2}{x^2}+cx^2(Ac+2bB)+b\log\left(x^2\right)(2Ac+bB)+\frac{1}{2}Bc^2x^4\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^7,x]
```

output
```
(-((A*b^2)/x^2) + c*(2*b*B + A*c)*x^2 + (B*c^2*x^4)/2 + b*(b*B + 2*A*c)*Lo
g[x^2])/2
```

3.19.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.19.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{B\,c^{2}x^{4}}{4}+\frac{A\,c^{2}x^{2}}{2}+Bbc\,x^{2}+b(2Ac+Bb)\ln\left(x\right)-\frac{A\,b^{2}}{2x^{2}}$	48
norman	$\frac{\left(\frac{1}{2}A\,c^{2}+Bbc\right)x^{8}-\frac{A\,b^{2}x^{4}}{2}+\frac{B\,c^{2}x^{10}}{4}}{x^{6}}+\left(2Abc+B\,b^{2}\right)\ln\left(x\right)$	54
parallelrisch	$\frac{B\,c^{2}x^{6}+2A\,c^{2}x^{4}+4x^{4}Bbc+8A\ln(x)x^{2}bc+4B\ln(x)x^{2}b^{2}-2b^{2}A}{4x^{2}}$	59
risch	$\frac{B\,c^{2}x^{4}}{4}+\frac{A\,c^{2}x^{2}}{2}+Bbc\,x^{2}+\frac{A^{2}c^{2}}{4B}+Abc+B\,b^{2}-\frac{A\,b^{2}}{2x^{2}}+2A\ln\left(x\right)bc+b^{2}B\ln\left(x\right)$	70

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/4*B*c^2*x^4+1/2*A*c^2*x^2+B*b*c*x^2+b*(2*A*c+B*b)*ln(x)-1/2*A*b^2/x^2
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7}\,dx = \frac{Bc^2x^6+2\,(2\,Bbc+Ac^2)x^4+4\,(Bb^2+2\,Abc)x^2\log{(x)}-2\,Ab^2}{4\,x^2}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^7,x, algorithm="fricas")
```

output
```
1/4*(B*c^2*x^6 + 2*(2*B*b*c + A*c^2)*x^4 + 4*(B*b^2 + 2*A*b*c)*x^2*log(x)
- 2*A*b^2)/x^2
```

3.19.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7}\,dx = -\frac{Ab^2}{2x^2}+\frac{Bc^2x^4}{4}+b(2Ac+Bb)\log{(x)}+x^2\left(\frac{Ac^2}{2}+Bbc\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**7,x)
```

output
```
-A*b**2/(2*x**2) + B*c**2*x**4/4 + b*(2*A*c + B*b)*log(x) + x**2*(A*c**2/2
 + B*b*c)
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7}\,dx = \frac{1}{4}\,Bc^2x^4+\frac{1}{2}\,(2\,Bbc+Ac^2)x^2 + \frac{1}{2}\,(Bb^2+2\,Abc)\log{(x^2)}-\frac{Ab^2}{2\,x^2}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^7,x, algorithm="maxima")
```

output
```
1/4*B*c^2*x^4 + 1/2*(2*B*b*c + A*c^2)*x^2 + 1/2*(B*b^2 + 2*A*b*c)*log(x^2)
 - 1/2*A*b^2/x^2
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7}\,dx = \frac{1}{4}Bc^2x^4 + Bbcx^2 + \frac{1}{2}Ac^2x^2 + \frac{1}{2}\left(Bb^2+2\,Abc\right)\log\left(x^2\right) - \frac{Bb^2x^2+2\,Abcx^2+Ab^2}{2\,x^2}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^7,x, algorithm="giac")
```

output
```
1/4*B*c^2*x^4 + B*b*c*x^2 + 1/2*A*c^2*x^2 + 1/2*(B*b^2 + 2*A*b*c)*log(x^2)
 - 1/2*(B*b^2*x^2 + 2*A*b*c*x^2 + A*b^2)/x^2
```

3.19.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7}\,dx = x^2\left(\frac{A\,c^2}{2}+B\,b\,c\right)+\ln\left(x\right)\,\left(B\,b^2+2\,A\,c\,b\right)-\frac{A\,b^2}{2\,x^2}+\frac{B\,c^2\,x^4}{4}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^7,x)
```

output
```
x^2*((A*c^2)/2 + B*b*c) + log(x)*(B*b^2 + 2*A*b*c) - (A*b^2)/(2*x^2) + (B*
c^2*x^4)/4
```

3.20 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^8} dx$

3.20.1 Optimal result

Integrand size = 24, antiderivative size = 48

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^8} dx = -\frac{Ab^2}{3x^3} - \frac{b(bB+2Ac)}{x} + c(2bB+Ac)x + \frac{1}{3}Bc^2x^3$$

output
```
-1/3*A*b^2/x^3-b*(2*A*c+B*b)/x+c*(A*c+2*B*b)*x+1/3*B*c^2*x^3
```

3.20.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^8} dx = -\frac{Ab^2}{3x^3} + \frac{-b^2B-2Abc}{x} + c(2bB+Ac)x + \frac{1}{3}Bc^2x^3$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^8,x]
```

output
```
-1/3*(A*b^2)/x^3 + (-(b^2*B) - 2*A*b*c)/x + c*(2*b*B + A*c)*x + (B*c^2*x^3
)/3
```

3.20.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^8}dx$$

$$\Big\downarrow \textbf{9}$$

$$\int \frac{(A+Bx^2)(b+cx^2)^2}{x^4}dx$$

$$\Big\downarrow \textbf{355}$$

$$\int \left(\frac{Ab^2}{x^4}+\frac{b(2Ac+bB)}{x^2}+c(Ac+2bB)+Bc^2x^2\right)dx$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{Ab^2}{3x^3}+cx(Ac+2bB)-\frac{b(2Ac+bB)}{x}+\frac{1}{3}Bc^2x^3$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^8,x]
```

output
```
-1/3*(A*b^2)/x^3 - (b*(b*B + 2*A*c))/x + c*(2*b*B + A*c)*x + (B*c^2*x^3)/3
```

3.20.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.20.4 Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{B\,c^{2} x^{3}}{3}+A\,c^{2} x+2 B b c x-\frac{A\,b^{2}}{3 x^{3}}-\frac{b \left(2 A c+B b\right)}{x}$	46
risch	$\frac{B\,c^{2} x^{3}}{3}+A\,c^{2} x+2 B b c x+\frac{\left(-2 A b c-B\,b^{2}\right) x^{2}-\frac{b^{2} A}{3}}{x^{3}}$	50
gosper	$-\frac{-B\,c^{2} x^{6}-3 A\,c^{2} x^{4}-6 x^{4} B b c+6 A b c\,x^{2}+3 b^{2} B\,x^{2}+b^{2} A}{3 x^{3}}$	55
norman	$\frac{\left(A\,c^{2}+2 B b c\right) x^{8}+\left(-2 A b c-B\,b^{2}\right) x^{6}-\frac{A\,b^{2} x^{4}}{3}+\frac{B\,c^{2} x^{10}}{3}}{x^{7}}$	55
parallelrisch	$\frac{B\,c^{2} x^{6}+3 A\,c^{2} x^{4}+6 x^{4} B b c-6 A b c\,x^{2}-3 b^{2} B\,x^{2}-b^{2} A}{3 x^{3}}$	55

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^2/x^8,x,method=_RETURNVERBOSE)
```

output
```
1/3*B*c^2*x^3+A*c^2*x+2*B*b*c*x-1/3*A*b^2/x^3-b*(2*A*c+B*b)/x
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^8}\,dx = \frac{Bc^2x^6+3\,(2\,Bbc+Ac^2)x^4-Ab^2-3\,(Bb^2+2\,Abc)x^2}{3\,x^3}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^8,x, algorithm="fricas")
```

output
```
1/3*(B*c^2*x^6 + 3*(2*B*b*c + A*c^2)*x^4 - A*b^2 - 3*(B*b^2 + 2*A*b*c)*x^2
)/x^3
```

3.20.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^8}\,dx = \frac{Bc^2x^3}{3} + x\left(Ac^2+2Bbc\right) + \frac{-Ab^2+x^2(-6Abc-3Bb^2)}{3x^3}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**8,x)
```

output
```
B*c**2*x**3/3 + x*(A*c**2 + 2*B*b*c) + (-A*b**2 + x**2*(-6*A*b*c - 3*B*b**
2))/(3*x**3)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^8}\,dx = \frac{1}{3}\,Bc^2x^3 + \left(2\,Bbc+Ac^2\right)x - \frac{Ab^2+3\,(Bb^2+2\,Abc)x^2}{3\,x^3}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^8,x, algorithm="maxima")
```

output
```
1/3*B*c^2*x^3 + (2*B*b*c + A*c^2)*x - 1/3*(A*b^2 + 3*(B*b^2 + 2*A*b*c)*x^2
)/x^3
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^8}\,dx = \frac{1}{3}\,Bc^2x^3 + 2\,Bbcx + Ac^2x - \frac{3\,Bb^2x^2+6\,Abcx^2+Ab^2}{3\,x^3}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^8,x, algorithm="giac")
```

output
```
1/3*B*c^2*x^3 + 2*B*b*c*x + A*c^2*x - 1/3*(3*B*b^2*x^2 + 6*A*b*c*x^2 + A*b
^2)/x^3
```

3.20.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^8}\,dx = x\left(A\,c^2+2\,B\,b\,c\right)-\frac{x^2\,(B\,b^2+2\,A\,c\,b)+\frac{A\,b^2}{3}}{x^3}+\frac{B\,c^2\,x^3}{3}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^8,x)
```

output
```
x*(A*c^2 + 2*B*b*c) - (x^2*(B*b^2 + 2*A*b*c) + (A*b^2)/3)/x^3 + (B*c^2*x^3
)/3
```

3.21 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9}\,dx$

3.21.1 Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9}\,dx = -\frac{Ab^2}{4x^4} - \frac{b(bB+2Ac)}{2x^2} + \frac{1}{2}Bc^2x^2 + c(2bB+Ac)\log(x)$$

output
```
-1/4*A*b^2/x^4-1/2*b*(2*A*c+B*b)/x^2+1/2*B*c^2*x^2+c*(A*c+2*B*b)*ln(x)
```

3.21.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9}\,dx = -\frac{Ab(b+4cx^2)+2Bx^2(b^2-c^2x^4)}{4x^4} + c(2bB+Ac)\log(x)$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^9,x]
```

output
```
-1/4*(A*b*(b + 4*c*x^2) + 2*B*x^2*(b^2 - c^2*x^4))/x^4 + c*(2*b*B + A*c)*L
og[x]
```

3.21.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx^2)(b+cx^2)^2}{x^5}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)(cx^2+b)^2}{x^6}dx^2$$

$$\Big\downarrow \text{ 85}$$

$$\frac{1}{2}\int \left(\frac{Ab^2}{x^6}+\frac{(bB+2Ac)b}{x^4}+Bc^2+\frac{c(2bB+Ac)}{x^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{Ab^2}{2x^4}-\frac{b(2Ac+bB)}{x^2}+c\log(x^2)(Ac+2bB)+Bc^2x^2\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^9,x]
```

output
```
(-1/2*(A*b^2)/x^4 - (b*(b*B + 2*A*c))/x^2 + B*c^2*x^2 + c*(2*b*B + A*c)*Lo
g[x^2])/2
```

3.21.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.21.4 Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{A\,b^{2}}{4x^{4}}-\frac{b(2Ac+Bb)}{2x^{2}}+\frac{B\,c^{2}x^{2}}{2}+c(Ac+2Bb)\ln\left(x\right)$	46
risch	$\frac{B\,c^{2}x^{2}}{2}+\frac{\left(-Abc-\frac{1}{2}B\,b^{2}\right)x^{2}-\frac{b^{2}A}{4}}{x^{4}}+A\ln\left(x\right)c^{2}+2B\ln\left(x\right)bc$	52
norman	$\frac{\left(-Abc-\frac{1}{2}B\,b^{2}\right)x^{6}-\frac{A\,b^{2}x^{4}}{4}+\frac{B\,c^{2}x^{10}}{2}}{x^{8}}+\left(A\,c^{2}+2Bbc\right)\ln\left(x\right)$	55
parallelrisch	$\frac{2B\,c^{2}x^{6}+4A\ln(x)x^{4}c^{2}+8B\ln(x)x^{4}bc-4Abc\,x^{2}-2b^{2}B\,x^{2}-b^{2}A}{4x^{4}}$	60

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^2/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/4*A*b^2/x^4-1/2*b*(2*A*c+B*b)/x^2+1/2*B*c^2*x^2+c*(A*c+2*B*b)*ln(x)
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9}\,dx$$
$$=\frac{2\,Bc^2x^6+4\,(2\,Bbc+Ac^2)x^4\log(x)-Ab^2-2\,(Bb^2+2\,Abc)x^2}{4\,x^4}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^9,x, algorithm="fricas")
```

output
```
1/4*(2*B*c^2*x^6 + 4*(2*B*b*c + A*c^2)*x^4*log(x) - A*b^2 - 2*(B*b^2 + 2*A
*b*c)*x^2)/x^4
```

3.21.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9}\,dx=\frac{Bc^2x^2}{2}+c(Ac+2Bb)\log(x)+\frac{-Ab^2+x^2(-4Abc-2Bb^2)}{4x^4}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**9,x)
```

output
```
B*c**2*x**2/2 + c*(A*c + 2*B*b)*log(x) + (-A*b**2 + x**2*(-4*A*b*c - 2*B*b
**2))/(4*x**4)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9}\,dx$$
$$=\frac{1}{2}\,Bc^2x^2+\frac{1}{2}\,(2\,Bbc+Ac^2)\log(x^2)-\frac{Ab^2+2\,(Bb^2+2\,Abc)x^2}{4\,x^4}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^9,x, algorithm="maxima")
```

output
```
1/2*B*c^2*x^2 + 1/2*(2*B*b*c + A*c^2)*log(x^2) - 1/4*(A*b^2 + 2*(B*b^2 + 2
*A*b*c)*x^2)/x^4
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.41

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9}\,dx = \frac{1}{2}\,Bc^2x^2 + \frac{1}{2}\left(2\,Bbc + Ac^2\right)\log\left(x^2\right) - \frac{6\,Bbcx^4 + 3\,Ac^2x^4 + 2\,Bb^2x^2 + 4\,Abcx^2 + Ab^2}{4\,x^4}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^9,x, algorithm="giac")
```

output
```
1/2*B*c^2*x^2 + 1/2*(2*B*b*c + A*c^2)*log(x^2) - 1/4*(6*B*b*c*x^4 + 3*A*c^
2*x^4 + 2*B*b^2*x^2 + 4*A*b*c*x^2 + A*b^2)/x^4
```

3.21.9 Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9}\,dx = \ln(x)\,\left(A\,c^2+2\,B\,b\,c\right) - \frac{x^2\left(\frac{B\,b^2}{2}+A\,c\,b\right)+\frac{A\,b^2}{4}}{x^4} + \frac{B\,c^2\,x^2}{2}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^9,x)
```

output
```
log(x)*(A*c^2 + 2*B*b*c) - (x^2*((B*b^2)/2 + A*b*c) + (A*b^2)/4)/x^4 + (B*
c^2*x^2)/2
```

3.22 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}}\,dx$

3.22.1 Optimal result

Integrand size = 24, antiderivative size = 48

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}}\,dx = -\frac{Ab^2}{5x^5} - \frac{b(bB+2Ac)}{3x^3} - \frac{c(2bB+Ac)}{x} + Bc^2x$$

output
```
-1/5*A*b^2/x^5-1/3*b*(2*A*c+B*b)/x^3-c*(A*c+2*B*b)/x+B*c^2*x
```

3.22.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}}\,dx = -\frac{Ab^2}{5x^5} - \frac{b(bB+2Ac)}{3x^3} - \frac{c(2bB+Ac)}{x} + Bc^2x$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^10,x]
```

output
```
-1/5*(A*b^2)/x^5 - (b*(b*B + 2*A*c))/(3*x^3) - (c*(2*b*B + A*c))/x + B*c^2
*x
```

3.22.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(A+Bx^2)(b+cx^2)^2}{x^6} dx$$

$$\Big\downarrow \; 355$$

$$\int \left(\frac{Ab^2}{x^6}+\frac{b(2Ac+bB)}{x^4}+\frac{c(Ac+2bB)}{x^2}+Bc^2\right) dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{Ab^2}{5x^5}-\frac{b(2Ac+bB)}{3x^3}-\frac{c(Ac+2bB)}{x}+Bc^2x$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^10,x]
```

output
```
-1/5*(A*b^2)/x^5 - (b*(b*B + 2*A*c))/(3*x^3) - (c*(2*b*B + A*c))/x + B*c^2
*x
```

3.22.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.22.4 Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{A\,b^{2}}{5x^{5}}-\frac{b(2Ac+Bb)}{3x^{3}}-\frac{c(Ac+2Bb)}{x}+B\,c^{2}x$	45
risch	$B\,c^{2}x+\frac{(-A\,c^{2}-2Bbc)x^{4}+(-\frac{2}{3}Abc-\frac{1}{3}B\,b^{2})x^{2}-\frac{b^{2}A}{5}}{x^{5}}$	51
norman	$\frac{(-\frac{2}{3}Abc-\frac{1}{3}B\,b^{2})x^{6}+(-A\,c^{2}-2Bbc)x^{8}+B\,c^{2}x^{10}-\frac{A\,b^{2}x^{4}}{5}}{x^{9}}$	55
gosper	$-\frac{-15B\,c^{2}x^{6}+15A\,c^{2}x^{4}+30x^{4}Bbc+10Abc\,x^{2}+5b^{2}B\,x^{2}+3b^{2}A}{15x^{5}}$	56
parallelrisch	$-\frac{-15B\,c^{2}x^{6}+15A\,c^{2}x^{4}+30x^{4}Bbc+10Abc\,x^{2}+5b^{2}B\,x^{2}+3b^{2}A}{15x^{5}}$	56

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^2/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/5*A*b^2/x^5-1/3*b*(2*A*c+B*b)/x^3-c*(A*c+2*B*b)/x+B*c^2*x
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^{10}}\,dx = \frac{15\,Bc^2x^6-15\,(2\,Bbc+Ac^2)x^4-3\,Ab^2-5\,(Bb^2+2\,Abc)x^2}{15\,x^5}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^10,x, algorithm="fricas")
```

output
```
1/15*(15*B*c^2*x^6 - 15*(2*B*b*c + A*c^2)*x^4 - 3*A*b^2 - 5*(B*b^2 + 2*A*b
*c)*x^2)/x^5
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}}\,dx$$
$$= Bc^2x + \frac{-3Ab^2 + x^4(-15Ac^2 - 30Bbc) + x^2(-10Abc - 5Bb^2)}{15x^5}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**10,x)
```

output
```
B*c**2*x + (-3*A*b**2 + x**4*(-15*A*c**2 - 30*B*b*c) + x**2*(-10*A*b*c - 5
*B*b**2))/(15*x**5)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}}\,dx = Bc^2x - \frac{15\,(2\,Bbc + Ac^2)x^4 + 3\,Ab^2 + 5\,(Bb^2 + 2\,Abc)x^2}{15\,x^5}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^10,x, algorithm="maxima")
```

output
```
B*c^2*x - 1/15*(15*(2*B*b*c + A*c^2)*x^4 + 3*A*b^2 + 5*(B*b^2 + 2*A*b*c)*x
^2)/x^5
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}}\,dx = Bc^2x - \frac{30\,Bbcx^4 + 15\,Ac^2x^4 + 5\,Bb^2x^2 + 10\,Abcx^2 + 3\,Ab^2}{15\,x^5}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^10,x, algorithm="giac")
```

output
```
B*c^2*x - 1/15*(30*B*b*c*x^4 + 15*A*c^2*x^4 + 5*B*b^2*x^2 + 10*A*b*c*x^2 +
 3*A*b^2)/x^5
```

3.22.9 Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}}\,dx = B\,c^2\,x - \frac{x^2\left(\frac{B\,b^2}{3}+\frac{2\,A\,c\,b}{3}\right)+x^4\,(A\,c^2+2\,B\,b\,c)+\frac{A\,b^2}{5}}{x^5}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^10,x)
```

output
```
B*c^2*x - (x^2*((B*b^2)/3 + (2*A*b*c)/3) + x^4*(A*c^2 + 2*B*b*c) + (A*b^2)
/5)/x^5
```

3.23 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{11}} dx$

3.23.1 Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{11}} dx = -\frac{Ab^2}{6x^6} - \frac{b(bB+2Ac)}{4x^4} - \frac{c(2bB+Ac)}{2x^2} + Bc^2\log(x)$$

output
```
-1/6*A*b^2/x^6-1/4*b*(2*A*c+B*b)/x^4-1/2*c*(A*c+2*B*b)/x^2+B*c^2*ln(x)
```

3.23.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{11}} dx = -\frac{3bBx^2(b+4cx^2)+2A(b^2+3bcx^2+3c^2x^4)}{12x^6} + Bc^2\log(x)$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^11,x]
```

output
```
-1/12*(3*b*B*x^2*(b + 4*c*x^2) + 2*A*(b^2 + 3*b*c*x^2 + 3*c^2*x^4))/x^6 +
B*c^2*Log[x]
```

3.23.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{11}}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx^2)(b+cx^2)^2}{x^7}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)(cx^2+b)^2}{x^8}dx^2$$

$$\Big\downarrow \text{ 85}$$

$$\frac{1}{2}\int \left(\frac{Ab^2}{x^8}+\frac{(bB+2Ac)b}{x^6}+\frac{Bc^2}{x^2}+\frac{c(2bB+Ac)}{x^4}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{Ab^2}{3x^6}-\frac{b(2Ac+bB)}{2x^4}-\frac{c(Ac+2bB)}{x^2}+Bc^2\log(x^2)\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^11,x]
```

output
```
(-1/3*(A*b^2)/x^6 - (b*(b*B + 2*A*c))/(2*x^4) - (c*(2*b*B + A*c))/x^2 + B*
c^2*Log[x^2])/2
```

3.23.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.23.4 Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{A\,b^{2}}{6x^{6}}-\frac{b(2Ac+Bb)}{4x^{4}}-\frac{c(Ac+2Bb)}{2x^{2}}+B\,c^{2}\ln\left(x\right)$	46
risch	$\frac{\left(-\frac{1}{2}A\,c^{2}-Bbc\right)x^{4}+\left(-\frac{1}{2}Abc-\frac{1}{4}B\,b^{2}\right)x^{2}-\frac{b^{2}A}{6}}{x^{6}}+B\,c^{2}\ln\left(x\right)$	52
norman	$\frac{\left(-\frac{1}{2}A\,c^{2}-Bbc\right)x^{8}+\left(-\frac{1}{2}Abc-\frac{1}{4}B\,b^{2}\right)x^{6}-\frac{A\,b^{2}x^{4}}{6}}{x^{10}}+B\,c^{2}\ln\left(x\right)$	55
parallelrisch	$-\frac{-12B\,c^{2}\ln\left(x\right)x^{6}+6A\,c^{2}x^{4}+12x^{4}Bbc+6Abc\,x^{2}+3b^{2}B\,x^{2}+2b^{2}A}{12x^{6}}$	58

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^2/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/6*A*b^2/x^6-1/4*b*(2*A*c+B*b)/x^4-1/2*c*(A*c+2*B*b)/x^2+B*c^2*ln(x)
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^{11}}\,dx = \frac{12\,Bc^2x^6\log\left(x\right)-6\,(2\,Bbc+Ac^2)x^4-2\,Ab^2-3\,(Bb^2+2\,Abc)x^2}{12\,x^6}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^11,x, algorithm="fricas")
```

output
```
1/12*(12*B*c^2*x^6*log(x) - 6*(2*B*b*c + A*c^2)*x^4 - 2*A*b^2 - 3*(B*b^2 +
 2*A*b*c)*x^2)/x^6
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^{11}}\,dx = Bc^2\log\left(x\right) + \frac{-2Ab^2+x^4(-6Ac^2-12Bbc)+x^2(-6Abc-3Bb^2)}{12x^6}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**11,x)
```

output
```
B*c**2*log(x) + (-2*A*b**2 + x**4*(-6*A*c**2 - 12*B*b*c) + x**2*(-6*A*b*c
- 3*B*b**2))/(12*x**6)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^{11}}\,dx = \frac{1}{2}\,Bc^2\log\left(x^2\right) - \frac{6\,(2\,Bbc+Ac^2)x^4+2\,Ab^2+3\,(Bb^2+2\,Abc)x^2}{12\,x^6}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^11,x, algorithm="maxima")
```

output
```
1/2*B*c^2*log(x^2) - 1/12*(6*(2*B*b*c + A*c^2)*x^4 + 2*A*b^2 + 3*(B*b^2 +
2*A*b*c)*x^2)/x^6
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{11}}\,dx$$
$$= \frac{1}{2}Bc^2\log\left(x^2\right) - \frac{11\,Bc^2x^6+12\,Bbcx^4+6\,Ac^2x^4+3\,Bb^2x^2+6\,Abcx^2+2\,Ab^2}{12\,x^6}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^11,x, algorithm="giac")
```

output
```
1/2*B*c^2*log(x^2) - 1/12*(11*B*c^2*x^6 + 12*B*b*c*x^4 + 6*A*c^2*x^4 + 3*B
*b^2*x^2 + 6*A*b*c*x^2 + 2*A*b^2)/x^6
```

3.23.9 Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{11}}\,dx = B\,c^2\,\ln(x) - \frac{x^2\left(\frac{B\,b^2}{4}+\frac{A\,c\,b}{2}\right)+x^4\left(\frac{A\,c^2}{2}+B\,b\,c\right)+\frac{A\,b^2}{6}}{x^6}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^11,x)
```

output
```
B*c^2*log(x) - (x^2*((B*b^2)/4 + (A*b*c)/2) + x^4*((A*c^2)/2 + B*b*c) + (A
*b^2)/6)/x^6
```

3.24 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{12}}\,dx$

3.24.1 Optimal result

Integrand size = 24, antiderivative size = 53

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{12}}\,dx = -\frac{Ab^2}{7x^7}-\frac{b(bB+2Ac)}{5x^5}-\frac{c(2bB+Ac)}{3x^3}-\frac{Bc^2}{x}$$

output
```
-1/7*A*b^2/x^7-1/5*b*(2*A*c+B*b)/x^5-1/3*c*(A*c+2*B*b)/x^3-B*c^2/x
```

3.24.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{12}}\,dx = -\frac{Ab^2}{7x^7}+\frac{-b^2B-2Abc}{5x^5}+\frac{-2bBc-Ac^2}{3x^3}-\frac{Bc^2}{x}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^12,x]
```

output
```
-1/7*(A*b^2)/x^7 + (-(b^2*B) - 2*A*b*c)/(5*x^5) + (-2*b*B*c - A*c^2)/(3*x^
3) - (B*c^2)/x
```

3.24.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{12}}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(A+Bx^2)(b+cx^2)^2}{x^8}dx$$

$$\Big\downarrow \; 355$$

$$\int \left(\frac{Ab^2}{x^8}+\frac{b(2Ac+bB)}{x^6}+\frac{c(Ac+2bB)}{x^4}+\frac{Bc^2}{x^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{Ab^2}{7x^7}-\frac{b(2Ac+bB)}{5x^5}-\frac{c(Ac+2bB)}{3x^3}-\frac{Bc^2}{x}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^12,x]
```

output
```
-1/7*(A*b^2)/x^7 - (b*(b*B + 2*A*c))/(5*x^5) - (c*(2*b*B + A*c))/(3*x^3) -
 (B*c^2)/x
```

3.24.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.24.4 Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{A\,b^{2}}{7x^{7}}-\frac{b(2Ac+Bb)}{5x^{5}}-\frac{c(Ac+2Bb)}{3x^{3}}-\frac{B\,c^{2}}{x}$	48
risch	$\frac{-B\,c^{2}x^{6}+\left(-\frac{1}{3}A\,c^{2}-\frac{2}{3}Bbc\right)x^{4}+\left(-\frac{2}{5}Abc-\frac{1}{5}B\,b^{2}\right)x^{2}-\frac{b^{2}A}{7}}{x^{7}}$	53
gosper	$-\frac{105B\,c^{2}x^{6}+35A\,c^{2}x^{4}+70x^{4}Bbc+42Abc\,x^{2}+21b^{2}B\,x^{2}+15b^{2}A}{105x^{7}}$	56
norman	$\frac{\left(-\frac{1}{3}A\,c^{2}-\frac{2}{3}Bbc\right)x^{8}+\left(-\frac{2}{5}Abc-\frac{1}{5}B\,b^{2}\right)x^{6}-\frac{A\,b^{2}x^{4}}{7}-B\,c^{2}x^{10}}{x^{11}}$	56
parallelrisch	$-\frac{105B\,c^{2}x^{6}+35A\,c^{2}x^{4}+70x^{4}Bbc+42Abc\,x^{2}+21b^{2}B\,x^{2}+15b^{2}A}{105x^{7}}$	56

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^2/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/7*A*b^2/x^7-1/5*b*(2*A*c+B*b)/x^5-1/3*c*(A*c+2*B*b)/x^3-B*c^2/x
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^{12}}\,dx = -\frac{105\,Bc^2x^6+35\,(2\,Bbc+Ac^2)x^4+15\,Ab^2+21\,(Bb^2+2\,Abc)x^2}{105\,x^7}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^12,x, algorithm="fricas")
```

output
```
-1/105*(105*B*c^2*x^6 + 35*(2*B*b*c + A*c^2)*x^4 + 15*A*b^2 + 21*(B*b^2 +
2*A*b*c)*x^2)/x^7
```

3.24.6 Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^{12}}\,dx = \frac{-15Ab^2-105Bc^2x^6+x^4(-35Ac^2-70Bbc)+x^2(-42Abc-21Bb^2)}{105x^7}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**12,x)
```

output
```
(-15*A*b**2 - 105*B*c**2*x**6 + x**4*(-35*A*c**2 - 70*B*b*c) + x**2*(-42*A
*b*c - 21*B*b**2))/(105*x**7)
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^{12}}\,dx = -\frac{105\,Bc^2x^6+35\,(2\,Bbc+Ac^2)x^4+15\,Ab^2+21\,(Bb^2+2\,Abc)x^2}{105\,x^7}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^12,x, algorithm="maxima")
```

output
```
-1/105*(105*B*c^2*x^6 + 35*(2*B*b*c + A*c^2)*x^4 + 15*A*b^2 + 21*(B*b^2 +
2*A*b*c)*x^2)/x^7
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^{12}}\,dx = -\frac{105\,Bc^2x^6+70\,Bbcx^4+35\,Ac^2x^4+21\,Bb^2x^2+42\,Abcx^2+15\,Ab^2}{105\,x^7}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^12,x, algorithm="giac")
```

output
```
-1/105*(105*B*c^2*x^6 + 70*B*b*c*x^4 + 35*A*c^2*x^4 + 21*B*b^2*x^2 + 42*A*
b*c*x^2 + 15*A*b^2)/x^7
```

3.24.9 Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{12}}\,dx=-\frac{x^2\left(\frac{B\,b^2}{5}+\frac{2\,A\,c\,b}{5}\right)+x^4\left(\frac{A\,c^2}{3}+\frac{2\,B\,b\,c}{3}\right)+\frac{A\,b^2}{7}+B\,c^2\,x^6}{x^7}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^12,x)
```

output
```
-(x^2*((B*b^2)/5 + (2*A*b*c)/5) + x^4*((A*c^2)/3 + (2*B*b*c)/3) + (A*b^2)/
7 + B*c^2*x^6)/x^7
```

3.25 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^2} dx$

3.25.1 Optimal result

Integrand size = 24, antiderivative size = 75

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^2} dx = \frac{1}{5}Ab^3x^5 + \frac{1}{7}b^2(bB+3Ac)x^7 + \frac{1}{3}bc(bB+Ac)x^9 + \frac{1}{11}c^2(3bB+Ac)x^{11} + \frac{1}{13}Bc^3x^{13}$$

output
```
1/5*A*b^3*x^5+1/7*b^2*(3*A*c+B*b)*x^7+1/3*b*c*(A*c+B*b)*x^9+1/11*c^2*(A*c+
3*B*b)*x^11+1/13*B*c^3*x^13
```

3.25.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^2} dx = \frac{1}{5}Ab^3x^5 + \frac{1}{7}b^2(bB+3Ac)x^7 + \frac{1}{3}bc(bB+Ac)x^9 + \frac{1}{11}c^2(3bB+Ac)x^{11} + \frac{1}{13}Bc^3x^{13}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^2,x]
```

output
```
(A*b^3*x^5)/5 + (b^2*(b*B + 3*A*c)*x^7)/7 + (b*c*(b*B + A*c)*x^9)/3 + (c^2
*(3*b*B + A*c)*x^11)/11 + (B*c^3*x^13)/13
```

3.25.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int x^4(A+Bx^2)(b+cx^2)^3\,dx$$

$$\Big\downarrow \; 355$$

$$\int \left(Ab^3x^4+b^2x^6(3Ac+bB)+c^2x^{10}(Ac+3bB)+3bcx^8(Ac+bB)+Bc^3x^{12}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{5}Ab^3x^5+\frac{1}{7}b^2x^7(3Ac+bB)+\frac{1}{11}c^2x^{11}(Ac+3bB)+\frac{1}{3}bcx^9(Ac+bB)+\frac{1}{13}Bc^3x^{13}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^2,x]
```

output
```
(A*b^3*x^5)/5 + (b^2*(b*B + 3*A*c)*x^7)/7 + (b*c*(b*B + A*c)*x^9)/3 + (c^2
*(3*b*B + A*c)*x^11)/11 + (B*c^3*x^13)/13
```

3.25.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.25.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

method	result
default	$\frac{B\,c^{3}x^{13}}{13}+\frac{\left(A\,c^{3}+3 B b\,c^{2}\right)x^{11}}{11}+\frac{\left(3 A b\,c^{2}+3 B\,b^{2} c\right)x^{9}}{9}+\frac{\left(3 b^{2} A c+B\,b^{3}\right)x^{7}}{7}+\frac{A\,b^{3}x^{5}}{5}$
risch	$\frac{1}{13}B\,c^{3}x^{13}+\frac{1}{11}x^{11}A\,c^{3}+\frac{3}{11}x^{11}B b\,c^{2}+\frac{1}{3}x^{9}A b\,c^{2}+\frac{1}{3}x^{9}B\,b^{2}c+\frac{3}{7}x^{7}b^{2}A c+\frac{1}{7}b^{3}B\,x^{7}+\frac{1}{5}A\,b^{3}x^{5}$
parallelrisch	$\frac{1}{13}B\,c^{3}x^{13}+\frac{1}{11}x^{11}A\,c^{3}+\frac{3}{11}x^{11}B b\,c^{2}+\frac{1}{3}x^{9}A b\,c^{2}+\frac{1}{3}x^{9}B\,b^{2}c+\frac{3}{7}x^{7}b^{2}A c+\frac{1}{7}b^{3}B\,x^{7}+\frac{1}{5}A\,b^{3}x^{5}$
norman	$\frac{\left(\frac{1}{11}A\,c^{3}+\frac{3}{11}B b\,c^{2}\right)x^{12}+\left(\frac{1}{3}A b\,c^{2}+\frac{1}{3}B\,b^{2}c\right)x^{10}+\left(\frac{3}{7}b^{2}A c+\frac{1}{7}B\,b^{3}\right)x^{8}+\frac{B\,c^{3}x^{14}}{13}+\frac{x^{6}b^{3}A}{5}}{x}$
gosper	$\frac{x^{5}\left(1155 B\,c^{3}x^{8}+1365 A\,c^{3}x^{6}+4095 x^{6}B b\,c^{2}+5005 A b\,c^{2}x^{4}+5005 x^{4}B\,b^{2}c+6435 A\,b^{2}c\,x^{2}+2145 b^{3}B\,x^{2}+3003 b^{3}A\right)}{15015}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/13*B*c^3*x^13+1/11*(A*c^3+3*B*b*c^2)*x^11+1/9*(3*A*b*c^2+3*B*b^2*c)*x^9+
1/7*(3*A*b^2*c+B*b^3)*x^7+1/5*A*b^3*x^5
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^2}\,dx = \frac{1}{13}\,Bc^3x^{13}+\frac{1}{11}\,(3\,Bbc^2+Ac^3)x^{11} + \frac{1}{3}\,(Bb^2c+Abc^2)x^9+\frac{1}{5}\,Ab^3x^5+\frac{1}{7}\,(Bb^3+3\,Ab^2c)x^7$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^2,x, algorithm="fricas")
```

output
```
1/13*B*c^3*x^13 + 1/11*(3*B*b*c^2 + A*c^3)*x^11 + 1/3*(B*b^2*c + A*b*c^2)*
x^9 + 1/5*A*b^3*x^5 + 1/7*(B*b^3 + 3*A*b^2*c)*x^7
```

3.25.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^2}\,dx = \frac{Ab^3x^5}{5} + \frac{Bc^3x^{13}}{13} + x^{11}\left(\frac{Ac^3}{11} + \frac{3Bbc^2}{11}\right) + x^9\left(\frac{Abc^2}{3} + \frac{Bb^2c}{3}\right) + x^7\cdot\left(\frac{3Ab^2c}{7} + \frac{Bb^3}{7}\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**2,x)
```

output
```
A*b**3*x**5/5 + B*c**3*x**13/13 + x**11*(A*c**3/11 + 3*B*b*c**2/11) + x**9
*(A*b*c**2/3 + B*b**2*c/3) + x**7*(3*A*b**2*c/7 + B*b**3/7)
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^2}\,dx = \frac{1}{13}\,Bc^3x^{13} + \frac{1}{11}\,(3\,Bbc^2 + Ac^3)x^{11} + \frac{1}{3}\,(Bb^2c + Abc^2)x^9 + \frac{1}{5}\,Ab^3x^5 + \frac{1}{7}\,(Bb^3 + 3\,Ab^2c)x^7$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^2,x, algorithm="maxima")
```

output
```
1/13*B*c^3*x^13 + 1/11*(3*B*b*c^2 + A*c^3)*x^11 + 1/3*(B*b^2*c + A*b*c^2)*
x^9 + 1/5*A*b^3*x^5 + 1/7*(B*b^3 + 3*A*b^2*c)*x^7
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^2}\,dx = \frac{1}{13}\,Bc^3x^{13} + \frac{3}{11}\,Bbc^2x^{11} + \frac{1}{11}\,Ac^3x^{11} + \frac{1}{3}\,Bb^2cx^9 + \frac{1}{3}\,Abc^2x^9 + \frac{1}{7}\,Bb^3x^7 + \frac{3}{7}\,Ab^2cx^7 + \frac{1}{5}\,Ab^3x^5$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^2,x, algorithm="giac")
```

output
```
1/13*B*c^3*x^13 + 3/11*B*b*c^2*x^11 + 1/11*A*c^3*x^11 + 1/3*B*b^2*c*x^9 +
1/3*A*b*c^2*x^9 + 1/7*B*b^3*x^7 + 3/7*A*b^2*c*x^7 + 1/5*A*b^3*x^5
```

3.25.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^2}\,dx = x^7\left(\frac{B\,b^3}{7}+\frac{3\,A\,c\,b^2}{7}\right)+x^{11}\left(\frac{A\,c^3}{11}+\frac{3\,B\,b\,c^2}{11}\right)+\frac{A\,b^3\,x^5}{5}+\frac{B\,c^3\,x^{13}}{13}+\frac{b\,c\,x^9\,(A\,c+B\,b)}{3}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^2,x)
```

output
```
x^7*((B*b^3)/7 + (3*A*b^2*c)/7) + x^11*((A*c^3)/11 + (3*B*b*c^2)/11) + (A*
b^3*x^5)/5 + (B*c^3*x^13)/13 + (b*c*x^9*(A*c + B*b))/3
```

3.26 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3}\,dx$

3.26.1 Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3}\,dx = \frac{b(bB-Ac)(b+cx^2)^4}{8c^3} - \frac{(2bB-Ac)(b+cx^2)^5}{10c^3} + \frac{B(b+cx^2)^6}{12c^3}$$

output
```
1/8*b*(-A*c+B*b)*(c*x^2+b)^4/c^3-1/10*(-A*c+2*B*b)*(c*x^2+b)^5/c^3+1/12*B*
(c*x^2+b)^6/c^3
```

3.26.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3}\,dx = \frac{1}{120}x^4\left(30Ab^3+20b^2(bB+3Ac)x^2+45bc(bB+Ac)x^4 + 12c^2(3bB+Ac)x^6+10Bc^3x^8\right)$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^3,x]
```

output
```
(x^4*(30*A*b^3 + 20*b^2*(b*B + 3*A*c)*x^2 + 45*b*c*(b*B + A*c)*x^4 + 12*c^
2*(3*b*B + A*c)*x^6 + 10*B*c^3*x^8))/120
```

3.26.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^3}{x^3}dx$$

$$\Big\downarrow \text{9}$$

$$\int x^3(A+Bx^2)\left(b+cx^2\right)^3dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int x^2(Bx^2+A)\left(cx^2+b\right)^3dx^2$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{2}\int \left(\frac{B\left(cx^2+b\right)^5}{c^2}+\frac{(Ac-2bB)\left(cx^2+b\right)^4}{c^2}+\frac{b(bB-Ac)\left(cx^2+b\right)^3}{c^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\left(b+cx^2\right)^5(2bB-Ac)}{5c^3}+\frac{b\left(b+cx^2\right)^4(bB-Ac)}{4c^3}+\frac{B\left(b+cx^2\right)^6}{6c^3}\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^3,x]
```

output
```
((b*(b*B - A*c)*(b + c*x^2)^4)/(4*c^3) - ((2*b*B - A*c)*(b + c*x^2)^5)/(5*
c^3) + (B*(b + c*x^2)^6)/(6*c^3))/2
```

3.26.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.26.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

method	result
default	$\frac{B\,c^3x^{12}}{12}+\frac{(A\,c^3+3Bb\,c^2)x^{10}}{10}+\frac{(3Ab\,c^2+3B\,b^2c)x^8}{8}+\frac{(3b^2Ac+B\,b^3)x^6}{6}+\frac{A\,b^3x^4}{4}$
risch	$\frac{1}{12}B\,c^3x^{12}+\frac{1}{10}x^{10}A\,c^3+\frac{3}{10}x^{10}Bb\,c^2+\frac{3}{8}x^8Ab\,c^2+\frac{3}{8}x^8B\,b^2c+\frac{1}{2}x^6b^2Ac+\frac{1}{6}b^3B\,x^6+\frac{1}{4}A\,b^3x^4$
parallelrisch	$\frac{1}{12}B\,c^3x^{12}+\frac{1}{10}x^{10}A\,c^3+\frac{3}{10}x^{10}Bb\,c^2+\frac{3}{8}x^8Ab\,c^2+\frac{3}{8}x^8B\,b^2c+\frac{1}{2}x^6b^2Ac+\frac{1}{6}b^3B\,x^6+\frac{1}{4}A\,b^3x^4$
norman	$\frac{\left(\frac{1}{10}A\,c^3+\frac{3}{10}Bb\,c^2\right)x^{12}+\left(\frac{3}{8}Ab\,c^2+\frac{3}{8}B\,b^2c\right)x^{10}+\left(\frac{1}{2}b^2Ac+\frac{1}{6}B\,b^3\right)x^8+\frac{B\,c^3x^{14}}{12}+\frac{x^6b^3A}{4}}{x^2}$
gosper	$\frac{x^4(10B\,c^3x^8+12A\,c^3x^6+36x^6Bb\,c^2+45Ab\,c^2x^4+45x^4B\,b^2c+60A\,b^2c\,x^2+20b^3B\,x^2+30b^3A)}{120}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*B*c^3*x^12+1/10*(A*c^3+3*B*b*c^2)*x^10+1/8*(3*A*b*c^2+3*B*b^2*c)*x^8+
1/6*(3*A*b^2*c+B*b^3)*x^6+1/4*A*b^3*x^4
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3}\,dx = \frac{1}{12}\,Bc^3x^{12} + \frac{1}{10}\,(3\,Bbc^2+Ac^3)x^{10} + \frac{3}{8}\,(Bb^2c+Abc^2)x^8 + \frac{1}{4}\,Ab^3x^4 + \frac{1}{6}\,(Bb^3+3\,Ab^2c)x^6$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^3,x, algorithm="fricas")
```

output
```
1/12*B*c^3*x^12 + 1/10*(3*B*b*c^2 + A*c^3)*x^10 + 3/8*(B*b^2*c + A*b*c^2)*
x^8 + 1/4*A*b^3*x^4 + 1/6*(B*b^3 + 3*A*b^2*c)*x^6
```

3.26.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3}\,dx = \frac{Ab^3x^4}{4} + \frac{Bc^3x^{12}}{12} + x^{10}\left(\frac{Ac^3}{10}+\frac{3Bbc^2}{10}\right) + x^8 \cdot \left(\frac{3Abc^2}{8}+\frac{3Bb^2c}{8}\right) + x^6\left(\frac{Ab^2c}{2}+\frac{Bb^3}{6}\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**3,x)
```

output
```
A*b**3*x**4/4 + B*c**3*x**12/12 + x**10*(A*c**3/10 + 3*B*b*c**2/10) + x**8
*(3*A*b*c**2/8 + 3*B*b**2*c/8) + x**6*(A*b**2*c/2 + B*b**3/6)
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3}\,dx = \frac{1}{12}Bc^3x^{12} + \frac{1}{10}(3\,Bbc^2+Ac^3)x^{10} + \frac{3}{8}(Bb^2c+Abc^2)x^8 + \frac{1}{4}Ab^3x^4 + \frac{1}{6}(Bb^3+3\,Ab^2c)x^6$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^3,x, algorithm="maxima")
```

output
```
1/12*B*c^3*x^12 + 1/10*(3*B*b*c^2 + A*c^3)*x^10 + 3/8*(B*b^2*c + A*b*c^2)*
x^8 + 1/4*A*b^3*x^4 + 1/6*(B*b^3 + 3*A*b^2*c)*x^6
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.13

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3}\,dx = \frac{1}{12}Bc^3x^{12} + \frac{3}{10}Bbc^2x^{10} + \frac{1}{10}Ac^3x^{10} + \frac{3}{8}Bb^2cx^8 + \frac{3}{8}Abc^2x^8 + \frac{1}{6}Bb^3x^6 + \frac{1}{2}Ab^2cx^6 + \frac{1}{4}Ab^3x^4$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^3,x, algorithm="giac")
```

output
```
1/12*B*c^3*x^12 + 3/10*B*b*c^2*x^10 + 1/10*A*c^3*x^10 + 3/8*B*b^2*c*x^8 +
3/8*A*b*c^2*x^8 + 1/6*B*b^3*x^6 + 1/2*A*b^2*c*x^6 + 1/4*A*b^3*x^4
```

3.26.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3}\,dx = x^6\left(\frac{B\,b^3}{6}+\frac{A\,c\,b^2}{2}\right) + x^{10}\left(\frac{A\,c^3}{10}+\frac{3\,B\,b\,c^2}{10}\right) + \frac{A\,b^3\,x^4}{4} + \frac{B\,c^3\,x^{12}}{12} + \frac{3\,b\,c\,x^8\,(A\,c+B\,b)}{8}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^3,x)
```

output
```
x^6*((B*b^3)/6 + (A*b^2*c)/2) + x^10*((A*c^3)/10 + (3*B*b*c^2)/10) + (A*b^
3*x^4)/4 + (B*c^3*x^12)/12 + (3*b*c*x^8*(A*c + B*b))/8
```

3.26. $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3}\,dx$

3.27 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4}\,dx$

3.27.1 Optimal result

Integrand size = 24, antiderivative size = 75

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4}\,dx = \frac{1}{3}Ab^3x^3+\frac{1}{5}b^2(bB+3Ac)x^5+\frac{3}{7}bc(bB+Ac)x^7+\frac{1}{9}c^2(3bB+Ac)x^9+\frac{1}{11}Bc^3x^{11}$$

output
```
1/3*A*b^3*x^3+1/5*b^2*(3*A*c+B*b)*x^5+3/7*b*c*(A*c+B*b)*x^7+1/9*c^2*(A*c+3
*B*b)*x^9+1/11*B*c^3*x^11
```

3.27.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4}\,dx = \frac{1}{3}Ab^3x^3+\frac{1}{5}b^2(bB+3Ac)x^5+\frac{3}{7}bc(bB+Ac)x^7+\frac{1}{9}c^2(3bB+Ac)x^9+\frac{1}{11}Bc^3x^{11}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^4,x]
```

output
```
(A*b^3*x^3)/3 + (b^2*(b*B + 3*A*c)*x^5)/5 + (3*b*c*(b*B + A*c)*x^7)/7 + (c
^2*(3*b*B + A*c)*x^9)/9 + (B*c^3*x^11)/11
```

3.27.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4}dx$$

$$\Big\downarrow \ 9$$

$$\int x^2(A+Bx^2)(b+cx^2)^3dx$$

$$\Big\downarrow \ 355$$

$$\int \left(Ab^3x^2+b^2x^4(3Ac+bB)+c^2x^8(Ac+3bB)+3bcx^6(Ac+bB)+Bc^3x^{10}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}Ab^3x^3+\frac{1}{5}b^2x^5(3Ac+bB)+\frac{1}{9}c^2x^9(Ac+3bB)+\frac{3}{7}bcx^7(Ac+bB)+\frac{1}{11}Bc^3x^{11}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^4,x]
```

output
```
(A*b^3*x^3)/3 + (b^2*(b*B + 3*A*c)*x^5)/5 + (3*b*c*(b*B + A*c)*x^7)/7 + (c
^2*(3*b*B + A*c)*x^9)/9 + (B*c^3*x^11)/11
```

3.27.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.27.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

method	result	s
default	$\frac{B\,c^{3}x^{11}}{11}+\frac{\left(A\,c^{3}+3Bb\,c^{2}\right)x^{9}}{9}+\frac{\left(3Ab\,c^{2}+3B\,b^{2}c\right)x^{7}}{7}+\frac{\left(3b^{2}Ac+B\,b^{3}\right)x^{5}}{5}+\frac{A\,b^{3}x^{3}}{3}$	7
risch	$\frac{1}{11}B\,c^{3}x^{11}+\frac{1}{9}x^{9}A\,c^{3}+\frac{1}{3}x^{9}Bb\,c^{2}+\frac{3}{7}x^{7}Ab\,c^{2}+\frac{3}{7}x^{7}B\,b^{2}c+\frac{3}{5}x^{5}b^{2}Ac+\frac{1}{5}b^{3}B\,x^{5}+\frac{1}{3}A\,b^{3}x^{3}$	7
parallelrisch	$\frac{1}{11}B\,c^{3}x^{11}+\frac{1}{9}x^{9}A\,c^{3}+\frac{1}{3}x^{9}Bb\,c^{2}+\frac{3}{7}x^{7}Ab\,c^{2}+\frac{3}{7}x^{7}B\,b^{2}c+\frac{3}{5}x^{5}b^{2}Ac+\frac{1}{5}b^{3}B\,x^{5}+\frac{1}{3}A\,b^{3}x^{3}$	7
norman	$\frac{\left(\frac{1}{9}A\,c^{3}+\frac{1}{3}Bb\,c^{2}\right)x^{12}+\left(\frac{3}{7}Ab\,c^{2}+\frac{3}{7}B\,b^{2}c\right)x^{10}+\left(\frac{3}{5}b^{2}Ac+\frac{1}{5}B\,b^{3}\right)x^{8}+\frac{B\,c^{3}x^{14}}{11}+\frac{x^{6}b^{3}A}{3}}{x^{3}}$	7
gosper	$\frac{x^{3}\left(315B\,c^{3}x^{8}+385A\,c^{3}x^{6}+1155x^{6}Bb\,c^{2}+1485Ab\,c^{2}x^{4}+1485x^{4}B\,b^{2}c+2079A\,b^{2}c\,x^{2}+693b^{3}B\,x^{2}+1155b^{3}A\right)}{3465}$	8

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/11*B*c^3*x^11+1/9*(A*c^3+3*B*b*c^2)*x^9+1/7*(3*A*b*c^2+3*B*b^2*c)*x^7+1/
5*(3*A*b^2*c+B*b^3)*x^5+1/3*A*b^3*x^3
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^3}{x^4}\,dx = \frac{1}{11}\,Bc^3x^{11}+\frac{1}{9}\left(3\,Bbc^2+Ac^3\right)x^9+\frac{3}{7}\left(Bb^2c+Abc^2\right)x^7 + \frac{1}{3}\,Ab^3x^3+\frac{1}{5}\left(Bb^3+3\,Ab^2c\right)x^5$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^4,x, algorithm="fricas")
```

output
```
1/11*B*c^3*x^11 + 1/9*(3*B*b*c^2 + A*c^3)*x^9 + 3/7*(B*b^2*c + A*b*c^2)*x^
7 + 1/3*A*b^3*x^3 + 1/5*(B*b^3 + 3*A*b^2*c)*x^5
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4}\,dx = \frac{Ab^3x^3}{3} + \frac{Bc^3x^{11}}{11} + x^9\left(\frac{Ac^3}{9} + \frac{Bbc^2}{3}\right) + x^7 \cdot \left(\frac{3Abc^2}{7} + \frac{3Bb^2c}{7}\right) + x^5 \cdot \left(\frac{3Ab^2c}{5} + \frac{Bb^3}{5}\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**4,x)
```

output
```
A*b**3*x**3/3 + B*c**3*x**11/11 + x**9*(A*c**3/9 + B*b*c**2/3) + x**7*(3*A
*b*c**2/7 + 3*B*b**2*c/7) + x**5*(3*A*b**2*c/5 + B*b**3/5)
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4}\,dx = \frac{1}{11}Bc^3x^{11} + \frac{1}{9}\left(3Bbc^2 + Ac^3\right)x^9 + \frac{3}{7}\left(Bb^2c + Abc^2\right)x^7 + \frac{1}{3}Ab^3x^3 + \frac{1}{5}\left(Bb^3 + 3Ab^2c\right)x^5$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^4,x, algorithm="maxima")
```

output
```
1/11*B*c^3*x^11 + 1/9*(3*B*b*c^2 + A*c^3)*x^9 + 3/7*(B*b^2*c + A*b*c^2)*x^
7 + 1/3*A*b^3*x^3 + 1/5*(B*b^3 + 3*A*b^2*c)*x^5
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4}\,dx = \frac{1}{11}Bc^3x^{11} + \frac{1}{3}Bbc^2x^9 + \frac{1}{9}Ac^3x^9 + \frac{3}{7}Bb^2cx^7 + \frac{3}{7}Abc^2x^7 + \frac{1}{5}Bb^3x^5 + \frac{3}{5}Ab^2cx^5 + \frac{1}{3}Ab^3x^3$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^4,x, algorithm="giac")
```

output
```
1/11*B*c^3*x^11 + 1/3*B*b*c^2*x^9 + 1/9*A*c^3*x^9 + 3/7*B*b^2*c*x^7 + 3/7*
A*b*c^2*x^7 + 1/5*B*b^3*x^5 + 3/5*A*b^2*c*x^5 + 1/3*A*b^3*x^3
```

3.27.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4}\,dx = x^5\left(\frac{B\,b^3}{5}+\frac{3\,A\,c\,b^2}{5}\right)+x^9\left(\frac{A\,c^3}{9}+\frac{B\,b\,c^2}{3}\right)+\frac{A\,b^3\,x^3}{3}+\frac{B\,c^3\,x^{11}}{11}+\frac{3\,b\,c\,x^7\,(A\,c+B\,b)}{7}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^4,x)
```

output
```
x^5*((B*b^3)/5 + (3*A*b^2*c)/5) + x^9*((A*c^3)/9 + (B*b*c^2)/3) + (A*b^3*x
^3)/3 + (B*c^3*x^11)/11 + (3*b*c*x^7*(A*c + B*b))/7
```

3.28 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5} dx$

3.28.1 Optimal result

Integrand size = 24, antiderivative size = 42

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5} dx = -\frac{(bB-Ac)(b+cx^2)^4}{8c^2}+\frac{B(b+cx^2)^5}{10c^2}$$

output
```
-1/8*(-A*c+B*b)*(c*x^2+b)^4/c^2+1/10*B*(c*x^2+b)^5/c^2
```

3.28.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.64

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5} dx = \frac{1}{40}x^2(20Ab^3+10b^2(bB+3Ac)x^2+20bc(bB+Ac)x^4 + 5c^2(3bB+Ac)x^6+4Bc^3x^8)$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^5,x]
```

output
```
(x^2*(20*A*b^3 + 10*b^2*(b*B + 3*A*c)*x^2 + 20*b*c*(b*B + A*c)*x^4 + 5*c^2
*(3*b*B + A*c)*x^6 + 4*B*c^3*x^8))/40
```

3.28.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5}dx$$

$$\Big\downarrow \text{9}$$

$$\int x(A+Bx^2)(b+cx^2)^3dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int (Bx^2+A)(cx^2+b)^3dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{B(cx^2+b)^4}{c}+\frac{(Ac-bB)(cx^2+b)^3}{c}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{B(b+cx^2)^5}{5c^2}-\frac{(b+cx^2)^4(bB-Ac)}{4c^2}\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^5,x]
```

output
```
(-1/4*((b*B - A*c)*(b + c*x^2)^4)/c^2 + (B*(b + c*x^2)^5)/(5*c^2))/2
```

3.28.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.28.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.81

method	result
default	$\frac{B\,c^3 x^{10}}{10}+\frac{(A\,c^3+3Bb\,c^2)x^8}{8}+\frac{(3Ab\,c^2+3B\,b^2c)x^6}{6}+\frac{(3b^2Ac+B\,b^3)x^4}{4}+\frac{A\,b^3x^2}{2}$
risch	$\frac{1}{10}B\,c^3x^{10}+\frac{1}{8}x^8A\,c^3+\frac{3}{8}x^8Bb\,c^2+\frac{1}{2}x^6Ab\,c^2+\frac{1}{2}x^6B\,b^2c+\frac{3}{4}x^4b^2Ac+\frac{1}{4}b^3B\,x^4+\frac{1}{2}A\,b^3x^2$
parallelrisch	$\frac{1}{10}B\,c^3x^{10}+\frac{1}{8}x^8A\,c^3+\frac{3}{8}x^8Bb\,c^2+\frac{1}{2}x^6Ab\,c^2+\frac{1}{2}x^6B\,b^2c+\frac{3}{4}x^4b^2Ac+\frac{1}{4}b^3B\,x^4+\frac{1}{2}A\,b^3x^2$
norman	$\frac{(\frac{1}{8}A\,c^3+\frac{3}{8}Bb\,c^2)x^{12}+(\frac{1}{2}Ab\,c^2+\frac{1}{2}B\,b^2c)x^{10}+(\frac{3}{4}b^2Ac+\frac{1}{4}B\,b^3)x^8+\frac{B\,c^3x^{14}}{10}+\frac{x^6b^3A}{2}}{x^4}$
gosper	$\frac{x^2(4B\,c^3x^8+5A\,c^3x^6+15x^6Bb\,c^2+20Ab\,c^2x^4+20x^4B\,b^2c+30A\,b^2c\,x^2+10b^3B\,x^2+20b^3A)}{40}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/10*B*c^3*x^10+1/8*(A*c^3+3*B*b*c^2)*x^8+1/6*(3*A*b*c^2+3*B*b^2*c)*x^6+1/
4*(3*A*b^2*c+B*b^3)*x^4+1/2*A*b^3*x^2
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.74

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5}\,dx = \frac{1}{10}\,Bc^3x^{10} + \frac{1}{8}\,(3\,Bbc^2+Ac^3)x^8 + \frac{1}{2}\,(Bb^2c+Abc^2)x^6 + \frac{1}{2}\,Ab^3x^2 + \frac{1}{4}\,(Bb^3+3\,Ab^2c)x^4$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^5,x, algorithm="fricas")
```

output
```
1/10*B*c^3*x^10 + 1/8*(3*B*b*c^2 + A*c^3)*x^8 + 1/2*(B*b^2*c + A*b*c^2)*x^
6 + 1/2*A*b^3*x^2 + 1/4*(B*b^3 + 3*A*b^2*c)*x^4
```

3.28.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(36) = 72.

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.90

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5}\,dx = \frac{Ab^3x^2}{2} + \frac{Bc^3x^{10}}{10} + x^8\left(\frac{Ac^3}{8}+\frac{3Bbc^2}{8}\right) + x^6\left(\frac{Abc^2}{2}+\frac{Bb^2c}{2}\right) + x^4\cdot\left(\frac{3Ab^2c}{4}+\frac{Bb^3}{4}\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**5,x)
```

output
```
A*b**3*x**2/2 + B*c**3*x**10/10 + x**8*(A*c**3/8 + 3*B*b*c**2/8) + x**6*(A
*b*c**2/2 + B*b**2*c/2) + x**4*(3*A*b**2*c/4 + B*b**3/4)
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.74

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5}\,dx = \frac{1}{10}\,Bc^3x^{10} + \frac{1}{8}\,(3\,Bbc^2+Ac^3)x^8 + \frac{1}{2}\,(Bb^2c+Abc^2)x^6 + \frac{1}{2}\,Ab^3x^2 + \frac{1}{4}\,(Bb^3+3\,Ab^2c)x^4$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^5,x, algorithm="maxima")
```

output
```
1/10*B*c^3*x^10 + 1/8*(3*B*b*c^2 + A*c^3)*x^8 + 1/2*(B*b^2*c + A*b*c^2)*x^
6 + 1/2*A*b^3*x^2 + 1/4*(B*b^3 + 3*A*b^2*c)*x^4
```

3.28.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(38) = 76$.

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.83

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5}\,dx = \frac{1}{10}Bc^3x^{10} + \frac{3}{8}Bbc^2x^8 + \frac{1}{8}Ac^3x^8 + \frac{1}{2}Bb^2cx^6 + \frac{1}{2}Abc^2x^6 + \frac{1}{4}Bb^3x^4 + \frac{3}{4}Ab^2cx^4 + \frac{1}{2}Ab^3x^2$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^5,x, algorithm="giac")
```

output
```
1/10*B*c^3*x^10 + 3/8*B*b*c^2*x^8 + 1/8*A*c^3*x^8 + 1/2*B*b^2*c*x^6 + 1/2*
A*b*c^2*x^6 + 1/4*B*b^3*x^4 + 3/4*A*b^2*c*x^4 + 1/2*A*b^3*x^2
```

3.28.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.64

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5}\,dx = x^4\left(\frac{B\,b^3}{4}+\frac{3\,A\,c\,b^2}{4}\right)+x^8\left(\frac{A\,c^3}{8}+\frac{3\,B\,b\,c^2}{8}\right) + \frac{A\,b^3\,x^2}{2}+\frac{B\,c^3\,x^{10}}{10}+\frac{b\,c\,x^6\,(A\,c+B\,b)}{2}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^5,x)
```

output
```
x^4*((B*b^3)/4 + (3*A*b^2*c)/4) + x^8*((A*c^3)/8 + (3*B*b*c^2)/8) + (A*b^3
*x^2)/2 + (B*c^3*x^10)/10 + (b*c*x^6*(A*c + B*b))/2
```

3.29 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6} \, dx$

3.29.1 Optimal result

Integrand size = 24, antiderivative size = 70

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6} \, dx = Ab^3x + \frac{1}{3}b^2(bB+3Ac)x^3 + \frac{3}{5}bc(bB+Ac)x^5 + \frac{1}{7}c^2(3bB+Ac)x^7 + \frac{1}{9}Bc^3x^9$$

output
```
A*b^3*x+1/3*b^2*(3*A*c+B*b)*x^3+3/5*b*c*(A*c+B*b)*x^5+1/7*c^2*(A*c+3*B*b)*
x^7+1/9*B*c^3*x^9
```

3.29.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6} \, dx = Ab^3x + \frac{1}{3}b^2(bB+3Ac)x^3 + \frac{3}{5}bc(bB+Ac)x^5 + \frac{1}{7}c^2(3bB+Ac)x^7 + \frac{1}{9}Bc^3x^9$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^6,x]
```

output
```
A*b^3*x + (b^2*(b*B + 3*A*c)*x^3)/3 + (3*b*c*(b*B + A*c)*x^5)/5 + (c^2*(3*
b*B + A*c)*x^7)/7 + (B*c^3*x^9)/9
```

3.29.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6}dx$$
$$\Big\downarrow \; 9$$
$$\int (A+Bx^2)(b+cx^2)^3dx$$
$$\Big\downarrow \; 290$$
$$\int \left(Ab^3+b^2x^2(3Ac+bB)+c^2x^6(Ac+3bB)+3bcx^4(Ac+bB)+Bc^3x^8\right)dx$$
$$\Big\downarrow \; 2009$$
$$Ab^3x+\frac{1}{3}b^2x^3(3Ac+bB)+\frac{1}{7}c^2x^7(Ac+3bB)+\frac{3}{5}bcx^5(Ac+bB)+\frac{1}{9}Bc^3x^9$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^6,x]
```

output
```
A*b^3*x + (b^2*(b*B + 3*A*c)*x^3)/3 + (3*b*c*(b*B + A*c)*x^5)/5 + (c^2*(3*
b*B + A*c)*x^7)/7 + (B*c^3*x^9)/9
```

3.29.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.29.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{B\,c^{3}x^{9}}{9}+\frac{\left(A\,c^{3}+3Bb\,c^{2}\right)x^{7}}{7}+\frac{\left(3Ab\,c^{2}+3B\,b^{2}c\right)x^{5}}{5}+\frac{\left(3b^{2}Ac+B\,b^{3}\right)x^{3}}{3}+A\,b^{3}x$	73
risch	$\frac{1}{9}B\,c^{3}x^{9}+\frac{1}{7}x^{7}A\,c^{3}+\frac{3}{7}x^{7}Bb\,c^{2}+\frac{3}{5}x^{5}Ab\,c^{2}+\frac{3}{5}x^{5}B\,b^{2}c+x^{3}b^{2}Ac+\frac{1}{3}b^{3}B\,x^{3}+A\,b^{3}x$	74
parallelrisch	$\frac{1}{9}B\,c^{3}x^{9}+\frac{1}{7}x^{7}A\,c^{3}+\frac{3}{7}x^{7}Bb\,c^{2}+\frac{3}{5}x^{5}Ab\,c^{2}+\frac{3}{5}x^{5}B\,b^{2}c+x^{3}b^{2}Ac+\frac{1}{3}b^{3}B\,x^{3}+A\,b^{3}x$	74
norman	$\frac{\left(\frac{1}{7}A\,c^{3}+\frac{3}{7}Bb\,c^{2}\right)x^{12}+\left(\frac{3}{5}Ab\,c^{2}+\frac{3}{5}B\,b^{2}c\right)x^{10}+\left(b^{2}Ac+\frac{1}{3}B\,b^{3}\right)x^{8}+x^{6}b^{3}A+\frac{B\,c^{3}x^{14}}{9}}{x^{5}}$	77
gosper	$\frac{x\left(35B\,c^{3}x^{8}+45A\,c^{3}x^{6}+135x^{6}Bb\,c^{2}+189Ab\,c^{2}x^{4}+189x^{4}B\,b^{2}c+315A\,b^{2}c\,x^{2}+105b^{3}B\,x^{2}+315b^{3}A\right)}{315}$	78

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/9*B*c^3*x^9+1/7*(A*c^3+3*B*b*c^2)*x^7+1/5*(3*A*b*c^2+3*B*b^2*c)*x^5+1/3*
(3*A*b^2*c+B*b^3)*x^3+A*b^3*x
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int\frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^6}\,dx=\frac{1}{9}\,Bc^3x^9+\frac{1}{7}\,(3\,Bbc^2+Ac^3)x^7+\frac{3}{5}\,(Bb^2c+Abc^2)\,x^5+Ab^3x+\frac{1}{3}\,(Bb^3+3\,Ab^2c)\,x^3$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^6,x, algorithm="fricas")
```

output
```
1/9*B*c^3*x^9 + 1/7*(3*B*b*c^2 + A*c^3)*x^7 + 3/5*(B*b^2*c + A*b*c^2)*x^5
+ A*b^3*x + 1/3*(B*b^3 + 3*A*b^2*c)*x^3
```

3.29.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6}\,dx = Ab^3x+\frac{Bc^3x^9}{9}+x^7\left(\frac{Ac^3}{7}+\frac{3Bbc^2}{7}\right)+x^5\cdot\left(\frac{3Abc^2}{5}+\frac{3Bb^2c}{5}\right)+x^3\left(Ab^2c+\frac{Bb^3}{3}\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**6,x)
```

output
```
A*b**3*x + B*c**3*x**9/9 + x**7*(A*c**3/7 + 3*B*b*c**2/7) + x**5*(3*A*b*c*
*2/5 + 3*B*b**2*c/5) + x**3*(A*b**2*c + B*b**3/3)
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6}\,dx = \frac{1}{9}Bc^3x^9+\frac{1}{7}(3Bbc^2+Ac^3)x^7+\frac{3}{5}(Bb^2c+Abc^2)x^5+Ab^3x+\frac{1}{3}(Bb^3+3Ab^2c)x^3$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^6,x, algorithm="maxima")
```

output
```
1/9*B*c^3*x^9 + 1/7*(3*B*b*c^2 + A*c^3)*x^7 + 3/5*(B*b^2*c + A*b*c^2)*x^5
+ A*b^3*x + 1/3*(B*b^3 + 3*A*b^2*c)*x^3
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6}\,dx = \frac{1}{9}Bc^3x^9+\frac{3}{7}Bbc^2x^7+\frac{1}{7}Ac^3x^7+\frac{3}{5}Bb^2cx^5+\frac{3}{5}Abc^2x^5+\frac{1}{3}Bb^3x^3+Ab^2cx^3+Ab^3x$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^6,x, algorithm="giac")
```

output
```
1/9*B*c^3*x^9 + 3/7*B*b*c^2*x^7 + 1/7*A*c^3*x^7 + 3/5*B*b^2*c*x^5 + 3/5*A*
b*c^2*x^5 + 1/3*B*b^3*x^3 + A*b^2*c*x^3 + A*b^3*x
```

3.29.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6}\,dx = x^3\left(\frac{B\,b^3}{3}+A\,c\,b^2\right)+x^7\left(\frac{A\,c^3}{7}+\frac{3\,B\,b\,c^2}{7}\right) + \frac{B\,c^3\,x^9}{9}+A\,b^3\,x+\frac{3\,b\,c\,x^5\,(A\,c+B\,b)}{5}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^6,x)
```

output
```
x^3*((B*b^3)/3 + A*b^2*c) + x^7*((A*c^3)/7 + (3*B*b*c^2)/7) + (B*c^3*x^9)/
9 + A*b^3*x + (3*b*c*x^5*(A*c + B*b))/5
```

3.30 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7}\,dx$

3.30.1 Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7}\,dx=\frac{3}{2}Ab^2cx^2+\frac{3}{4}Abc^2x^4+\frac{1}{6}Ac^3x^6+\frac{B(b+cx^2)^4}{8c}+Ab^3\log(x)$$

output
```
3/2*A*b^2*c*x^2+3/4*A*b*c^2*x^4+1/6*A*c^3*x^6+1/8*B*(c*x^2+b)^4/c+A*b^3*ln
(x)
```

3.30.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7}\,dx=\frac{1}{2}b^2(bB+3Ac)x^2+\frac{3}{4}bc(bB+Ac)x^4+\frac{1}{6}c^2(3bB+Ac)x^6+\frac{1}{8}Bc^3x^8+Ab^3\log(x)$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^7,x]
```

output
```
(b^2*(b*B + 3*A*c)*x^2)/2 + (3*b*c*(b*B + A*c)*x^4)/4 + (c^2*(3*b*B + A*c)
*x^6)/6 + (B*c^3*x^8)/8 + A*b^3*Log[x]
```

3.30.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {9, 354, 90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{(A+Bx^2)(b+cx^2)^3}{x}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)(cx^2+b)^3}{x^2}dx^2$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(A\int \frac{(cx^2+b)^3}{x^2}dx^2+\frac{B(b+cx^2)^4}{4c}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\left(A\int \left(c^3x^4+3bc^2x^2+3b^2c+\frac{b^3}{x^2}\right)dx^2+\frac{B(b+cx^2)^4}{4c}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(A\left(b^3\log(x^2)+3b^2cx^2+\frac{3}{2}bc^2x^4+\frac{c^3x^6}{3}\right)+\frac{B(b+cx^2)^4}{4c}\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^7,x]
```

output
```
((B*(b + c*x^2)^4)/(4*c) + A*(3*b^2*c*x^2 + (3*b*c^2*x^4)/2 + (c^3*x^6)/3
+ b^3*Log[x^2]))/2
```

3.30.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.30.4 Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.27

method	result	size
default	$\frac{B\,c^{3}x^{8}}{8}+\frac{A\,c^{3}x^{6}}{6}+\frac{x^{6}Bb\,c^{2}}{2}+\frac{3Ab\,c^{2}x^{4}}{4}+\frac{3x^{4}B\,b^{2}c}{4}+\frac{3A\,b^{2}c\,x^{2}}{2}+\frac{b^{3}B\,x^{2}}{2}+A\,b^{3}\ln\left(x\right)$	76
risch	$\frac{B\,c^{3}x^{8}}{8}+\frac{A\,c^{3}x^{6}}{6}+\frac{x^{6}Bb\,c^{2}}{2}+\frac{3Ab\,c^{2}x^{4}}{4}+\frac{3x^{4}B\,b^{2}c}{4}+\frac{3A\,b^{2}c\,x^{2}}{2}+\frac{b^{3}B\,x^{2}}{2}+A\,b^{3}\ln\left(x\right)$	76
parallelrisch	$\frac{B\,c^{3}x^{8}}{8}+\frac{A\,c^{3}x^{6}}{6}+\frac{x^{6}Bb\,c^{2}}{2}+\frac{3Ab\,c^{2}x^{4}}{4}+\frac{3x^{4}B\,b^{2}c}{4}+\frac{3A\,b^{2}c\,x^{2}}{2}+\frac{b^{3}B\,x^{2}}{2}+A\,b^{3}\ln\left(x\right)$	76
norman	$\frac{\left(\frac{1}{6}A\,c^{3}+\frac{1}{2}Bb\,c^{2}\right)x^{12}+\left(\frac{3}{4}Ab\,c^{2}+\frac{3}{4}B\,b^{2}c\right)x^{10}+\left(\frac{3}{2}b^{2}Ac+\frac{1}{2}B\,b^{3}\right)x^{8}+\frac{B\,c^{3}x^{14}}{8}}{x^{6}}+A\,b^{3}\ln\left(x\right)$	78

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/8*B*c^3*x^8+1/6*A*c^3*x^6+1/2*x^6*B*b*c^2+3/4*A*b*c^2*x^4+3/4*x^4*B*b^2*
c+3/2*A*b^2*c*x^2+1/2*b^3*B*x^2+A*b^3*ln(x)
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7}\,dx = \frac{1}{8}Bc^3x^8 + \frac{1}{6}(3\,Bbc^2 + Ac^3)x^6 + \frac{3}{4}(Bb^2c + Abc^2)x^4 + Ab^3\log(x) + \frac{1}{2}(Bb^3 + 3\,Ab^2c)x^2$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^7,x, algorithm="fricas")
```

output
```
1/8*B*c^3*x^8 + 1/6*(3*B*b*c^2 + A*c^3)*x^6 + 3/4*(B*b^2*c + A*b*c^2)*x^4
+ A*b^3*log(x) + 1/2*(B*b^3 + 3*A*b^2*c)*x^2
```

3.30.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.33

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7}\,dx = Ab^3\log(x) + \frac{Bc^3x^8}{8} + x^6\left(\frac{Ac^3}{6} + \frac{Bbc^2}{2}\right) + x^4 \cdot \left(\frac{3Abc^2}{4} + \frac{3Bb^2c}{4}\right) + x^2 \cdot \left(\frac{3Ab^2c}{2} + \frac{Bb^3}{2}\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**7,x)
```

output
```
A*b**3*log(x) + B*c**3*x**8/8 + x**6*(A*c**3/6 + B*b*c**2/2) + x**4*(3*A*b
*c**2/4 + 3*B*b**2*c/4) + x**2*(3*A*b**2*c/2 + B*b**3/2)
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7}\,dx = \frac{1}{8}Bc^3x^8 + \frac{1}{6}(3Bbc^2+Ac^3)x^6 + \frac{3}{4}(Bb^2c+Abc^2)x^4 + \frac{1}{2}Ab^3\log(x^2) + \frac{1}{2}(Bb^3+3Ab^2c)x^2$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^7,x, algorithm="maxima")
```

output
```
1/8*B*c^3*x^8 + 1/6*(3*B*b*c^2 + A*c^3)*x^6 + 3/4*(B*b^2*c + A*b*c^2)*x^4
+ 1/2*A*b^3*log(x^2) + 1/2*(B*b^3 + 3*A*b^2*c)*x^2
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7}\,dx = \frac{1}{8}Bc^3x^8 + \frac{1}{2}Bbc^2x^6 + \frac{1}{6}Ac^3x^6 + \frac{3}{4}Bb^2cx^4 + \frac{3}{4}Abc^2x^4 + \frac{1}{2}Bb^3x^2 + \frac{3}{2}Ab^2cx^2 + \frac{1}{2}Ab^3\log(x^2)$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^7,x, algorithm="giac")
```

output
```
1/8*B*c^3*x^8 + 1/2*B*b*c^2*x^6 + 1/6*A*c^3*x^6 + 3/4*B*b^2*c*x^4 + 3/4*A*
b*c^2*x^4 + 1/2*B*b^3*x^2 + 3/2*A*b^2*c*x^2 + 1/2*A*b^3*log(x^2)
```

3.30.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7}\,dx = x^2\left(\frac{B\,b^3}{2}+\frac{3\,A\,c\,b^2}{2}\right) + x^6\left(\frac{A\,c^3}{6}+\frac{B\,b\,c^2}{2}\right) + \frac{B\,c^3\,x^8}{8} + A\,b^3\,\ln(x) + \frac{3\,b\,c\,x^4\,(A\,c+B\,b)}{4}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^7,x)
```

output
```
x^2*((B*b^3)/2 + (3*A*b^2*c)/2) + x^6*((A*c^3)/6 + (B*b*c^2)/2) + (B*c^3*x
^8)/8 + A*b^3*log(x) + (3*b*c*x^4*(A*c + B*b))/4
```

3.30. $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7}\,dx$

3.31 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8}\,dx$

3.31.1 Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8}\,dx = -\frac{Ab^3}{x}+b^2(bB+3Ac)x+bc(bB+Ac)x^3 + \frac{1}{5}c^2(3bB+Ac)x^5+\frac{1}{7}Bc^3x^7$$

output
```
-A*b^3/x+b^2*(3*A*c+B*b)*x+b*c*(A*c+B*b)*x^3+1/5*c^2*(A*c+3*B*b)*x^5+1/7*B
*c^3*x^7
```

3.31.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8}\,dx = -\frac{Ab^3}{x}+b^2(bB+3Ac)x+bc(bB+Ac)x^3 + \frac{1}{5}c^2(3bB+Ac)x^5+\frac{1}{7}Bc^3x^7$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^8,x]
```

output
```
-((A*b^3)/x) + b^2*(b*B + 3*A*c)*x + b*c*(b*B + A*c)*x^3 + (c^2*(3*b*B + A
*c)*x^5)/5 + (B*c^3*x^7)/7
```

3.31.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{(A+Bx^2)(b+cx^2)^3}{x^2}dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{Ab^3}{x^2}+b^2(3Ac+bB)+c^2x^4(Ac+3bB)+3bcx^2(Ac+bB)+Bc^3x^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{Ab^3}{x}+b^2x(3Ac+bB)+\frac{1}{5}c^2x^5(Ac+3bB)+bcx^3(Ac+bB)+\frac{1}{7}Bc^3x^7$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^8,x]
```

output
```
-((A*b^3)/x) + b^2*(b*B + 3*A*c)*x + b*c*(b*B + A*c)*x^3 + (c^2*(3*b*B + A
*c)*x^5)/5 + (B*c^3*x^7)/7
```

3.31.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.31.4 Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{B\,c^{3} x^{7}}{7}+\frac{A\,c^{3} x^{5}}{5}+\frac{3 B b\,c^{2} x^{5}}{5}+A b\,c^{2} x^{3}+B\,b^{2} c\,x^{3}+3 A\,b^{2} c x+b^{3} B x-\frac{A\,b^{3}}{x}$	71
risch	$\frac{B\,c^{3} x^{7}}{7}+\frac{A\,c^{3} x^{5}}{5}+\frac{3 B b\,c^{2} x^{5}}{5}+A b\,c^{2} x^{3}+B\,b^{2} c\,x^{3}+3 A\,b^{2} c x+b^{3} B x-\frac{A\,b^{3}}{x}$	71
norman	$\frac{\left(\frac{1}{5} A\,c^{3}+\frac{3}{5} B b\,c^{2}\right) x^{12}+\left(A b\,c^{2}+B\,b^{2} c\right) x^{10}+\left(3 b^{2} A c+B\,b^{3}\right) x^{8}+\frac{B\,c^{3} x^{14}}{7}-x^{6} b^{3} A}{x^{7}}$	76
gosper	$-\frac{-5 B\,c^{3} x^{8}-7 A\,c^{3} x^{6}-21 x^{6} B b\,c^{2}-35 A b\,c^{2} x^{4}-35 x^{4} B\,b^{2} c-105 A\,b^{2} c\,x^{2}-35 b^{3} B\,x^{2}+35 b^{3} A}{35 x}$	80
parallelrisch	$\frac{5 B\,c^{3} x^{8}+7 A\,c^{3} x^{6}+21 x^{6} B b\,c^{2}+35 A b\,c^{2} x^{4}+35 x^{4} B\,b^{2} c+105 A\,b^{2} c\,x^{2}+35 b^{3} B\,x^{2}-35 b^{3} A}{35 x}$	80

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3/x^8,x,method=_RETURNVERBOSE)
```

output
```
1/7*B*c^3*x^7+1/5*A*c^3*x^5+3/5*B*b*c^2*x^5+A*b*c^2*x^3+B*b^2*c*x^3+3*A*b^
2*c*x+b^3*B*x-A*b^3/x
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^8}\,dx = \frac{5\,Bc^3x^8+7\,(3\,Bbc^2+Ac^3)x^6+35\,(Bb^2c+Abc^2)x^4-35\,Ab^3+35\,(Bb^3+3\,Ab^2c)x^2}{35\,x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^8,x, algorithm="fricas")
```

output
```
1/35*(5*B*c^3*x^8 + 7*(3*B*b*c^2 + A*c^3)*x^6 + 35*(B*b^2*c + A*b*c^2)*x^4
 - 35*A*b^3 + 35*(B*b^3 + 3*A*b^2*c)*x^2)/x
```

3.31.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8}\,dx = -\frac{Ab^3}{x} + \frac{Bc^3x^7}{7} + x^5\left(\frac{Ac^3}{5} + \frac{3Bbc^2}{5}\right) + x^3\left(Abc^2 + Bb^2c\right) + x\left(3Ab^2c + Bb^3\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**8,x)
```

output
```
-A*b**3/x + B*c**3*x**7/7 + x**5*(A*c**3/5 + 3*B*b*c**2/5) + x**3*(A*b*c**
2 + B*b**2*c) + x*(3*A*b**2*c + B*b**3)
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8}\,dx = \frac{1}{7}Bc^3x^7 + \frac{1}{5}\left(3Bbc^2 + Ac^3\right)x^5 + \left(Bb^2c + Abc^2\right)x^3 - \frac{Ab^3}{x} + \left(Bb^3 + 3Ab^2c\right)x$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^8,x, algorithm="maxima")
```

output
```
1/7*B*c^3*x^7 + 1/5*(3*B*b*c^2 + A*c^3)*x^5 + (B*b^2*c + A*b*c^2)*x^3 - A*
b^3/x + (B*b^3 + 3*A*b^2*c)*x
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8}\,dx = \frac{1}{7}Bc^3x^7 + \frac{3}{5}Bbc^2x^5 + \frac{1}{5}Ac^3x^5 + Bb^2cx^3 + Abc^2x^3 + Bb^3x + 3Ab^2cx - \frac{Ab^3}{x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^8,x, algorithm="giac")
```

output
```
1/7*B*c^3*x^7 + 3/5*B*b*c^2*x^5 + 1/5*A*c^3*x^5 + B*b^2*c*x^3 + A*b*c^2*x^
3 + B*b^3*x + 3*A*b^2*c*x - A*b^3/x
```

3.31.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8}\,dx = x\left(B\,b^3+3\,A\,c\,b^2\right)+x^5\left(\frac{A\,c^3}{5}+\frac{3\,B\,b\,c^2}{5}\right) - \frac{A\,b^3}{x}+\frac{B\,c^3\,x^7}{7}+b\,c\,x^3\,(A\,c+B\,b)$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^8,x)
```

output
```
x*(B*b^3 + 3*A*b^2*c) + x^5*((A*c^3)/5 + (3*B*b*c^2)/5) - (A*b^3)/x + (B*c
^3*x^7)/7 + b*c*x^3*(A*c + B*b)
```

3.32 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9} \, dx$

3.32.1 Optimal result

Integrand size = 24, antiderivative size = 71

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9} \, dx = -\frac{Ab^3}{2x^2} + \frac{3}{2}bc(bB+Ac)x^2 + \frac{1}{4}c^2(3bB+Ac)x^4 + \frac{1}{6}Bc^3x^6 + b^2(bB+3Ac)\log(x)$$

output
```
-1/2*A*b^3/x^2+3/2*b*c*(A*c+B*b)*x^2+1/4*c^2*(A*c+3*B*b)*x^4+1/6*B*c^3*x^6
+b^2*(3*A*c+B*b)*ln(x)
```

3.32.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9} \, dx = -\frac{Ab^3}{2x^2} + \frac{3}{2}bc(bB+Ac)x^2 + \frac{1}{4}c^2(3bB+Ac)x^4 + \frac{1}{6}Bc^3x^6 + \left(b^3B+3Ab^2c\right)\log(x)$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^9,x]
```

output
```
-1/2*(A*b^3)/x^2 + (3*b*c*(b*B + A*c)*x^2)/2 + (c^2*(3*b*B + A*c)*x^4)/4 +
 (B*c^3*x^6)/6 + (b^3*B + 3*A*b^2*c)*Log[x]
```

3.32.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(A+Bx^2)(b+cx^2)^3}{x^3}dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{(Bx^2+A)(cx^2+b)^3}{x^4}dx^2$$

$$\Big\downarrow \; 85$$

$$\frac{1}{2}\int \left(Bc^3x^4+c^2(3bB+Ac)x^2+3bc(bB+Ac)+\frac{b^2(bB+3Ac)}{x^2}+\frac{Ab^3}{x^4}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{Ab^3}{x^2}+b^2\log(x^2)(3Ac+bB)+\frac{1}{2}c^2x^4(Ac+3bB)+3bcx^2(Ac+bB)+\frac{1}{3}Bc^3x^6\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^9,x]
```

output
```
(-((A*b^3)/x^2) + 3*b*c*(b*B + A*c)*x^2 + (c^2*(3*b*B + A*c)*x^4)/2 + (B*c
^3*x^6)/3 + b^2*(b*B + 3*A*c)*Log[x^2])/2
```

3.32.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.32.4 Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{B\,c^{3}x^{6}}{6}+\frac{A\,c^{3}x^{4}}{4}+\frac{3Bb\,c^{2}x^{4}}{4}+\frac{3Ab\,c^{2}x^{2}}{2}+\frac{3B\,b^{2}c\,x^{2}}{2}+b^{2}(3Ac+Bb)\ln\left(x\right)-\frac{A\,b^{3}}{2x^{2}}$	73
risch	$\frac{B\,c^{3}x^{6}}{6}+\frac{A\,c^{3}x^{4}}{4}+\frac{3Bb\,c^{2}x^{4}}{4}+\frac{3Ab\,c^{2}x^{2}}{2}+\frac{3B\,b^{2}c\,x^{2}}{2}-\frac{A\,b^{3}}{2x^{2}}+3A\ln\left(x\right)b^{2}c+b^{3}B\ln\left(x\right)$	75
norman	$\frac{\left(\frac{1}{4}A\,c^{3}+\frac{3}{4}Bb\,c^{2}\right)x^{12}+\left(\frac{3}{2}Ab\,c^{2}+\frac{3}{2}B\,b^{2}c\right)x^{10}+\frac{B\,c^{3}x^{14}}{6}-\frac{x^{6}b^{3}A}{2}}{x^{8}}+\left(3b^{2}Ac+B\,b^{3}\right)\ln\left(x\right)$	78
parallelrisch	$\frac{2B\,c^{3}x^{8}+3A\,c^{3}x^{6}+9x^{6}Bb\,c^{2}+18Ab\,c^{2}x^{4}+18x^{4}B\,b^{2}c+36A\ln\left(x\right)x^{2}b^{2}c+12B\ln\left(x\right)x^{2}b^{3}-6b^{3}A}{12x^{2}}$	84

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/6*B*c^3*x^6+1/4*A*c^3*x^4+3/4*B*b*c^2*x^4+3/2*A*b*c^2*x^2+3/2*B*b^2*c*x^
2+b^2*(3*A*c+B*b)*ln(x)-1/2*A*b^3/x^2
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9}\,dx = \frac{2\,Bc^3x^8+3\,(3\,Bbc^2+Ac^3)x^6+18\,(Bb^2c+Abc^2)x^4-6\,Ab^3+12\,(Bb^3+3\,Ab^2c)x^2\log(x)}{12\,x^2}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^9,x, algorithm="fricas")
```

output
```
1/12*(2*B*c^3*x^8 + 3*(3*B*b*c^2 + A*c^3)*x^6 + 18*(B*b^2*c + A*b*c^2)*x^4
 - 6*A*b^3 + 12*(B*b^3 + 3*A*b^2*c)*x^2*log(x))/x^2
```

3.32.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9}\,dx = -\frac{Ab^3}{2x^2}+\frac{Bc^3x^6}{6}+b^2\cdot(3Ac+Bb)\log(x) + x^4\left(\frac{Ac^3}{4}+\frac{3Bbc^2}{4}\right)+x^2\cdot\left(\frac{3Abc^2}{2}+\frac{3Bb^2c}{2}\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**9,x)
```

output
```
-A*b**3/(2*x**2) + B*c**3*x**6/6 + b**2*(3*A*c + B*b)*log(x) + x**4*(A*c**
3/4 + 3*B*b*c**2/4) + x**2*(3*A*b*c**2/2 + 3*B*b**2*c/2)
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9}\,dx = \frac{1}{6}\,Bc^3x^6+\frac{1}{4}\,(3\,Bbc^2+Ac^3)x^4+\frac{3}{2}\,(Bb^2c+Abc^2)x^2 - \frac{Ab^3}{2\,x^2}+\frac{1}{2}\,(Bb^3+3\,Ab^2c)\log(x^2)$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^9,x, algorithm="maxima")
```

output
```
1/6*B*c^3*x^6 + 1/4*(3*B*b*c^2 + A*c^3)*x^4 + 3/2*(B*b^2*c + A*b*c^2)*x^2
- 1/2*A*b^3/x^2 + 1/2*(B*b^3 + 3*A*b^2*c)*log(x^2)
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.37

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9}\,dx = \frac{1}{6}Bc^3x^6 + \frac{3}{4}Bbc^2x^4 + \frac{1}{4}Ac^3x^4 + \frac{3}{2}Bb^2cx^2 + \frac{3}{2}Abc^2x^2 + \frac{1}{2}\left(Bb^3 + 3Ab^2c\right)\log\left(x^2\right) - \frac{Bb^3x^2 + 3Ab^2cx^2 + Ab^3}{2x^2}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^9,x, algorithm="giac")
```

output
```
1/6*B*c^3*x^6 + 3/4*B*b*c^2*x^4 + 1/4*A*c^3*x^4 + 3/2*B*b^2*c*x^2 + 3/2*A*
b*c^2*x^2 + 1/2*(B*b^3 + 3*A*b^2*c)*log(x^2) - 1/2*(B*b^3*x^2 + 3*A*b^2*c*
x^2 + A*b^3)/x^2
```

3.32.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9}\,dx = x^4\left(\frac{A\,c^3}{4} + \frac{3\,B\,b\,c^2}{4}\right) + \ln(x)\,\left(B\,b^3 + 3\,A\,c\,b^2\right) - \frac{A\,b^3}{2\,x^2} + \frac{B\,c^3\,x^6}{6} + \frac{3\,b\,c\,x^2\,(A\,c + B\,b)}{2}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^9,x)
```

output
```
x^4*((A*c^3)/4 + (3*B*b*c^2)/4) + log(x)*(B*b^3 + 3*A*b^2*c) - (A*b^3)/(2*
x^2) + (B*c^3*x^6)/6 + (3*b*c*x^2*(A*c + B*b))/2
```

3.33 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}}\,dx$

3.33.1 Optimal result

Integrand size = 24, antiderivative size = 69

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}}\,dx = -\frac{Ab^3}{3x^3}-\frac{b^2(bB+3Ac)}{x}+3bc(bB+Ac)x+\frac{1}{3}c^2(3bB+Ac)x^3+\frac{1}{5}Bc^3x^5$$

output
```
-1/3*A*b^3/x^3-b^2*(3*A*c+B*b)/x+3*b*c*(A*c+B*b)*x+1/3*c^2*(A*c+3*B*b)*x^3
+1/5*B*c^3*x^5
```

3.33.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}}\,dx = -\frac{Ab^3}{3x^3}+\frac{-b^3B-3Ab^2c}{x}+3bc(bB+Ac)x+\frac{1}{3}c^2(3bB+Ac)x^3+\frac{1}{5}Bc^3x^5$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^10,x]
```

output
```
-1/3*(A*b^3)/x^3 + (-(b^3*B) - 3*A*b^2*c)/x + 3*b*c*(b*B + A*c)*x + (c^2*(
3*b*B + A*c)*x^3)/3 + (B*c^3*x^5)/5
```

3.33.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}}dx$$

$$\Big\downarrow 9$$

$$\int \frac{(A+Bx^2)(b+cx^2)^3}{x^4}dx$$

$$\Big\downarrow 355$$

$$\int \left(\frac{Ab^3}{x^4}+\frac{b^2(3Ac+bB)}{x^2}+c^2x^2(Ac+3bB)+3bc(Ac+bB)+Bc^3x^4\right)dx$$

$$\Big\downarrow 2009$$

$$-\frac{Ab^3}{3x^3}-\frac{b^2(3Ac+bB)}{x}+\frac{1}{3}c^2x^3(Ac+3bB)+3bcx(Ac+bB)+\frac{1}{5}Bc^3x^5$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^10,x]
```

output
```
-1/3*(A*b^3)/x^3 - (b^2*(b*B + 3*A*c))/x + 3*b*c*(b*B + A*c)*x + (c^2*(3*b
*B + A*c)*x^3)/3 + (B*c^3*x^5)/5
```

3.33.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.33.4 Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{B\,c^{3}x^{5}}{5}+\frac{A\,c^{3}x^{3}}{3}+Bb\,c^{2}x^{3}+3Ab\,c^{2}x+3B\,b^{2}cx-\frac{A\,b^{3}}{3x^{3}}-\frac{b^{2}(3Ac+Bb)}{x}$	70
risch	$\frac{B\,c^{3}x^{5}}{5}+\frac{A\,c^{3}x^{3}}{3}+Bb\,c^{2}x^{3}+3Ab\,c^{2}x+3B\,b^{2}cx+\frac{(-3b^{2}Ac-B\,b^{3})x^{2}-\frac{b^{3}A}{3}}{x^{3}}$	74
norman	$\frac{(\frac{1}{3}A\,c^{3}+Bb\,c^{2})x^{12}+(3Ab\,c^{2}+3B\,b^{2}c)x^{10}+(-3b^{2}Ac-B\,b^{3})x^{8}+\frac{B\,c^{3}x^{14}}{5}-\frac{x^{6}b^{3}A}{3}}{x^{9}}$	78
gosper	$-\frac{-3B\,c^{3}x^{8}-5A\,c^{3}x^{6}-15x^{6}Bb\,c^{2}-45Ab\,c^{2}x^{4}-45x^{4}B\,b^{2}c+45A\,b^{2}c\,x^{2}+15b^{3}B\,x^{2}+5b^{3}A}{15x^{3}}$	80
parallelrisch	$\frac{3B\,c^{3}x^{8}+5A\,c^{3}x^{6}+15x^{6}Bb\,c^{2}+45Ab\,c^{2}x^{4}+45x^{4}B\,b^{2}c-45A\,b^{2}c\,x^{2}-15b^{3}B\,x^{2}-5b^{3}A}{15x^{3}}$	80

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/5*B*c^3*x^5+1/3*A*c^3*x^3+B*b*c^2*x^3+3*A*b*c^2*x+3*B*b^2*c*x-1/3*A*b^3/
x^3-b^2*(3*A*c+B*b)/x
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}}\,dx = \frac{3\,Bc^3x^8+5\,(3\,Bbc^2+Ac^3)x^6+45\,(Bb^2c+Abc^2)x^4-5\,Ab^3-15\,(Bb^3+3\,Ab^2c)x^2}{15\,x^3}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^10,x, algorithm="fricas")
```

output
```
1/15*(3*B*c^3*x^8 + 5*(3*B*b*c^2 + A*c^3)*x^6 + 45*(B*b^2*c + A*b*c^2)*x^4
 - 5*A*b^3 - 15*(B*b^3 + 3*A*b^2*c)*x^2)/x^3
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}}\,dx = \frac{Bc^3x^5}{5} + x^3\left(\frac{Ac^3}{3} + Bbc^2\right) + x(3Abc^2 + 3Bb^2c) + \frac{-Ab^3 + x^2(-9Ab^2c - 3Bb^3)}{3x^3}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**10,x)
```

output
```
B*c**3*x**5/5 + x**3*(A*c**3/3 + B*b*c**2) + x*(3*A*b*c**2 + 3*B*b**2*c) +
 (-A*b**3 + x**2*(-9*A*b**2*c - 3*B*b**3))/(3*x**3)
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}}\,dx = \frac{1}{5}Bc^3x^5 + \frac{1}{3}(3Bbc^2 + Ac^3)x^3 + 3(Bb^2c + Abc^2)x - \frac{Ab^3 + 3(Bb^3 + 3Ab^2c)x^2}{3x^3}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^10,x, algorithm="maxima")
```

output
```
1/5*B*c^3*x^5 + 1/3*(3*B*b*c^2 + A*c^3)*x^3 + 3*(B*b^2*c + A*b*c^2)*x - 1/
3*(A*b^3 + 3*(B*b^3 + 3*A*b^2*c)*x^2)/x^3
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}}\,dx = \frac{1}{5}Bc^3x^5 + Bbc^2x^3 + \frac{1}{3}Ac^3x^3 + 3Bb^2cx + 3Abc^2x - \frac{3Bb^3x^2 + 9Ab^2cx^2 + Ab^3}{3x^3}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^10,x, algorithm="giac")
```

output
```
1/5*B*c^3*x^5 + B*b*c^2*x^3 + 1/3*A*c^3*x^3 + 3*B*b^2*c*x + 3*A*b*c^2*x -
1/3*(3*B*b^3*x^2 + 9*A*b^2*c*x^2 + A*b^3)/x^3
```

3.33.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}}\,dx = x^3\left(\frac{A\,c^3}{3}+B\,b\,c^2\right)-\frac{\frac{A\,b^3}{3}+x^2\,(B\,b^3+3\,A\,c\,b^2)}{x^3}+\frac{B\,c^3\,x^5}{5}+3\,b\,c\,x\,(A\,c+B\,b)$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^10,x)
```

output
```
x^3*((A*c^3)/3 + B*b*c^2) - ((A*b^3)/3 + x^2*(B*b^3 + 3*A*b^2*c))/x^3 + (B
*c^3*x^5)/5 + 3*b*c*x*(A*c + B*b)
```

3.34 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}} \, dx$

3.34.1 Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}} \, dx = -\frac{Ab^3}{4x^4} - \frac{b^2(bB+3Ac)}{2x^2} + \frac{1}{2}c^2(3bB+Ac)x^2 + \frac{1}{4}Bc^3x^4 + 3bc(bB+Ac)\log(x)$$

output
```
-1/4*A*b^3/x^4-1/2*b^2*(3*A*c+B*b)/x^2+1/2*c^2*(A*c+3*B*b)*x^2+1/4*B*c^3*x
^4+3*b*c*(A*c+B*b)*ln(x)
```

3.34.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}} \, dx = \frac{-A(b^3+6b^2cx^2-2c^3x^6)+Bx^2(-2b^3+6bc^2x^4+c^3x^6)}{4x^4} + 3bc(bB+Ac)\log(x)$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^11,x]
```

output
```
(-(A*(b^3 + 6*b^2*c*x^2 - 2*c^3*x^6)) + B*x^2*(-2*b^3 + 6*b*c^2*x^4 + c^3*
x^6))/(4*x^4) + 3*b*c*(b*B + A*c)*Log[x]
```

3.34.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^3}{x^{11}}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx^2)\left(b+cx^2\right)^3}{x^5}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)\left(cx^2+b\right)^3}{x^6}dx^2$$

$$\Big\downarrow \text{ 85}$$

$$\frac{1}{2}\int \left(\frac{Ab^3}{x^6}+\frac{(bB+3Ac)b^2}{x^4}+\frac{3c(bB+Ac)b}{x^2}+Bc^3x^2+c^2(3bB+Ac)\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{Ab^3}{2x^4}-\frac{b^2(3Ac+bB)}{x^2}+c^2x^2(Ac+3bB)+3bc\log\left(x^2\right)(Ac+bB)+\frac{1}{2}Bc^3x^4\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^11,x]
```

output
```
(-1/2*(A*b^3)/x^4 - (b^2*(b*B + 3*A*c))/x^2 + c^2*(3*b*B + A*c)*x^2 + (B*c
^3*x^4)/2 + 3*b*c*(b*B + A*c)*Log[x^2])/2
```

3.34.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.34.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

method	result
default	$\frac{B\,c^{3}x^{4}}{4}+\frac{A\,c^{3}x^{2}}{2}+\frac{3Bb\,c^{2}x^{2}}{2}+3bc(Ac+Bb)\ln\left(x\right)-\frac{b^{2}(3Ac+Bb)}{2x^{2}}-\frac{A\,b^{3}}{4x^{4}}$
norman	$\frac{\left(\frac{1}{2}A\,c^{3}+\frac{3}{2}Bb\,c^{2}\right)x^{12}+\left(-\frac{3}{2}b^{2}Ac-\frac{1}{2}B\,b^{3}\right)x^{8}+\frac{B\,c^{3}x^{14}}{4}-\frac{x^{6}b^{3}A}{4}}{x^{10}}+\left(3Ab\,c^{2}+3B\,b^{2}c\right)\ln\left(x\right)$
parallelrisch	$\frac{B\,c^{3}x^{8}+2A\,c^{3}x^{6}+6x^{6}Bb\,c^{2}+12A\ln(x)x^{4}b\,c^{2}+12B\ln(x)x^{4}b^{2}c-6A\,b^{2}c\,x^{2}-2b^{3}B\,x^{2}-b^{3}A}{4x^{4}}$
risch	$\frac{B\,c^{3}x^{4}}{4}+\frac{A\,c^{3}x^{2}}{2}+\frac{3Bb\,c^{2}x^{2}}{2}+\frac{c^{3}A^{2}}{4B}+\frac{3Ab\,c^{2}}{2}+\frac{9B\,b^{2}c}{4}+\frac{\left(-\frac{3}{2}b^{2}Ac-\frac{1}{2}B\,b^{3}\right)x^{2}-\frac{b^{3}A}{4}}{x^{4}}+3A\ln\left(x\right)b\,c^{2}+3B$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3/x^11,x,method=_RETURNVERBOSE)
```

output
```
1/4*B*c^3*x^4+1/2*A*c^3*x^2+3/2*B*b*c^2*x^2+3*b*c*(A*c+B*b)*ln(x)-1/2*b^2*
(3*A*c+B*b)/x^2-1/4*A*b^3/x^4
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}}\,dx = \frac{Bc^3x^8+2\,(3\,Bbc^2+Ac^3)x^6+12\,(Bb^2c+Abc^2)x^4\log{(x)}-Ab^3-2\,(Bb^3+3\,Ab^2c)x^2}{4\,x^4}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^11,x, algorithm="fricas")
```

output
```
1/4*(B*c^3*x^8 + 2*(3*B*b*c^2 + A*c^3)*x^6 + 12*(B*b^2*c + A*b*c^2)*x^4*lo
g(x) - A*b^3 - 2*(B*b^3 + 3*A*b^2*c)*x^2)/x^4
```

3.34.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}}\,dx = \frac{Bc^3x^4}{4}+3bc(Ac+Bb)\log{(x)}+x^2\left(\frac{Ac^3}{2}+\frac{3Bbc^2}{2}\right)+\frac{-Ab^3+x^2(-6Ab^2c-2Bb^3)}{4x^4}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**11,x)
```

output
```
B*c**3*x**4/4 + 3*b*c*(A*c + B*b)*log(x) + x**2*(A*c**3/2 + 3*B*b*c**2/2)
+ (-A*b**3 + x**2*(-6*A*b**2*c - 2*B*b**3))/(4*x**4)
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}}\,dx = \frac{1}{4}Bc^3x^4 + \frac{1}{2}(3Bbc^2+Ac^3)x^2 + \frac{3}{2}(Bb^2c+Abc^2)\log(x^2) - \frac{Ab^3+2(Bb^3+3Ab^2c)x^2}{4x^4}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^11,x, algorithm="maxima")
```

output
```
1/4*B*c^3*x^4 + 1/2*(3*B*b*c^2 + A*c^3)*x^2 + 3/2*(B*b^2*c + A*b*c^2)*log(
x^2) - 1/4*(A*b^3 + 2*(B*b^3 + 3*A*b^2*c)*x^2)/x^4
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.36

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}}\,dx = \frac{1}{4}Bc^3x^4 + \frac{3}{2}Bbc^2x^2 + \frac{1}{2}Ac^3x^2 + \frac{3}{2}(Bb^2c+Abc^2)\log(x^2) - \frac{9Bb^2cx^4+9Abc^2x^4+2Bb^3x^2+6Ab^2cx^2+Ab^3}{4x^4}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^11,x, algorithm="giac")
```

output
```
1/4*B*c^3*x^4 + 3/2*B*b*c^2*x^2 + 1/2*A*c^3*x^2 + 3/2*(B*b^2*c + A*b*c^2)*
log(x^2) - 1/4*(9*B*b^2*c*x^4 + 9*A*b*c^2*x^4 + 2*B*b^3*x^2 + 6*A*b^2*c*x^
2 + A*b^3)/x^4
```

3.34.9 Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}}\,dx = \ln(x)\,\left(3\,B\,b^2\,c+3\,A\,b\,c^2\right) - \frac{\frac{A\,b^3}{4}+x^2\left(\frac{B\,b^3}{2}+\frac{3\,A\,c\,b^2}{2}\right)}{x^4} + x^2\left(\frac{A\,c^3}{2}+\frac{3\,B\,b\,c^2}{2}\right) + \frac{B\,c^3\,x^4}{4}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^11,x)
```

output
```
log(x)*(3*A*b*c^2 + 3*B*b^2*c) - ((A*b^3)/4 + x^2*((B*b^3)/2 + (3*A*b^2*c)
/2))/x^4 + x^2*((A*c^3)/2 + (3*B*b*c^2)/2) + (B*c^3*x^4)/4
```

3.34. $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}}\,dx$

3.35 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}}\,dx$

3.35.1 Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}}\,dx = -\frac{Ab^3}{5x^5}-\frac{b^2(bB+3Ac)}{3x^3}-\frac{3bc(bB+Ac)}{x}+c^2(3bB+Ac)x+\frac{1}{3}Bc^3x^3$$

output
```
-1/5*A*b^3/x^5-1/3*b^2*(3*A*c+B*b)/x^3-3*b*c*(A*c+B*b)/x+c^2*(A*c+3*B*b)*x
+1/3*B*c^3*x^3
```

3.35.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}}\,dx = -\frac{Ab^3}{5x^5}-\frac{b^2(bB+3Ac)}{3x^3}-\frac{3bc(bB+Ac)}{x}+c^2(3bB+Ac)x+\frac{1}{3}Bc^3x^3$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^12,x]
```

output
```
-1/5*(A*b^3)/x^5 - (b^2*(b*B + 3*A*c))/(3*x^3) - (3*b*c*(b*B + A*c))/x + c
^2*(3*b*B + A*c)*x + (B*c^3*x^3)/3
```

3.35.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(A+Bx^2)(b+cx^2)^3}{x^6}dx$$

$$\Big\downarrow \; 355$$

$$\int \left(\frac{Ab^3}{x^6}+\frac{b^2(3Ac+bB)}{x^4}+c^2(Ac+3bB)+\frac{3bc(Ac+bB)}{x^2}+Bc^3x^2\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{Ab^3}{5x^5}-\frac{b^2(3Ac+bB)}{3x^3}+c^2x(Ac+3bB)-\frac{3bc(Ac+bB)}{x}+\frac{1}{3}Bc^3x^3$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^12,x]
```

output
```
-1/5*(A*b^3)/x^5 - (b^2*(b*B + 3*A*c))/(3*x^3) - (3*b*c*(b*B + A*c))/x + c
^2*(3*b*B + A*c)*x + (B*c^3*x^3)/3
```

3.35.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.35.4 Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{B\,c^{3}x^{3}}{3}+A\,c^{3}x+3Bb\,c^{2}x-\frac{b^{2}(3Ac+Bb)}{3x^{3}}-\frac{3bc(Ac+Bb)}{x}-\frac{A\,b^{3}}{5x^{5}}$	64
risch	$\frac{B\,c^{3}x^{3}}{3}+A\,c^{3}x+3Bb\,c^{2}x+\frac{(-3Ab\,c^{2}-3B\,b^{2}c)x^{4}+(-b^{2}Ac-\frac{1}{3}B\,b^{3})x^{2}-\frac{b^{3}A}{5}}{x^{5}}$	73
norman	$\frac{(-b^{2}Ac-\frac{1}{3}B\,b^{3})x^{8}+(A\,c^{3}+3Bb\,c^{2})x^{12}+(-3Ab\,c^{2}-3B\,b^{2}c)x^{10}+\frac{B\,c^{3}x^{14}}{3}-\frac{x^{6}b^{3}A}{5}}{x^{11}}$	78
gosper	$-\frac{-5B\,c^{3}x^{8}-15A\,c^{3}x^{6}-45x^{6}Bb\,c^{2}+45Ab\,c^{2}x^{4}+45x^{4}B\,b^{2}c+15A\,b^{2}c\,x^{2}+5b^{3}B\,x^{2}+3b^{3}A}{15x^{5}}$	80
parallelrisch	$\frac{5B\,c^{3}x^{8}+15A\,c^{3}x^{6}+45x^{6}Bb\,c^{2}-45Ab\,c^{2}x^{4}-45x^{4}B\,b^{2}c-15A\,b^{2}c\,x^{2}-5b^{3}B\,x^{2}-3b^{3}A}{15x^{5}}$	80

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3/x^12,x,method=_RETURNVERBOSE)
```

output
```
1/3*B*c^3*x^3+A*c^3*x+3*B*b*c^2*x-1/3*b^2*(3*A*c+B*b)/x^3-3*b*c*(A*c+B*b)/
x-1/5*A*b^3/x^5
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^{12}}\,dx = \frac{5\,Bc^3x^8+15\,(3\,Bbc^2+Ac^3)x^6-45\,(Bb^2c+Abc^2)x^4-3\,Ab^3-5\,(Bb^3+3\,Ab^2c)x^2}{15\,x^5}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^12,x, algorithm="fricas")
```

output
```
1/15*(5*B*c^3*x^8 + 15*(3*B*b*c^2 + A*c^3)*x^6 - 45*(B*b^2*c + A*b*c^2)*x^
4 - 3*A*b^3 - 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^5
```

3.35.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}}\,dx = \frac{Bc^3x^3}{3} + x(Ac^3+3Bbc^2) + \frac{-3Ab^3+x^4(-45Abc^2-45Bb^2c)+x^2(-15Ab^2c-5Bb^3)}{15x^5}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**12,x)
```

output
```
B*c**3*x**3/3 + x*(A*c**3 + 3*B*b*c**2) + (-3*A*b**3 + x**4*(-45*A*b*c**2
- 45*B*b**2*c) + x**2*(-15*A*b**2*c - 5*B*b**3))/(15*x**5)
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}}\,dx = \frac{1}{3}Bc^3x^3 + (3Bbc^2+Ac^3)x - \frac{45(Bb^2c+Abc^2)x^4+3Ab^3+5(Bb^3+3Ab^2c)x^2}{15x^5}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^12,x, algorithm="maxima")
```

output
```
1/3*B*c^3*x^3 + (3*B*b*c^2 + A*c^3)*x - 1/15*(45*(B*b^2*c + A*b*c^2)*x^4 +
 3*A*b^3 + 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^5
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}}\,dx = \frac{1}{3}Bc^3x^3 + 3Bbc^2x + Ac^3x - \frac{45Bb^2cx^4+45Abc^2x^4+5Bb^3x^2+15Ab^2cx^2+3Ab^3}{15x^5}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^12,x, algorithm="giac")
```

output
```
1/3*B*c^3*x^3 + 3*B*b*c^2*x + A*c^3*x - 1/15*(45*B*b^2*c*x^4 + 45*A*b*c^2*
x^4 + 5*B*b^3*x^2 + 15*A*b^2*c*x^2 + 3*A*b^3)/x^5
```

3.35.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}}\,dx = x\left(A\,c^3+3\,B\,b\,c^2\right) - \frac{x^4\left(3\,B\,b^2\,c+3\,A\,b\,c^2\right)+\frac{A\,b^3}{5}+x^2\left(\frac{B\,b^3}{3}+A\,c\,b^2\right)}{x^5} + \frac{B\,c^3\,x^3}{3}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^12,x)
```

output
```
x*(A*c^3 + 3*B*b*c^2) - (x^4*(3*A*b*c^2 + 3*B*b^2*c) + (A*b^3)/5 + x^2*((B
*b^3)/3 + A*b^2*c))/x^5 + (B*c^3*x^3)/3
```

3.36 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}} \, dx$

3.36.1 Optimal result

Integrand size = 24, antiderivative size = 71

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}} \, dx = -\frac{Ab^3}{6x^6} - \frac{b^2(bB+3Ac)}{4x^4} - \frac{3bc(bB+Ac)}{2x^2} + \frac{1}{2}Bc^3x^2 + c^2(3bB+Ac)\log(x)$$

output
```
-1/6*A*b^3/x^6-1/4*b^2*(3*A*c+B*b)/x^4-3/2*b*c*(A*c+B*b)/x^2+1/2*B*c^3*x^2
+c^2*(A*c+3*B*b)*ln(x)
```

3.36.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}} \, dx = -\frac{Ab^3}{6x^6} - \frac{b^2(bB+3Ac)}{4x^4} - \frac{3bc(bB+Ac)}{2x^2} + \frac{1}{2}Bc^3x^2 + c^2(3bB+Ac)\log(x)$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^13,x]
```

output
```
-1/6*(A*b^3)/x^6 - (b^2*(b*B + 3*A*c))/(4*x^4) - (3*b*c*(b*B + A*c))/(2*x^
2) + (B*c^3*x^2)/2 + c^2*(3*b*B + A*c)*Log[x]
```

3.36.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {9, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{(A+Bx^2)(b+cx^2)^3}{x^7}dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2}\int \frac{(Bx^2+A)(cx^2+b)^3}{x^8}dx^2$$

$$\Big\downarrow \ 85$$

$$\frac{1}{2}\int \left(\frac{Ab^3}{x^8}+\frac{(bB+3Ac)b^2}{x^6}+\frac{3c(bB+Ac)b}{x^4}+Bc^3+\frac{c^2(3bB+Ac)}{x^2}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{Ab^3}{3x^6}-\frac{b^2(3Ac+bB)}{2x^4}+c^2\log\left(x^2\right)(Ac+3bB)-\frac{3bc(Ac+bB)}{x^2}+Bc^3x^2\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^13,x]
```

output
```
(-1/3*(A*b^3)/x^6 - (b^2*(b*B + 3*A*c))/(2*x^4) - (3*b*c*(b*B + A*c))/x^2
+ B*c^3*x^2 + c^2*(3*b*B + A*c)*Log[x^2])/2
```

3.36.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.36.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{A\,b^{3}}{6x^{6}}-\frac{b^{2}(3Ac+Bb)}{4x^{4}}-\frac{3bc(Ac+Bb)}{2x^{2}}+\frac{B\,c^{3}x^{2}}{2}+c^{2}(Ac+3Bb)\ln\left(x\right)$	64
risch	$\frac{B\,c^{3}x^{2}}{2}+\frac{\left(-\frac{3}{2}A b\,c^{2}-\frac{3}{2}B\,b^{2}c\right)x^{4}+\left(-\frac{3}{4}b^{2}Ac-\frac{1}{4}B\,b^{3}\right)x^{2}-\frac{b^{3}A}{6}}{x^{6}}+A\ln\left(x\right)c^{3}+3B\ln\left(x\right)b\,c^{2}$	75
norman	$\frac{\left(-\frac{3}{2}A b\,c^{2}-\frac{3}{2}B\,b^{2}c\right)x^{10}+\left(-\frac{3}{4}b^{2}Ac-\frac{1}{4}B\,b^{3}\right)x^{8}+\frac{B\,c^{3}x^{14}}{2}-\frac{x^{6}b^{3}A}{6}}{x^{12}}+\left(A\,c^{3}+3Bb\,c^{2}\right)\ln\left(x\right)$	78
parallelrisch	$\frac{6B\,c^{3}x^{8}+12A\ln\left(x\right)x^{6}c^{3}+36B\ln\left(x\right)x^{6}b\,c^{2}-18Ab\,c^{2}x^{4}-18x^{4}B\,b^{2}c-9A\,b^{2}c\,x^{2}-3b^{3}B\,x^{2}-2b^{3}A}{12x^{6}}$	84

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/6*A*b^3/x^6-1/4*b^2*(3*A*c+B*b)/x^4-3/2*b*c*(A*c+B*b)/x^2+1/2*B*c^3*x^2
+c^2*(A*c+3*B*b)*ln(x)
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^{13}}\,dx = \frac{6\,Bc^3x^8+12\,(3\,Bbc^2+Ac^3)x^6\log{(x)}-18\,(Bb^2c+Abc^2)x^4-2\,Ab^3-3\,(Bb^3+3\,Ab^2c)x^2}{12\,x^6}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^13,x, algorithm="fricas")
```

output
```
1/12*(6*B*c^3*x^8 + 12*(3*B*b*c^2 + A*c^3)*x^6*log(x) - 18*(B*b^2*c + A*b*
c^2)*x^4 - 2*A*b^3 - 3*(B*b^3 + 3*A*b^2*c)*x^2)/x^6
```

3.36.6 Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^{13}}\,dx = \frac{Bc^3x^2}{2}+c^2(Ac+3Bb)\log{(x)} + \frac{-2Ab^3+x^4(-18Abc^2-18Bb^2c)+x^2(-9Ab^2c-3Bb^3)}{12x^6}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**13,x)
```

output
```
B*c**3*x**2/2 + c**2*(A*c + 3*B*b)*log(x) + (-2*A*b**3 + x**4*(-18*A*b*c**
2 - 18*B*b**2*c) + x**2*(-9*A*b**2*c - 3*B*b**3))/(12*x**6)
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}}\,dx = \frac{1}{2}Bc^3x^2 + \frac{1}{2}\left(3\,Bbc^2 + Ac^3\right)\log\left(x^2\right) - \frac{18\,(Bb^2c + Abc^2)x^4 + 2\,Ab^3 + 3\,(Bb^3 + 3\,Ab^2c)x^2}{12\,x^6}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^13,x, algorithm="maxima")
```

output
```
1/2*B*c^3*x^2 + 1/2*(3*B*b*c^2 + A*c^3)*log(x^2) - 1/12*(18*(B*b^2*c + A*b
*c^2)*x^4 + 2*A*b^3 + 3*(B*b^3 + 3*A*b^2*c)*x^2)/x^6
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.39

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}}\,dx = \frac{1}{2}Bc^3x^2 + \frac{1}{2}\left(3\,Bbc^2 + Ac^3\right)\log\left(x^2\right) - \frac{33\,Bbc^2x^6 + 11\,Ac^3x^6 + 18\,Bb^2cx^4 + 18\,Abc^2x^4 + 3\,Bb^3x^2 + 9\,Ab^2cx^2 + 2\,Ab^3}{12\,x^6}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^13,x, algorithm="giac")
```

output
```
1/2*B*c^3*x^2 + 1/2*(3*B*b*c^2 + A*c^3)*log(x^2) - 1/12*(33*B*b*c^2*x^6 +
11*A*c^3*x^6 + 18*B*b^2*c*x^4 + 18*A*b*c^2*x^4 + 3*B*b^3*x^2 + 9*A*b^2*c*x
^2 + 2*A*b^3)/x^6
```

3.36.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}}\,dx = \ln(x)\,\left(A\,c^3+3\,B\,b\,c^2\right) - \frac{x^4\,\left(\frac{3\,B\,b^2\,c}{2}+\frac{3\,A\,b\,c^2}{2}\right)+\frac{A\,b^3}{6}+x^2\,\left(\frac{B\,b^3}{4}+\frac{3\,A\,c\,b^2}{4}\right)}{x^6}+\frac{B\,c^3\,x^2}{2}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^13,x)
```

output
```
log(x)*(A*c^3 + 3*B*b*c^2) - (x^4*((3*A*b*c^2)/2 + (3*B*b^2*c)/2) + (A*b^3
)/6 + x^2*((B*b^3)/4 + (3*A*b^2*c)/4))/x^6 + (B*c^3*x^2)/2
```

3.37 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}} dx$

3.37.1 Optimal result

Integrand size = 24, antiderivative size = 66

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}} dx = -\frac{Ab^3}{7x^7} - \frac{b^2(bB+3Ac)}{5x^5} - \frac{bc(bB+Ac)}{x^3} - \frac{c^2(3bB+Ac)}{x} + Bc^3x$$

output
```
-1/7*A*b^3/x^7-1/5*b^2*(3*A*c+B*b)/x^5-b*c*(A*c+B*b)/x^3-c^2*(A*c+3*B*b)/x
+B*c^3*x
```

3.37.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}} dx = -\frac{Ab^3}{7x^7} - \frac{b^2(bB+3Ac)}{5x^5} - \frac{bc(bB+Ac)}{x^3} - \frac{c^2(3bB+Ac)}{x} + Bc^3x$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^14,x]
```

output
```
-1/7*(A*b^3)/x^7 - (b^2*(b*B + 3*A*c))/(5*x^5) - (b*c*(b*B + A*c))/x^3 - (
c^2*(3*b*B + A*c))/x + B*c^3*x
```

3.37.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx^2)(b+cx^2)^3}{x^8}dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(\frac{Ab^3}{x^8}+\frac{b^2(3Ac+bB)}{x^6}+\frac{c^2(Ac+3bB)}{x^2}+\frac{3bc(Ac+bB)}{x^4}+Bc^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{Ab^3}{7x^7}-\frac{b^2(3Ac+bB)}{5x^5}-\frac{c^2(Ac+3bB)}{x}-\frac{bc(Ac+bB)}{x^3}+Bc^3x$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^14,x]
```

output
```
-1/7*(A*b^3)/x^7 - (b^2*(b*B + 3*A*c))/(5*x^5) - (b*c*(b*B + A*c))/x^3 - (
c^2*(3*b*B + A*c))/x + B*c^3*x
```

3.37.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.37.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{A\,b^{3}}{7x^{7}}-\frac{b^{2}(3Ac+Bb)}{5x^{5}}-\frac{bc(Ac+Bb)}{x^{3}}-\frac{c^{2}(Ac+3Bb)}{x}+B\,c^{3}x$	63
risch	$B\,c^{3}x+\frac{(-A\,c^{3}-3Bb\,c^{2})x^{6}+(-Ab\,c^{2}-B\,b^{2}c)x^{4}+(-\frac{3}{5}b^{2}Ac-\frac{1}{5}B\,b^{3})x^{2}-\frac{b^{3}A}{7}}{x^{7}}$	74
norman	$\frac{(-\frac{3}{5}b^{2}Ac-\frac{1}{5}B\,b^{3})x^{8}+(-A\,c^{3}-3Bb\,c^{2})x^{12}+(-Ab\,c^{2}-B\,b^{2}c)x^{10}+B\,c^{3}x^{14}-\frac{x^{6}b^{3}A}{7}}{x^{13}}$	78
gosper	$-\frac{-35B\,c^{3}x^{8}+35A\,c^{3}x^{6}+105x^{6}Bb\,c^{2}+35Ab\,c^{2}x^{4}+35x^{4}B\,b^{2}c+21A\,b^{2}c\,x^{2}+7b^{3}B\,x^{2}+5b^{3}A}{35x^{7}}$	80
parallelrisch	$-\frac{-35B\,c^{3}x^{8}+35A\,c^{3}x^{6}+105x^{6}Bb\,c^{2}+35Ab\,c^{2}x^{4}+35x^{4}B\,b^{2}c+21A\,b^{2}c\,x^{2}+7b^{3}B\,x^{2}+5b^{3}A}{35x^{7}}$	80

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/7*A*b^3/x^7-1/5*b^2*(3*A*c+B*b)/x^5-b*c*(A*c+B*b)/x^3-c^2*(A*c+3*B*b)/x
+B*c^3*x
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^{14}}\,dx = \frac{35\,Bc^3x^8-35\,(3\,Bbc^2+Ac^3)x^6-35\,(Bb^2c+Abc^2)x^4-5\,Ab^3-7\,(Bb^3+3\,Ab^2c)x^2}{35\,x^7}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^14,x, algorithm="fricas")
```

output
```
1/35*(35*B*c^3*x^8 - 35*(3*B*b*c^2 + A*c^3)*x^6 - 35*(B*b^2*c + A*b*c^2)*x
^4 - 5*A*b^3 - 7*(B*b^3 + 3*A*b^2*c)*x^2)/x^7
```

3.37.6 Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}}\,dx$$
$$= Bc^3x + \frac{-5Ab^3 + x^6(-35Ac^3 - 105Bbc^2) + x^4(-35Abc^2 - 35Bb^2c) + x^2(-21Ab^2c - 7Bb^3)}{35x^7}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**14,x)
```

output
```
B*c**3*x + (-5*A*b**3 + x**6*(-35*A*c**3 - 105*B*b*c**2) + x**4*(-35*A*b*c
**2 - 35*B*b**2*c) + x**2*(-21*A*b**2*c - 7*B*b**3))/(35*x**7)
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}}\,dx$$
$$= Bc^3x - \frac{35\,(3\,Bbc^2 + Ac^3)x^6 + 35\,(Bb^2c + Abc^2)x^4 + 5\,Ab^3 + 7\,(Bb^3 + 3\,Ab^2c)x^2}{35\,x^7}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^14,x, algorithm="maxima")
```

output
```
B*c^3*x - 1/35*(35*(3*B*b*c^2 + A*c^3)*x^6 + 35*(B*b^2*c + A*b*c^2)*x^4 +
5*A*b^3 + 7*(B*b^3 + 3*A*b^2*c)*x^2)/x^7
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}}\,dx$$
$$= Bc^3x - \frac{105\,Bbc^2x^6 + 35\,Ac^3x^6 + 35\,Bb^2cx^4 + 35\,Abc^2x^4 + 7\,Bb^3x^2 + 21\,Ab^2cx^2 + 5\,Ab^3}{35\,x^7}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^14,x, algorithm="giac")
```

output
```
B*c^3*x - 1/35*(105*B*b*c^2*x^6 + 35*A*c^3*x^6 + 35*B*b^2*c*x^4 + 35*A*b*c
^2*x^4 + 7*B*b^3*x^2 + 21*A*b^2*c*x^2 + 5*A*b^3)/x^7
```

3.37.9 Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}}\,dx$$
$$= B\,c^3\,x - \frac{x^4\,(B\,b^2\,c+A\,b\,c^2)+\frac{A\,b^3}{7}+x^2\left(\frac{B\,b^3}{5}+\frac{3\,A\,c\,b^2}{5}\right)+x^6\,(A\,c^3+3\,B\,b\,c^2)}{x^7}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^14,x)
```

output
```
B*c^3*x - (x^4*(A*b*c^2 + B*b^2*c) + (A*b^3)/7 + x^2*((B*b^3)/5 + (3*A*b^2
*c)/5) + x^6*(A*c^3 + 3*B*b*c^2))/x^7
```

3.38 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}} dx$

3.38.1 Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}} dx = -\frac{b^3B}{6x^6} - \frac{3b^2Bc}{4x^4} - \frac{3bBc^2}{2x^2} - \frac{A(b+cx^2)^4}{8bx^8} + Bc^3\log(x)$$

output
```
-1/6*b^3*B/x^6-3/4*b^2*B*c/x^4-3/2*b*B*c^2/x^2-1/8*A*(c*x^2+b)^4/b/x^8+B*c
^3*ln(x)
```

3.38.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}} dx = -\frac{2bBx^2(2b^2+9bcx^2+18c^2x^4)+3A(b^3+4b^2cx^2+6bc^2x^4+4c^3x^6)}{24x^8} + Bc^3\log(x)$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^15,x]
```

output
```
-1/24*(2*b*B*x^2*(2*b^2 + 9*b*c*x^2 + 18*c^2*x^4) + 3*A*(b^3 + 4*b^2*c*x^2
 + 6*b*c^2*x^4 + 4*c^3*x^6))/x^8 + B*c^3*Log[x]
```

3.38.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {9, 354, 87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{(A+Bx^2)(b+cx^2)^3}{x^9}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)(cx^2+b)^3}{x^{10}}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(B\int \frac{(cx^2+b)^3}{x^8}dx^2-\frac{A(b+cx^2)^4}{4bx^8}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\left(B\int \left(\frac{b^3}{x^8}+\frac{3cb^2}{x^6}+\frac{3c^2b}{x^4}+\frac{c^3}{x^2}\right)dx^2-\frac{A(b+cx^2)^4}{4bx^8}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(B\left(-\frac{b^3}{3x^6}-\frac{3b^2c}{2x^4}-\frac{3bc^2}{x^2}+c^3\log(x^2)\right)-\frac{A(b+cx^2)^4}{4bx^8}\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^15,x]
```

output
```
(-1/4*(A*(b + c*x^2)^4)/(b*x^8) + B*(-1/3*b^3/x^6 - (3*b^2*c)/(2*x^4) - (3
*b*c^2)/x^2 + c^3*Log[x^2]))/2
```

3.38.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.38.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

method	result	size
default	$B\,c^{3} \ln \left(x \right)-\frac{b^{2} \left(3 A c +B b \right)}{6 x^{6}}-\frac{b^{3} A}{8 x^{8}}-\frac{c^{2} \left(A c +3 B b \right)}{2 x^{2}}-\frac{3 b c \left(A c +B b \right)}{4 x^{4}}$	64
risch	$\frac{\left(-\frac{1}{2} A \,c^{3}-\frac{3}{2} B b \,c^{2}\right) x^{6}+\left(-\frac{3}{4} A b \,c^{2}-\frac{3}{4} B \,b^{2} c \right) x^{4}+\left(-\frac{1}{2} b^{2} A c -\frac{1}{6} B \,b^{3}\right) x^{2}-\frac{b^{3} A}{8}}{x^{8}}+B\,c^{3} \ln \left(x \right)$	75
norman	$\frac{\left(-\frac{1}{2} A \,c^{3}-\frac{3}{2} B b \,c^{2}\right) x^{12}+\left(-\frac{3}{4} A b \,c^{2}-\frac{3}{4} B \,b^{2} c \right) x^{10}+\left(-\frac{1}{2} b^{2} A c -\frac{1}{6} B \,b^{3}\right) x^{8}-\frac{x^{6} b^{3} A}{8}}{x^{14}}+B\,c^{3} \ln \left(x \right)$	78
parallelrisch	$-\frac{-24 B \,c^{3} \ln \left(x \right) x^{8}+12 A \,c^{3} x^{6}+36 x^{6} B b \,c^{2}+18 A b \,c^{2} x^{4}+18 x^{4} B \,b^{2} c +12 A \,b^{2} c \,x^{2}+4 b^{3} B \,x^{2}+3 b^{3} A}{24 x^{8}}$	82

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x,method=_RETURNVERBOSE)
```

output
```
B*c^3*ln(x)-1/6*b^2*(3*A*c+B*b)/x^6-1/8*b^3*A/x^8-1/2*c^2*(A*c+3*B*b)/x^2-
3/4*b*c*(A*c+B*b)/x^4
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}}\,dx = \frac{24\,Bc^3x^8\log(x) - 12\,(3\,Bbc^2 + Ac^3)x^6 - 18\,(Bb^2c + Abc^2)x^4 - 3\,Ab^3 - 4\,(Bb^3 + 3\,Ab^2c)x^2}{24\,x^8}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x, algorithm="fricas")
```

output
```
1/24*(24*B*c^3*x^8*log(x) - 12*(3*B*b*c^2 + A*c^3)*x^6 - 18*(B*b^2*c + A*b
*c^2)*x^4 - 3*A*b^3 - 4*(B*b^3 + 3*A*b^2*c)*x^2)/x^8
```

3.38.6 Sympy [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}}\,dx = Bc^3\log(x) + \frac{-3Ab^3 + x^6(-12Ac^3 - 36Bbc^2) + x^4(-18Abc^2 - 18Bb^2c) + x^2(-12Ab^2c - 4Bb^3)}{24x^8}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**15,x)
```

output
```
B*c**3*log(x) + (-3*A*b**3 + x**6*(-12*A*c**3 - 36*B*b*c**2) + x**4*(-18*A
*b*c**2 - 18*B*b**2*c) + x**2*(-12*A*b**2*c - 4*B*b**3))/(24*x**8)
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}}\,dx$$
$$= \frac{1}{2}Bc^3\log\left(x^2\right)$$
$$-\frac{12\,(3\,Bbc^2+Ac^3)x^6+18\,(Bb^2c+Abc^2)x^4+3\,Ab^3+4\,(Bb^3+3\,Ab^2c)x^2}{24\,x^8}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x, algorithm="maxima")
```

output
```
1/2*B*c^3*log(x^2) - 1/24*(12*(3*B*b*c^2 + A*c^3)*x^6 + 18*(B*b^2*c + A*b*
c^2)*x^4 + 3*A*b^3 + 4*(B*b^3 + 3*A*b^2*c)*x^2)/x^8
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}}\,dx = \frac{1}{2}Bc^3\log\left(x^2\right)$$
$$-\frac{25\,Bc^3x^8+36\,Bbc^2x^6+12\,Ac^3x^6+18\,Bb^2cx^4+18\,Abc^2x^4+4\,Bb^3x^2+12\,Ab^2cx^2+3\,Ab^3}{24\,x^8}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x, algorithm="giac")
```

output
```
1/2*B*c^3*log(x^2) - 1/24*(25*B*c^3*x^8 + 36*B*b*c^2*x^6 + 12*A*c^3*x^6 +
18*B*b^2*c*x^4 + 18*A*b*c^2*x^4 + 4*B*b^3*x^2 + 12*A*b^2*c*x^2 + 3*A*b^3)/
x^8
```

3.38.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}}\,dx$$
$$= B\,c^3\,\ln(x) - \frac{x^4\left(\frac{3\,B\,b^2\,c}{4}+\frac{3\,A\,b\,c^2}{4}\right)+\frac{A\,b^3}{8}+x^2\left(\frac{B\,b^3}{6}+\frac{A\,c\,b^2}{2}\right)+x^6\left(\frac{A\,c^3}{2}+\frac{3\,B\,b\,c^2}{2}\right)}{x^8}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^15,x)
```

output
```
B*c^3*log(x) - (x^4*((3*A*b*c^2)/4 + (3*B*b^2*c)/4) + (A*b^3)/8 + x^2*((B*
b^3)/6 + (A*b^2*c)/2) + x^6*((A*c^3)/2 + (3*B*b*c^2)/2))/x^8
```

3.39 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}}\,dx$

3.39.1 Optimal result

Integrand size = 24, antiderivative size = 73

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}}\,dx = -\frac{Ab^3}{9x^9}-\frac{b^2(bB+3Ac)}{7x^7}-\frac{3bc(bB+Ac)}{5x^5}-\frac{c^2(3bB+Ac)}{3x^3}-\frac{Bc^3}{x}$$

output
```
-1/9*A*b^3/x^9-1/7*b^2*(3*A*c+B*b)/x^7-3/5*b*c*(A*c+B*b)/x^5-1/3*c^2*(A*c+
3*B*b)/x^3-B*c^3/x
```

3.39.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}}\,dx = -\frac{Ab^3}{9x^9}-\frac{b^2(bB+3Ac)}{7x^7}-\frac{3bc(bB+Ac)}{5x^5}-\frac{c^2(3bB+Ac)}{3x^3}-\frac{Bc^3}{x}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^16,x]
```

output
```
-1/9*(A*b^3)/x^9 - (b^2*(b*B + 3*A*c))/(7*x^7) - (3*b*c*(b*B + A*c))/(5*x^
5) - (c^2*(3*b*B + A*c))/(3*x^3) - (B*c^3)/x
```

3.39.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(A+Bx^2)(b+cx^2)^3}{x^{10}}dx$$

$$\Big\downarrow \; 355$$

$$\int \left(\frac{Ab^3}{x^{10}}+\frac{b^2(3Ac+bB)}{x^8}+\frac{c^2(Ac+3bB)}{x^4}+\frac{3bc(Ac+bB)}{x^6}+\frac{Bc^3}{x^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{Ab^3}{9x^9}-\frac{b^2(3Ac+bB)}{7x^7}-\frac{c^2(Ac+3bB)}{3x^3}-\frac{3bc(Ac+bB)}{5x^5}-\frac{Bc^3}{x}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^16,x]
```

output
```
-1/9*(A*b^3)/x^9 - (b^2*(b*B + 3*A*c))/(7*x^7) - (3*b*c*(b*B + A*c))/(5*x^
5) - (c^2*(3*b*B + A*c))/(3*x^3) - (B*c^3)/x
```

3.39.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.39.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{A b^3}{9x^9}-\frac{b^2(3Ac+Bb)}{7x^7}-\frac{3bc(Ac+Bb)}{5x^5}-\frac{c^2(Ac+3Bb)}{3x^3}-\frac{B c^3}{x}$	66
risch	$\frac{-B c^3x^8+(-\frac{1}{3}A c^3-Bb c^2)x^6+(-\frac{3}{5}Ab c^2-\frac{3}{5}B b^2c)x^4+(-\frac{3}{7}b^2Ac-\frac{1}{7}B b^3)x^2-\frac{b^3A}{9}}{x^9}$	76
norman	$\frac{(-\frac{1}{3}A c^3-Bb c^2)x^{12}+(-\frac{3}{5}Ab c^2-\frac{3}{5}B b^2c)x^{10}+(-\frac{3}{7}b^2Ac-\frac{1}{7}B b^3)x^8-B c^3x^{14}-\frac{x^6b^3A}{9}}{x^{15}}$	79
gosper	$-\frac{315B c^3x^8+105A c^3x^6+315x^6Bb c^2+189Ab c^2x^4+189x^4B b^2c+135A b^2c x^2+45b^3B x^2+35b^3A}{315x^9}$	80
parallelrisch	$-\frac{315B c^3x^8+105A c^3x^6+315x^6Bb c^2+189Ab c^2x^4+189x^4B b^2c+135A b^2c x^2+45b^3B x^2+35b^3A}{315x^9}$	80

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/9*A*b^3/x^9-1/7*b^2*(3*A*c+B*b)/x^7-3/5*b*c*(A*c+B*b)/x^5-1/3*c^2*(A*c+
3*B*b)/x^3-B*c^3/x
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}}\,dx = -\frac{315\,Bc^3x^8+105\,(3\,Bbc^2+Ac^3)x^6+189\,(Bb^2c+Abc^2)x^4+35\,Ab^3+45\,(Bb^3+3\,Ab^2c)x^2}{315\,x^9}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^16,x, algorithm="fricas")
```

output
```
-1/315*(315*B*c^3*x^8 + 105*(3*B*b*c^2 + A*c^3)*x^6 + 189*(B*b^2*c + A*b*c
^2)*x^4 + 35*A*b^3 + 45*(B*b^3 + 3*A*b^2*c)*x^2)/x^9
```

3.39.6 Sympy [A] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}}\,dx = \frac{-35Ab^3-315Bc^3x^8+x^6(-105Ac^3-315Bbc^2)+x^4(-189Abc^2-189Bb^2c)+x^2(-135Ab^2c-45Bb^3)}{315x^9}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**16,x)
```

output
```
(-35*A*b**3 - 315*B*c**3*x**8 + x**6*(-105*A*c**3 - 315*B*b*c**2) + x**4*(
-189*A*b*c**2 - 189*B*b**2*c) + x**2*(-135*A*b**2*c - 45*B*b**3))/(315*x**
9)
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}}\,dx = -\frac{315\,Bc^3x^8+105\,(3\,Bbc^2+Ac^3)x^6+189\,(Bb^2c+Abc^2)x^4+35\,Ab^3+45\,(Bb^3+3\,Ab^2c)x^2}{315\,x^9}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^16,x, algorithm="maxima")
```

output
```
-1/315*(315*B*c^3*x^8 + 105*(3*B*b*c^2 + A*c^3)*x^6 + 189*(B*b^2*c + A*b*c
^2)*x^4 + 35*A*b^3 + 45*(B*b^3 + 3*A*b^2*c)*x^2)/x^9
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}}\,dx = -\frac{315\,Bc^3x^8+315\,Bbc^2x^6+105\,Ac^3x^6+189\,Bb^2cx^4+189\,Abc^2x^4+45\,Bb^3x^2+135\,Ab^2cx^2+35\,Ab^3}{315\,x^9}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^16,x, algorithm="giac")
```

output
```
-1/315*(315*B*c^3*x^8 + 315*B*b*c^2*x^6 + 105*A*c^3*x^6 + 189*B*b^2*c*x^4
+ 189*A*b*c^2*x^4 + 45*B*b^3*x^2 + 135*A*b^2*c*x^2 + 35*A*b^3)/x^9
```

3.39.9 Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}}\,dx = -\frac{x^4\left(\frac{3\,B\,b^2\,c}{5}+\frac{3\,A\,b\,c^2}{5}\right)+\frac{A\,b^3}{9}+x^2\left(\frac{B\,b^3}{7}+\frac{3\,A\,c\,b^2}{7}\right)+x^6\left(\frac{A\,c^3}{3}+B\,b\,c^2\right)+B\,c^3\,x^8}{x^9}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^16,x)
```

output
```
-(x^4*((3*A*b*c^2)/5 + (3*B*b^2*c)/5) + (A*b^3)/9 + x^2*((B*b^3)/7 + (3*A*
b^2*c)/7) + x^6*((A*c^3)/3 + B*b*c^2) + B*c^3*x^8)/x^9
```

3.40 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}}\,dx$

3.40.1 Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}}\,dx = -\frac{A(b+cx^2)^4}{10bx^{10}} - \frac{(5bB-Ac)(b+cx^2)^4}{40b^2x^8}$$

output
```
-1/10*A*(c*x^2+b)^4/b/x^10-1/40*(-A*c+5*B*b)*(c*x^2+b)^4/b^2/x^8
```

3.40.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}}\,dx$$
$$= -\frac{5Bx^2(b^3+4b^2cx^2+6bc^2x^4+4c^3x^6)+A(4b^3+15b^2cx^2+20bc^2x^4+10c^3x^6)}{40x^{10}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^17,x]
```

output
```
-1/40*(5*B*x^2*(b^3 + 4*b^2*c*x^2 + 6*b*c^2*x^4 + 4*c^3*x^6) + A*(4*b^3 +
15*b^2*c*x^2 + 20*b*c^2*x^4 + 10*c^3*x^6))/x^10
```

3.40.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx^2)(b+cx^2)^3}{x^{11}}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)(cx^2+b)^3}{x^{12}}dx^2$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{2}\left(\frac{(5bB-Ac)\int \frac{(cx^2+b)^3}{x^{10}}dx^2}{5b}-\frac{A(b+cx^2)^4}{5bx^{10}}\right)$$

$$\Big\downarrow \text{ 48}$$

$$\frac{1}{2}\left(-\frac{(b+cx^2)^4(5bB-Ac)}{20b^2x^8}-\frac{A(b+cx^2)^4}{5bx^{10}}\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^17,x]
```

output
```
(-1/5*(A*(b + c*x^2)^4)/(b*x^10) - ((5*b*B - A*c)*(b + c*x^2)^4)/(20*b^2*x
^8))/2
```

3.40.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.40.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

method	result	size
default	$-\frac{bc(Ac+Bb)}{2x^6}-\frac{b^2(3Ac+Bb)}{8x^8}-\frac{b^3A}{10x^{10}}-\frac{B\,c^3}{2x^2}-\frac{c^2(Ac+3Bb)}{4x^4}$	66
risch	$\frac{-\frac{B\,c^3x^8}{2}+(-\frac{1}{4}A\,c^3-\frac{3}{4}Bb\,c^2)x^6+(-\frac{1}{2}Ab\,c^2-\frac{1}{2}B\,b^2c)x^4+(-\frac{3}{8}b^2Ac-\frac{1}{8}B\,b^3)x^2-\frac{b^3A}{10}}{x^{10}}$	76
norman	$\frac{(-\frac{1}{4}A\,c^3-\frac{3}{4}Bb\,c^2)x^{12}+(-\frac{1}{2}Ab\,c^2-\frac{1}{2}B\,b^2c)x^{10}+(-\frac{3}{8}b^2Ac-\frac{1}{8}B\,b^3)x^8-\frac{B\,c^3x^{14}}{2}-\frac{x^6b^3A}{10}}{x^{16}}$	79
gosper	$-\frac{20B\,c^3x^8+10A\,c^3x^6+30x^6Bb\,c^2+20Ab\,c^2x^4+20x^4B\,b^2c+15A\,b^2c\,x^2+5b^3B\,x^2+4b^3A}{40x^{10}}$	80
parallelrisch	$-\frac{20B\,c^3x^8+10A\,c^3x^6+30x^6Bb\,c^2+20Ab\,c^2x^4+20x^4B\,b^2c+15A\,b^2c\,x^2+5b^3B\,x^2+4b^3A}{40x^{10}}$	80

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3/x^17,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*c*(A*c+B*b)/x^6-1/8*b^2*(3*A*c+B*b)/x^8-1/10*b^3*A/x^10-1/2*B*c^3/x
^2-1/4*c^2*(A*c+3*B*b)/x^4
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.53

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}}\,dx$$
$$= -\frac{20\,Bc^3x^8+10\,(3\,Bbc^2+Ac^3)x^6+20\,(Bb^2c+Abc^2)x^4+4\,Ab^3+5\,(Bb^3+3\,Ab^2c)x^2}{40\,x^{10}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^17,x, algorithm="fricas")
```

output
```
-1/40*(20*B*c^3*x^8 + 10*(3*B*b*c^2 + A*c^3)*x^6 + 20*(B*b^2*c + A*b*c^2)*
x^4 + 4*A*b^3 + 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^10
```

3.40.6 Sympy [A] (verification not implemented)

Time = 3.52 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}}\,dx$$
$$= \frac{-4Ab^3-20Bc^3x^8+x^6(-10Ac^3-30Bbc^2)+x^4(-20Abc^2-20Bb^2c)+x^2(-15Ab^2c-5Bb^3)}{40x^{10}}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**17,x)
```

output
```
(-4*A*b**3 - 20*B*c**3*x**8 + x**6*(-10*A*c**3 - 30*B*b*c**2) + x**4*(-20*
A*b*c**2 - 20*B*b**2*c) + x**2*(-15*A*b**2*c - 5*B*b**3))/(40*x**10)
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.53

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}}\,dx = -\frac{20\,Bc^3x^8+10\,(3\,Bbc^2+Ac^3)x^6+20\,(Bb^2c+Abc^2)x^4+4\,Ab^3+5\,(Bb^3+3\,Ab^2c)x^2}{40\,x^{10}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^17,x, algorithm="maxima")
```

output
```
-1/40*(20*B*c^3*x^8 + 10*(3*B*b*c^2 + A*c^3)*x^6 + 20*(B*b^2*c + A*b*c^2)*
x^4 + 4*A*b^3 + 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^10
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.61

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}}\,dx = -\frac{20\,Bc^3x^8+30\,Bbc^2x^6+10\,Ac^3x^6+20\,Bb^2cx^4+20\,Abc^2x^4+5\,Bb^3x^2+15\,Ab^2cx^2+4\,Ab^3}{40\,x^{10}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^17,x, algorithm="giac")
```

output
```
-1/40*(20*B*c^3*x^8 + 30*B*b*c^2*x^6 + 10*A*c^3*x^6 + 20*B*b^2*c*x^4 + 20*
A*b*c^2*x^4 + 5*B*b^3*x^2 + 15*A*b^2*c*x^2 + 4*A*b^3)/x^10
```

3.40.9 Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.55

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}}\,dx = -\frac{x^4\left(\frac{B\,b^2\,c}{2}+\frac{A\,b\,c^2}{2}\right)+\frac{A\,b^3}{10}+x^2\left(\frac{B\,b^3}{8}+\frac{3\,A\,c\,b^2}{8}\right)+x^6\left(\frac{A\,c^3}{4}+\frac{3\,B\,b\,c^2}{4}\right)+\frac{B\,c^3\,x^8}{2}}{x^{10}}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^17,x)
```

output
```
-(x^4*((A*b*c^2)/2 + (B*b^2*c)/2) + (A*b^3)/10 + x^2*((B*b^3)/8 + (3*A*b^2
*c)/8) + x^6*((A*c^3)/4 + (3*B*b*c^2)/4) + (B*c^3*x^8)/2)/x^10
```

3.40. $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}}\,dx$

3.41 $\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4}\,dx$

3.41.1 Optimal result

Integrand size = 24, antiderivative size = 119

$$\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{b^3(bB-Ac)x}{c^5}-\frac{b^2(bB-Ac)x^3}{3c^4}+\frac{b(bB-Ac)x^5}{5c^3}-\frac{(bB-Ac)x^7}{7c^2}+\frac{Bx^9}{9c}-\frac{b^{7/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{11/2}}$$

output
```
b^3*(-A*c+B*b)*x/c^5-1/3*b^2*(-A*c+B*b)*x^3/c^4+1/5*b*(-A*c+B*b)*x^5/c^3-1
/7*(-A*c+B*b)*x^7/c^2+1/9*B*x^9/c-b^(7/2)*(-A*c+B*b)*arctan(x*c^(1/2)/b^(1
/2))/c^(11/2)
```

3.41.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{b^3(bB-Ac)x}{c^5}-\frac{b^2(bB-Ac)x^3}{3c^4}+\frac{b(bB-Ac)x^5}{5c^3}+\frac{(-bB+Ac)x^7}{7c^2}+\frac{Bx^9}{9c}-\frac{b^{7/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{11/2}}$$

input
```
Integrate[(x^10*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(b^3*(b*B - A*c)*x)/c^5 - (b^2*(b*B - A*c)*x^3)/(3*c^4) + (b*(b*B - A*c)*x
^5)/(5*c^3) + ((-(b*B) + A*c)*x^7)/(7*c^2) + (B*x^9)/(9*c) - (b^(7/2)*(b*B
 - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(11/2)
```

3.41.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.78, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}\left(A+Bx^2\right)}{bx^2+cx^4}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^8\left(A+Bx^2\right)}{b+cx^2}dx$$

$$\Big\downarrow \; 363$$

$$\frac{Bx^9}{9c}-\frac{(bB-Ac)\int \frac{x^8}{cx^2+b}dx}{c}$$

$$\Big\downarrow \; 254$$

$$\frac{Bx^9}{9c}-\frac{(bB-Ac)\int \left(\frac{x^6}{c}-\frac{bx^4}{c^2}+\frac{b^2x^2}{c^3}+\frac{b^4}{c^4\left(cx^2+b\right)}-\frac{b^3}{c^4}\right)dx}{c}$$

$$\Big\downarrow \; 2009$$

$$\frac{Bx^9}{9c}-\frac{(bB-Ac)\left(\frac{b^{7/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}\right)}{c}$$

input
```
Int[(x^10*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(B*x^9)/(9*c) - ((b*B - A*c)*(-((b^3*x)/c^4) + (b^2*x^3)/(3*c^3) - (b*x^5)
/(5*c^2) + x^7/(7*c) + (b^(7/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(9/2)))/c
```

3.41.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.41.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03

method	result
default	$-\frac{-\frac{1}{9}B\,x^{9}c^{4}-\frac{1}{7}A\,c^{4}x^{7}+\frac{1}{7}Bb\,c^{3}x^{7}+\frac{1}{5}Ab\,c^{3}x^{5}-\frac{1}{5}B\,b^{2}c^{2}x^{5}-\frac{1}{3}A\,b^{2}c^{2}x^{3}+\frac{1}{3}B\,b^{3}c\,x^{3}+A\,b^{3}cx-B\,b^{4}x}{c^{5}}+\frac{b^{4}(Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c^{5}\sqrt{bc}}$
risch	$\frac{B\,x^{9}}{9c}+\frac{A\,x^{7}}{7c}-\frac{Bb\,x^{7}}{7c^{2}}-\frac{Ab\,x^{5}}{5c^{2}}+\frac{B\,b^{2}x^{5}}{5c^{3}}+\frac{A\,b^{2}x^{3}}{3c^{3}}-\frac{B\,b^{3}x^{3}}{3c^{4}}-\frac{A\,b^{3}x}{c^{4}}+\frac{B\,b^{4}x}{c^{5}}+\frac{\sqrt{-bc}\,b^{3}\ln(-\sqrt{-bc}\,x+b)A}{2c^{5}}-\sqrt{-bc}$

input
```
int(x^10*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/c^5*(-1/9*B*x^9*c^4-1/7*A*c^4*x^7+1/7*B*b*c^3*x^7+1/5*A*b*c^3*x^5-1/5*B
*b^2*c^2*x^5-1/3*A*b^2*c^2*x^3+1/3*B*b^3*c*x^3+A*b^3*c*x-B*b^4*x)+b^4*(A*c
-B*b)/c^5/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.30

$$\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4}\,dx$$
$$= \left[\frac{70\,Bc^4x^9 - 90\,(Bbc^3 - Ac^4)x^7 + 126\,(Bb^2c^2 - Abc^3)x^5 - 210\,(Bb^3c - Ab^2c^2)x^3 - 315\,(Bb^4 - Ab^3c)\sqrt{-}}{630\,c^5}\right.$$

input
```
integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
[1/630*(70*B*c^4*x^9 - 90*(B*b*c^3 - A*c^4)*x^7 + 126*(B*b^2*c^2 - A*b*c^3
)*x^5 - 210*(B*b^3*c - A*b^2*c^2)*x^3 - 315*(B*b^4 - A*b^3*c)*sqrt(-b/c)*l
og((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 630*(B*b^4 - A*b^3*c)*x)/
c^5, 1/315*(35*B*c^4*x^9 - 45*(B*b*c^3 - A*c^4)*x^7 + 63*(B*b^2*c^2 - A*b*
c^3)*x^5 - 105*(B*b^3*c - A*b^2*c^2)*x^3 - 315*(B*b^4 - A*b^3*c)*sqrt(b/c)
*arctan(c*x*sqrt(b/c)/b) + 315*(B*b^4 - A*b^3*c)*x)/c^5]
```

3.41.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.71

$$\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx^9}{9c} + x^7\left(\frac{A}{7c} - \frac{Bb}{7c^2}\right) + x^5\left(-\frac{Ab}{5c^2} + \frac{Bb^2}{5c^3}\right) + x^3\left(\frac{Ab^2}{3c^3} - \frac{Bb^3}{3c^4}\right) + x\left(-\frac{Ab^3}{c^4} + \frac{Bb^4}{c^5}\right) + \frac{\sqrt{-\frac{b^7}{c^{11}}}(-Ac+Bb)\log\left(-\frac{c^5\sqrt{-\frac{b^7}{c^{11}}}(-Ac+Bb)}{-Ab^3c+Bb^4} + x\right)}{2} - \frac{\sqrt{-\frac{b^7}{c^{11}}}(-Ac+Bb)\log\left(\frac{c^5\sqrt{-\frac{b^7}{c^{11}}}(-Ac+Bb)}{-Ab^3c+Bb^4} + x\right)}{2}$$

input
```
integrate(x**10*(B*x**2+A)/(c*x**4+b*x**2),x)
```

output
```
B*x**9/(9*c) + x**7*(A/(7*c) - B*b/(7*c**2)) + x**5*(-A*b/(5*c**2) + B*b**
2/(5*c**3)) + x**3*(A*b**2/(3*c**3) - B*b**3/(3*c**4)) + x*(-A*b**3/c**4 +
 B*b**4/c**5) + sqrt(-b**7/c**11)*(-A*c + B*b)*log(-c**5*sqrt(-b**7/c**11)
*(-A*c + B*b)/(-A*b**3*c + B*b**4) + x)/2 - sqrt(-b**7/c**11)*(-A*c + B*b)
*log(c**5*sqrt(-b**7/c**11)*(-A*c + B*b)/(-A*b**3*c + B*b**4) + x)/2
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04

$$\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{(Bb^5-Ab^4c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^5} + \frac{35\,Bc^4x^9-45\,(Bbc^3-Ac^4)x^7+63\,(Bb^2c^2-Abc^3)x^5-105\,(Bb^3c-Ab^2c^2)x^3+315\,(Bb^4-Ab^3c)x}{315\,c^5}$$

input
```
integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-(B*b^5 - A*b^4*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) + 1/315*(35*B*c^4
*x^9 - 45*(B*b*c^3 - A*c^4)*x^7 + 63*(B*b^2*c^2 - A*b*c^3)*x^5 - 105*(B*b^
3*c - A*b^2*c^2)*x^3 + 315*(B*b^4 - A*b^3*c)*x)/c^5
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.12

$$\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{(Bb^5-Ab^4c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^5} + \frac{35\,Bc^8x^9-45\,Bbc^7x^7+45\,Ac^8x^7+63\,Bb^2c^6x^5-63\,Abc^7x^5-105\,Bb^3c^5x^3+105\,Ab^2c^6x^3+315\,Bb^4c^4}{315\,c^9}$$

input
```
integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-(B*b^5 - A*b^4*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) + 1/315*(35*B*c^8
*x^9 - 45*B*b*c^7*x^7 + 45*A*c^8*x^7 + 63*B*b^2*c^6*x^5 - 63*A*b*c^7*x^5 -
 105*B*b^3*c^5*x^3 + 105*A*b^2*c^6*x^3 + 315*B*b^4*c^4*x - 315*A*b^3*c^5*x
)/c^9
```

3.41.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.21

$$\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4}\,dx = x^7\left(\frac{A}{7\,c}-\frac{B\,b}{7\,c^2}\right)+\frac{B\,x^9}{9\,c}+\frac{b^2\,x^3\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{3\,c^2} - \frac{b^{7/2}\,\mathrm{atan}\left(\frac{b^{7/2}\,\sqrt{c}\,x\,(A\,c-B\,b)}{B\,b^5-A\,b^4\,c}\right)\,(A\,c-B\,b)}{c^{11/2}} - \frac{b\,x^5\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{5\,c}-\frac{b^3\,x\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{c^3}$$

input
```
int((x^10*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

output
```
x^7*(A/(7*c) - (B*b)/(7*c^2)) + (B*x^9)/(9*c) + (b^2*x^3*(A/c - (B*b)/c^2)
)/(3*c^2) - (b^(7/2)*atan((b^(7/2)*c^(1/2)*x*(A*c - B*b))/(B*b^5 - A*b^4*c
))*(A*c - B*b))/c^(11/2) - (b*x^5*(A/c - (B*b)/c^2))/(5*c) - (b^3*x*(A/c -
 (B*b)/c^2))/c^3
```

3.42 $\int \frac{x^9(A+Bx^2)}{bx^2+cx^4}\,dx$

3.42.1 Optimal result

Integrand size = 24, antiderivative size = 96

$$\int \frac{x^9(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{b^2(bB-Ac)x^2}{2c^4}+\frac{b(bB-Ac)x^4}{4c^3}-\frac{(bB-Ac)x^6}{6c^2}+\frac{Bx^8}{8c}+\frac{b^3(bB-Ac)\log(b+cx^2)}{2c^5}$$

output
```
-1/2*b^2*(-A*c+B*b)*x^2/c^4+1/4*b*(-A*c+B*b)*x^4/c^3-1/6*(-A*c+B*b)*x^6/c^
2+1/8*B*x^8/c+1/2*b^3*(-A*c+B*b)*ln(c*x^2+b)/c^5
```

3.42.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int \frac{x^9(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{cx^2(-12b^3B+6b^2c(2A+Bx^2)-2bc^2x^2(3A+2Bx^2)+c^3x^4(4A+3Bx^2))+12b^3(bB-Ac)\log(b+cx^2)}{24c^5}$$

input
```
Integrate[(x^9*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(c*x^2*(-12*b^3*B + 6*b^2*c*(2*A + B*x^2) - 2*b*c^2*x^2*(3*A + 2*B*x^2) +
c^3*x^4*(4*A + 3*B*x^2)) + 12*b^3*(b*B - A*c)*Log[b + c*x^2])/(24*c^5)
```

3.42.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9 (A+Bx^2)}{bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^7 (A+Bx^2)}{b+cx^2}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{x^6 (Bx^2+A)}{cx^2+b}dx^2$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{2}\int \left(\frac{Bx^6}{c}+\frac{(Ac-bB)x^4}{c^2}+\frac{b(bB-Ac)x^2}{c^3}-\frac{b^2(bB-Ac)}{c^4}+\frac{b^3(bB-Ac)}{c^4 (cx^2+b)}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{b^3(bB-Ac)\log (b+cx^2)}{c^5}-\frac{b^2x^2(bB-Ac)}{c^4}+\frac{bx^4(bB-Ac)}{2c^3}-\frac{x^6(bB-Ac)}{3c^2}+\frac{Bx^8}{4c}\right)$$

input
```
Int[(x^9*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(-((b^2*(b*B - A*c)*x^2)/c^4) + (b*(b*B - A*c)*x^4)/(2*c^3) - ((b*B - A*c)
*x^6)/(3*c^2) + (B*x^8)/(4*c) + (b^3*(b*B - A*c)*Log[b + c*x^2])/c^5)/2
```

3.42.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.42.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

method	result
norman	$\frac{\frac{B\,x^9}{8c}+\frac{(Ac-Bb)x^7}{6c^2}-\frac{b(Ac-Bb)x^5}{4c^3}+\frac{b^2(Ac-Bb)x^3}{2c^4}}{x}-\frac{b^3(Ac-Bb)\ln\left(c\,x^2+b\right)}{2c^5}$
default	$\frac{\frac{1}{4}B\,c^3x^8+\frac{1}{3}A\,c^3x^6-\frac{1}{3}x^6Bb\,c^2-\frac{1}{2}Ab\,c^2x^4+\frac{1}{2}x^4B\,b^2c+A\,b^2c\,x^2-b^3B\,x^2}{2c^4}-\frac{b^3(Ac-Bb)\ln\left(c\,x^2+b\right)}{2c^5}$
parallelrisch	$-\frac{-3B\,x^8c^4-4A\,x^6c^4+4B\,x^6b\,c^3+6A\,x^4b\,c^3-6B\,x^4b^2c^2-12A\,x^2b^2c^2+12B\,x^2b^3c+12A\ln\left(c\,x^2+b\right)b^3c-12B\ln\left(c\,x^2+b\right)b^4}{24c^5}$
risch	$\frac{B\,x^8}{8c}+\frac{A\,x^6}{6c}-\frac{x^6Bb}{6c^2}-\frac{Ab\,x^4}{4c^2}+\frac{x^4B\,b^2}{4c^3}+\frac{A\,b^2x^2}{2c^3}-\frac{b^3B\,x^2}{2c^4}-\frac{b^3\ln\left(c\,x^2+b\right)A}{2c^4}+\frac{b^4\ln\left(c\,x^2+b\right)B}{2c^5}$

input
```
int(x^9*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
(1/8*B*x^9/c+1/6/c^2*(A*c-B*b)*x^7-1/4*b/c^3*(A*c-B*b)*x^5+1/2*b^2*(A*c-B*
b)/c^4*x^3)/x-1/2*b^3*(A*c-B*b)/c^5*ln(c*x^2+b)
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.02

$$\int \frac{x^9(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{3\,Bc^4x^8-4\,(Bbc^3-Ac^4)x^6+6\,(Bb^2c^2-Abc^3)x^4-12\,(Bb^3c-Ab^2c^2)x^2+12\,(Bb^4-Ab^3c)\log\left(cx^2+b\right)}{24\,c^5}$$

input
```
integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/24*(3*B*c^4*x^8 - 4*(B*b*c^3 - A*c^4)*x^6 + 6*(B*b^2*c^2 - A*b*c^3)*x^4
- 12*(B*b^3*c - A*b^2*c^2)*x^2 + 12*(B*b^4 - A*b^3*c)*log(c*x^2 + b))/c^5
```

3.42.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98

$$\int \frac{x^9(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx^8}{8c}+\frac{b^3(-Ac+Bb)\log\left(b+cx^2\right)}{2c^5}+x^6\left(\frac{A}{6c}-\frac{Bb}{6c^2}\right)+x^4\left(-\frac{Ab}{4c^2}+\frac{Bb^2}{4c^3}\right)+x^2\left(\frac{Ab^2}{2c^3}-\frac{Bb^3}{2c^4}\right)$$

input
```
integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2),x)
```

output
```
B*x**8/(8*c) + b**3*(-A*c + B*b)*log(b + c*x**2)/(2*c**5) + x**6*(A/(6*c)
- B*b/(6*c**2)) + x**4*(-A*b/(4*c**2) + B*b**2/(4*c**3)) + x**2*(A*b**2/(2
*c**3) - B*b**3/(2*c**4))
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01

$$\int \frac{x^9(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{3\,Bc^3x^8-4\,(Bbc^2-Ac^3)x^6+6\,(Bb^2c-Abc^2)x^4-12\,(Bb^3-Ab^2c)x^2}{24\,c^4}+\frac{(Bb^4-Ab^3c)\log\left(cx^2+b\right)}{2\,c^5}$$

input
```
integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/24*(3*B*c^3*x^8 - 4*(B*b*c^2 - A*c^3)*x^6 + 6*(B*b^2*c - A*b*c^2)*x^4 -
12*(B*b^3 - A*b^2*c)*x^2)/c^4 + 1/2*(B*b^4 - A*b^3*c)*log(c*x^2 + b)/c^5
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.05

$$\int \frac{x^9(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{3\,Bc^3x^8-4\,Bbc^2x^6+4\,Ac^3x^6+6\,Bb^2cx^4-6\,Abc^2x^4-12\,Bb^3x^2+12\,Ab^2cx^2}{24\,c^4} + \frac{(Bb^4-Ab^3c)\log\left(|cx^2+b|\right)}{2\,c^5}$$

input
```
integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/24*(3*B*c^3*x^8 - 4*B*b*c^2*x^6 + 4*A*c^3*x^6 + 6*B*b^2*c*x^4 - 6*A*b*c^
2*x^4 - 12*B*b^3*x^2 + 12*A*b^2*c*x^2)/c^4 + 1/2*(B*b^4 - A*b^3*c)*log(abs
(c*x^2 + b))/c^5
```

3.42.9 Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.04

$$\int \frac{x^9(A+Bx^2)}{bx^2+cx^4}\,dx = x^6\left(\frac{A}{6\,c}-\frac{B\,b}{6\,c^2}\right)+\frac{B\,x^8}{8\,c}+\frac{\ln\left(c\,x^2+b\right)\,\left(B\,b^4-A\,b^3\,c\right)}{2\,c^5} + \frac{b^2\,x^2\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{2\,c^2}-\frac{b\,x^4\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{4\,c}$$

input
```
int((x^9*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

output
```
x^6*(A/(6*c) - (B*b)/(6*c^2)) + (B*x^8)/(8*c) + (log(b + c*x^2)*(B*b^4 - A
*b^3*c))/(2*c^5) + (b^2*x^2*(A/c - (B*b)/c^2))/(2*c^2) - (b*x^4*(A/c - (B*
b)/c^2))/(4*c)
```

3.43 $\int \frac{x^8(A+Bx^2)}{bx^2+cx^4}\,dx$

3.43.1 Optimal result

Integrand size = 24, antiderivative size = 98

$$\int \frac{x^8(A+Bx^2)}{bx^2+cx^4}\,dx=-\frac{b^2(bB-Ac)x}{c^4}+\frac{b(bB-Ac)x^3}{3c^3}-\frac{(bB-Ac)x^5}{5c^2}+\frac{Bx^7}{7c}+\frac{b^{5/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}}$$

output
```
-b^2*(-A*c+B*b)*x/c^4+1/3*b*(-A*c+B*b)*x^3/c^3-1/5*(-A*c+B*b)*x^5/c^2+1/7*
B*x^7/c+b^(5/2)*(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/c^(9/2)
```

3.43.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{x^8(A+Bx^2)}{bx^2+cx^4}\,dx=-\frac{b^2(bB-Ac)x}{c^4}+\frac{b(bB-Ac)x^3}{3c^3}+\frac{(-bB+Ac)x^5}{5c^2}+\frac{Bx^7}{7c}+\frac{b^{5/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}}$$

input
```
Integrate[(x^8*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
-((b^2*(b*B - A*c)*x)/c^4) + (b*(b*B - A*c)*x^3)/(3*c^3) + ((-(b*B) + A*c)
*x^5)/(5*c^2) + (B*x^7)/(7*c) + (b^(5/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/c^(9/2)
```

3.43.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(A+Bx^2)}{bx^2+cx^4}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^6(A+Bx^2)}{b+cx^2}dx$$

$$\Big\downarrow \; 363$$

$$\frac{Bx^7}{7c}-\frac{(bB-Ac)\int \frac{x^6}{cx^2+b}dx}{c}$$

$$\Big\downarrow \; 254$$

$$\frac{Bx^7}{7c}-\frac{(bB-Ac)\int \left(\frac{x^4}{c}-\frac{bx^2}{c^2}-\frac{b^3}{c^3(cx^2+b)}+\frac{b^2}{c^3}\right)dx}{c}$$

$$\Big\downarrow \; 2009$$

$$\frac{Bx^7}{7c}-\frac{(bB-Ac)\left(-\frac{b^{5/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}}+\frac{b^2x}{c^3}-\frac{bx^3}{3c^2}+\frac{x^5}{5c}\right)}{c}$$

input
```
Int[(x^8*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(B*x^7)/(7*c) - ((b*B - A*c)*((b^2*x)/c^3 - (b*x^3)/(3*c^2) + x^5/(5*c) -
(b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(7/2)))/c
```

3.43.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.43.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

method	result
default	$\frac{\frac{1}{7}B c^3x^7+\frac{1}{5}A c^3x^5-\frac{1}{5}Bb c^2x^5-\frac{1}{3}Ab c^2x^3+\frac{1}{3}B b^2c x^3+A b^2cx-b^3Bx}{c^4}-\frac{b^3(Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c^4\sqrt{bc}}$
risch	$\frac{B x^7}{7c}+\frac{A x^5}{5c}-\frac{Bb x^5}{5c^2}-\frac{Ab x^3}{3c^2}+\frac{B b^2x^3}{3c^3}+\frac{A b^2x}{c^3}-\frac{b^3Bx}{c^4}+\frac{\sqrt{-bc}\,b^2\ln(-\sqrt{-bc}\,x-b)A}{2c^4}-\frac{\sqrt{-bc}\,b^3\ln(-\sqrt{-bc}\,x-b)B}{2c^5}$

input
```
int(x^8*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(1/7*B*c^3*x^7+1/5*A*c^3*x^5-1/5*B*b*c^2*x^5-1/3*A*b*c^2*x^3+1/3*B*b
^2*c*x^3+A*b^2*c*x-b^3*B*x)-b^3*(A*c-B*b)/c^4/(b*c)^(1/2)*arctan(c*x/(b*c)
^(1/2))
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.33

$$\int \frac{x^8(A+Bx^2)}{bx^2+cx^4}\,dx$$
$$=\left[\frac{30\,Bc^3x^7-42\,(Bbc^2-Ac^3)x^5+70\,(Bb^2c-Abc^2)x^3-105\,(Bb^3-Ab^2c)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)-2}{210\,c^4}\right.$$

input
```
integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
[1/210*(30*B*c^3*x^7 - 42*(B*b*c^2 - A*c^3)*x^5 + 70*(B*b^2*c - A*b*c^2)*x
^3 - 105*(B*b^3 - A*b^2*c)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(
c*x^2 + b)) - 210*(B*b^3 - A*b^2*c)*x)/c^4, 1/105*(15*B*c^3*x^7 - 21*(B*b*
c^2 - A*c^3)*x^5 + 35*(B*b^2*c - A*b*c^2)*x^3 + 105*(B*b^3 - A*b^2*c)*sqrt
(b/c)*arctan(c*x*sqrt(b/c)/b) - 105*(B*b^3 - A*b^2*c)*x)/c^4]
```

3.43.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(87) = 174.

Time = 0.22 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.84

$$\int \frac{x^8(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{Bx^7}{7c}+x^5\left(\frac{A}{5c}-\frac{Bb}{5c^2}\right)+x^3\left(-\frac{Ab}{3c^2}+\frac{Bb^2}{3c^3}\right)+x\left(\frac{Ab^2}{c^3}-\frac{Bb^3}{c^4}\right)$$
$$-\frac{\sqrt{-\frac{b^5}{c^9}}(-Ac+Bb)\log\left(-\frac{c^4\sqrt{-\frac{b^5}{c^9}}(-Ac+Bb)}{-Ab^2c+Bb^3}+x\right)}{2}$$
$$+\frac{\sqrt{-\frac{b^5}{c^9}}(-Ac+Bb)\log\left(\frac{c^4\sqrt{-\frac{b^5}{c^9}}(-Ac+Bb)}{-Ab^2c+Bb^3}+x\right)}{2}$$

input
```
integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2),x)
```

output
```
B*x**7/(7*c) + x**5*(A/(5*c) - B*b/(5*c**2)) + x**3*(-A*b/(3*c**2) + B*b**
2/(3*c**3)) + x*(A*b**2/c**3 - B*b**3/c**4) - sqrt(-b**5/c**9)*(-A*c + B*b
)*log(-c**4*sqrt(-b**5/c**9)*(-A*c + B*b)/(-A*b**2*c + B*b**3) + x)/2 + sq
rt(-b**5/c**9)*(-A*c + B*b)*log(c**4*sqrt(-b**5/c**9)*(-A*c + B*b)/(-A*b**
2*c + B*b**3) + x)/2
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{x^8(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{(Bb^4-Ab^3c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^4} + \frac{15\,Bc^3x^7-21\,(Bbc^2-Ac^3)x^5+35\,(Bb^2c-Abc^2)x^3-105\,(Bb^3-Ab^2c)x}{105\,c^4}$$

input
```
integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
(B*b^4 - A*b^3*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/105*(15*B*c^3*
x^7 - 21*(B*b*c^2 - A*c^3)*x^5 + 35*(B*b^2*c - A*b*c^2)*x^3 - 105*(B*b^3 -
 A*b^2*c)*x)/c^4
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10

$$\int \frac{x^8(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{(Bb^4-Ab^3c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^4} + \frac{15\,Bc^6x^7-21\,Bbc^5x^5+21\,Ac^6x^5+35\,Bb^2c^4x^3-35\,Abc^5x^3-105\,Bb^3c^3x+105\,Ab^2c^4x}{105\,c^7}$$

input
```
integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
(B*b^4 - A*b^3*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/105*(15*B*c^6*
x^7 - 21*B*b*c^5*x^5 + 21*A*c^6*x^5 + 35*B*b^2*c^4*x^3 - 35*A*b*c^5*x^3 -
105*B*b^3*c^3*x + 105*A*b^2*c^4*x)/c^7
```

3.43.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.20

$$\int \frac{x^8(A+Bx^2)}{bx^2+cx^4}\,dx = x^5\left(\frac{A}{5\,c}-\frac{B\,b}{5\,c^2}\right)+\frac{B\,x^7}{7\,c}+\frac{b^{5/2}\,\mathrm{atan}\left(\frac{b^{5/2}\,\sqrt{c}\,x\,(A\,c-B\,b)}{B\,b^4-A\,b^3\,c}\right)\,(A\,c-B\,b)}{c^{9/2}} - \frac{b\,x^3\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{3\,c}+\frac{b^2\,x\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{c^2}$$

input
```
int((x^8*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

output
```
x^5*(A/(5*c) - (B*b)/(5*c^2)) + (B*x^7)/(7*c) + (b^(5/2)*atan((b^(5/2)*c^(
1/2)*x*(A*c - B*b))/(B*b^4 - A*b^3*c))*(A*c - B*b))/c^(9/2) - (b*x^3*(A/c
- (B*b)/c^2))/(3*c) + (b^2*x*(A/c - (B*b)/c^2))/c^2
```

3.44 $\int \frac{x^7(A+Bx^2)}{bx^2+cx^4}\,dx$

3.44.1 Optimal result

Integrand size = 24, antiderivative size = 75

$$\int \frac{x^7(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{b(bB-Ac)x^2}{2c^3}-\frac{(bB-Ac)x^4}{4c^2}+\frac{Bx^6}{6c}-\frac{b^2(bB-Ac)\log(b+cx^2)}{2c^4}$$

output
```
1/2*b*(-A*c+B*b)*x^2/c^3-1/4*(-A*c+B*b)*x^4/c^2+1/6*B*x^6/c-1/2*b^2*(-A*c+
B*b)*ln(c*x^2+b)/c^4
```

3.44.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{x^7(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{cx^2(6b^2B-3bc(2A+Bx^2)+c^2x^2(3A+2Bx^2))+6b^2(-bB+Ac)\log(b+cx^2)}{12c^4}$$

input
```
Integrate[(x^7*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(c*x^2*(6*b^2*B - 3*b*c*(2*A + B*x^2) + c^2*x^2*(3*A + 2*B*x^2)) + 6*b^2*(
-(b*B) + A*c)*Log[b + c*x^2])/(12*c^4)
```

3.44.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 (A+Bx^2)}{bx^2+cx^4} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^5 (A+Bx^2)}{b+cx^2}dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{x^4 (Bx^2+A)}{cx^2+b}dx^2$$

$$\Big\downarrow \; 86$$

$$\frac{1}{2}\int \left(\frac{Bx^4}{c}+\frac{(Ac-bB)x^2}{c^2}+\frac{b(bB-Ac)}{c^3}-\frac{b^2(bB-Ac)}{c^3 (cx^2+b)}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{b^2(bB-Ac)\log\left(b+cx^2\right)}{c^4}+\frac{bx^2(bB-Ac)}{c^3}-\frac{x^4(bB-Ac)}{2c^2}+\frac{Bx^6}{3c}\right)$$

input
```
Int[(x^7*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
((b*(b*B - A*c)*x^2)/c^3 - ((b*B - A*c)*x^4)/(2*c^2) + (B*x^6)/(3*c) - (b^
2*(b*B - A*c)*Log[b + c*x^2])/c^4)/2
```

3.44.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.44.4 Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

method	result	size
norman	$\frac{\frac{B\,x^{7}}{6c}+\frac{(Ac-Bb)x^{5}}{4c^{2}}-\frac{b(Ac-Bb)x^{3}}{2c^{3}}}{x}+\frac{b^{2}(Ac-Bb)\ln\left(c\,x^{2}+b\right)}{2c^{4}}$	73
default	$-\frac{-\frac{1}{3}B\,c^{2}x^{6}-\frac{1}{2}A\,c^{2}x^{4}+\frac{1}{2}x^{4}Bbc+Abc\,x^{2}-b^{2}B\,x^{2}}{2c^{3}}+\frac{b^{2}(Ac-Bb)\ln\left(c\,x^{2}+b\right)}{2c^{4}}$	74
parallelrisch	$\frac{2B\,c^{3}x^{6}+3A\,c^{3}x^{4}-3Bb\,c^{2}x^{4}-6Ab\,c^{2}x^{2}+6B\,b^{2}c\,x^{2}+6A\ln\left(c\,x^{2}+b\right)b^{2}c-6B\ln\left(c\,x^{2}+b\right)b^{3}}{12c^{4}}$	84
risch	$\frac{B\,x^{6}}{6c}+\frac{A\,x^{4}}{4c}-\frac{x^{4}Bb}{4c^{2}}-\frac{Ab\,x^{2}}{2c^{2}}+\frac{b^{2}B\,x^{2}}{2c^{3}}+\frac{b^{2}\ln\left(c\,x^{2}+b\right)A}{2c^{3}}-\frac{b^{3}\ln\left(c\,x^{2}+b\right)B}{2c^{4}}$	86

input
```
int(x^7*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
(1/6*B*x^7/c+1/4/c^2*(A*c-B*b)*x^5-1/2*b/c^3*(A*c-B*b)*x^3)/x+1/2*b^2*(A*c
-B*b)/c^4*ln(c*x^2+b)
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{x^7(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2\,Bc^3x^6-3\,(Bbc^2-Ac^3)x^4+6\,(Bb^2c-Abc^2)x^2-6\,(Bb^3-Ab^2c)\log\left(cx^2+b\right)}{12\,c^4}$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/12*(2*B*c^3*x^6 - 3*(B*b*c^2 - A*c^3)*x^4 + 6*(B*b^2*c - A*b*c^2)*x^2 -
6*(B*b^3 - A*b^2*c)*log(c*x^2 + b))/c^4
```

3.44.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int \frac{x^7(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx^6}{6c}-\frac{b^2(-Ac+Bb)\log\left(b+cx^2\right)}{2c^4}+x^4\left(\frac{A}{4c}-\frac{Bb}{4c^2}\right)+x^2\left(-\frac{Ab}{2c^2}+\frac{Bb^2}{2c^3}\right)$$

input
```
integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2),x)
```

output
```
B*x**6/(6*c) - b**2*(-A*c + B*b)*log(b + c*x**2)/(2*c**4) + x**4*(A/(4*c)
- B*b/(4*c**2)) + x**2*(-A*b/(2*c**2) + B*b**2/(2*c**3))
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{x^7(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2\,Bc^2x^6-3\,(Bbc-Ac^2)x^4+6\,(Bb^2-Abc)x^2}{12\,c^3} - \frac{(Bb^3-Ab^2c)\log\left(cx^2+b\right)}{2\,c^4}$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/12*(2*B*c^2*x^6 - 3*(B*b*c - A*c^2)*x^4 + 6*(B*b^2 - A*b*c)*x^2)/c^3 - 1
/2*(B*b^3 - A*b^2*c)*log(c*x^2 + b)/c^4
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int \frac{x^7(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2\,Bc^2x^6 - 3\,Bbcx^4 + 3\,Ac^2x^4 + 6\,Bb^2x^2 - 6\,Abcx^2}{12\,c^3} - \frac{(Bb^3 - Ab^2c)\log\left(|cx^2+b|\right)}{2\,c^4}$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/12*(2*B*c^2*x^6 - 3*B*b*c*x^4 + 3*A*c^2*x^4 + 6*B*b^2*x^2 - 6*A*b*c*x^2)
/c^3 - 1/2*(B*b^3 - A*b^2*c)*log(abs(c*x^2 + b))/c^4
```

3.44.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int \frac{x^7(A+Bx^2)}{bx^2+cx^4}\,dx = x^4\left(\frac{A}{4\,c} - \frac{B\,b}{4\,c^2}\right) + \frac{B\,x^6}{6\,c} - \frac{\ln\left(c\,x^2+b\right)\,\left(B\,b^3 - A\,b^2\,c\right)}{2\,c^4} - \frac{b\,x^2\,\left(\frac{A}{c} - \frac{B\,b}{c^2}\right)}{2\,c}$$

input
```
int((x^7*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

output
```
x^4*(A/(4*c) - (B*b)/(4*c^2)) + (B*x^6)/(6*c) - (log(b + c*x^2)*(B*b^3 - A
*b^2*c))/(2*c^4) - (b*x^2*(A/c - (B*b)/c^2))/(2*c)
```

3.45 $\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx$

3.45.1 Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{b(bB-Ac)x}{c^3}-\frac{(bB-Ac)x^3}{3c^2}+\frac{Bx^5}{5c}-\frac{b^{3/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}}$$

output
```
b*(-A*c+B*b)*x/c^3-1/3*(-A*c+B*b)*x^3/c^2+1/5*B*x^5/c-b^(3/2)*(-A*c+B*b)*a
rctan(x*c^(1/2)/b^(1/2))/c^(7/2)
```

3.45.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{b(bB-Ac)x}{c^3}+\frac{(-bB+Ac)x^3}{3c^2}+\frac{Bx^5}{5c}-\frac{b^{3/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}}$$

input
```
Integrate[(x^6*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(b*(b*B - A*c)*x)/c^3 + ((-(b*B) + A*c)*x^3)/(3*c^2) + (B*x^5)/(5*c) - (b^
(3/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(7/2)
```

3.45.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^4(A+Bx^2)}{b+cx^2}dx$$

$$\Big\downarrow \; 363$$

$$\frac{Bx^5}{5c}-\frac{(bB-Ac)\int \frac{x^4}{cx^2+b}dx}{c}$$

$$\Big\downarrow \; 254$$

$$\frac{Bx^5}{5c}-\frac{(bB-Ac)\int \left(\frac{b^2}{c^2(cx^2+b)}-\frac{b}{c^2}+\frac{x^2}{c}\right)dx}{c}$$

$$\Big\downarrow \; 2009$$

$$\frac{Bx^5}{5c}-\frac{(bB-Ac)\left(\frac{b^{3/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{bx}{c^2}+\frac{x^3}{3c}\right)}{c}$$

input
```
Int[(x^6*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(B*x^5)/(5*c) - ((b*B - A*c)*(-((b*x)/c^2) + x^3/(3*c) + (b^(3/2)*ArcTan[(
Sqrt[c]*x)/Sqrt[b]])/c^(5/2)))/c
```

3.45.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.45.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

method	result
default	$-\frac{-\frac{1}{5}B\,c^{2}x^{5}-\frac{1}{3}A\,c^{2}x^{3}+\frac{1}{3}Bbc\,x^{3}+Abcx-b^{2}Bx}{c^{3}}+\frac{b^{2}(Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c^{3}\sqrt{bc}}$
risch	$\frac{B\,x^{5}}{5c}+\frac{A\,x^{3}}{3c}-\frac{Bb\,x^{3}}{3c^{2}}-\frac{Abx}{c^{2}}+\frac{b^{2}Bx}{c^{3}}+\frac{\sqrt{-bc}\,b\ln(-\sqrt{-bc}\,x+b)A}{2c^{3}}-\frac{\sqrt{-bc}\,b^{2}\ln(-\sqrt{-bc}\,x+b)B}{2c^{4}}-\frac{\sqrt{-bc}\,b\ln(\sqrt{-bc}\,x+b)}{2c^{3}}$

input
```
int(x^6*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/c^3*(-1/5*B*c^2*x^5-1/3*A*c^2*x^3+1/3*B*b*c*x^3+A*b*c*x-b^2*B*x)+b^2*(A
*c-B*b)/c^3/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.31

$$\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx = \left[\frac{6\,Bc^2x^5-10\,(Bbc-Ac^2)x^3-15\,(Bb^2-Abc)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+30\,(Bb^2-Abc)x}{30\,c^3}, \frac{3\,Bc^2x^5-}{}\right.$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
[1/30*(6*B*c^2*x^5 - 10*(B*b*c - A*c^2)*x^3 - 15*(B*b^2 - A*b*c)*sqrt(-b/c
)*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 30*(B*b^2 - A*b*c)*x)/
c^3, 1/15*(3*B*c^2*x^5 - 5*(B*b*c - A*c^2)*x^3 - 15*(B*b^2 - A*b*c)*sqrt(b
/c)*arctan(c*x*sqrt(b/c)/b) + 15*(B*b^2 - A*b*c)*x)/c^3]
```

3.45.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(68) = 136.

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.99

$$\begin{aligned}\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx &= \frac{Bx^5}{5c}+x^3\left(\frac{A}{3c}-\frac{Bb}{3c^2}\right)+x\left(-\frac{Ab}{c^2}+\frac{Bb^2}{c^3}\right)\\ &\quad+\frac{\sqrt{-\frac{b^3}{c^7}}(-Ac+Bb)\log\left(-\frac{c^3\sqrt{-\frac{b^3}{c^7}}(-Ac+Bb)}{-Abc+Bb^2}+x\right)}{2}\\ &\quad-\frac{\sqrt{-\frac{b^3}{c^7}}(-Ac+Bb)\log\left(\frac{c^3\sqrt{-\frac{b^3}{c^7}}(-Ac+Bb)}{-Abc+Bb^2}+x\right)}{2}\end{aligned}$$

input
```
integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2),x)
```

output
```
B*x**5/(5*c) + x**3*(A/(3*c) - B*b/(3*c**2)) + x*(-A*b/c**2 + B*b**2/c**3)
 + sqrt(-b**3/c**7)*(-A*c + B*b)*log(-c**3*sqrt(-b**3/c**7)*(-A*c + B*b)/(
-A*b*c + B*b**2) + x)/2 - sqrt(-b**3/c**7)*(-A*c + B*b)*log(c**3*sqrt(-b**
3/c**7)*(-A*c + B*b)/(-A*b*c + B*b**2) + x)/2
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{(Bb^3-Ab^2c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^3} + \frac{3\,Bc^2x^5-5\,(Bbc-Ac^2)x^3+15\,(Bb^2-Abc)x}{15\,c^3}$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-(B*b^3 - A*b^2*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/15*(3*B*c^2*x
^5 - 5*(B*b*c - A*c^2)*x^3 + 15*(B*b^2 - A*b*c)*x)/c^3
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{(Bb^3-Ab^2c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^3} + \frac{3\,Bc^4x^5-5\,Bbc^3x^3+5\,Ac^4x^3+15\,Bb^2c^2x-15\,Abc^3x}{15\,c^5}$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-(B*b^3 - A*b^2*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/15*(3*B*c^4*x
^5 - 5*B*b*c^3*x^3 + 5*A*c^4*x^3 + 15*B*b^2*c^2*x - 15*A*b*c^3*x)/c^5
```

3.45.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.25

$$\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx = x^3\left(\frac{A}{3\,c}-\frac{B\,b}{3\,c^2}\right)+\frac{B\,x^5}{5\,c} - \frac{b^{3/2}\,\mathrm{atan}\left(\frac{b^{3/2}\,\sqrt{c}\,x\,(A\,c-B\,b)}{B\,b^3-A\,b^2\,c}\right)\,(A\,c-B\,b)}{c^{7/2}} - \frac{b\,x\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{c}$$

input
```
int((x^6*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

output
```
x^3*(A/(3*c) - (B*b)/(3*c^2)) + (B*x^5)/(5*c) - (b^(3/2)*atan((b^(3/2)*c^(
1/2)*x*(A*c - B*b))/(B*b^3 - A*b^2*c))*(A*c - B*b))/c^(7/2) - (b*x*(A/c -
(B*b)/c^2))/c
```

3.45. $\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx$

3.46 $\int \frac{x^5(A+Bx^2)}{bx^2+cx^4}\,dx$

3.46.1 Optimal result

Integrand size = 24, antiderivative size = 54

$$\int \frac{x^5(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{b(bB-Ac)\log(b+cx^2)}{2c^3}$$

output
```
-1/2*(-A*c+B*b)*x^2/c^2+1/4*B*x^4/c+1/2*b*(-A*c+B*b)*ln(c*x^2+b)/c^3
```

3.46.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{x^5(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{cx^2(-2bB+2Ac+Bcx^2)+2b(bB-Ac)\log(b+cx^2)}{4c^3}$$

input
```
Integrate[(x^5*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(c*x^2*(-2*b*B + 2*A*c + B*c*x^2) + 2*b*(b*B - A*c)*Log[b + c*x^2])/(4*c^3
)
```

3.46.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(A+B x^2\right)}{b x^2+c x^4} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^3 \left(A+B x^2\right)}{b+c x^2}dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2} \int \frac{x^2 \left(B x^2+A\right)}{c x^2+b}dx^2$$

$$\Big\downarrow \; 86$$

$$\frac{1}{2} \int \left(\frac{B x^2}{c}+\frac{A c-b B}{c^2}+\frac{b (b B-A c)}{c^2 \left(c x^2+b\right)}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2} \left(\frac{b (b B-A c) \log \left(b+c x^2\right)}{c^3}-\frac{x^2 (b B-A c)}{c^2}+\frac{B x^4}{2 c}\right)$$

input
```
Int[(x^5*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(-(((b*B - A*c)*x^2)/c^2) + (B*x^4)/(2*c) + (b*(b*B - A*c)*Log[b + c*x^2])
/c^3)/2
```

3.46.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.46.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\frac{1}{2}B c\,x^{4}+A c\,x^{2}-b B\,x^{2}}{2c^{2}}-\frac{b(Ac-Bb)\ln\left(c\,x^{2}+b\right)}{2c^{3}}$	50
norman	$\frac{\frac{B\,x^{5}}{4c}+\frac{(Ac-Bb)x^{3}}{2c^{2}}}{x}-\frac{b(Ac-Bb)\ln\left(c\,x^{2}+b\right)}{2c^{3}}$	54
parallelrisch	$-\frac{-B\,c^{2}x^{4}-2A\,c^{2}x^{2}+2Bbc\,x^{2}+2A\ln\left(c\,x^{2}+b\right)bc-2B\ln\left(c\,x^{2}+b\right)b^{2}}{4c^{3}}$	60
risch	$\frac{B\,x^{4}}{4c}+\frac{A\,x^{2}}{2c}-\frac{Bb\,x^{2}}{2c^{2}}+\frac{A^{2}}{4Bc}-\frac{Ab}{2c^{2}}+\frac{B\,b^{2}}{4c^{3}}-\frac{b\ln\left(c\,x^{2}+b\right)A}{2c^{2}}+\frac{b^{2}\ln\left(c\,x^{2}+b\right)B}{2c^{3}}$	89

input
```
int(x^5*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/c^2*(1/2*B*c*x^4+A*c*x^2-b*B*x^2)-1/2*b/c^3*(A*c-B*b)*ln(c*x^2+b)
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{x^5(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bc^2x^4 - 2\,(Bbc - Ac^2)x^2 + 2\,(Bb^2 - Abc)\log\left(cx^2+b\right)}{4\,c^3}$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/4*(B*c^2*x^4 - 2*(B*b*c - A*c^2)*x^2 + 2*(B*b^2 - A*b*c)*log(c*x^2 + b))
/c^3
```

3.46.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{x^5(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx^4}{4c} + \frac{b(-Ac+Bb)\log\left(b+cx^2\right)}{2c^3} + x^2\left(\frac{A}{2c} - \frac{Bb}{2c^2}\right)$$

input
```
integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2),x)
```

output
```
B*x**4/(4*c) + b*(-A*c + B*b)*log(b + c*x**2)/(2*c**3) + x**2*(A/(2*c) - B
*b/(2*c**2))
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{x^5(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bcx^4 - 2\,(Bb - Ac)x^2}{4\,c^2} + \frac{(Bb^2 - Abc)\log\left(cx^2+b\right)}{2\,c^3}$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/4*(B*c*x^4 - 2*(B*b - A*c)*x^2)/c^2 + 1/2*(B*b^2 - A*b*c)*log(c*x^2 + b)
/c^3
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{x^5(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bcx^4-2\,Bbx^2+2\,Acx^2}{4\,c^2} + \frac{(Bb^2-Abc)\log\left(|cx^2+b|\right)}{2\,c^3}$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/4*(B*c*x^4 - 2*B*b*x^2 + 2*A*c*x^2)/c^2 + 1/2*(B*b^2 - A*b*c)*log(abs(c*
x^2 + b))/c^3
```

3.46.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{x^5(A+Bx^2)}{bx^2+cx^4}\,dx = x^2\left(\frac{A}{2\,c}-\frac{B\,b}{2\,c^2}\right) + \frac{\ln\left(c\,x^2+b\right)\,\left(B\,b^2-A\,b\,c\right)}{2\,c^3} + \frac{B\,x^4}{4\,c}$$

input
```
int((x^5*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

output
```
x^2*(A/(2*c) - (B*b)/(2*c^2)) + (log(b + c*x^2)*(B*b^2 - A*b*c))/(2*c^3) +
 (B*x^4)/(4*c)
```

3.47 $\int \frac{x^4(A+Bx^2)}{bx^2+cx^4}\,dx$

3.47.1 Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{x^4(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{(bB-Ac)x}{c^2}+\frac{Bx^3}{3c}+\frac{\sqrt{b}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}$$

output
```
-(-A*c+B*b)*x/c^2+1/3*B*x^3/c+(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))*b^(1/2)
/c^(5/2)
```

3.47.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{x^4(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{(-bB+Ac)x}{c^2}+\frac{Bx^3}{3c}+\frac{\sqrt{b}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}$$

input
```
Integrate[(x^4*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
((-(b*B) + A*c)*x)/c^2 + (B*x^3)/(3*c) + (Sqrt[b]*(b*B - A*c)*ArcTan[(Sqrt
[c]*x)/Sqrt[b]])/c^(5/2)
```

3.47.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 363, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (A+Bx^2)}{bx^2+cx^4} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^2 (A+Bx^2)}{b+cx^2} dx$$

$$\Big\downarrow \; 363$$

$$\frac{Bx^3}{3c}-\frac{(bB-Ac)\int \frac{x^2}{cx^2+b}dx}{c}$$

$$\Big\downarrow \; 262$$

$$\frac{Bx^3}{3c}-\frac{(bB-Ac)\left(\frac{x}{c}-\frac{b\int \frac{1}{cx^2+b}dx}{c}\right)}{c}$$

$$\Big\downarrow \; 218$$

$$\frac{Bx^3}{3c}-\frac{(bB-Ac)\left(\frac{x}{c}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{3/2}}\right)}{c}$$

input
```
Int[(x^4*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(B*x^3)/(3*c) - ((b*B - A*c)*(x/c - (Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/
c^(3/2)))/c
```

3.47.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.47.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

method	result
default	$\frac{\frac{1}{3}Bc\,x^{3}+Acx-bBx}{c^{2}}-\frac{b(Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c^{2}\sqrt{bc}}$
risch	$\frac{B\,x^{3}}{3c}+\frac{Ax}{c}-\frac{bBx}{c^{2}}+\frac{\sqrt{-bc}\,\ln(-\sqrt{-bc}\,x-b)A}{2c^{2}}-\frac{\sqrt{-bc}\,\ln(-\sqrt{-bc}\,x-b)Bb}{2c^{3}}-\frac{\sqrt{-bc}\,\ln(\sqrt{-bc}\,x-b)A}{2c^{2}}+\frac{\sqrt{-bc}\,\ln(\sqrt{-bc}\,x-}{2c^{3}}$

input
```
int(x^4*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/3*B*c*x^3+A*c*x-b*B*x)-b*(A*c-B*b)/c^2/(b*c)^(1/2)*arctan(c*x/(b*
c)^(1/2))
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.22

$$\int \frac{x^4(A+Bx^2)}{bx^2+cx^4}\,dx$$
$$= \left[\frac{2\,Bcx^3 - 3\,(Bb-Ac)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right) - 6\,(Bb-Ac)x}{6\,c^2}, \frac{Bcx^3 + 3\,(Bb-Ac)\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right.}{3\,c^2}\right.$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
[1/6*(2*B*c*x^3 - 3*(B*b - A*c)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) -
 b)/(c*x^2 + b)) - 6*(B*b - A*c)*x)/c^2, 1/3*(B*c*x^3 + 3*(B*b - A*c)*sqrt
(b/c)*arctan(c*x*sqrt(b/c)/b) - 3*(B*b - A*c)*x)/c^2]
```

3.47.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.55

$$\int \frac{x^4(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx^3}{3c} + x\left(\frac{A}{c} - \frac{Bb}{c^2}\right) - \frac{\sqrt{-\frac{b}{c^5}}(-Ac+Bb)\log\left(-c^2\sqrt{-\frac{b}{c^5}}+x\right)}{2} + \frac{\sqrt{-\frac{b}{c^5}}(-Ac+Bb)\log\left(c^2\sqrt{-\frac{b}{c^5}}+x\right)}{2}$$

input
```
integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2),x)
```

output
```
B*x**3/(3*c) + x*(A/c - B*b/c**2) - sqrt(-b/c**5)*(-A*c + B*b)*log(-c**2*s
qrt(-b/c**5) + x)/2 + sqrt(-b/c**5)*(-A*c + B*b)*log(c**2*sqrt(-b/c**5) +
x)/2
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{x^4(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{(Bb^2-Abc)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^2} + \frac{Bcx^3-3\,(Bb-Ac)x}{3\,c^2}$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
(B*b^2 - A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/3*(B*c*x^3 - 3*(
B*b - A*c)*x)/c^2
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{x^4(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{(Bb^2-Abc)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^2} + \frac{Bc^2x^3-3\,Bbcx+3\,Ac^2x}{3\,c^3}$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
(B*b^2 - A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/3*(B*c^2*x^3 - 3
*B*b*c*x + 3*A*c^2*x)/c^3
```

3.47.9 Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \frac{x^4(A+Bx^2)}{bx^2+cx^4}\,dx = x\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)+\frac{B\,x^3}{3\,c}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c}\,x\,(A\,c-B\,b)}{B\,b^2-A\,b\,c}\right)\,(A\,c-B\,b)}{c^{5/2}}$$

input
```
int((x^4*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

output
```
x*(A/c - (B*b)/c^2) + (B*x^3)/(3*c) + (b^(1/2)*atan((b^(1/2)*c^(1/2)*x*(A*
c - B*b))/(B*b^2 - A*b*c))*(A*c - B*b))/c^(5/2)
```

3.48 $\int \frac{x^3(A+Bx^2)}{bx^2+cx^4}\,dx$

3.48.1 Optimal result

Integrand size = 24, antiderivative size = 35

$$\int \frac{x^3(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx^2}{2c} - \frac{(bB-Ac)\log(b+cx^2)}{2c^2}$$

output
```
1/2*B*x^2/c-1/2*(-A*c+B*b)*ln(c*x^2+b)/c^2
```

3.48.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^3(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bcx^2+(-bB+Ac)\log(b+cx^2)}{2c^2}$$

input
```
Integrate[(x^3*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(B*c*x^2 + (-(b*B) + A*c)*Log[b + c*x^2])/(2*c^2)
```

3.48.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (A+Bx^2)}{bx^2+cx^4}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x (A+Bx^2)}{b+cx^2}dx$$

$$\Big\downarrow \ 353$$

$$\frac{1}{2}\int \frac{Bx^2+A}{cx^2+b}dx^2$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2}\int \left(\frac{B}{c}+\frac{Ac-bB}{c\left(cx^2+b\right)}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{Bx^2}{c}-\frac{(bB-Ac)\log\left(b+cx^2\right)}{c^2}\right)$$

input
```
Int[(x^3*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
((B*x^2)/c - ((b*B - A*c)*Log[b + c*x^2])/c^2)/2
```

3.48.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.48.4 Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{B\,x^2}{2c}+\frac{(Ac-Bb)\ln(c\,x^2+b)}{2c^2}$	32
norman	$\frac{B\,x^2}{2c}+\frac{(Ac-Bb)\ln(c\,x^2+b)}{2c^2}$	32
parallelrisch	$\frac{Bc\,x^2+A\ln(c\,x^2+b)c-B\ln(c\,x^2+b)b}{2c^2}$	36
risch	$\frac{B\,x^2}{2c}+\frac{\ln(c\,x^2+b)A}{2c}-\frac{\ln(c\,x^2+b)Bb}{2c^2}$	40

input
```
int(x^3*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*B*x^2/c+1/2/c^2*(A*c-B*b)*ln(c*x^2+b)
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{x^3(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{Bcx^2-(Bb-Ac)\log\left(cx^2+b\right)}{2\,c^2}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/2*(B*c*x^2 - (B*b - A*c)*log(c*x^2 + b))/c^2
```

3.48.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x^3(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{Bx^2}{2c}-\frac{(-Ac+Bb)\log{(b+cx^2)}}{2c^2}$$

input
```
integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2),x)
```

output
```
B*x**2/(2*c) - (-A*c + B*b)*log(b + c*x**2)/(2*c**2)
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^3(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{Bx^2}{2\,c}-\frac{(Bb-Ac)\log{(cx^2+b)}}{2\,c^2}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/2*B*x^2/c - 1/2*(B*b - A*c)*log(c*x^2 + b)/c^2
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{x^3(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{Bx^2}{2\,c}-\frac{(Bb-Ac)\log{(|cx^2+b|)}}{2\,c^2}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/2*B*x^2/c - 1/2*(B*b - A*c)*log(abs(c*x^2 + b))/c^2
```

3.48.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^3(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{B\,x^2}{2\,c} + \frac{\ln\left(c\,x^2+b\right)\,\left(A\,c-B\,b\right)}{2\,c^2}$$

input
```
int((x^3*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

output
```
(B*x^2)/(2*c) + (log(b + c*x^2)*(A*c - B*b))/(2*c^2)
```

3.49 $\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}\,dx$

3.49.1 Optimal result

Integrand size = 24, antiderivative size = 40

$$\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{Bx}{c}-\frac{(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}c^{3/2}}$$

output
```
B*x/c-(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/c^(3/2)/b^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{Bx}{c}-\frac{(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}c^{3/2}}$$

input
```
Integrate[(x^2*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(B*x)/c - ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(Sqrt[b]*c^(3/2))
```

3.49.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {9, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{A+Bx^2}{b+cx^2}dx$$

$$\Big\downarrow \; 299$$

$$\frac{Bx}{c}-\frac{(bB-Ac)\int \frac{1}{cx^2+b}dx}{c}$$

$$\Big\downarrow \; 218$$

$$\frac{Bx}{c}-\frac{(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}c^{3/2}}$$

input
```
Int[(x^2*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(B*x)/c - ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(Sqrt[b]*c^(3/2))
```

3.49.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.49.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{Bx}{c}+\frac{(Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c\sqrt{bc}}$	34
risch	$\frac{Bx}{c}-\frac{\ln(cx+\sqrt{-bc})A}{2\sqrt{-bc}}+\frac{\ln(cx+\sqrt{-bc})Bb}{2c\sqrt{-bc}}+\frac{\ln(-cx+\sqrt{-bc})A}{2\sqrt{-bc}}-\frac{\ln(-cx+\sqrt{-bc})Bb}{2c\sqrt{-bc}}$	98

input
```
int(x^2*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
B*x/c+(A*c-B*b)/c/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.48

$$\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}\,dx = \left[\frac{2\,Bbcx+(Bb-Ac)\sqrt{-bc}\log\left(\frac{cx^2-2\,\sqrt{-bc}x-b}{cx^2+b}\right)}{2\,bc^2}, \frac{Bbcx-(Bb-Ac)\sqrt{bc}\arctan\left(\frac{\sqrt{bc}x}{b}\right)}{bc^2}\right]$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
[1/2*(2*B*b*c*x + (B*b - A*c)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/
(c*x^2 + b)))/(b*c^2), (B*b*c*x - (B*b - A*c)*sqrt(b*c)*arctan(sqrt(b*c)*x
/b))/(b*c^2)]
```

3.49.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(34) = 68$.

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.05

$$\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx}{c} + \frac{\sqrt{-\frac{1}{bc^3}}(-Ac+Bb)\log\left(-bc\sqrt{-\frac{1}{bc^3}}+x\right)}{2} - \frac{\sqrt{-\frac{1}{bc^3}}(-Ac+Bb)\log\left(bc\sqrt{-\frac{1}{bc^3}}+x\right)}{2}$$

input
```
integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2),x)
```

output
```
B*x/c + sqrt(-1/(b*c**3))*(-A*c + B*b)*log(-b*c*sqrt(-1/(b*c**3)) + x)/2 -
 sqrt(-1/(b*c**3))*(-A*c + B*b)*log(b*c*sqrt(-1/(b*c**3)) + x)/2
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx}{c} - \frac{(Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c}$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
B*x/c - (B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c)
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx}{c} - \frac{(Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c}$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
B*x/c - (B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c)
```

3.49.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{B\,x}{c} + \frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(A\,c-B\,b)}{\sqrt{b}\,c^{3/2}}$$

input
```
int((x^2*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

output
```
(B*x)/c + (atan((c^(1/2)*x)/b^(1/2))*(A*c - B*b))/(b^(1/2)*c^(3/2))
```

3.50 $\int \frac{x(A+Bx^2)}{bx^2+cx^4}\,dx$

3.50.1 Optimal result

Integrand size = 22, antiderivative size = 34

$$\int \frac{x(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{A\log(x)}{b}+\frac{(bB-Ac)\log(b+cx^2)}{2bc}$$

output
```
A*ln(x)/b+1/2*(-A*c+B*b)*ln(c*x^2+b)/b/c
```

3.50.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{x(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{A\log(x)}{b}+\frac{(bB-Ac)\log(b+cx^2)}{2bc}$$

input
```
Integrate[(x*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(A*Log[x])/b + ((b*B - A*c)*Log[b + c*x^2])/(2*b*c)
```

3.50.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^2)}{bx^2+cx^4}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{A+Bx^2}{x(b+cx^2)}dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^2(cx^2+b)}dx^2$$

$$\Big\downarrow \; 86$$

$$\frac{1}{2}\int \left(\frac{A}{bx^2}+\frac{bB-Ac}{b(cx^2+b)}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{(bB-Ac)\log(b+cx^2)}{bc}+\frac{A\log(x^2)}{b}\right)$$

input
```
Int[(x*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
((A*Log[x^2])/b + ((b*B - A*c)*Log[b + c*x^2])/(b*c))/2
```

3.50.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.50.4 Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{A\ln(x)}{b}-\frac{(Ac-Bb)\ln(c\,x^2+b)}{2bc}$	33
norman	$\frac{A\ln(x)}{b}-\frac{(Ac-Bb)\ln(c\,x^2+b)}{2bc}$	33
risch	$\frac{A\ln(x)}{b}-\frac{\ln(c\,x^2+b)A}{2b}+\frac{\ln(c\,x^2+b)B}{2c}$	37
parallelrisch	$\frac{2A\ln(x)c-A\ln(c\,x^2+b)c+B\ln(c\,x^2+b)b}{2bc}$	39

input
```
int(x*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
A*ln(x)/b-1/2*(A*c-B*b)/b/c*ln(c*x^2+b)
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{x(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2\,Ac\log(x)+(Bb-Ac)\log(cx^2+b)}{2\,bc}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/2*(2*A*c*log(x) + (B*b - A*c)*log(c*x^2 + b))/(b*c)
```

3.50.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{x(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{A\log(x)}{b} + \frac{(-Ac+Bb)\log\left(\frac{b}{c}+x^2\right)}{2bc}$$

input
```
integrate(x*(B*x**2+A)/(c*x**4+b*x**2),x)
```

output
```
A*log(x)/b + (-A*c + B*b)*log(b/c + x**2)/(2*b*c)
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{x(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{A\log(x^2)}{2\,b} + \frac{(Bb-Ac)\log(cx^2+b)}{2\,bc}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/2*A*log(x^2)/b + 1/2*(B*b - A*c)*log(c*x^2 + b)/(b*c)
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{x(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{A\log(|x|)}{b} + \frac{(Bb-Ac)\log(|cx^2+b|)}{2\,bc}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
A*log(abs(x))/b + 1/2*(B*b - A*c)*log(abs(c*x^2 + b))/(b*c)
```

3.50.9 Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{x(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{A\,\ln(x)}{b} - \frac{\ln(c\,x^2+b)\,(A\,c-B\,b)}{2\,b\,c}$$

input
```
int((x*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

output
```
(A*log(x))/b - (log(b + c*x^2)*(A*c - B*b))/(2*b*c)
```

3.51 $\int \frac{A+Bx^2}{bx^2+cx^4} \, dx$

3.51.1 Optimal result

Integrand size = 21, antiderivative size = 42

$$\int \frac{A+Bx^2}{bx^2+cx^4} \, dx = -\frac{A}{bx}+\frac{(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}$$

output
```
-A/b/x+(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(3/2)/c^(1/2)
```

3.51.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{bx^2+cx^4} \, dx = -\frac{A}{bx}+\frac{(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}$$

input
```
Integrate[(A + B*x^2)/(b*x^2 + c*x^4),x]
```

output
```
-(A/(b*x)) + ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

3.51.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2026, 359, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{bx^2+cx^4} \, dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{A+Bx^2}{x^2 (b+cx^2)} dx$$

$$\Big\downarrow \text{ 359}$$

$$\frac{(bB-Ac)\int \frac{1}{cx^2+b}dx}{b} - \frac{A}{bx}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}} - \frac{A}{bx}$$

input
```
Int[(A + B*x^2)/(b*x^2 + c*x^4),x]
```

output
```
-(A/(b*x)) + ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

3.51.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] && !MonomialQ[Px, x] && (ILtQ[p, 0] || !PolyQ[u, x])
```

3.51.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	si
default	$-\frac{A}{bx}+\frac{(-Ac+Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{b\sqrt{bc}}$	37
risch	$-\frac{A}{bx}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^3c\_Z^2+A^2c^2-2ABbc+B^2b^2\right)}\_R\ln\left(\left(3\_R^2b^3c+2A^2c^2-4ABbc+2B^2b^2\right)x+\left(b^2Ac-B\,b^3\right)\_R\right)\right)}{2}$	99

input
```
int((B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-A/b/x+(-A*c+B*b)/b/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.50

$$\int \frac{A+Bx^2}{bx^2+cx^4}\,dx = \left[\frac{(Bb-Ac)\sqrt{-bc}x\log\left(\frac{cx^2+2\sqrt{-bc}x-b}{cx^2+b}\right)-2\,Abc}{2\,b^2cx}, \frac{(Bb-Ac)\sqrt{bc}x\arctan\left(\frac{\sqrt{bc}x}{b}\right)-Abc}{b^2cx}\right]$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
[1/2*((B*b - A*c)*sqrt(-b*c)*x*log((c*x^2 + 2*sqrt(-b*c)*x - b)/(c*x^2 + b
)) - 2*A*b*c)/(b^2*c*x), ((B*b - A*c)*sqrt(b*c)*x*arctan(sqrt(b*c)*x/b) -
A*b*c)/(b^2*c*x)]
```

3.51.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(34) = 68$.

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.95

$$\int \frac{A+Bx^2}{bx^2+cx^4}\,dx = -\frac{A}{bx} - \frac{\sqrt{-\frac{1}{b^3c}}(-Ac+Bb)\log\left(-b^2\sqrt{-\frac{1}{b^3c}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{b^3c}}(-Ac+Bb)\log\left(b^2\sqrt{-\frac{1}{b^3c}}+x\right)}{2}$$

input
```
integrate((B*x**2+A)/(c*x**4+b*x**2),x)
```

output
```
-A/(b*x) - sqrt(-1/(b**3*c))*(-A*c + B*b)*log(-b**2*sqrt(-1/(b**3*c)) + x)
/2 + sqrt(-1/(b**3*c))*(-A*c + B*b)*log(b**2*sqrt(-1/(b**3*c)) + x)/2
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^2}{bx^2+cx^4}\,dx = \frac{(Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b} - \frac{A}{bx}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
(B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) - A/(b*x)
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^2}{bx^2+cx^4}\,dx = \frac{(Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b} - \frac{A}{bx}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
(B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) - A/(b*x)
```

3.51.9 Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^2}{bx^2+cx^4} \, dx = -\frac{A}{b\,x} - \frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(A\,c-B\,b)}{b^{3/2}\,\sqrt{c}}$$

input
```
int((A + B*x^2)/(b*x^2 + c*x^4),x)
```

output
```
- A/(b*x) - (atan((c^(1/2)*x)/b^(1/2))*(A*c - B*b))/(b^(3/2)*c^(1/2))
```

3.52 $\int \frac{A+Bx^2}{bx^2-cx^4}\,dx$

3.52.1 Optimal result

Integrand size = 22, antiderivative size = 41

$$\int \frac{A+Bx^2}{bx^2-cx^4}\,dx=-\frac{A}{bx}+\frac{(bB+Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}$$

output
```
-A/b/x+(A*c+B*b)*arctanh(x*c^(1/2)/b^(1/2))/b^(3/2)/c^(1/2)
```

3.52.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{bx^2-cx^4}\,dx=-\frac{A}{bx}+\frac{(bB+Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}$$

input
```
Integrate[(A + B*x^2)/(b*x^2 - c*x^4),x]
```

output
```
-(A/(b*x)) + ((b*B + A*c)*ArcTanh[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

3.52.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2026, 359, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{bx^2-cx^4}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{A+Bx^2}{x^2\left(b-cx^2\right)}dx$$

$$\Big\downarrow \text{359}$$

$$\frac{(Ac+bB)\int \frac{1}{b-cx^2}dx}{b}-\frac{A}{bx}$$

$$\Big\downarrow \text{221}$$

$$\frac{(Ac+bB)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}-\frac{A}{bx}$$

input
```
Int[(A + B*x^2)/(b*x^2 - c*x^4),x]
```

output
```
-(A/(b*x)) + ((b*B + A*c)*ArcTanh[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

3.52.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.52.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

method	result	si
default	$-\frac{(-Ac-Bb)\operatorname{arctanh}\left(\frac{cx}{\sqrt{bc}}\right)}{b\sqrt{bc}}-\frac{A}{bx}$	39
risch	$-\frac{A}{bx}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(b^3c\_Z^2-A^2c^2-2ABbc-B^2b^2\right)}\_R\ln\left(\left(3\_R^2b^3c-2A^2c^2-4ABbc-2B^2b^2\right)x+\left(b^2Ac+B\,b^3\right)\_R\right)\right)}{2}$	10

input
```
int((B*x^2+A)/(-c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-(-A*c-B*b)/b/(b*c)^(1/2)*arctanh(c*x/(b*c)^(1/2))-A/b/x
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.51

$$\int \frac{A+Bx^2}{bx^2-cx^4}\,dx=\left[\frac{(Bb+Ac)\sqrt{bc}x\log\left(\frac{cx^2+2\sqrt{bc}x+b}{cx^2-b}\right)-2\,Abc}{2\,b^2cx},\right.$$
$$\left.-\frac{(Bb+Ac)\sqrt{-bc}x\arctan\left(\frac{\sqrt{-bc}x}{b}\right)+Abc}{b^2cx}\right]$$

input
```
integrate((B*x^2+A)/(-c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
[1/2*((B*b + A*c)*sqrt(b*c)*x*log((c*x^2 + 2*sqrt(b*c)*x + b)/(c*x^2 - b))
 - 2*A*b*c)/(b^2*c*x), -((B*b + A*c)*sqrt(-b*c)*x*arctan(sqrt(-b*c)*x/b) +
 A*b*c)/(b^2*c*x)]
```

3.52.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(34) = 68$.

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.83

$$\int \frac{A+Bx^2}{bx^2-cx^4}\,dx = -\frac{A}{bx} - \frac{\sqrt{\frac{1}{b^3c}}(Ac+Bb)\log\left(-b^2\sqrt{\frac{1}{b^3c}}+x\right)}{2} + \frac{\sqrt{\frac{1}{b^3c}}(Ac+Bb)\log\left(b^2\sqrt{\frac{1}{b^3c}}+x\right)}{2}$$

input
```
integrate((B*x**2+A)/(-c*x**4+b*x**2),x)
```

output
```
-A/(b*x) - sqrt(1/(b**3*c))*(A*c + B*b)*log(-b**2*sqrt(1/(b**3*c)) + x)/2
+ sqrt(1/(b**3*c))*(A*c + B*b)*log(b**2*sqrt(1/(b**3*c)) + x)/2
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx^2}{bx^2-cx^4}\,dx = -\frac{(Bb+Ac)\log\left(\frac{cx-\sqrt{bc}}{cx+\sqrt{bc}}\right)}{2\sqrt{bc}b} - \frac{A}{bx}$$

input
```
integrate((B*x^2+A)/(-c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-1/2*(B*b + A*c)*log((c*x - sqrt(b*c))/(c*x + sqrt(b*c)))/(sqrt(b*c)*b) -
A/(b*x)
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^2}{bx^2-cx^4}\,dx = -\frac{(Bb+Ac)\arctan\left(\frac{cx}{\sqrt{-bc}}\right)}{\sqrt{-bc}b} - \frac{A}{bx}$$

input
```
integrate((B*x^2+A)/(-c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-(B*b + A*c)*arctan(c*x/sqrt(-b*c))/(sqrt(-b*c)*b) - A/(b*x)
```

3.52.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx^2}{bx^2-cx^4}\,dx=\frac{\operatorname{atanh}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,\left(A\,c+B\,b\right)}{b^{3/2}\,\sqrt{c}}-\frac{A}{b\,x}$$

input
```
int((A + B*x^2)/(b*x^2 - c*x^4),x)
```

output
```
(atanh((c^(1/2)*x)/b^(1/2))*(A*c + B*b))/(b^(3/2)*c^(1/2)) - A/(b*x)
```

3.53 $\int \frac{A+Bx^2}{x(bx^2+cx^4)}\,dx$

3.53.1 Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \frac{A+Bx^2}{x(bx^2+cx^4)}\,dx=-\frac{A}{2bx^2}+\frac{(bB-Ac)\log(x)}{b^2}-\frac{(bB-Ac)\log(b+cx^2)}{2b^2}$$

output
```
-1/2*A/b/x^2+(-A*c+B*b)*ln(x)/b^2-1/2*(-A*c+B*b)*ln(c*x^2+b)/b^2
```

3.53.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x(bx^2+cx^4)}\,dx=-\frac{A}{2bx^2}+\frac{(bB-Ac)\log(x)}{b^2}+\frac{(-bB+Ac)\log(b+cx^2)}{2b^2}$$

input
```
Integrate[(A + B*x^2)/(x*(b*x^2 + c*x^4)),x]
```

output
```
-1/2*A/(b*x^2) + ((b*B - A*c)*Log[x])/b^2 + ((-(b*B) + A*c)*Log[b + c*x^2]
)/(2*b^2)
```

3.53.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{A+Bx^2}{x^3\left(b+cx^2\right)}dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^4\left(cx^2+b\right)}dx^2$$

$$\Big\downarrow \; 86$$

$$\frac{1}{2}\int \left(\frac{A}{bx^4}-\frac{c(bB-Ac)}{b^2\left(cx^2+b\right)}+\frac{bB-Ac}{b^2x^2}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{\log\left(x^2\right)(bB-Ac)}{b^2}-\frac{(bB-Ac)\log\left(b+cx^2\right)}{b^2}-\frac{A}{bx^2}\right)$$

input
```
Int[(A + B*x^2)/(x*(b*x^2 + c*x^4)),x]
```

output
```
(-(A/(b*x^2)) + ((b*B - A*c)*Log[x^2])/b^2 - ((b*B - A*c)*Log[b + c*x^2])/
b^2)/2
```

3.53.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.53.4 Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{A}{2b\,x^{2}}+\frac{(-Ac+Bb)\ln(x)}{b^{2}}+\frac{(Ac-Bb)\ln\left(c\,x^{2}+b\right)}{2b^{2}}$	46
norman	$-\frac{A}{2b\,x^{2}}-\frac{(Ac-Bb)\ln(x)}{b^{2}}+\frac{(Ac-Bb)\ln\left(c\,x^{2}+b\right)}{2b^{2}}$	47
parallelrisch	$-\frac{2A\ln(x)x^{2}c-A\ln\left(c\,x^{2}+b\right)x^{2}c-2B\ln(x)x^{2}b+B\ln\left(c\,x^{2}+b\right)x^{2}b+Ab}{2x^{2}b^{2}}$	60
risch	$-\frac{A}{2b\,x^{2}}-\frac{\ln(x)Ac}{b^{2}}+\frac{\ln(x)B}{b}+\frac{\ln\left(-c\,x^{2}-b\right)Ac}{2b^{2}}-\frac{\ln\left(-c\,x^{2}-b\right)B}{2b}$	62

input
```
int((B*x^2+A)/x/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/b/x^2+(-A*c+B*b)*ln(x)/b^2+1/2*(A*c-B*b)/b^2*ln(c*x^2+b)
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)}\,dx = -\frac{(Bb-Ac)x^2\log\left(cx^2+b\right)-2\,(Bb-Ac)x^2\log\left(x\right)+Ab}{2\,b^2x^2}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
-1/2*((B*b - A*c)*x^2*log(c*x^2 + b) - 2*(B*b - A*c)*x^2*log(x) + A*b)/(b^
2*x^2)
```

3.53.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)}\,dx = -\frac{A}{2bx^2}+\frac{(-Ac+Bb)\log\left(x\right)}{b^2}-\frac{(-Ac+Bb)\log\left(\frac{b}{c}+x^2\right)}{2b^2}$$

input
```
integrate((B*x**2+A)/x/(c*x**4+b*x**2),x)
```

output
```
-A/(2*b*x**2) + (-A*c + B*b)*log(x)/b**2 - (-A*c + B*b)*log(b/c + x**2)/(2
*b**2)
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)}\,dx = -\frac{(Bb-Ac)\log\left(cx^2+b\right)}{2\,b^2}+\frac{(Bb-Ac)\log\left(x^2\right)}{2\,b^2}-\frac{A}{2\,bx^2}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-1/2*(B*b - A*c)*log(c*x^2 + b)/b^2 + 1/2*(B*b - A*c)*log(x^2)/b^2 - 1/2*A
/(b*x^2)
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

$$\int \frac{A+Bx^2}{x(bx^2+cx^4)}\,dx = \frac{(Bb-Ac)\log(x^2)}{2\,b^2} - \frac{(Bbc-Ac^2)\log(|cx^2+b|)}{2\,b^2c} - \frac{Bbx^2-Acx^2+Ab}{2\,b^2x^2}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/2*(B*b - A*c)*log(x^2)/b^2 - 1/2*(B*b*c - A*c^2)*log(abs(c*x^2 + b))/(b^
2*c) - 1/2*(B*b*x^2 - A*c*x^2 + A*b)/(b^2*x^2)
```

3.53.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{x(bx^2+cx^4)}\,dx = \frac{\ln(c\,x^2+b)\,(A\,c-B\,b)}{2\,b^2} - \frac{A}{2\,b\,x^2} - \frac{\ln(x)\,(A\,c-B\,b)}{b^2}$$

input
```
int((A + B*x^2)/(x*(b*x^2 + c*x^4)),x)
```

output
```
(log(b + c*x^2)*(A*c - B*b))/(2*b^2) - A/(2*b*x^2) - (log(x)*(A*c - B*b))/
b^2
```

3.54 $\int \frac{A+Bx^2}{x^2(bx^2+cx^4)} dx$

3.54.1 Optimal result

Integrand size = 24, antiderivative size = 61

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)}\,dx=-\frac{A}{3bx^3}-\frac{bB-Ac}{b^2x}-\frac{\sqrt{c}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}}$$

output
```
-1/3*A/b/x^3+(A*c-B*b)/b^2/x-(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))*c^(1/2)/
b^(5/2)
```

3.54.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)}\,dx=-\frac{A}{3bx^3}+\frac{-bB+Ac}{b^2x}-\frac{\sqrt{c}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}}$$

input
```
Integrate[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)),x]
```

output
```
-1/3*A/(b*x^3) + (-(b*B) + A*c)/(b^2*x) - (Sqrt[c]*(b*B - A*c)*ArcTan[(Sqr
t[c]*x)/Sqrt[b]])/b^(5/2)
```

3.54.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 359, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{A+Bx^2}{x^4\left(b+cx^2\right)}dx$$

$$\Big\downarrow \; 359$$

$$\frac{(bB-Ac)\int \frac{1}{x^2(cx^2+b)}dx}{b}-\frac{A}{3bx^3}$$

$$\Big\downarrow \; 264$$

$$\frac{(bB-Ac)\left(-\frac{c\int \frac{1}{cx^2+b}dx}{b}-\frac{1}{bx}\right)}{b}-\frac{A}{3bx^3}$$

$$\Big\downarrow \; 218$$

$$\frac{(bB-Ac)\left(-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}\right)}{b}-\frac{A}{3bx^3}$$

input
```
Int[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)),x]
```

output
```
-1/3*A/(b*x^3) + ((b*B - A*c)*(-(1/(b*x)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/b^(3/2)))/b
```

3.54.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.54.4 Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result
default	$-\frac{A}{3b\,x^{3}}-\frac{-Ac+Bb}{x\,b^{2}}+\frac{c(Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{b^{2}\sqrt{bc}}$
risch	$\frac{\frac{(Ac-Bb)x^{2}}{b^{2}}-\frac{A}{3b}}{x^{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^{5}\_Z^{2}+A^{2}c^{3}-2ABb\,c^{2}+B^{2}b^{2}c\right)}\_R\ln\left(\left(3\_R^{2}b^{5}+2A^{2}c^{3}-4ABb\,c^{2}+2B^{2}b^{2}c\right)x+\left(-A\,b^{3}c+B\,b^{4}\right.\right.\right.}{2}$

input
```
int((B*x^2+A)/x^2/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*A/b/x^3-(-A*c+B*b)/x/b^2+c*(A*c-B*b)/b^2/(b*c)^(1/2)*arctan(c*x/(b*c)
^(1/2))
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.21

$$\int \frac{A+Bx^2}{x^2(bx^2+cx^4)}\,dx = \left[-\frac{3\,(Bb-Ac)x^3\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)+6\,(Bb-Ac)x^2+2\,Ab}{6\,b^2x^3},\; -\frac{3\,(Bb-Ac)x^3\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)+3\,(Bb-Ac)x^2+Ab}{3\,b^2x^3}\right]$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
[-1/6*(3*(B*b - A*c)*x^3*sqrt(-c/b)*log((c*x^2 + 2*b*x*sqrt(-c/b) - b)/(c*
x^2 + b)) + 6*(B*b - A*c)*x^2 + 2*A*b)/(b^2*x^3), -1/3*(3*(B*b - A*c)*x^3*
sqrt(c/b)*arctan(x*sqrt(c/b)) + 3*(B*b - A*c)*x^2 + A*b)/(b^2*x^3)]
```

3.54.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(49) = 98$.

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.11

$$\int \frac{A+Bx^2}{x^2(bx^2+cx^4)}\,dx = \frac{\sqrt{-\frac{c}{b^5}}(-Ac+Bb)\log\left(-\frac{b^3\sqrt{-\frac{c}{b^5}}(-Ac+Bb)}{-Ac^2+Bbc}+x\right)}{2} - \frac{\sqrt{-\frac{c}{b^5}}(-Ac+Bb)\log\left(\frac{b^3\sqrt{-\frac{c}{b^5}}(-Ac+Bb)}{-Ac^2+Bbc}+x\right)}{2} + \frac{-Ab+x^2\cdot(3Ac-3Bb)}{3b^2x^3}$$

input
```
integrate((B*x**2+A)/x**2/(c*x**4+b*x**2),x)
```

output
```
sqrt(-c/b**5)*(-A*c + B*b)*log(-b**3*sqrt(-c/b**5)*(-A*c + B*b)/(-A*c**2 +
 B*b*c) + x)/2 - sqrt(-c/b**5)*(-A*c + B*b)*log(b**3*sqrt(-c/b**5)*(-A*c +
 B*b)/(-A*c**2 + B*b*c) + x)/2 + (-A*b + x**2*(3*A*c - 3*B*b))/(3*b**2*x**
3)
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)}\,dx = -\frac{\left(Bbc-Ac^2\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b^2} - \frac{3\,(Bb-Ac)x^2+Ab}{3\,b^2x^3}$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-(B*b*c - A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) - 1/3*(3*(B*b - A*c
)*x^2 + A*b)/(b^2*x^3)
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)}\,dx = -\frac{\left(Bbc-Ac^2\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b^2} - \frac{3\,Bbx^2-3\,Acx^2+Ab}{3\,b^2x^3}$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-(B*b*c - A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) - 1/3*(3*B*b*x^2 -
3*A*c*x^2 + A*b)/(b^2*x^3)
```

3.54.9 Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)}\,dx = \frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,\left(A\,c-B\,b\right)}{b^{5/2}} - \frac{\frac{A}{3\,b}-\frac{x^2\,\left(A\,c-B\,b\right)}{b^2}}{x^3}$$

input
```
int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)),x)
```

output
```
(c^(1/2)*atan((c^(1/2)*x)/b^(1/2))*(A*c - B*b))/b^(5/2) - (A/(3*b) - (x^2*
(A*c - B*b))/b^2)/x^3
```

3.55 $\int \frac{A+Bx^2}{x^3(bx^2+cx^4)}\,dx$

3.55.1 Optimal result

Integrand size = 24, antiderivative size = 70

$$\int \frac{A+Bx^2}{x^3(bx^2+cx^4)}\,dx=-\frac{A}{4bx^4}-\frac{bB-Ac}{2b^2x^2}-\frac{c(bB-Ac)\log(x)}{b^3}+\frac{c(bB-Ac)\log(b+cx^2)}{2b^3}$$

output
```
-1/4*A/b/x^4+1/2*(A*c-B*b)/b^2/x^2-c*(-A*c+B*b)*ln(x)/b^3+1/2*c*(-A*c+B*b)
*ln(c*x^2+b)/b^3
```

3.55.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x^3(bx^2+cx^4)}\,dx =\frac{-b(Ab+2bBx^2-2Acx^2)+4c(-bB+Ac)x^4\log(x)+2c(bB-Ac)x^4\log(b+cx^2)}{4b^3x^4}$$

input
```
Integrate[(A + B*x^2)/(x^3*(b*x^2 + c*x^4)),x]
```

output
```
(-(b*(A*b + 2*b*B*x^2 - 2*A*c*x^2)) + 4*c*(-(b*B) + A*c)*x^4*Log[x] + 2*c*
(b*B - A*c)*x^4*Log[b + c*x^2])/(4*b^3*x^4)
```

3.55.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^3 \left(bx^2+cx^4\right)} \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{A+Bx^2}{x^5 \left(b+cx^2\right)}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2} \int \frac{Bx^2+A}{x^6 \left(cx^2+b\right)}dx^2$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{2} \int \left(\frac{(bB-Ac)c^2}{b^3 \left(cx^2+b\right)}-\frac{(bB-Ac)c}{b^3x^2}+\frac{bB-Ac}{b^2x^4}+\frac{A}{bx^6}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(-\frac{c \log \left(x^2\right) (bB-Ac)}{b^3}+\frac{c(bB-Ac) \log \left(b+cx^2\right)}{b^3}-\frac{bB-Ac}{b^2x^2}-\frac{A}{2bx^4}\right)$$

input
```
Int[(A + B*x^2)/(x^3*(b*x^2 + c*x^4)),x]
```

output
```
(-1/2*A/(b*x^4) - (b*B - A*c)/(b^2*x^2) - (c*(b*B - A*c)*Log[x^2])/b^3 + (
c*(b*B - A*c)*Log[b + c*x^2])/b^3)/2
```

3.55.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.55.4 Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{A}{4b\,x^{4}}-\frac{-Ac+Bb}{2x^{2}b^{2}}+\frac{c(Ac-Bb)\ln(x)}{b^{3}}-\frac{c(Ac-Bb)\ln(c\,x^{2}+b)}{2b^{3}}$	64
norman	$\frac{-\frac{A}{4b}+\frac{(Ac-Bb)x^{2}}{2b^{2}}}{x^{4}}+\frac{c(Ac-Bb)\ln(x)}{b^{3}}-\frac{c(Ac-Bb)\ln(c\,x^{2}+b)}{2b^{3}}$	66
risch	$\frac{-\frac{A}{4b}+\frac{(Ac-Bb)x^{2}}{2b^{2}}}{x^{4}}+\frac{c^{2}\ln(x)A}{b^{3}}-\frac{c\ln(x)B}{b^{2}}-\frac{c^{2}\ln(c\,x^{2}+b)A}{2b^{3}}+\frac{c\ln(c\,x^{2}+b)B}{2b^{2}}$	80
parallelrisch	$\frac{4A\ln(x)x^{4}c^{2}-2A\ln(c\,x^{2}+b)x^{4}c^{2}-4B\ln(x)x^{4}bc+2B\ln(c\,x^{2}+b)x^{4}bc+2Abc\,x^{2}-2b^{2}B\,x^{2}-b^{2}A}{4b^{3}x^{4}}$	87

input
```
int((B*x^2+A)/x^3/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*A/b/x^4-1/2*(-A*c+B*b)/x^2/b^2+c*(A*c-B*b)/b^3*ln(x)-1/2*c*(A*c-B*b)/
b^3*ln(c*x^2+b)
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)}\,dx = \frac{2\left(Bbc-Ac^2\right)x^4\log\left(cx^2+b\right)-4\left(Bbc-Ac^2\right)x^4\log\left(x\right)-Ab^2-2\left(Bb^2-Abc\right)x^2}{4\,b^3x^4}$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/4*(2*(B*b*c - A*c^2)*x^4*log(c*x^2 + b) - 4*(B*b*c - A*c^2)*x^4*log(x) -
 A*b^2 - 2*(B*b^2 - A*b*c)*x^2)/(b^3*x^4)
```

3.55.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)}\,dx = \frac{-Ab+x^2\cdot(2Ac-2Bb)}{4b^2x^4}-\frac{c(-Ac+Bb)\log\left(x\right)}{b^3}+\frac{c(-Ac+Bb)\log\left(\frac{b}{c}+x^2\right)}{2b^3}$$

input
```
integrate((B*x**2+A)/x**3/(c*x**4+b*x**2),x)
```

output
```
(-A*b + x**2*(2*A*c - 2*B*b))/(4*b**2*x**4) - c*(-A*c + B*b)*log(x)/b**3 +
 c*(-A*c + B*b)*log(b/c + x**2)/(2*b**3)
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)}\,dx = \frac{\left(Bbc-Ac^2\right)\log\left(cx^2+b\right)}{2\,b^3}-\frac{\left(Bbc-Ac^2\right)\log\left(x^2\right)}{2\,b^3}-\frac{2\left(Bb-Ac\right)x^2+Ab}{4\,b^2x^4}$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/2*(B*b*c - A*c^2)*log(c*x^2 + b)/b^3 - 1/2*(B*b*c - A*c^2)*log(x^2)/b^3
- 1/4*(2*(B*b - A*c)*x^2 + A*b)/(b^2*x^4)
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.43

$$\int \frac{A+Bx^2}{x^3(bx^2+cx^4)}\,dx = -\frac{(Bbc-Ac^2)\log(x^2)}{2\,b^3} + \frac{(Bbc^2-Ac^3)\log(|cx^2+b|)}{2\,b^3c} + \frac{3\,Bbcx^4-3\,Ac^2x^4-2\,Bb^2x^2+2\,Abcx^2-Ab^2}{4\,b^3x^4}$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-1/2*(B*b*c - A*c^2)*log(x^2)/b^3 + 1/2*(B*b*c^2 - A*c^3)*log(abs(c*x^2 +
b))/(b^3*c) + 1/4*(3*B*b*c*x^4 - 3*A*c^2*x^4 - 2*B*b^2*x^2 + 2*A*b*c*x^2 -
 A*b^2)/(b^3*x^4)
```

3.55.9 Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x^3(bx^2+cx^4)}\,dx = \frac{\ln(x)\,(A\,c^2-B\,b\,c)}{b^3} - \frac{\ln(c\,x^2+b)\,(A\,c^2-B\,b\,c)}{2\,b^3} - \frac{\frac{A}{4\,b}-\frac{x^2\,(A\,c-B\,b)}{2\,b^2}}{x^4}$$

input
```
int((A + B*x^2)/(x^3*(b*x^2 + c*x^4)),x)
```

output
```
(log(x)*(A*c^2 - B*b*c))/b^3 - (log(b + c*x^2)*(A*c^2 - B*b*c))/(2*b^3) -
(A/(4*b) - (x^2*(A*c - B*b))/(2*b^2))/x^4
```

3.56 $\int \frac{A+Bx^2}{x^4(bx^2+cx^4)}\,dx$

3.56.1 Optimal result

Integrand size = 24, antiderivative size = 78

$$\int \frac{A+Bx^2}{x^4\left(bx^2+cx^4\right)}\,dx=-\frac{A}{5bx^5}-\frac{bB-Ac}{3b^2x^3}+\frac{c(bB-Ac)}{b^3x}+\frac{c^{3/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{7/2}}$$

output
```
-1/5*A/b/x^5+1/3*(A*c-B*b)/b^2/x^3+c*(-A*c+B*b)/b^3/x+c^(3/2)*(-A*c+B*b)*a
rctan(x*c^(1/2)/b^(1/2))/b^(7/2)
```

3.56.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x^4\left(bx^2+cx^4\right)}\,dx=-\frac{A}{5bx^5}+\frac{-bB+Ac}{3b^2x^3}+\frac{c(bB-Ac)}{b^3x}+\frac{c^{3/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{7/2}}$$

input
```
Integrate[(A + B*x^2)/(x^4*(b*x^2 + c*x^4)),x]
```

output
```
-1/5*A/(b*x^5) + (-(b*B) + A*c)/(3*b^2*x^3) + (c*(b*B - A*c))/(b^3*x) + (c
^(3/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(7/2)
```

3.56.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {9, 359, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^4\left(bx^2+cx^4\right)}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{A+Bx^2}{x^6\left(b+cx^2\right)}dx$$

$$\Big\downarrow \; 359$$

$$\frac{(bB-Ac)\int \frac{1}{x^4(cx^2+b)}dx}{b}-\frac{A}{5bx^5}$$

$$\Big\downarrow \; 264$$

$$\frac{(bB-Ac)\left(-\frac{c\int \frac{1}{x^2\left(cx^2+b\right)}dx}{b}-\frac{1}{3bx^3}\right)}{b}-\frac{A}{5bx^5}$$

$$\Big\downarrow \; 264$$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{c\int \frac{1}{cx^2+b}dx}{b}-\frac{1}{bx}\right)}{b}-\frac{1}{3bx^3}\right)}{b}-\frac{A}{5bx^5}$$

$$\Big\downarrow \; 218$$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}\right)}{b}-\frac{1}{3bx^3}\right)}{b}-\frac{A}{5bx^5}$$

input
```
Int[(A + B*x^2)/(x^4*(b*x^2 + c*x^4)),x]
```

output
```
-1/5*A/(b*x^5) + ((b*B - A*c)*(-1/3*1/(b*x^3) - (c*(-(1/(b*x)) - (Sqrt[c]*
ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2)))/b))/b
```

3.56.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.56.4 Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

method	result
default	$-\frac{A}{5b\,x^{5}}-\frac{-Ac+Bb}{3x^{3}b^{2}}-\frac{c(Ac-Bb)}{b^{3}x}-\frac{c^{2}(Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{b^{3}\sqrt{bc}}$
risch	$\frac{-\frac{c(Ac-Bb)x^{4}}{b^{3}}+\frac{(Ac-Bb)x^{2}}{3b^{2}}-\frac{A}{5b}}{x^{5}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^{7}\_Z^{2}+A^{2}c^{5}-2ABb\,c^{4}+B^{2}b^{2}c^{3}\right)}\_R\ln\left(\left(3\_R^{2}b^{7}+2A^{2}c^{5}-4ABb\,c^{4}+2B^{2}b^{2}c^{3}\right)\right.\right.}{2}$

input
```
int((B*x^2+A)/x^4/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*A/b/x^5-1/3*(-A*c+B*b)/x^3/b^2-c*(A*c-B*b)/b^3/x-c^2*(A*c-B*b)/b^3/(b
*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.36

$$\int \frac{A+Bx^2}{x^4\,(bx^2+cx^4)}\,dx = \left[-\frac{15\,(Bbc-Ac^2)x^5\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)-30\,(Bbc-Ac^2)x^4+6\,Ab^2+10\,(Bb^2-Abc)x^2}{30\,b^3x^5}, \frac{15\,(Bbc\right.$$

input
```
integrate((B*x^2+A)/x^4/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
[-1/30*(15*(B*b*c - A*c^2)*x^5*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) -
b)/(c*x^2 + b)) - 30*(B*b*c - A*c^2)*x^4 + 6*A*b^2 + 10*(B*b^2 - A*b*c)*x^
2)/(b^3*x^5), 1/15*(15*(B*b*c - A*c^2)*x^5*sqrt(c/b)*arctan(x*sqrt(c/b)) +
 15*(B*b*c - A*c^2)*x^4 - 3*A*b^2 - 5*(B*b^2 - A*b*c)*x^2)/(b^3*x^5)]
```

3.56.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(68) = 136.

Time = 0.25 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.09

$$\int \frac{A+Bx^2}{x^4\,(bx^2+cx^4)}\,dx = -\frac{\sqrt{-\frac{c^3}{b^7}}(-Ac+Bb)\log\left(-\frac{b^4\sqrt{-\frac{c^3}{b^7}}(-Ac+Bb)}{-Ac^3+Bbc^2}+x\right)}{2} + \frac{\sqrt{-\frac{c^3}{b^7}}(-Ac+Bb)\log\left(\frac{b^4\sqrt{-\frac{c^3}{b^7}}(-Ac+Bb)}{-Ac^3+Bbc^2}+x\right)}{2} + \frac{-3Ab^2+x^4(-15Ac^2+15Bbc)+x^2\cdot(5Abc-5Bb^2)}{15b^3x^5}$$

input
```
integrate((B*x**2+A)/x**4/(c*x**4+b*x**2),x)
```

output
```
-sqrt(-c**3/b**7)*(-A*c + B*b)*log(-b**4*sqrt(-c**3/b**7)*(-A*c + B*b)/(-A
*c**3 + B*b*c**2) + x)/2 + sqrt(-c**3/b**7)*(-A*c + B*b)*log(b**4*sqrt(-c*
*3/b**7)*(-A*c + B*b)/(-A*c**3 + B*b*c**2) + x)/2 + (-3*A*b**2 + x**4*(-15
*A*c**2 + 15*B*b*c) + x**2*(5*A*b*c - 5*B*b**2))/(15*b**3*x**5)
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2}{x^4\,(bx^2+cx^4)}\,dx = \frac{(Bbc^2-Ac^3)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b^3} + \frac{15\,(Bbc-Ac^2)x^4-3\,Ab^2-5\,(Bb^2-Abc)x^2}{15\,b^3x^5}$$

input
```
integrate((B*x^2+A)/x^4/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
(B*b*c^2 - A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) + 1/15*(15*(B*b*c
- A*c^2)*x^4 - 3*A*b^2 - 5*(B*b^2 - A*b*c)*x^2)/(b^3*x^5)
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2}{x^4\,(bx^2+cx^4)}\,dx = \frac{(Bbc^2-Ac^3)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b^3} + \frac{15\,Bbcx^4-15\,Ac^2x^4-5\,Bb^2x^2+5\,Abcx^2-3\,Ab^2}{15\,b^3x^5}$$

input
```
integrate((B*x^2+A)/x^4/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
(B*b*c^2 - A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) + 1/15*(15*B*b*c*x
^4 - 15*A*c^2*x^4 - 5*B*b^2*x^2 + 5*A*b*c*x^2 - 3*A*b^2)/(b^3*x^5)
```

3.56.9 Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^2}{x^4\,(bx^2+cx^4)}\,dx = -\frac{\frac{A}{5\,b}-\frac{x^2\,(A\,c-B\,b)}{3\,b^2}+\frac{c\,x^4\,(A\,c-B\,b)}{b^3}}{x^5} - \frac{c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(A\,c-B\,b)}{b^{7/2}}$$

input
```
int((A + B*x^2)/(x^4*(b*x^2 + c*x^4)),x)
```

output
```
- (A/(5*b) - (x^2*(A*c - B*b))/(3*b^2) + (c*x^4*(A*c - B*b))/b^3)/x^5 - (c
^(3/2)*atan((c^(1/2)*x)/b^(1/2))*(A*c - B*b))/b^(7/2)
```

3.56. $\int \frac{A+Bx^2}{x^4(bx^2+cx^4)}\,dx$

3.57 $\int \frac{A+Bx^2}{x^5(bx^2+cx^4)}\,dx$

3.57.1 Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)}\,dx=-\frac{A}{6bx^6}-\frac{bB-Ac}{4b^2x^4}+\frac{c(bB-Ac)}{2b^3x^2}+\frac{c^2(bB-Ac)\log(x)}{b^4}-\frac{c^2(bB-Ac)\log\left(b+cx^2\right)}{2b^4}$$

output
```
-1/6*A/b/x^6+1/4*(A*c-B*b)/b^2/x^4+1/2*c*(-A*c+B*b)/b^3/x^2+c^2*(-A*c+B*b)
*ln(x)/b^4-1/2*c^2*(-A*c+B*b)*ln(c*x^2+b)/b^4
```

3.57.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)}\,dx=-\frac{A}{6bx^6}+\frac{-bB+Ac}{4b^2x^4}+\frac{c(bB-Ac)}{2b^3x^2}+\frac{\left(bBc^2-Ac^3\right)\log(x)}{b^4}+\frac{\left(-bBc^2+Ac^3\right)\log\left(b+cx^2\right)}{2b^4}$$

input
```
Integrate[(A + B*x^2)/(x^5*(b*x^2 + c*x^4)),x]
```

output
```
-1/6*A/(b*x^6) + (-(b*B) + A*c)/(4*b^2*x^4) + (c*(b*B - A*c))/(2*b^3*x^2)
+ ((b*B*c^2 - A*c^3)*Log[x])/b^4 + ((-(b*B*c^2) + A*c^3)*Log[b + c*x^2])/(
2*b^4)
```

3.57.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^5 \left(bx^2+cx^4\right)} \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{A+Bx^2}{x^7 \left(b+cx^2\right)}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2} \int \frac{Bx^2+A}{x^8 \left(cx^2+b\right)}dx^2$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{2} \int \left(-\frac{(bB-Ac)c^3}{b^4 \left(cx^2+b\right)}+\frac{(bB-Ac)c^2}{b^4x^2}-\frac{(bB-Ac)c}{b^3x^4}+\frac{bB-Ac}{b^2x^6}+\frac{A}{bx^8}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(\frac{c^2 \log \left(x^2\right) (bB-Ac)}{b^4}-\frac{c^2 (bB-Ac) \log \left(b+cx^2\right)}{b^4}+\frac{c(bB-Ac)}{b^3x^2}-\frac{bB-Ac}{2b^2x^4}-\frac{A}{3bx^6}\right)$$

input
```
Int[(A + B*x^2)/(x^5*(b*x^2 + c*x^4)),x]
```

output
```
(-1/3*A/(b*x^6) - (b*B - A*c)/(2*b^2*x^4) + (c*(b*B - A*c))/(b^3*x^2) + (c
^2*(b*B - A*c)*Log[x^2])/b^4 - (c^2*(b*B - A*c)*Log[b + c*x^2])/b^4)/2
```

3.57.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.57.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.93

method	result
default	$-\frac{A}{6b\,x^{6}}-\frac{-Ac+Bb}{4x^{4}b^{2}}-\frac{c(Ac-Bb)}{2b^{3}x^{2}}-\frac{c^{2}(Ac-Bb)\ln(x)}{b^{4}}+\frac{c^{2}(Ac-Bb)\ln\left(c\,x^{2}+b\right)}{2b^{4}}$
norman	$\frac{-\frac{A}{6b}+\frac{(Ac-Bb)x^{2}}{4b^{2}}-\frac{c(Ac-Bb)x^{4}}{2b^{3}}}{x^{6}}-\frac{c^{2}(Ac-Bb)\ln(x)}{b^{4}}+\frac{c^{2}(Ac-Bb)\ln\left(c\,x^{2}+b\right)}{2b^{4}}$
risch	$\frac{-\frac{A}{6b}+\frac{(Ac-Bb)x^{2}}{4b^{2}}-\frac{c(Ac-Bb)x^{4}}{2b^{3}}}{x^{6}}-\frac{c^{3}\ln(x)A}{b^{4}}+\frac{c^{2}\ln(x)B}{b^{3}}+\frac{c^{3}\ln\left(-c\,x^{2}-b\right)A}{2b^{4}}-\frac{c^{2}\ln\left(-c\,x^{2}-b\right)B}{2b^{3}}$
parallelrisch	$-\frac{12A\ln(x)x^{6}c^{3}-6A\ln\left(c\,x^{2}+b\right)x^{6}c^{3}-12B\ln(x)x^{6}b\,c^{2}+6B\ln\left(c\,x^{2}+b\right)x^{6}b\,c^{2}+6Ab\,c^{2}x^{4}-6x^{4}B\,b^{2}c-3A\,b^{2}c\,x^{2}+3b^{3}B\,x^{2}+2b^{3}}{12b^{4}x^{6}}$

input
```
int((B*x^2+A)/x^5/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*A/b/x^6-1/4*(-A*c+B*b)/x^4/b^2-1/2*c*(A*c-B*b)/b^3/x^2-c^2*(A*c-B*b)/
b^4*ln(x)+1/2*c^2*(A*c-B*b)/b^4*ln(c*x^2+b)
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^2}{x^5\,(bx^2+cx^4)}\,dx = -\frac{6\,(Bbc^2-Ac^3)x^6\log\left(cx^2+b\right)-12\,(Bbc^2-Ac^3)x^6\log\left(x\right)-6\,(Bb^2c-Abc^2)x^4+2\,Ab^3+3\,(Bb^3-A}{12\,b^4x^6}$$

input
```
integrate((B*x^2+A)/x^5/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
-1/12*(6*(B*b*c^2 - A*c^3)*x^6*log(c*x^2 + b) - 12*(B*b*c^2 - A*c^3)*x^6*l
og(x) - 6*(B*b^2*c - A*b*c^2)*x^4 + 2*A*b^3 + 3*(B*b^3 - A*b^2*c)*x^2)/(b^
4*x^6)
```

3.57.6 Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2}{x^5\,(bx^2+cx^4)}\,dx = \frac{-2Ab^2+x^4(-6Ac^2+6Bbc)+x^2\cdot(3Abc-3Bb^2)}{12b^3x^6} + \frac{c^2(-Ac+Bb)\log\left(x\right)}{b^4} - \frac{c^2(-Ac+Bb)\log\left(\frac{b}{c}+x^2\right)}{2b^4}$$

input
```
integrate((B*x**2+A)/x**5/(c*x**4+b*x**2),x)
```

output
```
(-2*A*b**2 + x**4*(-6*A*c**2 + 6*B*b*c) + x**2*(3*A*b*c - 3*B*b**2))/(12*b
**3*x**6) + c**2*(-A*c + B*b)*log(x)/b**4 - c**2*(-A*c + B*b)*log(b/c + x*
*2)/(2*b**4)
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2}{x^5\,(bx^2+cx^4)}\,dx = -\frac{(Bbc^2-Ac^3)\log\left(cx^2+b\right)}{2\,b^4} + \frac{(Bbc^2-Ac^3)\log\left(x^2\right)}{2\,b^4} + \frac{6\,(Bbc-Ac^2)x^4-2\,Ab^2-3\,(Bb^2-Abc)x^2}{12\,b^3x^6}$$

input
```
integrate((B*x^2+A)/x^5/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-1/2*(B*b*c^2 - A*c^3)*log(c*x^2 + b)/b^4 + 1/2*(B*b*c^2 - A*c^3)*log(x^2)
/b^4 + 1/12*(6*(B*b*c - A*c^2)*x^4 - 2*A*b^2 - 3*(B*b^2 - A*b*c)*x^2)/(b^3
*x^6)
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx^2}{x^5(bx^2+cx^4)}\,dx = \frac{(Bbc^2-Ac^3)\log(x^2)}{2\,b^4} - \frac{(Bbc^3-Ac^4)\log(|cx^2+b|)}{2\,b^4c} - \frac{11\,Bbc^2x^6-11\,Ac^3x^6-6\,Bb^2cx^4+6\,Abc^2x^4+3\,Bb^3x^2-3\,Ab^2cx^2+2\,Ab^3}{12\,b^4x^6}$$

input
```
integrate((B*x^2+A)/x^5/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/2*(B*b*c^2 - A*c^3)*log(x^2)/b^4 - 1/2*(B*b*c^3 - A*c^4)*log(abs(c*x^2 +
 b))/(b^4*c) - 1/12*(11*B*b*c^2*x^6 - 11*A*c^3*x^6 - 6*B*b^2*c*x^4 + 6*A*b
*c^2*x^4 + 3*B*b^3*x^2 - 3*A*b^2*c*x^2 + 2*A*b^3)/(b^4*x^6)
```

3.57.9 Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x^5(bx^2+cx^4)}\,dx = \frac{\ln(c\,x^2+b)\,(A\,c^3-B\,b\,c^2)}{2\,b^4} - \frac{\frac{A}{6\,b}-\frac{x^2\,(A\,c-B\,b)}{4\,b^2}+\frac{c\,x^4\,(A\,c-B\,b)}{2\,b^3}}{x^6} - \frac{\ln(x)\,(A\,c^3-B\,b\,c^2)}{b^4}$$

input
```
int((A + B*x^2)/(x^5*(b*x^2 + c*x^4)),x)
```

output
```
(log(b + c*x^2)*(A*c^3 - B*b*c^2))/(2*b^4) - (A/(6*b) - (x^2*(A*c - B*b))/
(4*b^2) + (c*x^4*(A*c - B*b))/(2*b^3))/x^6 - (log(x)*(A*c^3 - B*b*c^2))/b^
4
```

3.58 $\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.58.1 Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{b^2(4bB-3Ac)x}{c^5}+\frac{b(3bB-2Ac)x^3}{3c^4}-\frac{(2bB-Ac)x^5}{5c^3}+\frac{Bx^7}{7c^2}-\frac{b^3(bB-Ac)x}{2c^5\,(b+cx^2)}+\frac{b^{5/2}(9bB-7Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{11/2}}$$

output
```
-b^2*(-3*A*c+4*B*b)*x/c^5+1/3*b*(-2*A*c+3*B*b)*x^3/c^4-1/5*(-A*c+2*B*b)*x^
5/c^3+1/7*B*x^7/c^2-1/2*b^3*(-A*c+B*b)*x/c^5/(c*x^2+b)+1/2*b^(5/2)*(-7*A*c
+9*B*b)*arctan(x*c^(1/2)/b^(1/2))/c^(11/2)
```

3.58.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.01

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{b^2(4bB-3Ac)x}{c^5}+\frac{b(3bB-2Ac)x^3}{3c^4}+\frac{(-2bB+Ac)x^5}{5c^3}+\frac{Bx^7}{7c^2}+\frac{(-b^4B+Ab^3c)\,x}{2c^5\,(b+cx^2)}+\frac{b^{5/2}(9bB-7Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{11/2}}$$

input
```
Integrate[(x^12*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
-((b^2*(4*b*B - 3*A*c)*x)/c^5) + (b*(3*b*B - 2*A*c)*x^3)/(3*c^4) + ((-2*b*
B + A*c)*x^5)/(5*c^3) + (B*x^7)/(7*c^2) + ((-(b^4*B) + A*b^3*c)*x)/(2*c^5*
(b + c*x^2)) + (b^(5/2)*(9*b*B - 7*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^
(11/2))
```

3.58.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {9, 360, 25, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{x^8(A+Bx^2)}{(b+cx^2)^2}dx$$

$$\Big\downarrow \text{360}$$

$$-\frac{\int -\frac{2Bc^4x^8-2c^3(bB-Ac)x^6+2bc^2(bB-Ac)x^4-2b^2c(bB-Ac)x^2+b^3(bB-Ac)}{cx^2+b}dx}{2c^5}-\frac{b^3x(bB-Ac)}{2c^5(b+cx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2Bc^4x^8-2c^3(bB-Ac)x^6+2bc^2(bB-Ac)x^4-2b^2c(bB-Ac)x^2+b^3(bB-Ac)}{cx^2+b}dx}{2c^5}-\frac{b^3x(bB-Ac)}{2c^5(b+cx^2)}$$

$$\Big\downarrow \text{2341}$$

$$\frac{\int \left(2Bc^3x^6-2c^2(2bB-Ac)x^4+2bc(3bB-2Ac)x^2-2b^2(4bB-3Ac)+\frac{9b^4B-7Ab^3c}{cx^2+b}\right)dx}{2c^5}-\frac{b^3x(bB-Ac)}{2c^5(b+cx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^{5/2}(9bB-7Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{c}}-2b^2x(4bB-3Ac)-\frac{2}{5}c^2x^5(2bB-Ac)+\frac{2}{3}bcx^3(3bB-2Ac)+\frac{2}{7}Bc^3x^7}{2c^5}-\frac{b^3x(bB-Ac)}{2c^5(b+cx^2)}$$

input
```
Int[(x^12*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(b^3*(b*B - A*c)*x)/(c^5*(b + c*x^2)) + (-2*b^2*(4*b*B - 3*A*c)*x + (
2*b*c*(3*b*B - 2*A*c)*x^3)/3 - (2*c^2*(2*b*B - A*c)*x^5)/5 + (2*B*c^3*x^7)
/7 + (b^(5/2)*(9*b*B - 7*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/Sqrt[c])/(2*c^5
)
```

3.58.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.58.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

method	result
default	$\frac{\frac{1}{7}Bc^3x^7+\frac{1}{5}Ac^3x^5-\frac{2}{5}Bbc^2x^5-\frac{2}{3}Abc^2x^3+Bb^2cx^3+3Ab^2cx-4b^3Bx}{c^5}-\frac{b^3\left(\frac{\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)x}{cx^2+b}+\frac{(7Ac-9Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{c^5}$
risch	$\frac{Bx^7}{7c^2}+\frac{Ax^5}{5c^2}-\frac{2Bbx^5}{5c^3}-\frac{2Abx^3}{3c^3}+\frac{Bb^2x^3}{c^4}+\frac{3Ab^2x}{c^4}-\frac{4b^3Bx}{c^5}+\frac{\left(\frac{1}{2}Ab^3c-\frac{1}{2}Bb^4\right)x}{c^5(cx^2+b)}+\frac{7\sqrt{-bc}\,b^2\ln(-\sqrt{-bc}\,x-b)A}{4c^5}-$

input
```
int(x^12*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(1/7*B*c^3*x^7+1/5*A*c^3*x^5-2/5*B*b*c^2*x^5-2/3*A*b*c^2*x^3+B*b^2*c
*x^3+3*A*b^2*c*x-4*b^3*B*x)-b^3/c^5*((-1/2*A*c+1/2*B*b)*x/(c*x^2+b)+1/2*(7
*A*c-9*B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.63

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$
$$=\left[\frac{60\,Bc^4x^9-12\,(9\,Bbc^3-7\,Ac^4)x^7+28\,(9\,Bb^2c^2-7\,Abc^3)x^5-140\,(9\,Bb^3c-7\,Ab^2c^2)x^3-105\,(9\,Bb^4-}{420\,(c^6x^2+bc^5)}\right.$$

input
```
integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
[1/420*(60*B*c^4*x^9 - 12*(9*B*b*c^3 - 7*A*c^4)*x^7 + 28*(9*B*b^2*c^2 - 7*
A*b*c^3)*x^5 - 140*(9*B*b^3*c - 7*A*b^2*c^2)*x^3 - 105*(9*B*b^4 - 7*A*b^3*
c + (9*B*b^3*c - 7*A*b^2*c^2)*x^2)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c
) - b)/(c*x^2 + b)) - 210*(9*B*b^4 - 7*A*b^3*c)*x)/(c^6*x^2 + b*c^5), 1/21
0*(30*B*c^4*x^9 - 6*(9*B*b*c^3 - 7*A*c^4)*x^7 + 14*(9*B*b^2*c^2 - 7*A*b*c^
3)*x^5 - 70*(9*B*b^3*c - 7*A*b^2*c^2)*x^3 + 105*(9*B*b^4 - 7*A*b^3*c + (9*
B*b^3*c - 7*A*b^2*c^2)*x^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - 105*(9*B*b
^4 - 7*A*b^3*c)*x)/(c^6*x^2 + b*c^5)]
```

3.58.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.79

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bx^7}{7c^2} + x^5\left(\frac{A}{5c^2} - \frac{2Bb}{5c^3}\right) + x^3\left(-\frac{2Ab}{3c^3} + \frac{Bb^2}{c^4}\right) + x\left(\frac{3Ab^2}{c^4} - \frac{4Bb^3}{c^5}\right) + \frac{x(Ab^3c - Bb^4)}{2bc^5 + 2c^6x^2} - \frac{\sqrt{-\frac{b^5}{c^{11}}}(-7Ac+9Bb)\log\left(-\frac{c^5\sqrt{-\frac{b^5}{c^{11}}}(-7Ac+9Bb)}{-7Ab^2c+9Bb^3} + x\right)}{4} + \frac{\sqrt{-\frac{b^5}{c^{11}}}(-7Ac+9Bb)\log\left(\frac{c^5\sqrt{-\frac{b^5}{c^{11}}}(-7Ac+9Bb)}{-7Ab^2c+9Bb^3} + x\right)}{4}$$

input
```
integrate(x**12*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
B*x**7/(7*c**2) + x**5*(A/(5*c**2) - 2*B*b/(5*c**3)) + x**3*(-2*A*b/(3*c**
3) + B*b**2/c**4) + x*(3*A*b**2/c**4 - 4*B*b**3/c**5) + x*(A*b**3*c - B*b*
*4)/(2*b*c**5 + 2*c**6*x**2) - sqrt(-b**5/c**11)*(-7*A*c + 9*B*b)*log(-c**
5*sqrt(-b**5/c**11)*(-7*A*c + 9*B*b)/(-7*A*b**2*c + 9*B*b**3) + x)/4 + sqr
t(-b**5/c**11)*(-7*A*c + 9*B*b)*log(c**5*sqrt(-b**5/c**11)*(-7*A*c + 9*B*b
)/(-7*A*b**2*c + 9*B*b**3) + x)/4
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.02

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(Bb^4 - Ab^3c)x}{2\,(c^6x^2 + bc^5)} + \frac{(9\,Bb^4 - 7\,Ab^3c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^5} + \frac{15\,Bc^3x^7 - 21\,(2\,Bbc^2 - Ac^3)x^5 + 35\,(3\,Bb^2c - 2\,Abc^2)x^3 - 105\,(4\,Bb^3 - 3\,Ab^2c)x}{105\,c^5}$$

input
```
integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*b^4 - A*b^3*c)*x/(c^6*x^2 + b*c^5) + 1/2*(9*B*b^4 - 7*A*b^3*c)*arc
tan(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) + 1/105*(15*B*c^3*x^7 - 21*(2*B*b*c^2 -
 A*c^3)*x^5 + 35*(3*B*b^2*c - 2*A*b*c^2)*x^3 - 105*(4*B*b^3 - 3*A*b^2*c)*x
)/c^5
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(9\,Bb^4-7\,Ab^3c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^5} - \frac{Bb^4x-Ab^3cx}{2\,(cx^2+b)c^5} + \frac{15\,Bc^{12}x^7-42\,Bbc^{11}x^5+21\,Ac^{12}x^5+105\,Bb^2c^{10}x^3-70\,Abc^{11}x^3-420\,Bb^3c^9x+315\,Ab^2c^{10}x}{105\,c^{14}}$$

input
```
integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/2*(9*B*b^4 - 7*A*b^3*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) - 1/2*(B*b
^4*x - A*b^3*c*x)/((c*x^2 + b)*c^5) + 1/105*(15*B*c^12*x^7 - 42*B*b*c^11*x
^5 + 21*A*c^12*x^5 + 105*B*b^2*c^10*x^3 - 70*A*b*c^11*x^3 - 420*B*b^3*c^9*
x + 315*A*b^2*c^10*x)/c^14
```

3.58.9 Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.53

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = x\left(\frac{2\,b\left(\frac{2\,b\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)}{c}+\frac{B\,b^2}{c^4}\right)}{c}-\frac{b^2\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)}{c^2}\right) + x^5\left(\frac{A}{5\,c^2}-\frac{2\,B\,b}{5\,c^3}\right) - x^3\left(\frac{2\,b\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)}{3\,c}+\frac{B\,b^2}{3\,c^4}\right) + \frac{B\,x^7}{7\,c^2} - \frac{x\left(\frac{B\,b^4}{2}-\frac{A\,b^3\,c}{2}\right)}{c^6\,x^2+b\,c^5} + \frac{b^{5/2}\,\mathrm{atan}\left(\frac{b^{5/2}\,\sqrt{c}\,x\,(7\,A\,c-9\,B\,b)}{9\,B\,b^4-7\,A\,b^3\,c}\right)\,(7\,A\,c-9\,B\,b)}{2\,c^{11/2}}$$

input
```
int((x^12*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output
```
x*((2*b*((2*b*(A/c^2 - (2*B*b)/c^3))/c + (B*b^2)/c^4))/c - (b^2*(A/c^2 - (
2*B*b)/c^3))/c^2) + x^5*(A/(5*c^2) - (2*B*b)/(5*c^3)) - x^3*((2*b*(A/c^2 -
 (2*B*b)/c^3))/(3*c) + (B*b^2)/(3*c^4)) + (B*x^7)/(7*c^2) - (x*((B*b^4)/2
- (A*b^3*c)/2))/(b*c^5 + c^6*x^2) + (b^(5/2)*atan((b^(5/2)*c^(1/2)*x*(7*A*
c - 9*B*b))/(9*B*b^4 - 7*A*b^3*c))*(7*A*c - 9*B*b))/(2*c^(11/2))
```

3.58. $\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.59 $\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.59.1 Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{b(3bB-2Ac)x^2}{2c^4} - \frac{(2bB-Ac)x^4}{4c^3} + \frac{Bx^6}{6c^2} - \frac{b^3(bB-Ac)}{2c^5(b+cx^2)} - \frac{b^2(4bB-3Ac)\log(b+cx^2)}{2c^5}$$

output
```
1/2*b*(-2*A*c+3*B*b)*x^2/c^4-1/4*(-A*c+2*B*b)*x^4/c^3+1/6*B*x^6/c^2-1/2*b^
3*(-A*c+B*b)/c^5/(c*x^2+b)-1/2*b^2*(-3*A*c+4*B*b)*ln(c*x^2+b)/c^5
```

3.59.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{6bc(3bB-2Ac)x^2 + 3c^2(-2bB+Ac)x^4 + 2Bc^3x^6 + \frac{6b^3(-bB+Ac)}{b+cx^2} + 6b^2(-4bB+3Ac)\log(b+cx^2)}{12c^5}$$

input
```
Integrate[(x^11*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
(6*b*c*(3*b*B - 2*A*c)*x^2 + 3*c^2*(-2*b*B + A*c)*x^4 + 2*B*c^3*x^6 + (6*b
^3*(-(b*B) + A*c))/(b + c*x^2) + 6*b^2*(-4*b*B + 3*A*c)*Log[b + c*x^2])/(1
2*c^5)
```

3.59.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^7\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{x^6\left(Bx^2+A\right)}{\left(cx^2+b\right)^2}dx^2$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{2}\int \left(\frac{Bx^4}{c^2}+\frac{(Ac-2bB)x^2}{c^3}+\frac{b(3bB-2Ac)}{c^4}-\frac{b^2(4bB-3Ac)}{c^4\left(cx^2+b\right)}+\frac{b^3(bB-Ac)}{c^4\left(cx^2+b\right)^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{b^3(bB-Ac)}{c^5\left(b+cx^2\right)}-\frac{b^2(4bB-3Ac)\log\left(b+cx^2\right)}{c^5}+\frac{bx^2(3bB-2Ac)}{c^4}-\frac{x^4(2bB-Ac)}{2c^3}+\frac{Bx^6}{3c^2}\right)$$

input
```
Int[(x^11*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
((b*(3*b*B - 2*A*c)*x^2)/c^4 - ((2*b*B - A*c)*x^4)/(2*c^3) + (B*x^6)/(3*c^
2) - (b^3*(b*B - A*c))/(c^5*(b + c*x^2)) - (b^2*(4*b*B - 3*A*c)*Log[b + c*
x^2])/c^5)/2
```

3.59.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.59.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98

method	result
default	$-\frac{-\frac{B\,c^{2} x^{6}}{6}+\frac{\left(-A\,c^{2}+2 B b c \right) x^{4}}{4}+\frac{\left(2 A b c -3 B\,b^{2}\right) x^{2}}{2}}{c^{4}}+\frac{b^{2} \left(\frac{b \left(A c -B b \right)}{c \left(c\,x^{2}+b \right)}+\frac{\left(3 A c -4 B b \right) \ln \left(c\,x^{2}+b \right)}{c}\right)}{2 c^{4}}$
norman	$\frac{\frac{B\,x^{11}}{6 c}+\frac{\left(3 A c -4 B b \right) x^{9}}{12 c^{2}}-\frac{b \left(3 A c -4 B b \right) x^{7}}{4 c^{3}}+\frac{b \left(3 b^{2} A c -4 B\,b^{3}\right) x^{3}}{2 c^{5}}}{x^{3} \left(c\,x^{2}+b \right)}+\frac{b^{2} \left(3 A c -4 B b \right) \ln \left(c\,x^{2}+b \right)}{2 c^{5}}$
risch	$\frac{B\,x^{6}}{6 c^{2}}+\frac{A\,x^{4}}{4 c^{2}}-\frac{x^{4} B b}{2 c^{3}}-\frac{A b\,x^{2}}{c^{3}}+\frac{3 b^{2} B\,x^{2}}{2 c^{4}}+\frac{b^{3} A}{2 c^{4} \left(c\,x^{2}+b \right)}-\frac{b^{4} B}{2 c^{5} \left(c\,x^{2}+b \right)}+\frac{3 b^{2} \ln \left(c\,x^{2}+b \right) A}{2 c^{4}}-\frac{2 b^{3} \ln \left(c\,x^{2}+b \right) B}{c^{5}}$
parallelrisch	$\frac{2 B\,x^{8} c^{4}+3 A\,x^{6} c^{4}-4 B\,x^{6} b\,c^{3}-9 A\,x^{4} b\,c^{3}+12 B\,x^{4} b^{2} c^{2}+18 A \ln \left(c\,x^{2}+b \right) x^{2} b^{2} c^{2}-24 B \ln \left(c\,x^{2}+b \right) x^{2} b^{3} c+18 A \ln \left(c\,x^{2}+b \right) b^{3} c-24}{12 c^{5} \left(c\,x^{2}+b \right)}$

input
```
int(x^11*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/c^4*(-1/6*B*c^2*x^6+1/4*(-A*c^2+2*B*b*c)*x^4+1/2*(2*A*b*c-3*B*b^2)*x^2)
+1/2*b^2/c^4*(b*(A*c-B*b)/c/(c*x^2+b)+(3*A*c-4*B*b)/c*ln(c*x^2+b))
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.41

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$
$$= \frac{2\,Bc^4x^8 - (4\,Bbc^3 - 3\,Ac^4)x^6 - 6\,Bb^4 + 6\,Ab^3c + 3\,(4\,Bb^2c^2 - 3\,Abc^3)x^4 + 6\,(3\,Bb^3c - 2\,Ab^2c^2)x^2 - 6\,(4}{12\,(c^6x^2 + bc^5)}$$

input
```
integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/12*(2*B*c^4*x^8 - (4*B*b*c^3 - 3*A*c^4)*x^6 - 6*B*b^4 + 6*A*b^3*c + 3*(4
*B*b^2*c^2 - 3*A*b*c^3)*x^4 + 6*(3*B*b^3*c - 2*A*b^2*c^2)*x^2 - 6*(4*B*b^4
 - 3*A*b^3*c + (4*B*b^3*c - 3*A*b^2*c^2)*x^2)*log(c*x^2 + b))/(c^6*x^2 + b
*c^5)
```

3.59.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bx^6}{6c^2} - \frac{b^2(-3Ac+4Bb)\log{(b+cx^2)}}{2c^5} + x^4\left(\frac{A}{4c^2} - \frac{Bb}{2c^3}\right) + x^2\left(-\frac{Ab}{c^3} + \frac{3Bb^2}{2c^4}\right) + \frac{Ab^3c - Bb^4}{2bc^5 + 2c^6x^2}$$

input
```
integrate(x**11*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
B*x**6/(6*c**2) - b**2*(-3*A*c + 4*B*b)*log(b + c*x**2)/(2*c**5) + x**4*(A
/(4*c**2) - B*b/(2*c**3)) + x**2*(-A*b/c**3 + 3*B*b**2/(2*c**4)) + (A*b**3
*c - B*b**4)/(2*b*c**5 + 2*c**6*x**2)
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{Bb^4-Ab^3c}{2\,(c^6x^2+bc^5)} + \frac{2\,Bc^2x^6-3\,(2\,Bbc-Ac^2)x^4+6\,(3\,Bb^2-2\,Abc)x^2}{12\,c^4} - \frac{(4\,Bb^3-3\,Ab^2c)\log\left(cx^2+b\right)}{2\,c^5}$$

input
```
integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*b^4 - A*b^3*c)/(c^6*x^2 + b*c^5) + 1/12*(2*B*c^2*x^6 - 3*(2*B*b*c
- A*c^2)*x^4 + 6*(3*B*b^2 - 2*A*b*c)*x^2)/c^4 - 1/2*(4*B*b^3 - 3*A*b^2*c)*
log(c*x^2 + b)/c^5
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.29

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(4\,Bb^3-3\,Ab^2c)\log\left(|cx^2+b|\right)}{2\,c^5} + \frac{2\,Bc^4x^6-6\,Bbc^3x^4+3\,Ac^4x^4+18\,Bb^2c^2x^2-12\,Abc^3x^2}{12\,c^6} + \frac{4\,Bb^3cx^2-3\,Ab^2c^2x^2+3\,Bb^4-2\,Ab^3c}{2\,(cx^2+b)c^5}$$

input
```
integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-1/2*(4*B*b^3 - 3*A*b^2*c)*log(abs(c*x^2 + b))/c^5 + 1/12*(2*B*c^4*x^6 - 6
*B*b*c^3*x^4 + 3*A*c^4*x^4 + 18*B*b^2*c^2*x^2 - 12*A*b*c^3*x^2)/c^6 + 1/2*
(4*B*b^3*c*x^2 - 3*A*b^2*c^2*x^2 + 3*B*b^4 - 2*A*b^3*c)/((c*x^2 + b)*c^5)
```

3.59.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = x^4\left(\frac{A}{4\,c^2}-\frac{B\,b}{2\,c^3}\right)-x^2\left(\frac{b\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)}{c}+\frac{B\,b^2}{2\,c^4}\right)+\frac{B\,x^6}{6\,c^2}-\frac{\ln\left(c\,x^2+b\right)\,\left(4\,B\,b^3-3\,A\,b^2\,c\right)}{2\,c^5}-\frac{B\,b^4-A\,b^3\,c}{2\,c\,\left(c^5\,x^2+b\,c^4\right)}$$

input
```
int((x^11*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output
```
x^4*(A/(4*c^2) - (B*b)/(2*c^3)) - x^2*((b*(A/c^2 - (2*B*b)/c^3))/c + (B*b^
2)/(2*c^4)) + (B*x^6)/(6*c^2) - (log(b + c*x^2)*(4*B*b^3 - 3*A*b^2*c))/(2*
c^5) - (B*b^4 - A*b^3*c)/(2*c*(b*c^4 + c^5*x^2))
```

3.60 $\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.60.1 Optimal result

Integrand size = 24, antiderivative size = 110

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{b(3bB-2Ac)x}{c^4} - \frac{(2bB-Ac)x^3}{3c^3} + \frac{Bx^5}{5c^2} + \frac{b^2(bB-Ac)x}{2c^4(b+cx^2)} - \frac{b^{3/2}(7bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}}$$

output
```
b*(-2*A*c+3*B*b)*x/c^4-1/3*(-A*c+2*B*b)*x^3/c^3+1/5*B*x^5/c^2+1/2*b^2*(-A*
c+B*b)*x/c^4/(c*x^2+b)-1/2*b^(3/2)*(-5*A*c+7*B*b)*arctan(x*c^(1/2)/b^(1/2)
)/c^(9/2)
```

3.60.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{b(3bB-2Ac)x}{c^4} + \frac{(-2bB+Ac)x^3}{3c^3} + \frac{Bx^5}{5c^2} - \frac{(-b^3B+Ab^2c)\,x}{2c^4(b+cx^2)} - \frac{b^{3/2}(7bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}}$$

input
```
Integrate[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
(b*(3*b*B - 2*A*c)*x)/c^4 + ((-2*b*B + A*c)*x^3)/(3*c^3) + (B*x^5)/(5*c^2)
 - ((-(b^3*B) + A*b^2*c)*x)/(2*c^4*(b + c*x^2)) - (b^(3/2)*(7*b*B - 5*A*c)
*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

3.60.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 360, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^6(A+Bx^2)}{(b+cx^2)^2}dx$$

$$\Big\downarrow \text{ 360}$$

$$\frac{b^2x(bB-Ac)}{2c^4(b+cx^2)}-\frac{\int \frac{-2Bc^3x^6+2c^2(bB-Ac)x^4-2bc(bB-Ac)x^2+b^2(bB-Ac)}{cx^2+b}dx}{2c^4}$$

$$\Big\downarrow \text{ 2341}$$

$$\frac{b^2x(bB-Ac)}{2c^4(b+cx^2)}-\frac{\int \left(-2Bc^2x^4+2c(2bB-Ac)x^2-2b(3bB-2Ac)+\frac{7b^3B-5Ab^2c}{cx^2+b}\right)dx}{2c^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^2x(bB-Ac)}{2c^4(b+cx^2)}-\frac{\frac{b^{3/2}(7bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{c}}+\frac{2}{3}cx^3(2bB-Ac)-2bx(3bB-2Ac)-\frac{2}{5}Bc^2x^5}{2c^4}$$

input
```
Int[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
(b^2*(b*B - A*c)*x)/(2*c^4*(b + c*x^2)) - (-2*b*(3*b*B - 2*A*c)*x + (2*c*(
2*b*B - A*c)*x^3)/3 - (2*B*c^2*x^5)/5 + (b^(3/2)*(7*b*B - 5*A*c)*ArcTan[(S
qrt[c]*x)/Sqrt[b]])/Sqrt[c])/(2*c^4)
```

3.60.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.60.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

method	result
default	$-\frac{-\frac{1}{5}B\,c^{2}x^{5}-\frac{1}{3}A\,c^{2}x^{3}+\frac{2}{3}Bbc\,x^{3}+2Abcx-3b^{2}Bx}{c^{4}}+\frac{b^{2}\left(\frac{\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)x}{c\,x^{2}+b}+\frac{(5Ac-7Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{c^{4}}$
risch	$\frac{B\,x^{5}}{5c^{2}}+\frac{A\,x^{3}}{3c^{2}}-\frac{2Bb\,x^{3}}{3c^{3}}-\frac{2Abx}{c^{3}}+\frac{3b^{2}Bx}{c^{4}}+\frac{\left(-\frac{1}{2}b^{2}Ac+\frac{1}{2}B\,b^{3}\right)x}{c^{4}(c\,x^{2}+b)}+\frac{5\sqrt{-bc}\,b\ln\left(-\sqrt{-bc}\,x+b\right)A}{4c^{4}}-\frac{7\sqrt{-bc}\,b^{2}\ln\left(-\sqrt{-bc}\,x+\right.}{4c^{5}}$

input
```
int(x^10*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/c^4*(-1/5*B*c^2*x^5-1/3*A*c^2*x^3+2/3*B*b*c*x^3+2*A*b*c*x-3*b^2*B*x)+b^
2/c^4*((-1/2*A*c+1/2*B*b)*x/(c*x^2+b)+1/2*(5*A*c-7*B*b)/(b*c)^(1/2)*arctan
(c*x/(b*c)^(1/2)))
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.71

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$
$$= \left[\frac{12\,Bc^3x^7-4\,(7\,Bbc^2-5\,Ac^3)x^5+20\,(7\,Bb^2c-5\,Abc^2)x^3-15\,(7\,Bb^3-5\,Ab^2c+(7\,Bb^2c-5\,Abc^2)x^2)}{60\,(c^5x^2+bc^4)}\right.$$

input
```
integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
[1/60*(12*B*c^3*x^7 - 4*(7*B*b*c^2 - 5*A*c^3)*x^5 + 20*(7*B*b^2*c - 5*A*b*
c^2)*x^3 - 15*(7*B*b^3 - 5*A*b^2*c + (7*B*b^2*c - 5*A*b*c^2)*x^2)*sqrt(-b/
c)*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 30*(7*B*b^3 - 5*A*b^2
*c)*x)/(c^5*x^2 + b*c^4), 1/30*(6*B*c^3*x^7 - 2*(7*B*b*c^2 - 5*A*c^3)*x^5
+ 10*(7*B*b^2*c - 5*A*b*c^2)*x^3 - 15*(7*B*b^3 - 5*A*b^2*c + (7*B*b^2*c -
5*A*b*c^2)*x^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) + 15*(7*B*b^3 - 5*A*b^2*
c)*x)/(c^5*x^2 + b*c^4)]
```

3.60.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(104) = 208$.

Time = 0.39 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.92

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bx^5}{5c^2}+x^3\left(\frac{A}{3c^2}-\frac{2Bb}{3c^3}\right)+x\left(-\frac{2Ab}{c^3}+\frac{3Bb^2}{c^4}\right)+\frac{x(-Ab^2c+Bb^3)}{2bc^4+2c^5x^2}$$
$$+\frac{\sqrt{-\frac{b^3}{c^9}}(-5Ac+7Bb)\log\left(-\frac{c^4\sqrt{-\frac{b^3}{c^9}}(-5Ac+7Bb)}{-5Abc+7Bb^2}+x\right)}{4}$$
$$-\frac{\sqrt{-\frac{b^3}{c^9}}(-5Ac+7Bb)\log\left(\frac{c^4\sqrt{-\frac{b^3}{c^9}}(-5Ac+7Bb)}{-5Abc+7Bb^2}+x\right)}{4}$$

input
```
integrate(x**10*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
B*x**5/(5*c**2) + x**3*(A/(3*c**2) - 2*B*b/(3*c**3)) + x*(-2*A*b/c**3 + 3*
B*b**2/c**4) + x*(-A*b**2*c + B*b**3)/(2*b*c**4 + 2*c**5*x**2) + sqrt(-b**
3/c**9)*(-5*A*c + 7*B*b)*log(-c**4*sqrt(-b**3/c**9)*(-5*A*c + 7*B*b)/(-5*A
*b*c + 7*B*b**2) + x)/4 - sqrt(-b**3/c**9)*(-5*A*c + 7*B*b)*log(c**4*sqrt(
-b**3/c**9)*(-5*A*c + 7*B*b)/(-5*A*b*c + 7*B*b**2) + x)/4
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb^3-Ab^2c)x}{2\,(c^5x^2+bc^4)} - \frac{(7\,Bb^3-5\,Ab^2c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^4} + \frac{3\,Bc^2x^5-5\,(2\,Bbc-Ac^2)x^3+15\,(3\,Bb^2-2\,Abc)x}{15\,c^4}$$

input
```
integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b^3 - A*b^2*c)*x/(c^5*x^2 + b*c^4) - 1/2*(7*B*b^3 - 5*A*b^2*c)*arct
an(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/15*(3*B*c^2*x^5 - 5*(2*B*b*c - A*c^2
)*x^3 + 15*(3*B*b^2 - 2*A*b*c)*x)/c^4
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(7\,Bb^3-5\,Ab^2c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^4} + \frac{Bb^3x-Ab^2cx}{2\,(cx^2+b)c^4} + \frac{3\,Bc^8x^5-10\,Bbc^7x^3+5\,Ac^8x^3+45\,Bb^2c^6x-30\,Abc^7x}{15\,c^{10}}$$

input
```
integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-1/2*(7*B*b^3 - 5*A*b^2*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/2*(B*
b^3*x - A*b^2*c*x)/((c*x^2 + b)*c^4) + 1/15*(3*B*c^8*x^5 - 10*B*b*c^7*x^3
+ 5*A*c^8*x^3 + 45*B*b^2*c^6*x - 30*A*b*c^7*x)/c^10
```

3.60.9 Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.28

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = x^3\left(\frac{A}{3\,c^2}-\frac{2\,B\,b}{3\,c^3}\right)-x\left(\frac{2\,b\,\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)}{c}+\frac{B\,b^2}{c^4}\right)+\frac{B\,x^5}{5\,c^2}+\frac{x\,\left(\frac{B\,b^3}{2}-\frac{A\,b^2\,c}{2}\right)}{c^5\,x^2+b\,c^4}-\frac{b^{3/2}\,\mathrm{atan}\left(\frac{b^{3/2}\,\sqrt{c}\,x\,(5\,A\,c-7\,B\,b)}{7\,B\,b^3-5\,A\,b^2\,c}\right)\,(5\,A\,c-7\,B\,b)}{2\,c^{9/2}}$$

input
```
int((x^10*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output
```
x^3*(A/(3*c^2) - (2*B*b)/(3*c^3)) - x*((2*b*(A/c^2 - (2*B*b)/c^3))/c + (B*
b^2)/c^4) + (B*x^5)/(5*c^2) + (x*((B*b^3)/2 - (A*b^2*c)/2))/(b*c^4 + c^5*x
^2) - (b^(3/2)*atan((b^(3/2)*c^(1/2)*x*(5*A*c - 7*B*b))/(7*B*b^3 - 5*A*b^2
*c))*(5*A*c - 7*B*b))/(2*c^(9/2))
```

3.61 $\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.61.1 Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(2bB-Ac)x^2}{2c^3}+\frac{Bx^4}{4c^2}+\frac{b^2(bB-Ac)}{2c^4(b+cx^2)}+\frac{b(3bB-2Ac)\log(b+cx^2)}{2c^4}$$

output
```
-1/2*(-A*c+2*B*b)*x^2/c^3+1/4*B*x^4/c^2+1/2*b^2*(-A*c+B*b)/c^4/(c*x^2+b)+1
/2*b*(-2*A*c+3*B*b)*ln(c*x^2+b)/c^4
```

3.61.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{2c(-2bB+Ac)x^2+Bc^2x^4+\frac{2b^2(bB-Ac)}{b+cx^2}+2b(3bB-2Ac)\log(b+cx^2)}{4c^4}$$

input
```
Integrate[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
(2*c*(-2*b*B + A*c)*x^2 + B*c^2*x^4 + (2*b^2*(b*B - A*c))/(b + c*x^2) + 2*
b*(3*b*B - 2*A*c)*Log[b + c*x^2])/(4*c^4)
```

3.61.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9 (A+Bx^2)}{(bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^5 (A+Bx^2)}{(b+cx^2)^2}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{x^4 (Bx^2+A)}{(cx^2+b)^2}dx^2$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{2}\int \left(-\frac{(bB-Ac)b^2}{c^3 (cx^2+b)^2}+\frac{(3bB-2Ac)b}{c^3 (cx^2+b)}+\frac{Bx^2}{c^2}+\frac{Ac-2bB}{c^3}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{b^2(bB-Ac)}{c^4 (b+cx^2)}+\frac{b(3bB-2Ac)\log(b+cx^2)}{c^4}-\frac{x^2(2bB-Ac)}{c^3}+\frac{Bx^4}{2c^2}\right)$$

input
```
Int[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
(-(((2*b*B - A*c)*x^2)/c^3) + (B*x^4)/(2*c^2) + (b^2*(b*B - A*c))/(c^4*(b
+ c*x^2)) + (b*(3*b*B - 2*A*c)*Log[b + c*x^2])/c^4)/2
```

3.61.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.61.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

method	result
default	$\frac{\left(B c \,x^{2}+A c -2 B b \right)^{2}}{4 c^{4} B}-\frac{b \left(\frac{b \left(A c -B b \right)}{c \left(c \,x^{2}+b \right)}+\frac{\left(2 A c -3 B b \right) \ln \left(c \,x^{2}+b \right)}{c}\right)}{2 c^{3}}$
norman	$\frac{\frac{B \,x^{9}}{4 c}+\frac{\left(2 A c -3 B b \right) x^{7}}{4 c^{2}}-\frac{b \left(2 A b c -3 B \,b^{2}\right) x^{3}}{2 c^{4}}}{x^{3} \left(c \,x^{2}+b \right)}-\frac{b \left(2 A c -3 B b \right) \ln \left(c \,x^{2}+b \right)}{2 c^{4}}$
parallelrisch	$-\frac{-B \,c^{3} x^{6}-2 A \,c^{3} x^{4}+3 B b \,c^{2} x^{4}+4 A \ln \left(c \,x^{2}+b \right) x^{2} b \,c^{2}-6 B \ln \left(c \,x^{2}+b \right) x^{2} b^{2} c +4 A \ln \left(c \,x^{2}+b \right) b^{2} c -6 B \ln \left(c \,x^{2}+b \right) b^{3}+4 b^{2} A c -6 B}{4 c^{4} \left(c \,x^{2}+b \right)}$
risch	$\frac{B \,x^{4}}{4 c^{2}}+\frac{A \,x^{2}}{2 c^{2}}-\frac{B b \,x^{2}}{c^{3}}+\frac{A^{2}}{4 c^{2} B}-\frac{A b}{c^{3}}+\frac{B \,b^{2}}{c^{4}}-\frac{b^{2} A}{2 c^{3} \left(c \,x^{2}+b \right)}+\frac{b^{3} B}{2 c^{4} \left(c \,x^{2}+b \right)}-\frac{b \ln \left(c \,x^{2}+b \right) A}{c^{3}}+\frac{3 b^{2} \ln \left(c \,x^{2}+b \right) B}{2 c^{4}}$

input
```
int(x^9*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(B*c*x^2+A*c-2*B*b)^2/c^4/B-1/2*b/c^3*(b*(A*c-B*b)/c/(c*x^2+b)+(2*A*c-
3*B*b)/c*ln(c*x^2+b))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.46

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$
$$= \frac{Bc^3x^6 - (3\,Bbc^2 - 2\,Ac^3)x^4 + 2\,Bb^3 - 2\,Ab^2c - 2\,(2\,Bb^2c - Abc^2)x^2 + 2\,(3\,Bb^3 - 2\,Ab^2c + (3\,Bb^2c - 2\,A}{4\,(c^5x^2 + bc^4)}$$

input
```
integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/4*(B*c^3*x^6 - (3*B*b*c^2 - 2*A*c^3)*x^4 + 2*B*b^3 - 2*A*b^2*c - 2*(2*B*
b^2*c - A*b*c^2)*x^2 + 2*(3*B*b^3 - 2*A*b^2*c + (3*B*b^2*c - 2*A*b*c^2)*x^
2)*log(c*x^2 + b))/(c^5*x^2 + b*c^4)
```

3.61.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bx^4}{4c^2} + \frac{b(-2Ac+3Bb)\log{(b+cx^2)}}{2c^4} + x^2\left(\frac{A}{2c^2} - \frac{Bb}{c^3}\right) + \frac{-Ab^2c+Bb^3}{2bc^4+2c^5x^2}$$

input
```
integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
B*x**4/(4*c**2) + b*(-2*A*c + 3*B*b)*log(b + c*x**2)/(2*c**4) + x**2*(A/(2
*c**2) - B*b/c**3) + (-A*b**2*c + B*b**3)/(2*b*c**4 + 2*c**5*x**2)
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bb^3-Ab^2c}{2\,(c^5x^2+bc^4)} + \frac{Bcx^4-2\,(2\,Bb-Ac)x^2}{4\,c^3} + \frac{(3\,Bb^2-2\,Abc)\log\left(cx^2+b\right)}{2\,c^4}$$

input
```
integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b^3 - A*b^2*c)/(c^5*x^2 + b*c^4) + 1/4*(B*c*x^4 - 2*(2*B*b - A*c)*x
^2)/c^3 + 1/2*(3*B*b^2 - 2*A*b*c)*log(c*x^2 + b)/c^4
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.28

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(3\,Bb^2-2\,Abc)\log\left(|cx^2+b|\right)}{2\,c^4} + \frac{Bc^2x^4-4\,Bbcx^2+2\,Ac^2x^2}{4\,c^4} - \frac{3\,Bb^2cx^2-2\,Abc^2x^2+2\,Bb^3-Ab^2c}{2\,(cx^2+b)c^4}$$

input
```
integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/2*(3*B*b^2 - 2*A*b*c)*log(abs(c*x^2 + b))/c^4 + 1/4*(B*c^2*x^4 - 4*B*b*c
*x^2 + 2*A*c^2*x^2)/c^4 - 1/2*(3*B*b^2*c*x^2 - 2*A*b*c^2*x^2 + 2*B*b^3 - A
*b^2*c)/((c*x^2 + b)*c^4)
```

3.61.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = x^2\left(\frac{A}{2\,c^2}-\frac{B\,b}{c^3}\right) + \frac{\ln\left(c\,x^2+b\right)\,\left(3\,B\,b^2-2\,A\,b\,c\right)}{2\,c^4} + \frac{B\,x^4}{4\,c^2} + \frac{B\,b^3-A\,b^2\,c}{2\,c\,\left(c^4\,x^2+b\,c^3\right)}$$

input
```
int((x^9*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output
```
x^2*(A/(2*c^2) - (B*b)/c^3) + (log(b + c*x^2)*(3*B*b^2 - 2*A*b*c))/(2*c^4)
 + (B*x^4)/(4*c^2) + (B*b^3 - A*b^2*c)/(2*c*(b*c^3 + c^4*x^2))
```

3.61. $\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.62 $\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.62.1 Optimal result

Integrand size = 24, antiderivative size = 89

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(2bB-Ac)x}{c^3}+\frac{Bx^3}{3c^2}-\frac{b(bB-Ac)x}{2c^3\,(b+cx^2)}+\frac{\sqrt{b}(5bB-3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{7/2}}$$

output
```
-(-A*c+2*B*b)*x/c^3+1/3*B*x^3/c^2-1/2*b*(-A*c+B*b)*x/c^3/(c*x^2+b)+1/2*(-3
*A*c+5*B*b)*arctan(x*c^(1/2)/b^(1/2))*b^(1/2)/c^(7/2)
```

3.62.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(-2bB+Ac)x}{c^3}+\frac{Bx^3}{3c^2}+\frac{(-b^2B+Abc)\,x}{2c^3\,(b+cx^2)}+\frac{\sqrt{b}(5bB-3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{7/2}}$$

input
```
Integrate[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
((-2*b*B + A*c)*x)/c^3 + (B*x^3)/(3*c^2) + ((-(b^2*B) + A*b*c)*x)/(2*c^3*(
b + c*x^2)) + (Sqrt[b]*(5*b*B - 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(
7/2))
```

3.62.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {9, 360, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (A+Bx^2)}{(bx^2+cx^4)^2} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^4 (A+Bx^2)}{(b+cx^2)^2} dx$$

$$\Big\downarrow \; 360$$

$$-\frac{\int -\frac{2Bc^2x^4-2c(bB-Ac)x^2+b(bB-Ac)}{cx^2+b}dx}{2c^3}-\frac{bx(bB-Ac)}{2c^3 (b+cx^2)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{2Bc^2x^4-2c(bB-Ac)x^2+b(bB-Ac)}{cx^2+b}dx}{2c^3}-\frac{bx(bB-Ac)}{2c^3 (b+cx^2)}$$

$$\Big\downarrow \; 1467$$

$$\frac{\int \left(2Bcx^2-2(2bB-Ac)+\frac{5b^2B-3Abc}{cx^2+b}\right)dx}{2c^3}-\frac{bx(bB-Ac)}{2c^3 (b+cx^2)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\sqrt{b}(5bB-3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{c}}-2x(2bB-Ac)+\frac{2}{3}Bcx^3}{2c^3}-\frac{bx(bB-Ac)}{2c^3 (b+cx^2)}$$

input
```
Int[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(b*(b*B - A*c)*x)/(c^3*(b + c*x^2)) + (-2*(2*b*B - A*c)*x + (2*B*c*x^
3)/3 + (Sqrt[b]*(5*b*B - 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/Sqrt[c])/(2*c
^3)
```

3.62.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.62.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.84

method	result
default	$\frac{\frac{1}{3}Bc\,x^3+Acx-2bBx}{c^3}-\frac{b\left(\frac{\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)x}{c\,x^2+b}+\frac{(3Ac-5Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{c^3}$
risch	$\frac{B\,x^3}{3c^2}+\frac{Ax}{c^2}-\frac{2bBx}{c^3}+\frac{\left(\frac{1}{2}Abc-\frac{1}{2}B\,b^2\right)x}{c^3(c\,x^2+b)}+\frac{3\sqrt{-bc}\,\ln\left(-\sqrt{-bc}\,x-b\right)A}{4c^3}-\frac{5\sqrt{-bc}\,\ln\left(-\sqrt{-bc}\,x-b\right)Bb}{4c^4}-\frac{3\sqrt{-bc}\,\ln\left(\sqrt{-bc}\,x-b\right)}{4c^3}$

input
```
int(x^8*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/3*B*c*x^3+A*c*x-2*b*B*x)-b/c^3*((-1/2*A*c+1/2*B*b)*x/(c*x^2+b)+1/
2*(3*A*c-5*B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.70

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \left[\frac{4\,Bc^2x^5-4\,(5\,Bbc-3\,Ac^2)x^3-3\,(5\,Bb^2-3\,Abc+(5\,Bbc-3\,Ac^2)x^2)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)-6\,(5\right.}{12\,(c^4x^2+bc^3)}$$

input
```
integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*B*c^2*x^5 - 4*(5*B*b*c - 3*A*c^2)*x^3 - 3*(5*B*b^2 - 3*A*b*c + (5
*B*b*c - 3*A*c^2)*x^2)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^
2 + b)) - 6*(5*B*b^2 - 3*A*b*c)*x)/(c^4*x^2 + b*c^3), 1/6*(2*B*c^2*x^5 - 2
*(5*B*b*c - 3*A*c^2)*x^3 + 3*(5*B*b^2 - 3*A*b*c + (5*B*b*c - 3*A*c^2)*x^2)
*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - 3*(5*B*b^2 - 3*A*b*c)*x)/(c^4*x^2 + b
*c^3)]
```

3.62.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.45

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bx^3}{3c^2}+x\left(\frac{A}{c^2}-\frac{2Bb}{c^3}\right)+\frac{x(Abc-Bb^2)}{2bc^3+2c^4x^2} - \frac{\sqrt{-\frac{b}{c^7}}(-3Ac+5Bb)\log\left(-c^3\sqrt{-\frac{b}{c^7}}+x\right)}{4} + \frac{\sqrt{-\frac{b}{c^7}}(-3Ac+5Bb)\log\left(c^3\sqrt{-\frac{b}{c^7}}+x\right)}{4}$$

input
```
integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
B*x**3/(3*c**2) + x*(A/c**2 - 2*B*b/c**3) + x*(A*b*c - B*b**2)/(2*b*c**3 +
 2*c**4*x**2) - sqrt(-b/c**7)*(-3*A*c + 5*B*b)*log(-c**3*sqrt(-b/c**7) + x
)/4 + sqrt(-b/c**7)*(-3*A*c + 5*B*b)*log(c**3*sqrt(-b/c**7) + x)/4
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(Bb^2-Abc)x}{2\,(c^4x^2+bc^3)} + \frac{(5\,Bb^2-3\,Abc)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^3} + \frac{Bcx^3-3\,(2\,Bb-Ac)x}{3\,c^3}$$

input
```
integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*b^2 - A*b*c)*x/(c^4*x^2 + b*c^3) + 1/2*(5*B*b^2 - 3*A*b*c)*arctan(
c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/3*(B*c*x^3 - 3*(2*B*b - A*c)*x)/c^3
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(5\,Bb^2-3\,Abc)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^3} - \frac{Bb^2x-Abcx}{2\,(cx^2+b)c^3} + \frac{Bc^4x^3-6\,Bbc^3x+3\,Ac^4x}{3\,c^6}$$

input
```
integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/2*(5*B*b^2 - 3*A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) - 1/2*(B*b^2
*x - A*b*c*x)/((c*x^2 + b)*c^3) + 1/3*(B*c^4*x^3 - 6*B*b*c^3*x + 3*A*c^4*x
)/c^6
```

3.62.9 Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = x\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)-\frac{x\left(\frac{B\,b^2}{2}-\frac{A\,b\,c}{2}\right)}{c^4\,x^2+b\,c^3}+\frac{B\,x^3}{3\,c^2}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c}\,x\,(3\,A\,c-5\,B\,b)}{5\,B\,b^2-3\,A\,b\,c}\right)\,(3\,A\,c-5\,B\,b)}{2\,c^{7/2}}$$

input

```
int((x^8*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output

```
x*(A/c^2 - (2*B*b)/c^3) - (x*((B*b^2)/2 - (A*b*c)/2))/(b*c^3 + c^4*x^2) +
(B*x^3)/(3*c^2) + (b^(1/2)*atan((b^(1/2)*c^(1/2)*x*(3*A*c - 5*B*b))/(5*B*b
^2 - 3*A*b*c))*(3*A*c - 5*B*b))/(2*c^(7/2))
```

3.63 $\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.63.1 Optimal result

Integrand size = 24, antiderivative size = 61

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bx^2}{2c^2} - \frac{b(bB-Ac)}{2c^3(b+cx^2)} - \frac{(2bB-Ac)\log(b+cx^2)}{2c^3}$$

output
```
1/2*B*x^2/c^2-1/2*b*(-A*c+B*b)/c^3/(c*x^2+b)-1/2*(-A*c+2*B*b)*ln(c*x^2+b)/
c^3
```

3.63.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bcx^2 + \frac{b(-bB+Ac)}{b+cx^2} + (-2bB+Ac)\log(b+cx^2)}{2c^3}$$

input
```
Integrate[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
(B*c*x^2 + (b*(-(b*B) + A*c))/(b + c*x^2) + (-2*b*B + A*c)*Log[b + c*x^2])
/(2*c^3)
```

3.63.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 (A+Bx^2)}{(bx^2+cx^4)^2} \, dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^3 (A+Bx^2)}{(b+cx^2)^2} dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2} \int \frac{x^2 (Bx^2+A)}{(cx^2+b)^2} dx^2$$

$$\Big\downarrow \ 86$$

$$\frac{1}{2} \int \left(\frac{B}{c^2} + \frac{Ac-2bB}{c^2 (cx^2+b)} + \frac{b(bB-Ac)}{c^2 (cx^2+b)^2}\right) dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{b(bB-Ac)}{c^3 (b+cx^2)} - \frac{(2bB-Ac)\log(b+cx^2)}{c^3} + \frac{Bx^2}{c^2}\right)$$

input
```
Int[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
((B*x^2)/c^2 - (b*(b*B - A*c))/(c^3*(b + c*x^2)) - ((2*b*B - A*c)*Log[b +
c*x^2])/c^3)/2
```

3.63.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.63.4 Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{B\,x^{2}}{2c^{2}}+\frac{\frac{b(Ac-Bb)}{c\left(c\,x^{2}+b\right)}+\frac{(Ac-2Bb)\ln\left(c\,x^{2}+b\right)}{c}}{2c^{2}}$	59
norman	$\frac{\frac{B\,x^{7}}{2c}+\frac{b(Ac-2Bb)x^{3}}{2c^{3}}}{x^{3}(c\,x^{2}+b)}+\frac{(Ac-2Bb)\ln(c\,x^{2}+b)}{2c^{3}}$	63
risch	$\frac{B\,x^{2}}{2c^{2}}+\frac{bA}{2c^{2}(c\,x^{2}+b)}-\frac{b^{2}B}{2c^{3}(c\,x^{2}+b)}+\frac{\ln(c\,x^{2}+b)A}{2c^{2}}-\frac{\ln(c\,x^{2}+b)Bb}{c^{3}}$	74
parallelrisch	$\frac{B\,c^{2}x^{4}+A\ln(c\,x^{2}+b)x^{2}c^{2}-2B\ln(c\,x^{2}+b)x^{2}bc+A\ln(c\,x^{2}+b)bc-2B\ln(c\,x^{2}+b)b^{2}+Abc-2B\,b^{2}}{2c^{3}(c\,x^{2}+b)}$	92

input
```
int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*B*x^2/c^2+1/2/c^2*(b*(A*c-B*b)/c/(c*x^2+b)+1/c*(A*c-2*B*b)*ln(c*x^2+b)
)
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.33

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bc^2x^4+Bbcx^2-Bb^2+Abc-(2\,Bb^2-Abc+(2\,Bbc-Ac^2)x^2)\log(cx^2+b)}{2\,(c^4x^2+bc^3)}$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/2*(B*c^2*x^4 + B*b*c*x^2 - B*b^2 + A*b*c - (2*B*b^2 - A*b*c + (2*B*b*c -
 A*c^2)*x^2)*log(c*x^2 + b))/(c^4*x^2 + b*c^3)
```

3.63.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bx^2}{2c^2}+\frac{Abc-Bb^2}{2bc^3+2c^4x^2}-\frac{(-Ac+2Bb)\log(b+cx^2)}{2c^3}$$

input
```
integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
B*x**2/(2*c**2) + (A*b*c - B*b**2)/(2*b*c**3 + 2*c**4*x**2) - (-A*c + 2*B*
b)*log(b + c*x**2)/(2*c**3)
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bx^2}{2\,c^2} - \frac{Bb^2-Abc}{2\,(c^4x^2+bc^3)} - \frac{(2\,Bb-Ac)\log{(cx^2+b)}}{2\,c^3}$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/2*B*x^2/c^2 - 1/2*(B*b^2 - A*b*c)/(c^4*x^2 + b*c^3) - 1/2*(2*B*b - A*c)*
log(c*x^2 + b)/c^3
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bx^2}{2\,c^2} - \frac{(2\,Bb-Ac)\log{(|cx^2+b|)}}{2\,c^3} + \frac{2\,Bbcx^2-Ac^2x^2+Bb^2}{2\,(cx^2+b)c^3}$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/2*B*x^2/c^2 - 1/2*(2*B*b - A*c)*log(abs(c*x^2 + b))/c^3 + 1/2*(2*B*b*c*x
^2 - A*c^2*x^2 + B*b^2)/((c*x^2 + b)*c^3)
```

3.63.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{B\,x^2}{2\,c^2} + \frac{\ln{(c\,x^2+b)}\,(A\,c-2\,B\,b)}{2\,c^3} - \frac{B\,b^2-A\,b\,c}{2\,c\,(c^3\,x^2+b\,c^2)}$$

input
```
int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output
```
(B*x^2)/(2*c^2) + (log(b + c*x^2)*(A*c - 2*B*b))/(2*c^3) - (B*b^2 - A*b*c)
/(2*c*(b*c^2 + c^3*x^2))
```

3.64 $\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.64.1 Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=\frac{Bx}{c^2}+\frac{(bB-Ac)x}{2c^2(b+cx^2)}-\frac{(3bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{5/2}}$$

output
```
B*x/c^2+1/2*(-A*c+B*b)*x/c^2/(c*x^2+b)-1/2*(-A*c+3*B*b)*arctan(x*c^(1/2)/b
^(1/2))/c^(5/2)/b^(1/2)
```

3.64.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=\frac{Bx}{c^2}-\frac{(-bB+Ac)x}{2c^2(b+cx^2)}-\frac{(3bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{5/2}}$$

input
```
Integrate[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
(B*x)/c^2 - ((-(b*B) + A*c)*x)/(2*c^2*(b + c*x^2)) - ((3*b*B - A*c)*ArcTan
[(Sqrt[c]*x)/Sqrt[b]])/(2*Sqrt[b]*c^(5/2))
```

3.64.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {9, 360, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (A+Bx^2)}{(bx^2+cx^4)^2} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^2 (A+Bx^2)}{(b+cx^2)^2} dx$$

$$\Big\downarrow \; 360$$

$$\frac{x(bB-Ac)}{2c^2 (b+cx^2)} - \frac{\int \frac{-2Bcx^2+bB-Ac}{cx^2+b} dx}{2c^2}$$

$$\Big\downarrow \; 299$$

$$\frac{x(bB-Ac)}{2c^2 (b+cx^2)} - \frac{(3bB-Ac)\int \frac{1}{cx^2+b} dx - 2Bx}{2c^2}$$

$$\Big\downarrow \; 218$$

$$\frac{x(bB-Ac)}{2c^2 (b+cx^2)} - \frac{\frac{(3bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{c}} - 2Bx}{2c^2}$$

input
```
Int[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
((b*B - A*c)*x)/(2*c^2*(b + c*x^2)) - (-2*B*x + ((3*b*B - A*c)*ArcTan[(Sqr
t[c]*x)/Sqrt[b]])/(Sqrt[b]*Sqrt[c]))/(2*c^2)
```

3.64.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

3.64.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{Bx}{c^{2}}+\frac{\frac{\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)x}{c\,x^{2}+b}+\frac{(Ac-3Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}}{c^{2}}$	57
risch	$\frac{Bx}{c^{2}}+\frac{\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)x}{c^{2}(c\,x^{2}+b)}-\frac{\ln\left(cx+\sqrt{-bc}\right)A}{4c\sqrt{-bc}}+\frac{3\ln\left(cx+\sqrt{-bc}\right)Bb}{4c^{2}\sqrt{-bc}}+\frac{\ln\left(-cx+\sqrt{-bc}\right)A}{4c\sqrt{-bc}}-\frac{3\ln\left(-cx+\sqrt{-bc}\right)Bb}{4c^{2}\sqrt{-bc}}$	127

input
```
int(x^6*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
B*x/c^2+1/c^2*((-1/2*A*c+1/2*B*b)*x/(c*x^2+b)+1/2*(A*c-3*B*b)/(b*c)^(1/2)*
arctan(c*x/(b*c)^(1/2)))
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 208, normalized size of antiderivative = 3.06

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$
$$= \left[\frac{4\,Bbc^2x^3 + (3\,Bb^2 - Abc + (3\,Bbc - Ac^2)x^2)\sqrt{-bc}\log\left(\frac{cx^2-2\sqrt{-bc}x-b}{cx^2+b}\right) + 2\,(3\,Bb^2c - Abc^2)x}{4\,(bc^4x^2 + b^2c^3)}, \frac{2\,Bbc^2x^3 \cdots}{\cdots}\right.$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*B*b*c^2*x^3 + (3*B*b^2 - A*b*c + (3*B*b*c - A*c^2)*x^2)*sqrt(-b*c)
*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)) + 2*(3*B*b^2*c - A*b*c^2)*x
)/(b*c^4*x^2 + b^2*c^3), 1/2*(2*B*b*c^2*x^3 - (3*B*b^2 - A*b*c + (3*B*b*c
- A*c^2)*x^2)*sqrt(b*c)*arctan(sqrt(b*c)*x/b) + (3*B*b^2*c - A*b*c^2)*x)/(
b*c^4*x^2 + b^2*c^3)]
```

3.64.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.68

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bx}{c^2} + \frac{x(-Ac+Bb)}{2bc^2+2c^3x^2} + \frac{\sqrt{-\frac{1}{bc^5}}(-Ac+3Bb)\log\left(-bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{4} - \frac{\sqrt{-\frac{1}{bc^5}}(-Ac+3Bb)\log\left(bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{4}$$

input
```
integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
B*x/c**2 + x*(-A*c + B*b)/(2*b*c**2 + 2*c**3*x**2) + sqrt(-1/(b*c**5))*(-A
*c + 3*B*b)*log(-b*c**2*sqrt(-1/(b*c**5)) + x)/4 - sqrt(-1/(b*c**5))*(-A*c
 + 3*B*b)*log(b*c**2*sqrt(-1/(b*c**5)) + x)/4
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb-Ac)x}{2\,(c^3x^2+bc^2)} + \frac{Bx}{c^2} - \frac{(3\,Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^2}$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b - A*c)*x/(c^3*x^2 + b*c^2) + B*x/c^2 - 1/2*(3*B*b - A*c)*arctan(c
*x/sqrt(b*c))/(sqrt(b*c)*c^2)
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bx}{c^2} - \frac{(3\,Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^2} + \frac{Bbx-Acx}{2\,(cx^2+b)c^2}$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
B*x/c^2 - 1/2*(3*B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/2*(B
*b*x - A*c*x)/((c*x^2 + b)*c^2)
```

3.64.9 Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{B\,x}{c^2} - \frac{x\,\left(\frac{A\,c}{2}-\frac{B\,b}{2}\right)}{c^3\,x^2+b\,c^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,\left(A\,c-3\,B\,b\right)}{2\,\sqrt{b}\,c^{5/2}}$$

input
```
int((x^6*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output
```
(B*x)/c^2 - (x*((A*c)/2 - (B*b)/2))/(b*c^2 + c^3*x^2) + (atan((c^(1/2)*x)/
b^(1/2))*(A*c - 3*B*b))/(2*b^(1/2)*c^(5/2))
```

3.65 $\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.65.1 Optimal result

Integrand size = 24, antiderivative size = 41

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{bB-Ac}{2c^2(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^2}$$

output
```
1/2*(-A*c+B*b)/c^2/(c*x^2+b)+1/2*B*ln(c*x^2+b)/c^2
```

3.65.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{bB-Ac}{2c^2(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^2}$$

input
```
Integrate[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
(b*B - A*c)/(2*c^2*(b + c*x^2)) + (B*Log[b + c*x^2])/(2*c^2)
```

3.65.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx^2)}{(bx^2+cx^4)^2} \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x (A+Bx^2)}{(b+cx^2)^2}dx$$

$$\Big\downarrow \text{ 353}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{(cx^2+b)^2}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{B}{c(cx^2+b)}+\frac{Ac-bB}{c(cx^2+b)^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{bB-Ac}{c^2(b+cx^2)}+\frac{B\log(b+cx^2)}{c^2}\right)$$

input
```
Int[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
((b*B - A*c)/(c^2*(b + c*x^2)) + (B*Log[b + c*x^2])/c^2)/2
```

3.65.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.65.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{Ac-Bb}{2c^2(c\,x^2+b)}+\frac{B\ln(c\,x^2+b)}{2c^2}$	38
norman	$-\frac{Ac-Bb}{2c^2(c\,x^2+b)}+\frac{B\ln(c\,x^2+b)}{2c^2}$	38
risch	$-\frac{A}{2c(c\,x^2+b)}+\frac{Bb}{2c^2(c\,x^2+b)}+\frac{B\ln(c\,x^2+b)}{2c^2}$	47
parallelrisch	$-\frac{-B\ln(c\,x^2+b)x^2c-B\ln(c\,x^2+b)b+Ac-Bb}{2c^2(c\,x^2+b)}$	50

input
```
int(x^5*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^2*(A*c-B*b)/(c*x^2+b)+1/2*B*ln(c*x^2+b)/c^2
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=\frac{Bb-Ac+(Bcx^2+Bb)\log(cx^2+b)}{2(c^3x^2+bc^2)}$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/2*(B*b - A*c + (B*c*x^2 + B*b)*log(c*x^2 + b))/(c^3*x^2 + b*c^2)
```

3.65.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=\frac{B\log(b+cx^2)}{2c^2}+\frac{-Ac+Bb}{2bc^2+2c^3x^2}$$

input
```
integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
B*log(b + c*x**2)/(2*c**2) + (-A*c + B*b)/(2*b*c**2 + 2*c**3*x**2)
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=\frac{Bb-Ac}{2\,(c^3x^2+bc^2)}+\frac{B\log(cx^2+b)}{2\,c^2}$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b - A*c)/(c^3*x^2 + b*c^2) + 1/2*B*log(c*x^2 + b)/c^2
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=\frac{B\log(|cx^2+b|)}{2\,c^2}-\frac{Bx^2+A}{2\,(cx^2+b)c}$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/2*B*log(abs(c*x^2 + b))/c^2 - 1/2*(B*x^2 + A)/((c*x^2 + b)*c)
```

3.65.9 Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{B\,\ln\left(c\,x^2+b\right)}{2\,c^2}-\frac{A\,c-B\,b}{2\,c^2\,\left(c\,x^2+b\right)}$$

input
```
int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output
```
(B*log(b + c*x^2))/(2*c^2) - (A*c - B*b)/(2*c^2*(b + c*x^2))
```

3.66 $\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.66.1 Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(bB-Ac)x}{2bc(b+cx^2)} + \frac{(bB+Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}}$$

output
```
-1/2*(-A*c+B*b)*x/b/c/(c*x^2+b)+1/2*(A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/b^
(3/2)/c^(3/2)
```

3.66.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(bB-Ac)x}{2bc(b+cx^2)} + \frac{(bB+Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}}$$

input
```
Integrate[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*((b*B - A*c)*x)/(b*c*(b + c*x^2)) + ((b*B + A*c)*ArcTan[(Sqrt[c]*x)/S
qrt[b]])/(2*b^(3/2)*c^(3/2))
```

3.66.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {9, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{A+Bx^2}{(b+cx^2)^2}dx$$

$$\Big\downarrow \; 298$$

$$\frac{(Ac+bB)\int \frac{1}{cx^2+b}dx}{2bc} - \frac{x(bB-Ac)}{2bc\,(b+cx^2)}$$

$$\Big\downarrow \; 218$$

$$\frac{(Ac+bB)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}} - \frac{x(bB-Ac)}{2bc\,(b+cx^2)}$$

input
```
Int[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*((b*B - A*c)*x)/(b*c*(b + c*x^2)) + ((b*B + A*c)*ArcTan[(Sqrt[c]*x)/S
qrt[b]])/(2*b^(3/2)*c^(3/2))
```

3.66.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.66.4 Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{(Ac-Bb)x}{2bc(c\,x^2+b)}+\frac{(Ac+Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2bc\sqrt{bc}}$	57
risch	$\frac{(Ac-Bb)x}{2bc(c\,x^2+b)}-\frac{\ln\left(cx+\sqrt{-bc}\right)A}{4\sqrt{-bc}\,b}-\frac{\ln\left(cx+\sqrt{-bc}\right)B}{4\sqrt{-bc}\,c}+\frac{\ln\left(-cx+\sqrt{-bc}\right)A}{4\sqrt{-bc}\,b}+\frac{\ln\left(-cx+\sqrt{-bc}\right)B}{4\sqrt{-bc}\,c}$	122

input
```
int(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(A*c-B*b)/b/c*x/(c*x^2+b)+1/2*(A*c+B*b)/b/c/(b*c)^(1/2)*arctan(c*x/(b*
c)^(1/2))
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.89

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$
$$= \left[-\frac{(Bb^2+Abc+(Bbc+Ac^2)x^2)\sqrt{-bc}\log\left(\frac{cx^2-2\sqrt{-bc}x-b}{cx^2+b}\right)+2\,(Bb^2c-Abc^2)x}{4\,(b^2c^3x^2+b^3c^2)},\frac{(Bb^2+Abc+(Bbc+Ac\right.$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*((B*b^2 + A*b*c + (B*b*c + A*c^2)*x^2)*sqrt(-b*c)*log((c*x^2 - 2*sqr
t(-b*c)*x - b)/(c*x^2 + b)) + 2*(B*b^2*c - A*b*c^2)*x)/(b^2*c^3*x^2 + b^3*
c^2), 1/2*((B*b^2 + A*b*c + (B*b*c + A*c^2)*x^2)*sqrt(b*c)*arctan(sqrt(b*c
)*x/b) - (B*b^2*c - A*b*c^2)*x)/(b^2*c^3*x^2 + b^3*c^2)]
```

3.66.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(54) = 108$.

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.78

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{x(Ac-Bb)}{2b^2c+2bc^2x^2} - \frac{\sqrt{-\frac{1}{b^3c^3}}(Ac+Bb)\log\left(-b^2c\sqrt{-\frac{1}{b^3c^3}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{b^3c^3}}(Ac+Bb)\log\left(b^2c\sqrt{-\frac{1}{b^3c^3}}+x\right)}{4}$$

input
```
integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
x*(A*c - B*b)/(2*b**2*c + 2*b*c**2*x**2) - sqrt(-1/(b**3*c**3))*(A*c + B*b
)*log(-b**2*c*sqrt(-1/(b**3*c**3)) + x)/4 + sqrt(-1/(b**3*c**3))*(A*c + B*
b)*log(b**2*c*sqrt(-1/(b**3*c**3)) + x)/4
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(Bb-Ac)x}{2(bc^2x^2+b^2c)} + \frac{(Bb+Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}bc}$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*b - A*c)*x/(b*c^2*x^2 + b^2*c) + 1/2*(B*b + A*c)*arctan(c*x/sqrt(b
*c))/(sqrt(b*c)*b*c)
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb+Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}bc} - \frac{Bbx-Acx}{2(cx^2+b)bc}$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/2*(B*b + A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b*c) - 1/2*(B*b*x - A*c*x
)/((c*x^2 + b)*b*c)
```

3.66.9 Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(A\,c+B\,b)}{2\,b^{3/2}\,c^{3/2}} + \frac{x\,(A\,c-B\,b)}{2\,b\,c\,(c\,x^2+b)}$$

input
```
int((x^4*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output
```
(atan((c^(1/2)*x)/b^(1/2))*(A*c + B*b))/(2*b^(3/2)*c^(3/2)) + (x*(A*c - B*
b))/(2*b*c*(b + c*x^2))
```

3.67 $\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.67.1 Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{bB-Ac}{2bc\,(b+cx^2)}+\frac{A\log(x)}{b^2}-\frac{A\log(b+cx^2)}{2b^2}$$

output
```
1/2*(A*c-B*b)/b/c/(c*x^2+b)+A*ln(x)/b^2-1/2*A*ln(c*x^2+b)/b^2
```

3.67.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\frac{b(-bB+Ac)}{c(b+cx^2)}+2A\log(x)-A\log(b+cx^2)}{2b^2}$$

input
```
Integrate[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
((b*(-(b*B) + A*c))/(c*(b + c*x^2)) + 2*A*Log[x] - A*Log[b + c*x^2])/(2*b^
2)
```

3.67.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (A+Bx^2)}{(bx^2+cx^4)^2} \, dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{A+Bx^2}{x(b+cx^2)^2}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^2(cx^2+b)^2}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(-\frac{cA}{b^2(cx^2+b)}+\frac{A}{b^2x^2}+\frac{bB-Ac}{b(cx^2+b)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{A\log(b+cx^2)}{b^2}+\frac{A\log(x^2)}{b^2}-\frac{bB-Ac}{bc(b+cx^2)}\right)$$

input
```
Int[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
(-((b*B - A*c)/(b*c*(b + c*x^2))) + (A*Log[x^2])/b^2 - (A*Log[b + c*x^2])/
b^2)/2
```

3.67.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.67.4 Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{A\ln(x)}{b^2}-\frac{-\frac{b(Ac-Bb)}{c\left(c\,x^2+b\right)}+A\ln\left(c\,x^2+b\right)}{2b^2}$	48
norman	$-\frac{(Ac-Bb)x^2}{2b^2(c\,x^2+b)}+\frac{A\ln(x)}{b^2}-\frac{A\ln\left(c\,x^2+b\right)}{2b^2}$	48
risch	$\frac{A}{2b(c\,x^2+b)}-\frac{B}{2c(c\,x^2+b)}+\frac{A\ln(x)}{b^2}-\frac{A\ln\left(c\,x^2+b\right)}{2b^2}$	53
parallelrisch	$\frac{2A\ln(x)x^2c-A\ln\left(c\,x^2+b\right)x^2c-Ac\,x^2+bB\,x^2+2Ab\ln(x)-A\ln\left(c\,x^2+b\right)b}{2b^2\left(c\,x^2+b\right)}$	71

input
```
int(x^3*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
A*ln(x)/b^2-1/2/b^2*(-b*(A*c-B*b)/c/(c*x^2+b)+A*ln(c*x^2+b))
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{Bb^2 - Abc + (Ac^2x^2 + Abc)\log(cx^2+b) - 2(Ac^2x^2 + Abc)\log(x)}{2(b^2c^2x^2 + b^3c)}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(B*b^2 - A*b*c + (A*c^2*x^2 + A*b*c)*log(c*x^2 + b) - 2*(A*c^2*x^2 +
A*b*c)*log(x))/(b^2*c^2*x^2 + b^3*c)
```

3.67.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{A\log(x)}{b^2} - \frac{A\log\left(\frac{b}{c}+x^2\right)}{2b^2} + \frac{Ac-Bb}{2b^2c+2bc^2x^2}$$

input
```
integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
A*log(x)/b**2 - A*log(b/c + x**2)/(2*b**2) + (A*c - B*b)/(2*b**2*c + 2*b*c
**2*x**2)
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{Bb-Ac}{2(bc^2x^2+b^2c)} - \frac{A\log(cx^2+b)}{2b^2} + \frac{A\log(x^2)}{2b^2}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*b - A*c)/(b*c^2*x^2 + b^2*c) - 1/2*A*log(c*x^2 + b)/b^2 + 1/2*A*lo
g(x^2)/b^2
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{A\log(|cx^2+b|)}{2\,b^2} + \frac{A\log(|x|)}{b^2} - \frac{Bb^2-Abc}{2\,(cx^2+b)b^2c}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-1/2*A*log(abs(c*x^2 + b))/b^2 + A*log(abs(x))/b^2 - 1/2*(B*b^2 - A*b*c)/(
(c*x^2 + b)*b^2*c)
```

3.67.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{A\,\ln(x)}{b^2} - \frac{A\,\ln(c\,x^2+b)}{2\,b^2} + \frac{A\,c-B\,b}{2\,b\,c\,(c\,x^2+b)}$$

input
```
int((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output
```
(A*log(x))/b^2 - (A*log(b + c*x^2))/(2*b^2) + (A*c - B*b)/(2*b*c*(b + c*x^
2))
```

3.68 $\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.68.1 Optimal result

Integrand size = 24, antiderivative size = 70

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{A}{b^2x}+\frac{(bB-Ac)x}{2b^2(b+cx^2)}+\frac{(bB-3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{c}}$$

output
```
-A/b^2/x+1/2*(-A*c+B*b)*x/b^2/(c*x^2+b)+1/2*(-3*A*c+B*b)*arctan(x*c^(1/2)/
b^(1/2))/b^(5/2)/c^(1/2)
```

3.68.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{A}{b^2x}+\frac{(bB-Ac)x}{2b^2(b+cx^2)}+\frac{(bB-3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{c}}$$

input
```
Integrate[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
-(A/(b^2*x)) + ((b*B - A*c)*x)/(2*b^2*(b + c*x^2)) + ((b*B - 3*A*c)*ArcTan
[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(5/2)*Sqrt[c])
```

3.68.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {9, 361, 25, 27, 359, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^2)}{(bx^2+cx^4)^2} \, dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{A+Bx^2}{x^2 (b+cx^2)^2}dx$$

$$\Big\downarrow \ 361$$

$$\frac{x(bB-Ac)}{2b^2 (b+cx^2)}-\frac{1}{2}\int -\frac{(bB-Ac)x^2+2Ab}{b^2x^2 (cx^2+b)}dx$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\int \frac{(bB-Ac)x^2+2Ab}{b^2x^2 (cx^2+b)}dx+\frac{x(bB-Ac)}{2b^2 (b+cx^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(bB-Ac)x^2+2Ab}{x^2(cx^2+b)}dx}{2b^2}+\frac{x(bB-Ac)}{2b^2 (b+cx^2)}$$

$$\Big\downarrow \ 359$$

$$\frac{(bB-3Ac)\int \frac{1}{cx^2+b}dx-\frac{2A}{x}}{2b^2}+\frac{x(bB-Ac)}{2b^2 (b+cx^2)}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{(bB-3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{c}}-\frac{2A}{x}}{2b^2}+\frac{x(bB-Ac)}{2b^2 (b+cx^2)}$$

input
```
Int[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
((b*B - A*c)*x)/(2*b^2*(b + c*x^2)) + ((-2*A)/x + ((b*B - 3*A*c)*ArcTan[(S
qrt[c]*x)/Sqrt[b]])/(Sqrt[b]*Sqrt[c]))/(2*b^2)
```

3.68.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

3.68.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{A}{b^{2} x}-\frac{\frac{\left(\frac{A c}{2}-\frac{B b}{2}\right) x}{c\,x^{2}+b}+\frac{(3 A c-B b) \arctan\left(\frac{c x}{\sqrt{b c}}\right)}{2 \sqrt{b c}}}{b^{2}}$	62
risch	$\frac{-\frac{(3 A c-B b) x^{2}}{2 b^{2}}-\frac{A}{b}}{\left(c\,x^{2}+b\right) x}-\frac{3 \ln \left(-\sqrt{-b c}\,x-b\right) A c}{4 \sqrt{-b c}\,b^{2}}+\frac{\ln \left(-\sqrt{-b c}\,x-b\right) B}{4 \sqrt{-b c}\,b}+\frac{3 \ln \left(-\sqrt{-b c}\,x+b\right) A c}{4 \sqrt{-b c}\,b^{2}}-\frac{\ln \left(-\sqrt{-b c}\,x+b\right) B}{4 \sqrt{-b c}\,b}$	141

input
```
int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-A/b^2/x-1/b^2*((1/2*A*c-1/2*B*b)*x/(c*x^2+b)+1/2*(3*A*c-B*b)/(b*c)^(1/2)*
arctan(c*x/(b*c)^(1/2)))
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.00

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$
$$=\left[-\frac{4\,Ab^2c-2\,(Bb^2c-3\,Abc^2)x^2-((Bbc-3\,Ac^2)x^3+(Bb^2-3\,Abc)x)\sqrt{-bc}\log\left(\frac{cx^2+2\,\sqrt{-bc}x-b}{cx^2+b}\right)}{4\,(b^3c^2x^3+b^4cx)},\right.$$
$$\left.-\frac{2\,Ab^2c-(Bb^2c-3\,Abc^2)x^2-((Bbc-3\,Ac^2)x^3+(Bb^2-3\,Abc)x)\sqrt{bc}\arctan\left(\frac{\sqrt{bc}x}{b}\right)}{2\,(b^3c^2x^3+b^4cx)}\right]$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*A*b^2*c - 2*(B*b^2*c - 3*A*b*c^2)*x^2 - ((B*b*c - 3*A*c^2)*x^3 +
(B*b^2 - 3*A*b*c)*x)*sqrt(-b*c)*log((c*x^2 + 2*sqrt(-b*c)*x - b)/(c*x^2 +
b)))/(b^3*c^2*x^3 + b^4*c*x), -1/2*(2*A*b^2*c - (B*b^2*c - 3*A*b*c^2)*x^2
- ((B*b*c - 3*A*c^2)*x^3 + (B*b^2 - 3*A*b*c)*x)*sqrt(b*c)*arctan(sqrt(b*c)
*x/b))/(b^3*c^2*x^3 + b^4*c*x)]
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.63

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{\sqrt{-\frac{1}{b^5c}}(-3Ac+Bb)\log\left(-b^3\sqrt{-\frac{1}{b^5c}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{b^5c}}(-3Ac+Bb)\log\left(b^3\sqrt{-\frac{1}{b^5c}}+x\right)}{4} + \frac{-2Ab+x^2(-3Ac+Bb)}{2b^3x+2b^2cx^3}$$

input
```
integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
-sqrt(-1/(b**5*c))*(-3*A*c + B*b)*log(-b**3*sqrt(-1/(b**5*c)) + x)/4 + sqr
t(-1/(b**5*c))*(-3*A*c + B*b)*log(b**3*sqrt(-1/(b**5*c)) + x)/4 + (-2*A*b
+ x**2*(-3*A*c + B*b))/(2*b**3*x + 2*b**2*c*x**3)
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb-3\,Ac)x^2-2\,Ab}{2\,(b^2cx^3+b^3x)} + \frac{(Bb-3\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^2}$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/2*((B*b - 3*A*c)*x^2 - 2*A*b)/(b^2*c*x^3 + b^3*x) + 1/2*(B*b - 3*A*c)*ar
ctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb-3\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^2} + \frac{Bbx^2-3\,Acx^2-2\,Ab}{2\,(cx^3+bx)b^2}$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/2*(B*b - 3*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/2*(B*b*x^2 - 3
*A*c*x^2 - 2*A*b)/((c*x^3 + b*x)*b^2)
```

3.68.9 Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{\frac{A}{b}+\frac{x^2\,(3\,A\,c-B\,b)}{2\,b^2}}{c\,x^3+b\,x} - \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(3\,A\,c-B\,b)}{2\,b^{5/2}\,\sqrt{c}}$$

input
```
int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output
```
- (A/b + (x^2*(3*A*c - B*b))/(2*b^2))/(b*x + c*x^3) - (atan((c^(1/2)*x)/b^
(1/2))*(3*A*c - B*b))/(2*b^(5/2)*c^(1/2))
```

3.69 $\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.69.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{A}{2b^2x^2}+\frac{bB-Ac}{2b^2(b+cx^2)}+\frac{(bB-2Ac)\log(x)}{b^3}-\frac{(bB-2Ac)\log(b+cx^2)}{2b^3}$$

output
```
-1/2*A/b^2/x^2+1/2*(-A*c+B*b)/b^2/(c*x^2+b)+(-2*A*c+B*b)*ln(x)/b^3-1/2*(-2
*A*c+B*b)*ln(c*x^2+b)/b^3
```

3.69.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{-\frac{Ab}{x^2}+\frac{b(bB-Ac)}{b+cx^2}+2(bB-2Ac)\log(x)+(-bB+2Ac)\log(b+cx^2)}{2b^3}$$

input
```
Integrate[(x*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
(-((A*b)/x^2) + (b*(b*B - A*c))/(b + c*x^2) + 2*(b*B - 2*A*c)*Log[x] + (-(
b*B) + 2*A*c)*Log[b + c*x^2])/(2*b^3)
```

3.69.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{A+Bx^2}{x^3(b+cx^2)^2}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^4(cx^2+b)^2}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{A}{b^2x^4}-\frac{c(bB-2Ac)}{b^3(cx^2+b)}+\frac{bB-2Ac}{b^3x^2}-\frac{c(bB-Ac)}{b^2(cx^2+b)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\log(x^2)(bB-2Ac)}{b^3}-\frac{(bB-2Ac)\log(b+cx^2)}{b^3}+\frac{bB-Ac}{b^2(b+cx^2)}-\frac{A}{b^2x^2}\right)$$

input
```
Int[(x*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
(-(A/(b^2*x^2)) + (b*B - A*c)/(b^2*(b + c*x^2)) + ((b*B - 2*A*c)*Log[x^2])
/b^3 - ((b*B - 2*A*c)*Log[b + c*x^2])/b^3)/2
```

3.69.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.69.4 Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

method	result
default	$-\frac{A}{2b^{2}x^{2}}+\frac{\left(-2Ac+Bb\right)\ln\left(x\right)}{b^{3}}+\frac{c\left(\frac{\left(2Ac-Bb\right)\ln\left(c\,x^{2}+b\right)}{c}-\frac{b\left(Ac-Bb\right)}{c\left(c\,x^{2}+b\right)}\right)}{2b^{3}}$
norman	$\frac{-\frac{Ax}{2b}+\frac{c\left(2Ac-Bb\right)x^{5}}{2b^{3}}}{x^{3}\left(c\,x^{2}+b\right)}-\frac{\left(2Ac-Bb\right)\ln\left(x\right)}{b^{3}}+\frac{\left(2Ac-Bb\right)\ln\left(c\,x^{2}+b\right)}{2b^{3}}$
risch	$\frac{-\frac{\left(2Ac-Bb\right)x^{2}}{2b^{2}}-\frac{A}{2b}}{x^{2}\left(c\,x^{2}+b\right)}-\frac{2\ln\left(x\right)Ac}{b^{3}}+\frac{\ln\left(x\right)B}{b^{2}}+\frac{\ln\left(-c\,x^{2}-b\right)Ac}{b^{3}}-\frac{\ln\left(-c\,x^{2}-b\right)B}{2b^{2}}$
parallelrisch	$-\frac{4A\ln\left(x\right)x^{4}c^{2}-2A\ln\left(c\,x^{2}+b\right)x^{4}c^{2}-2B\ln\left(x\right)x^{4}bc+B\ln\left(c\,x^{2}+b\right)x^{4}bc-2A\,c^{2}x^{4}+x^{4}Bbc+4A\ln\left(x\right)x^{2}bc-2A\ln\left(c\,x^{2}+b\right)x^{2}bc-}{2b^{3}x^{2}\left(c\,x^{2}+b\right)}$

input
```
int(x*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/b^2/x^2+(-2*A*c+B*b)*ln(x)/b^3+1/2/b^3*c*((2*A*c-B*b)/c*ln(c*x^2+b)
-b*(A*c-B*b)/c/(c*x^2+b))
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{Ab^2-(Bb^2-2\,Abc)x^2+((Bbc-2\,Ac^2)x^4+(Bb^2-2\,Abc)x^2)\log(cx^2+b)-2\,((Bbc-2\,Ac^2)x^4+(}{2\,(b^3cx^4+b^4x^2)}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(A*b^2 - (B*b^2 - 2*A*b*c)*x^2 + ((B*b*c - 2*A*c^2)*x^4 + (B*b^2 - 2*
A*b*c)*x^2)*log(c*x^2 + b) - 2*((B*b*c - 2*A*c^2)*x^4 + (B*b^2 - 2*A*b*c)*
x^2)*log(x))/(b^3*c*x^4 + b^4*x^2)
```

3.69.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{-Ab+x^2(-2Ac+Bb)}{2b^3x^2+2b^2cx^4}+\frac{(-2Ac+Bb)\log(x)}{b^3} - \frac{(-2Ac+Bb)\log\left(\frac{b}{c}+x^2\right)}{2b^3}$$

input
```
integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
(-A*b + x**2*(-2*A*c + B*b))/(2*b**3*x**2 + 2*b**2*c*x**4) + (-2*A*c + B*b
)*log(x)/b**3 - (-2*A*c + B*b)*log(b/c + x**2)/(2*b**3)
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb-2\,Ac)x^2-Ab}{2\,(b^2cx^4+b^3x^2)} - \frac{(Bb-2\,Ac)\log{(cx^2+b)}}{2\,b^3} + \frac{(Bb-2\,Ac)\log{(x^2)}}{2\,b^3}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/2*((B*b - 2*A*c)*x^2 - A*b)/(b^2*c*x^4 + b^3*x^2) - 1/2*(B*b - 2*A*c)*lo
g(c*x^2 + b)/b^3 + 1/2*(B*b - 2*A*c)*log(x^2)/b^3
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb-2\,Ac)\log{(|x|)}}{b^3} + \frac{Bbx^2-2\,Acx^2-Ab}{2\,(cx^4+bx^2)b^2} - \frac{(Bbc-2\,Ac^2)\log{(|cx^2+b|)}}{2\,b^3c}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
(B*b - 2*A*c)*log(abs(x))/b^3 + 1/2*(B*b*x^2 - 2*A*c*x^2 - A*b)/((c*x^4 +
b*x^2)*b^2) - 1/2*(B*b*c - 2*A*c^2)*log(abs(c*x^2 + b))/(b^3*c)
```

3.69.9 Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\ln{(c\,x^2+b)}\,\,(2\,A\,c-B\,b)}{2\,b^3} - \frac{\frac{A}{2\,b}+\frac{x^2\,(2\,A\,c-B\,b)}{2\,b^2}}{c\,x^4+b\,x^2} - \frac{\ln{(x)}\,\,(2\,A\,c-B\,b)}{b^3}$$

input
```
int((x*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output
```
(log(b + c*x^2)*(2*A*c - B*b))/(2*b^3) - (A/(2*b) + (x^2*(2*A*c - B*b))/(2
*b^2))/(b*x^2 + c*x^4) - (log(x)*(2*A*c - B*b))/b^3
```

3.70 $\int \frac{A+Bx^2}{(bx^2+cx^4)^2}\,dx$

3.70.1 Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^2}\,dx=-\frac{A}{3b^2x^3}-\frac{bB-2Ac}{b^3x}-\frac{c(bB-Ac)x}{2b^3(b+cx^2)}-\frac{\sqrt{c}(3bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}}$$

output
```
-1/3*A/b^2/x^3+(2*A*c-B*b)/b^3/x-1/2*c*(-A*c+B*b)*x/b^3/(c*x^2+b)-1/2*(-5*
A*c+3*B*b)*arctan(x*c^(1/2)/b^(1/2))*c^(1/2)/b^(7/2)
```

3.70.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^2}\,dx=-\frac{A}{3b^2x^3}+\frac{-bB+2Ac}{b^3x}-\frac{c(bB-Ac)x}{2b^3(b+cx^2)}-\frac{\sqrt{c}(3bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}}$$

input
```
Integrate[(A + B*x^2)/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/3*A/(b^2*x^3) + (-(b*B) + 2*A*c)/(b^3*x) - (c*(b*B - A*c)*x)/(2*b^3*(b
+ c*x^2)) - (Sqrt[c]*(3*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/
2))
```

3.70.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2026, 361, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{A+Bx^2}{x^4(b+cx^2)^2}dx$$

$$\Big\downarrow \text{361}$$

$$-\frac{1}{2}c\int -\frac{-\frac{(bB-Ac)x^4}{b^3}+\frac{2(bB-Ac)x^2}{b^2c}+\frac{2A}{bc}}{x^4(cx^2+b)}dx-\frac{cx(bB-Ac)}{2b^3(b+cx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}c\int \frac{-\frac{(bB-Ac)x^4}{b^3}+\frac{2(bB-Ac)x^2}{b^2c}+\frac{2A}{bc}}{x^4(cx^2+b)}dx-\frac{cx(bB-Ac)}{2b^3(b+cx^2)}$$

$$\Big\downarrow \text{1584}$$

$$\frac{1}{2}c\int \left(\frac{2A}{b^2cx^4}+\frac{5Ac-3bB}{b^3(cx^2+b)}+\frac{2(bB-2Ac)}{b^3cx^2}\right)dx-\frac{cx(bB-Ac)}{2b^3(b+cx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}c\left(-\frac{(3bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{7/2}\sqrt{c}}-\frac{2(bB-2Ac)}{b^3cx}-\frac{2A}{3b^2cx^3}\right)-\frac{cx(bB-Ac)}{2b^3(b+cx^2)}$$

input
```
Int[(A + B*x^2)/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(c*(b*B - A*c)*x)/(b^3*(b + c*x^2)) + (c*((-2*A)/(3*b^2*c*x^3) - (2*(
b*B - 2*A*c))/(b^3*c*x) - ((3*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(b
^(7/2)*Sqrt[c])))/2
```

3.70.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.70.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

method	result
default	$-\frac{A}{3b^2x^3}-\frac{-2Ac+Bb}{x\,b^3}+\frac{c\left(\frac{\left(\frac{Ac}{2}-\frac{Bb}{2}\right)x}{c\,x^2+b}+\frac{(5Ac-3Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{b^3}$
risch	$\frac{\frac{c(5Ac-3Bb)x^4}{2b^3}+\frac{(5Ac-3Bb)x^2}{3b^2}-\frac{A}{3b}}{x^3(c\,x^2+b)}+\frac{5\sqrt{-bc}\,\ln(-cx-\sqrt{-bc})Ac}{4b^4}-\frac{3\sqrt{-bc}\,\ln(-cx-\sqrt{-bc})B}{4b^3}-\frac{5\sqrt{-bc}\,\ln(-cx+\sqrt{-bc})Ac}{4b^4}+$

input
```
int((B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A/b^2/x^3-(-2*A*c+B*b)/x/b^3+1/b^3*c*((1/2*A*c-1/2*B*b)*x/(c*x^2+b)+1
/2*(5*A*c-3*B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.78

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^2}\,dx$$

$$= \left[-\frac{6\,(3\,Bbc-5\,Ac^2)x^4+4\,Ab^2+4\,(3\,Bb^2-5\,Abc)x^2+3\,((3\,Bbc-5\,Ac^2)x^5+(3\,Bb^2-5\,Abc)x^3)\sqrt{-\frac{c}{b}}\,\mathrm{l}}{12\,(b^3cx^5+b^4x^3)}\right.$$

$$-\frac{3\,(3\,Bbc-5\,Ac^2)x^4+2\,Ab^2+2\,(3\,Bb^2-5\,Abc)x^2+3\,((3\,Bbc-5\,Ac^2)x^5+(3\,Bb^2-5\,Abc)x^3)\sqrt{\frac{c}{b}}\,\mathrm{arc}}{6\,(b^3cx^5+b^4x^3)}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
[-1/12*(6*(3*B*b*c - 5*A*c^2)*x^4 + 4*A*b^2 + 4*(3*B*b^2 - 5*A*b*c)*x^2 +
3*((3*B*b*c - 5*A*c^2)*x^5 + (3*B*b^2 - 5*A*b*c)*x^3)*sqrt(-c/b)*log((c*x^
2 + 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b^3*c*x^5 + b^4*x^3), -1/6*(3*(3*
B*b*c - 5*A*c^2)*x^4 + 2*A*b^2 + 2*(3*B*b^2 - 5*A*b*c)*x^2 + 3*((3*B*b*c -
 5*A*c^2)*x^5 + (3*B*b^2 - 5*A*b*c)*x^3)*sqrt(c/b)*arctan(x*sqrt(c/b)))/(b
^3*c*x^5 + b^4*x^3)]
```

3.70.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(82) = 164.

Time = 0.31 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.04

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^2}\,dx = \frac{\sqrt{-\frac{c}{b^7}}(-5Ac+3Bb)\log\left(-\frac{b^4\sqrt{-\frac{c}{b^7}}(-5Ac+3Bb)}{-5Ac^2+3Bbc}+x\right)}{4} - \frac{\sqrt{-\frac{c}{b^7}}(-5Ac+3Bb)\log\left(\frac{b^4\sqrt{-\frac{c}{b^7}}(-5Ac+3Bb)}{-5Ac^2+3Bbc}+x\right)}{4} + \frac{-2Ab^2+x^4\cdot(15Ac^2-9Bbc)+x^2\cdot(10Abc-6Bb^2)}{6b^4x^3+6b^3cx^5}$$

input
```
integrate((B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
sqrt(-c/b**7)*(-5*A*c + 3*B*b)*log(-b**4*sqrt(-c/b**7)*(-5*A*c + 3*B*b)/(-
5*A*c**2 + 3*B*b*c) + x)/4 - sqrt(-c/b**7)*(-5*A*c + 3*B*b)*log(b**4*sqrt(
-c/b**7)*(-5*A*c + 3*B*b)/(-5*A*c**2 + 3*B*b*c) + x)/4 + (-2*A*b**2 + x**4
*(15*A*c**2 - 9*B*b*c) + x**2*(10*A*b*c - 6*B*b**2))/(6*b**4*x**3 + 6*b**3
*c*x**5)
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^2}\,dx = -\frac{3\,(3\,Bbc-5\,Ac^2)x^4+2\,Ab^2+2\,(3\,Bb^2-5\,Abc)x^2}{6\,(b^3cx^5+b^4x^3)} - \frac{(3\,Bbc-5\,Ac^2)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^3}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*(3*B*b*c - 5*A*c^2)*x^4 + 2*A*b^2 + 2*(3*B*b^2 - 5*A*b*c)*x^2)/(b^
3*c*x^5 + b^4*x^3) - 1/2*(3*B*b*c - 5*A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b
*c)*b^3)
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^2}\,dx = -\frac{(3\,Bbc-5\,Ac^2)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^3} - \frac{Bbcx-Ac^2x}{2\,(cx^2+b)b^3} - \frac{3\,Bbx^2-6\,Acx^2+Ab}{3\,b^3x^3}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-1/2*(3*B*b*c - 5*A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/2*(B*b*
c*x - A*c^2*x)/((c*x^2 + b)*b^3) - 1/3*(3*B*b*x^2 - 6*A*c*x^2 + A*b)/(b^3*
x^3)
```

3.70.9 Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^2}\,dx = \frac{\frac{x^2\,(5\,A\,c-3\,B\,b)}{3\,b^2}-\frac{A}{3\,b}+\frac{c\,x^4\,(5\,A\,c-3\,B\,b)}{2\,b^3}}{c\,x^5+b\,x^3} + \frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(5\,A\,c-3\,B\,b)}{2\,b^{7/2}}$$

input
```
int((A + B*x^2)/(b*x^2 + c*x^4)^2,x)
```

output
```
((x^2*(5*A*c - 3*B*b))/(3*b^2) - A/(3*b) + (c*x^4*(5*A*c - 3*B*b))/(2*b^3)
)/(b*x^3 + c*x^5) + (c^(1/2)*atan((c^(1/2)*x)/b^(1/2))*(5*A*c - 3*B*b))/(2
*b^(7/2))
```

3.71 $\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2} \, dx$

3.71.1 Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2} \, dx = -\frac{A}{4b^2x^4}-\frac{bB-2Ac}{2b^3x^2}-\frac{c(bB-Ac)}{2b^3\left(b+cx^2\right)} -\frac{c(2bB-3Ac)\log(x)}{b^4}+\frac{c(2bB-3Ac)\log\left(b+cx^2\right)}{2b^4}$$

output
```
-1/4*A/b^2/x^4+1/2*(2*A*c-B*b)/b^3/x^2-1/2*c*(-A*c+B*b)/b^3/(c*x^2+b)-c*(-
3*A*c+2*B*b)*ln(x)/b^4+1/2*c*(-3*A*c+2*B*b)*ln(c*x^2+b)/b^4
```

3.71.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2} \, dx = -\frac{\frac{Ab^2}{x^4}+\frac{2b(bB-2Ac)}{x^2}+\frac{2bc(bB-Ac)}{b+cx^2}-4c(-2bB+3Ac)\log(x)+2c(-2bB+3Ac)\log\left(b+cx^2\right)}{4b^4}$$

input
```
Integrate[(A + B*x^2)/(x*(b*x^2 + c*x^4)^2),x]
```

output
```
-1/4*((A*b^2)/x^4 + (2*b*(b*B - 2*A*c))/x^2 + (2*b*c*(b*B - A*c))/(b + c*x
^2) - 4*c*(-2*b*B + 3*A*c)*Log[x] + 2*c*(-2*b*B + 3*A*c)*Log[b + c*x^2])/b
^4
```

3.71.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{A+Bx^2}{x^5\left(b+cx^2\right)^2}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^6\left(cx^2+b\right)^2}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{(2bB-3Ac)c^2}{b^4\left(cx^2+b\right)}+\frac{(bB-Ac)c^2}{b^3\left(cx^2+b\right)^2}-\frac{(2bB-3Ac)c}{b^4x^2}+\frac{bB-2Ac}{b^3x^4}+\frac{A}{b^2x^6}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{c\log\left(x^2\right)(2bB-3Ac)}{b^4}+\frac{c(2bB-3Ac)\log\left(b+cx^2\right)}{b^4}-\frac{c(bB-Ac)}{b^3\left(b+cx^2\right)}-\frac{bB-2Ac}{b^3x^2}-\frac{A}{2b^2x^4}\right)$$

input
```
Int[(A + B*x^2)/(x*(b*x^2 + c*x^4)^2),x]
```

output
```
(-1/2*A/(b^2*x^4) - (b*B - 2*A*c)/(b^3*x^2) - (c*(b*B - A*c))/(b^3*(b + c*
x^2)) - (c*(2*b*B - 3*A*c)*Log[x^2])/b^4 + (c*(2*b*B - 3*A*c)*Log[b + c*x^
2])/b^4)/2
```

3.71.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.71.4 Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

method	result
default	$-\frac{A}{4b^2x^4}-\frac{-2Ac+Bb}{2x^2b^3}+\frac{c(3Ac-2Bb)\ln(x)}{b^4}-\frac{c^2\left(\frac{(3Ac-2Bb)\ln\left(c\,x^2+b\right)}{c}-\frac{b(Ac-Bb)}{c\left(c\,x^2+b\right)}\right)}{2b^4}$
norman	$\frac{-\frac{A}{4b}+\frac{(3Ac-2Bb)x^2}{4b^2}-\frac{c\left(3A\,c^2-2Bbc\right)x^6}{2b^4}}{x^4(c\,x^2+b)}+\frac{c(3Ac-2Bb)\ln(x)}{b^4}-\frac{c(3Ac-2Bb)\ln(c\,x^2+b)}{2b^4}$
risch	$\frac{\frac{c(3Ac-2Bb)x^4}{2b^3}+\frac{(3Ac-2Bb)x^2}{4b^2}-\frac{A}{4b}}{x^4(c\,x^2+b)}+\frac{3c^2\ln(x)A}{b^4}-\frac{2c\ln(x)B}{b^3}-\frac{3c^2\ln(c\,x^2+b)A}{2b^4}+\frac{c\ln(c\,x^2+b)B}{b^3}$
parallelrisch	$\frac{12A\ln(x)x^6c^3-6A\ln(c\,x^2+b)x^6c^3-8B\ln(x)x^6b\,c^2+4B\ln(c\,x^2+b)x^6b\,c^2-6A\,c^3x^6+4x^6Bb\,c^2+12A\ln(x)x^4b\,c^2-6A\ln(c\,x^2+}{4b^4x^4(c\,x^2+b)}$

input
```
int((B*x^2+A)/x/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*A/b^2/x^4-1/2*(-2*A*c+B*b)/x^2/b^3+c*(3*A*c-2*B*b)/b^4*ln(x)-1/2/b^4*
c^2*((3*A*c-2*B*b)/c*ln(c*x^2+b)-b*(A*c-B*b)/c/(c*x^2+b))
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.59

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2}\,dx = -\frac{2\,(2\,Bb^2c-3\,Abc^2)x^4+Ab^3+(2\,Bb^3-3\,Ab^2c)x^2-2\,((2\,Bbc^2-3\,Ac^3)x^6+(2\,Bb^2c-3\,Abc^2)x^4)\log(}{4\,(b^4cx^6+b^5x^4)}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*(2*B*b^2*c - 3*A*b*c^2)*x^4 + A*b^3 + (2*B*b^3 - 3*A*b^2*c)*x^2 -
2*((2*B*b*c^2 - 3*A*c^3)*x^6 + (2*B*b^2*c - 3*A*b*c^2)*x^4)*log(c*x^2 + b)
 + 4*((2*B*b*c^2 - 3*A*c^3)*x^6 + (2*B*b^2*c - 3*A*b*c^2)*x^4)*log(x))/(b^
4*c*x^6 + b^5*x^4)
```

3.71.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2}\,dx = \frac{-Ab^2+x^4\cdot(6Ac^2-4Bbc)+x^2\cdot(3Abc-2Bb^2)}{4b^4x^4+4b^3cx^6} - \frac{c(-3Ac+2Bb)\log(x)}{b^4} + \frac{c(-3Ac+2Bb)\log\left(\frac{b}{c}+x^2\right)}{2b^4}$$

input
```
integrate((B*x**2+A)/x/(c*x**4+b*x**2)**2,x)
```

output
```
(-A*b**2 + x**4*(6*A*c**2 - 4*B*b*c) + x**2*(3*A*b*c - 2*B*b**2))/(4*b**4*
x**4 + 4*b**3*c*x**6) - c*(-3*A*c + 2*B*b)*log(x)/b**4 + c*(-3*A*c + 2*B*b
)*log(b/c + x**2)/(2*b**4)
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^2}{x\,(bx^2+cx^4)^2}\,dx = -\frac{2\,(2\,Bbc-3\,Ac^2)x^4+Ab^2+(2\,Bb^2-3\,Abc)x^2}{4\,(b^3cx^6+b^4x^4)} + \frac{(2\,Bbc-3\,Ac^2)\log{(cx^2+b)}}{2\,b^4} - \frac{(2\,Bbc-3\,Ac^2)\log{(x^2)}}{2\,b^4}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*(2*B*b*c - 3*A*c^2)*x^4 + A*b^2 + (2*B*b^2 - 3*A*b*c)*x^2)/(b^3*c*
x^6 + b^4*x^4) + 1/2*(2*B*b*c - 3*A*c^2)*log(c*x^2 + b)/b^4 - 1/2*(2*B*b*c
 - 3*A*c^2)*log(x^2)/b^4
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.55

$$\int \frac{A+Bx^2}{x\,(bx^2+cx^4)^2}\,dx = -\frac{(2\,Bbc-3\,Ac^2)\log{(x^2)}}{2\,b^4} + \frac{(2\,Bbc^2-3\,Ac^3)\log{(|cx^2+b|)}}{2\,b^4c} - \frac{2\,Bbc^2x^2-3\,Ac^3x^2+3\,Bb^2c-4\,Abc^2}{2\,(cx^2+b)b^4} + \frac{6\,Bbcx^4-9\,Ac^2x^4-2\,Bb^2x^2+4\,Abcx^2-Ab^2}{4\,b^4x^4}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-1/2*(2*B*b*c - 3*A*c^2)*log(x^2)/b^4 + 1/2*(2*B*b*c^2 - 3*A*c^3)*log(abs(
c*x^2 + b))/(b^4*c) - 1/2*(2*B*b*c^2*x^2 - 3*A*c^3*x^2 + 3*B*b^2*c - 4*A*b
*c^2)/((c*x^2 + b)*b^4) + 1/4*(6*B*b*c*x^4 - 9*A*c^2*x^4 - 2*B*b^2*x^2 + 4
*A*b*c*x^2 - A*b^2)/(b^4*x^4)
```

3.71.9 Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2}\,dx=\frac{\frac{x^2\,\left(3\,A\,c-2\,B\,b\right)}{4\,b^2}-\frac{A}{4\,b}+\frac{c\,x^4\,\left(3\,A\,c-2\,B\,b\right)}{2\,b^3}}{c\,x^6+b\,x^4}-\frac{\ln\left(c\,x^2+b\right)\,\left(3\,A\,c^2-2\,B\,b\,c\right)}{2\,b^4}+\frac{\ln\left(x\right)\,\left(3\,A\,c^2-2\,B\,b\,c\right)}{b^4}$$

input
```
int((A + B*x^2)/(x*(b*x^2 + c*x^4)^2),x)
```

output
```
((x^2*(3*A*c - 2*B*b))/(4*b^2) - A/(4*b) + (c*x^4*(3*A*c - 2*B*b))/(2*b^3)
)/(b*x^4 + c*x^6) - (log(b + c*x^2)*(3*A*c^2 - 2*B*b*c))/(2*b^4) + (log(x)
*(3*A*c^2 - 2*B*b*c))/b^4
```

3.72 $\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2}\,dx$

3.72.1 Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2}\,dx=-\frac{A}{5b^2x^5}-\frac{bB-2Ac}{3b^3x^3}+\frac{c(2bB-3Ac)}{b^4x}+\frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)}+\frac{c^{3/2}(5bB-7Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}}$$

output
```
-1/5*A/b^2/x^5+1/3*(2*A*c-B*b)/b^3/x^3+c*(-3*A*c+2*B*b)/b^4/x+1/2*c^2*(-A*
c+B*b)*x/b^4/(c*x^2+b)+1/2*c^(3/2)*(-7*A*c+5*B*b)*arctan(x*c^(1/2)/b^(1/2)
)/b^(9/2)
```

3.72.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2}\,dx=-\frac{A}{5b^2x^5}+\frac{-bB+2Ac}{3b^3x^3}+\frac{c(2bB-3Ac)}{b^4x}+\frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)}+\frac{c^{3/2}(5bB-7Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}}$$

input
```
Integrate[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)^2),x]
```

output
```
-1/5*A/(b^2*x^5) + (-(b*B) + 2*A*c)/(3*b^3*x^3) + (c*(2*b*B - 3*A*c))/(b^4
*x) + (c^2*(b*B - A*c)*x)/(2*b^4*(b + c*x^2)) + (c^(3/2)*(5*b*B - 7*A*c)*A
rcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(9/2))
```

3.72.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {9, 361, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{A+Bx^2}{x^6\left(b+cx^2\right)^2}dx$$

$$\Big\downarrow \text{ 361}$$

$$\frac{c^2x(bB-Ac)}{2b^4\left(b+cx^2\right)}-\frac{1}{2}c^2\int -\frac{\frac{(bB-Ac)x^6}{b^4}-\frac{2(bB-Ac)x^4}{b^3c}+\frac{2(bB-Ac)x^2}{b^2c^2}+\frac{2A}{bc^2}}{x^6\left(cx^2+b\right)}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}c^2\int \frac{\frac{(bB-Ac)x^6}{b^4}-\frac{2(bB-Ac)x^4}{b^3c}+\frac{2(bB-Ac)x^2}{b^2c^2}+\frac{2A}{bc^2}}{x^6\left(cx^2+b\right)}dx+\frac{c^2x(bB-Ac)}{2b^4\left(b+cx^2\right)}$$

$$\Big\downarrow \text{ 2333}$$

$$\frac{1}{2}c^2\int \left(\frac{2A}{b^2c^2x^6}+\frac{5bB-7Ac}{b^4\left(cx^2+b\right)}-\frac{2(2bB-3Ac)}{b^4cx^2}+\frac{2(bB-2Ac)}{b^3c^2x^4}\right)dx+\frac{c^2x(bB-Ac)}{2b^4\left(b+cx^2\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}c^2\left(\frac{(5bB-7Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{9/2}\sqrt{c}}+\frac{2(2bB-3Ac)}{b^4cx}-\frac{2(bB-2Ac)}{3b^3c^2x^3}-\frac{2A}{5b^2c^2x^5}\right)+\frac{c^2x(bB-Ac)}{2b^4\left(b+cx^2\right)}$$

input
```
Int[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)^2),x]
```

output
```
(c^2*(b*B - A*c)*x)/(2*b^4*(b + c*x^2)) + (c^2*((-2*A)/(5*b^2*c^2*x^5) - (
2*(b*B - 2*A*c))/(3*b^3*c^2*x^3) + (2*(2*b*B - 3*A*c))/(b^4*c*x) + ((5*b*B
 - 7*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(b^(9/2)*Sqrt[c])))/2
```

3.72.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.72.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.89

method	result
default	$-\frac{A}{5b^2x^5}-\frac{-2Ac+Bb}{3x^3b^3}-\frac{c(3Ac-2Bb)}{b^4x}-\frac{c^2\left(\frac{\left(\frac{Ac}{2}-\frac{Bb}{2}\right)x}{c\,x^2+b}+\frac{(7Ac-5Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{b^4}$
risch	$\frac{-\frac{c^2(7Ac-5Bb)x^6}{2b^4}-\frac{c(7Ac-5Bb)x^4}{3b^3}+\frac{(7Ac-5Bb)x^2}{15b^2}-\frac{A}{5b}}{x^5(c\,x^2+b)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^9\_Z^2+49A^2c^5-70ABb\,c^4+25B^2b^2c^3\right)}\_R\ln\left(\left(3\_R^2b^9+98\right.\right.\right.}{4}$

input
```
int((B*x^2+A)/x^2/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5*A/b^2/x^5-1/3*(-2*A*c+B*b)/x^3/b^3-c*(3*A*c-2*B*b)/b^4/x-1/b^4*c^2*((
1/2*A*c-1/2*B*b)*x/(c*x^2+b)+1/2*(7*A*c-5*B*b)/(b*c)^(1/2)*arctan(c*x/(b*c
)^(1/2)))
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.77

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2}\,dx$$
$$=\left[\frac{30\,(5\,Bbc^2-7\,Ac^3)x^6+20\,(5\,Bb^2c-7\,Abc^2)x^4-12\,Ab^3-4\,(5\,Bb^3-7\,Ab^2c)x^2-15\,((5\,Bbc^2-7\,Ac^3)\right.}{60\,(b^4cx^7+b^5x^5)}$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
[1/60*(30*(5*B*b*c^2 - 7*A*c^3)*x^6 + 20*(5*B*b^2*c - 7*A*b*c^2)*x^4 - 12*
A*b^3 - 4*(5*B*b^3 - 7*A*b^2*c)*x^2 - 15*((5*B*b*c^2 - 7*A*c^3)*x^7 + (5*B
*b^2*c - 7*A*b*c^2)*x^5)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*
x^2 + b)))/(b^4*c*x^7 + b^5*x^5), 1/30*(15*(5*B*b*c^2 - 7*A*c^3)*x^6 + 10*
(5*B*b^2*c - 7*A*b*c^2)*x^4 - 6*A*b^3 - 2*(5*B*b^3 - 7*A*b^2*c)*x^2 + 15*(
(5*B*b*c^2 - 7*A*c^3)*x^7 + (5*B*b^2*c - 7*A*b*c^2)*x^5)*sqrt(c/b)*arctan(
x*sqrt(c/b)))/(b^4*c*x^7 + b^5*x^5)]
```

3.72.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(104) = 208$.

Time = 0.36 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.96

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2}\,dx$$
$$= -\frac{\sqrt{-\frac{c^3}{b^9}}(-7Ac+5Bb)\log\left(-\frac{b^5\sqrt{-\frac{c^3}{b^9}}(-7Ac+5Bb)}{-7Ac^3+5Bbc^2}+x\right)}{4}$$
$$+\frac{\sqrt{-\frac{c^3}{b^9}}(-7Ac+5Bb)\log\left(\frac{b^5\sqrt{-\frac{c^3}{b^9}}(-7Ac+5Bb)}{-7Ac^3+5Bbc^2}+x\right)}{4}$$
$$+\frac{-6Ab^3+x^6(-105Ac^3+75Bbc^2)+x^4(-70Abc^2+50Bb^2c)+x^2\cdot(14Ab^2c-10Bb^3)}{30b^5x^5+30b^4cx^7}$$

input
```
integrate((B*x**2+A)/x**2/(c*x**4+b*x**2)**2,x)
```

output
```
-sqrt(-c**3/b**9)*(-7*A*c + 5*B*b)*log(-b**5*sqrt(-c**3/b**9)*(-7*A*c + 5*
B*b)/(-7*A*c**3 + 5*B*b*c**2) + x)/4 + sqrt(-c**3/b**9)*(-7*A*c + 5*B*b)*l
og(b**5*sqrt(-c**3/b**9)*(-7*A*c + 5*B*b)/(-7*A*c**3 + 5*B*b*c**2) + x)/4
+ (-6*A*b**3 + x**6*(-105*A*c**3 + 75*B*b*c**2) + x**4*(-70*A*b*c**2 + 50*
B*b**2*c) + x**2*(14*A*b**2*c - 10*B*b**3))/(30*b**5*x**5 + 30*b**4*c*x**7
)
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2}\,dx$$
$$= \frac{15\,(5\,Bbc^2-7\,Ac^3)x^6+10\,(5\,Bb^2c-7\,Abc^2)x^4-6\,Ab^3-2\,(5\,Bb^3-7\,Ab^2c)x^2}{30\,(b^4cx^7+b^5x^5)}$$
$$+\frac{(5\,Bbc^2-7\,Ac^3)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^4}$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/30*(15*(5*B*b*c^2 - 7*A*c^3)*x^6 + 10*(5*B*b^2*c - 7*A*b*c^2)*x^4 - 6*A*
b^3 - 2*(5*B*b^3 - 7*A*b^2*c)*x^2)/(b^4*c*x^7 + b^5*x^5) + 1/2*(5*B*b*c^2
- 7*A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4)
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2}{x^2(bx^2+cx^4)^2}\,dx = \frac{(5\,Bbc^2-7\,Ac^3)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^4} + \frac{Bbc^2x-Ac^3x}{2\,(cx^2+b)b^4} + \frac{30\,Bbcx^4-45\,Ac^2x^4-5\,Bb^2x^2+10\,Abcx^2-3\,Ab^2}{15\,b^4x^5}$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/2*(5*B*b*c^2 - 7*A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) + 1/2*(B*b
*c^2*x - A*c^3*x)/((c*x^2 + b)*b^4) + 1/15*(30*B*b*c*x^4 - 45*A*c^2*x^4 -
5*B*b^2*x^2 + 10*A*b*c*x^2 - 3*A*b^2)/(b^4*x^5)
```

3.72.9 Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{x^2(bx^2+cx^4)^2}\,dx = -\frac{\frac{A}{5\,b}-\frac{x^2\,(7\,A\,c-5\,B\,b)}{15\,b^2}+\frac{c^2\,x^6\,(7\,A\,c-5\,B\,b)}{2\,b^4}+\frac{c\,x^4\,(7\,A\,c-5\,B\,b)}{3\,b^3}}{c\,x^7+b\,x^5} - \frac{c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(7\,A\,c-5\,B\,b)}{2\,b^{9/2}}$$

input
```
int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)^2),x)
```

output
```
- (A/(5*b) - (x^2*(7*A*c - 5*B*b))/(15*b^2) + (c^2*x^6*(7*A*c - 5*B*b))/(2
*b^4) + (c*x^4*(7*A*c - 5*B*b))/(3*b^3))/(b*x^5 + c*x^7) - (c^(3/2)*atan((
c^(1/2)*x)/b^(1/2))*(7*A*c - 5*B*b))/(2*b^(9/2))
```

3.73 $\int \frac{x^{14}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.73.1 Optimal result

Integrand size = 24, antiderivative size = 140

$$\int \frac{x^{14}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3b(2bB-Ac)x}{c^5} - \frac{(3bB-Ac)x^3}{3c^4} + \frac{Bx^5}{5c^3} - \frac{b^3(bB-Ac)x}{4c^5(b+cx^2)^2} + \frac{b^2(17bB-13Ac)x}{8c^5(b+cx^2)} - \frac{7b^{3/2}(9bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{11/2}}$$

output
```
3*b*(-A*c+2*B*b)*x/c^5-1/3*(-A*c+3*B*b)*x^3/c^4+1/5*B*x^5/c^3-1/4*b^3*(-A*
c+B*b)*x/c^5/(c*x^2+b)^2+1/8*b^2*(-13*A*c+17*B*b)*x/c^5/(c*x^2+b)-7/8*b^(3
/2)*(-5*A*c+9*B*b)*arctan(x*c^(1/2)/b^(1/2))/c^(11/2)
```

3.73.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.95

$$\int \frac{x^{14}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{x(945b^4B-525b^3c(A-3Bx^2)+8c^4x^6(5A+3Bx^2)-8bc^3x^4(35A+9Bx^2)+7b^2c^2x^2(-125A+72Bx^2)}{120c^5(b+cx^2)^2} - \frac{7b^{3/2}(9bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{11/2}}$$

input
```
Integrate[(x^14*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
(x*(945*b^4*B - 525*b^3*c*(A - 3*B*x^2) + 8*c^4*x^6*(5*A + 3*B*x^2) - 8*b*
c^3*x^4*(35*A + 9*B*x^2) + 7*b^2*c^2*x^2*(-125*A + 72*B*x^2)))/(120*c^5*(b
 + c*x^2)^2) - (7*b^(3/2)*(9*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*
c^(11/2))
```

3.73.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {9, 360, 25, 2345, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{x^8\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}dx$$

$$\Big\downarrow \text{360}$$

$$-\frac{\int -\frac{4Bc^4x^8-4c^3(bB-Ac)x^6+4bc^2(bB-Ac)x^4-4b^2c(bB-Ac)x^2+b^3(bB-Ac)}{\left(cx^2+b\right)^2}dx}{4c^5}-\frac{b^3x(bB-Ac)}{4c^5\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4Bc^4x^8-4c^3(bB-Ac)x^6+4bc^2(bB-Ac)x^4-4b^2c(bB-Ac)x^2+b^3(bB-Ac)}{\left(cx^2+b\right)^2}dx}{4c^5}-\frac{b^3x(bB-Ac)}{4c^5\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{b^2x(17bB-13Ac)}{2\left(b+cx^2\right)}-\frac{\int \frac{-8bBc^3x^6+8bc^2(2bB-Ac)x^4-8b^2c(3bB-2Ac)x^2+b^3(15bB-11Ac)}{cx^2+b}dx}{2b}}{4c^5}-\frac{b^3x(bB-Ac)}{4c^5\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{2341}$$

$$\frac{\frac{b^2x(17bB-13Ac)}{2\left(b+cx^2\right)}-\frac{\int \left(-8bBc^2x^4+8bc(3bB-Ac)x^2-24b^2(2bB-Ac)+\frac{7\left(9b^4B-5Ab^3c\right)}{cx^2+b}\right)dx}{2b}}{4c^5}-\frac{b^3x(bB-Ac)}{4c^5\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2x(17bB-13Ac)}{2(b+cx^2)}-\frac{\frac{7b^{5/2}(9bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{c}}-24b^2x(2bB-Ac)+\frac{8}{3}bcx^3(3bB-Ac)-\frac{8}{5}bBc^2x^5}{2b}}{4c^5}-\frac{b^3x(bB-Ac)}{4c^5\left(b+cx^2\right)^2}$$

input
```
Int[(x^14*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*(b^3*(b*B - A*c)*x)/(c^5*(b + c*x^2)^2) + ((b^2*(17*b*B - 13*A*c)*x)/
(2*(b + c*x^2)) - (-24*b^2*(2*b*B - A*c)*x + (8*b*c*(3*b*B - A*c)*x^3)/3 -
 (8*b*B*c^2*x^5)/5 + (7*b^(5/2)*(9*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]
])/Sqrt[c])/(2*b))/(4*c^5)
```

3.73.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.73.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.85

method	result
default	$-\frac{-\frac{1}{5}B\,c^2x^5-\frac{1}{3}A\,c^2x^3+Bbc\,x^3+3Abcx-6b^2Bx}{c^5}+\frac{b^2\left(\frac{\left(-\frac{13}{8}A\,c^2+\frac{17}{8}Bbc\right)x^3-\frac{b(11Ac-15Bb)x}{8}}{\left(c\,x^2+b\right)^2}+\frac{7(5Ac-9Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{c^5}$
risch	$\frac{B\,x^5}{5c^3}+\frac{A\,x^3}{3c^3}-\frac{Bb\,x^3}{c^4}-\frac{3Abx}{c^4}+\frac{6b^2Bx}{c^5}+\frac{\left(-\frac{13}{8}b^2A\,c^2+\frac{17}{8}B\,b^3c\right)x^3-\frac{b^3(11Ac-15Bb)x}{8}}{c^5(c\,x^2+b)^2}+\frac{35\sqrt{-bc}\,b\ln(-\sqrt{-bc}\,x+b)A}{16c^5}-$

input
```
int(x^14*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/c^5*(-1/5*B*c^2*x^5-1/3*A*c^2*x^3+B*b*c*x^3+3*A*b*c*x-6*b^2*B*x)+b^2/c^
5*(((-13/8*A*c^2+17/8*B*b*c)*x^3-1/8*b*(11*A*c-15*B*b)*x)/(c*x^2+b)^2+7/8*
(5*A*c-9*B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.97

$$\int \frac{x^{14}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$

$$=\left[\frac{48\,Bc^4x^9-16\,(9\,Bbc^3-5\,Ac^4)x^7+112\,(9\,Bb^2c^2-5\,Abc^3)x^5+350\,(9\,Bb^3c-5\,Ab^2c^2)x^3-105\,(9\,Bb^4-}{240\,(c^7x^4-}\right.$$

input
```
integrate(x^14*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
[1/240*(48*B*c^4*x^9 - 16*(9*B*b*c^3 - 5*A*c^4)*x^7 + 112*(9*B*b^2*c^2 - 5
*A*b*c^3)*x^5 + 350*(9*B*b^3*c - 5*A*b^2*c^2)*x^3 - 105*(9*B*b^4 - 5*A*b^3
*c + (9*B*b^2*c^2 - 5*A*b*c^3)*x^4 + 2*(9*B*b^3*c - 5*A*b^2*c^2)*x^2)*sqrt
(-b/c)*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 210*(9*B*b^4 - 5*
A*b^3*c)*x)/(c^7*x^4 + 2*b*c^6*x^2 + b^2*c^5), 1/120*(24*B*c^4*x^9 - 8*(9*
B*b*c^3 - 5*A*c^4)*x^7 + 56*(9*B*b^2*c^2 - 5*A*b*c^3)*x^5 + 175*(9*B*b^3*c
 - 5*A*b^2*c^2)*x^3 - 105*(9*B*b^4 - 5*A*b^3*c + (9*B*b^2*c^2 - 5*A*b*c^3)
*x^4 + 2*(9*B*b^3*c - 5*A*b^2*c^2)*x^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b)
+ 105*(9*B*b^4 - 5*A*b^3*c)*x)/(c^7*x^4 + 2*b*c^6*x^2 + b^2*c^5)]
```

3.73.6 Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.80

$$\int \frac{x^{14}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{Bx^5}{5c^3}+x^3\left(\frac{A}{3c^3}-\frac{Bb}{c^4}\right)+x\left(-\frac{3Ab}{c^4}+\frac{6Bb^2}{c^5}\right) + \frac{7\sqrt{-\frac{b^3}{c^{11}}}(-5Ac+9Bb)\log\left(-\frac{7c^5\sqrt{-\frac{b^3}{c^{11}}}(-5Ac+9Bb)}{-35Abc+63Bb^2}+x\right)}{16} - \frac{7\sqrt{-\frac{b^3}{c^{11}}}(-5Ac+9Bb)\log\left(\frac{7c^5\sqrt{-\frac{b^3}{c^{11}}}(-5Ac+9Bb)}{-35Abc+63Bb^2}+x\right)}{16} + \frac{x^3(-13Ab^2c^2+17Bb^3c)+x(-11Ab^3c+15Bb^4)}{8b^2c^5+16bc^6x^2+8c^7x^4}$$

input
```
integrate(x**14*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
B*x**5/(5*c**3) + x**3*(A/(3*c**3) - B*b/c**4) + x*(-3*A*b/c**4 + 6*B*b**2
/c**5) + 7*sqrt(-b**3/c**11)*(-5*A*c + 9*B*b)*log(-7*c**5*sqrt(-b**3/c**11
)*(-5*A*c + 9*B*b)/(-35*A*b*c + 63*B*b**2) + x)/16 - 7*sqrt(-b**3/c**11)*(
-5*A*c + 9*B*b)*log(7*c**5*sqrt(-b**3/c**11)*(-5*A*c + 9*B*b)/(-35*A*b*c +
 63*B*b**2) + x)/16 + (x**3*(-13*A*b**2*c**2 + 17*B*b**3*c) + x*(-11*A*b**
3*c + 15*B*b**4))/(8*b**2*c**5 + 16*b*c**6*x**2 + 8*c**7*x**4)
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05

$$\int \frac{x^{14}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(17\,Bb^3c-13\,Ab^2c^2)x^3+(15\,Bb^4-11\,Ab^3c)x}{8\,(c^7x^4+2\,bc^6x^2+b^2c^5)} - \frac{7\,(9\,Bb^3-5\,Ab^2c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^5} + \frac{3\,Bc^2x^5-5\,(3\,Bbc-Ac^2)x^3+45\,(2\,Bb^2-Abc)x}{15\,c^5}$$

input
```
integrate(x^14*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/8*((17*B*b^3*c - 13*A*b^2*c^2)*x^3 + (15*B*b^4 - 11*A*b^3*c)*x)/(c^7*x^4
 + 2*b*c^6*x^2 + b^2*c^5) - 7/8*(9*B*b^3 - 5*A*b^2*c)*arctan(c*x/sqrt(b*c)
)/(sqrt(b*c)*c^5) + 1/15*(3*B*c^2*x^5 - 5*(3*B*b*c - A*c^2)*x^3 + 45*(2*B*
b^2 - A*b*c)*x)/c^5
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99

$$\int \frac{x^{14}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{7\,(9\,Bb^3-5\,Ab^2c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^5} + \frac{17\,Bb^3cx^3-13\,Ab^2c^2x^3+15\,Bb^4x-11\,Ab^3cx}{8\,(cx^2+b)^2c^5} + \frac{3\,Bc^{12}x^5-15\,Bbc^{11}x^3+5\,Ac^{12}x^3+90\,Bb^2c^{10}x-45\,Abc^{11}x}{15\,c^{15}}$$

input
```
integrate(x^14*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-7/8*(9*B*b^3 - 5*A*b^2*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) + 1/8*(17
*B*b^3*c*x^3 - 13*A*b^2*c^2*x^3 + 15*B*b^4*x - 11*A*b^3*c*x)/((c*x^2 + b)^
2*c^5) + 1/15*(3*B*c^12*x^5 - 15*B*b*c^11*x^3 + 5*A*c^12*x^3 + 90*B*b^2*c^
10*x - 45*A*b*c^11*x)/c^15
```

3.73.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.26

$$\int \frac{x^{14}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{x\left(\frac{15\,B\,b^4}{8}-\frac{11\,A\,b^3\,c}{8}\right)-x^3\left(\frac{13\,A\,b^2\,c^2}{8}-\frac{17\,B\,b^3\,c}{8}\right)}{b^2\,c^5+2\,b\,c^6\,x^2+c^7\,x^4} - x\left(\frac{3\,b\left(\frac{A}{c^3}-\frac{3\,B\,b}{c^4}\right)}{c}+\frac{3\,B\,b^2}{c^5}\right)+x^3\left(\frac{A}{3\,c^3}-\frac{B\,b}{c^4}\right) + \frac{B\,x^5}{5\,c^3}-\frac{7\,b^{3/2}\,\mathrm{atan}\left(\frac{b^{3/2}\,\sqrt{c}\,x\,(5\,A\,c-9\,B\,b)}{9\,B\,b^3-5\,A\,b^2\,c}\right)\,(5\,A\,c-9\,B\,b)}{8\,c^{11/2}}$$

input
```
int((x^14*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
(x*((15*B*b^4)/8 - (11*A*b^3*c)/8) - x^3*((13*A*b^2*c^2)/8 - (17*B*b^3*c)/
8))/(b^2*c^5 + c^7*x^4 + 2*b*c^6*x^2) - x*((3*b*(A/c^3 - (3*B*b)/c^4))/c +
 (3*B*b^2)/c^5) + x^3*(A/(3*c^3) - (B*b)/c^4) + (B*x^5)/(5*c^3) - (7*b^(3/
2)*atan((b^(3/2)*c^(1/2)*x*(5*A*c - 9*B*b))/(9*B*b^3 - 5*A*b^2*c))*(5*A*c
- 9*B*b))/(8*c^(11/2))
```

3.74 $\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.74.1 Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{(3bB-Ac)x^2}{2c^4}+\frac{Bx^4}{4c^3}-\frac{b^3(bB-Ac)}{4c^5(b+cx^2)^2} + \frac{b^2(4bB-3Ac)}{2c^5(b+cx^2)}+\frac{3b(2bB-Ac)\log(b+cx^2)}{2c^5}$$

output
```
-1/2*(-A*c+3*B*b)*x^2/c^4+1/4*B*x^4/c^3-1/4*b^3*(-A*c+B*b)/c^5/(c*x^2+b)^2
+1/2*b^2*(-3*A*c+4*B*b)/c^5/(c*x^2+b)+3/2*b*(-A*c+2*B*b)*ln(c*x^2+b)/c^5
```

3.74.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{2c(-3bB+Ac)x^2+Bc^2x^4+\frac{b^3(-bB+Ac)}{(b+cx^2)^2}+\frac{2b^2(4bB-3Ac)}{b+cx^2}+6b(2bB-Ac)\log(b+cx^2)}{4c^5}$$

input
```
Integrate[(x^13*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
(2*c*(-3*b*B + A*c)*x^2 + B*c^2*x^4 + (b^3*(-(b*B) + A*c))/(b + c*x^2)^2 +
 (2*b^2*(4*b*B - 3*A*c))/(b + c*x^2) + 6*b*(2*b*B - A*c)*Log[b + c*x^2])/(
4*c^5)
```

3.74.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13} \left(A+B x^2\right)}{\left(b x^2+c x^4\right)^3} \, dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^7 \left(A+B x^2\right)}{\left(b+c x^2\right)^3}dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2} \int \frac{x^6 \left(B x^2+A\right)}{\left(c x^2+b\right)^3}dx^2$$

$$\Big\downarrow \ 86$$

$$\frac{1}{2} \int \left(\frac{(b B-A c) b^3}{c^4 \left(c x^2+b\right)^3}-\frac{(4 b B-3 A c) b^2}{c^4 \left(c x^2+b\right)^2}+\frac{3 (2 b B-A c) b}{c^4 \left(c x^2+b\right)}+\frac{B x^2}{c^3}+\frac{A c-3 b B}{c^4}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2} \left(-\frac{b^3 (b B-A c)}{2 c^5 \left(b+c x^2\right)^2}+\frac{b^2 (4 b B-3 A c)}{c^5 \left(b+c x^2\right)}+\frac{3 b (2 b B-A c) \log \left(b+c x^2\right)}{c^5}-\frac{x^2 (3 b B-A c)}{c^4}+\frac{B x^4}{2 c^3}\right)$$

input
```
Int[(x^13*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
(-(((3*b*B - A*c)*x^2)/c^4) + (B*x^4)/(2*c^3) - (b^3*(b*B - A*c))/(2*c^5*(
b + c*x^2)^2) + (b^2*(4*b*B - 3*A*c))/(c^5*(b + c*x^2)) + (3*b*(2*b*B - A*
c)*Log[b + c*x^2])/c^5)/2
```

3.74.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.74.4 Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.92

method	result
default	$\frac{\left(B c\,x^{2}+A c-3 B b\right)^{2}}{4 c^{5} B}-\frac{b\left(\frac{\left(3 A c-6 B b\right) \ln\left(c\,x^{2}+b\right)}{c}-\frac{b^{2} \left(A c-B b\right)}{2 c \left(c\,x^{2}+b\right)^{2}}+\frac{b \left(3 A c-4 B b\right)}{c \left(c\,x^{2}+b\right)}\right)}{2 c^{4}}$
norman	$\frac{\frac{B\,x^{13}}{4 c}+\frac{\left(A c-2 B b\right) x^{11}}{2 c^{2}}-\frac{b\left(3 A b c-6 B\,b^{2}\right) x^{7}}{c^{4}}-\frac{b^{2} \left(9 A b c-18 B\,b^{2}\right) x^{5}}{4 c^{5}}}{x^{5} \left(c\,x^{2}+b\right)^{2}}-\frac{3 b \left(A c-2 B b\right) \ln\left(c\,x^{2}+b\right)}{2 c^{5}}$
risch	$\frac{B\,x^{4}}{4 c^{3}}+\frac{A\,x^{2}}{2 c^{3}}-\frac{3 B b\,x^{2}}{2 c^{4}}+\frac{A^{2}}{4 c^{3} B}-\frac{3 A b}{2 c^{4}}+\frac{9 B\,b^{2}}{4 c^{5}}+\frac{\left(-\frac{3}{2} b^{2} A c+2 B\,b^{3}\right) x^{2}-\frac{b^{3} \left(5 A c-7 B b\right)}{4 c}}{c^{4} \left(c\,x^{2}+b\right)^{2}}-\frac{3 b \ln\left(c\,x^{2}+b\right) A}{2 c^{4}}+\frac{3 b^{2} \ln($
parallelrisch	$-\frac{-B\,x^{8} c^{4}-2 A\,x^{6} c^{4}+4 B\,x^{6} b\,c^{3}+6 A \ln\left(c\,x^{2}+b\right) x^{4} b\,c^{3}-12 B \ln\left(c\,x^{2}+b\right) x^{4} b^{2} c^{2}+12 A \ln\left(c\,x^{2}+b\right) x^{2} b^{2} c^{2}-24 B \ln\left(c\,x^{2}+b\right) x^{2} b^{3} c}{4 c^{5} \left(c\,x^{2}+b\right)^{2}}$

input
```
int(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(B*c*x^2+A*c-3*B*b)^2/c^5/B-1/2*b/c^4*((3*A*c-6*B*b)/c*ln(c*x^2+b)-1/2
*b^2*(A*c-B*b)/c/(c*x^2+b)^2+b*(3*A*c-4*B*b)/c/(c*x^2+b))
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.61

$$\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$
$$= \frac{Bc^4x^8 - 2\,(2\,Bbc^3 - Ac^4)x^6 + 7\,Bb^4 - 5\,Ab^3c - (11\,Bb^2c^2 - 4\,Abc^3)x^4 + 2\,(Bb^3c - 2\,Ab^2c^2)x^2 + 6\,(2\,Bb^4}{4\,(c^7x^4 + 2\,bc^6x^2 + b^2c^5)}$$

input
```
integrate(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/4*(B*c^4*x^8 - 2*(2*B*b*c^3 - A*c^4)*x^6 + 7*B*b^4 - 5*A*b^3*c - (11*B*b
^2*c^2 - 4*A*b*c^3)*x^4 + 2*(B*b^3*c - 2*A*b^2*c^2)*x^2 + 6*(2*B*b^4 - A*b
^3*c + (2*B*b^2*c^2 - A*b*c^3)*x^4 + 2*(2*B*b^3*c - A*b^2*c^2)*x^2)*log(c*
x^2 + b))/(c^7*x^4 + 2*b*c^6*x^2 + b^2*c^5)
```

3.74.6 Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07

$$\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{Bx^4}{4c^3} + \frac{3b(-Ac+2Bb)\log{(b+cx^2)}}{2c^5} + x^2\left(\frac{A}{2c^3} - \frac{3Bb}{2c^4}\right) + \frac{-5Ab^3c + 7Bb^4 + x^2(-6Ab^2c^2 + 8Bb^3c)}{4b^2c^5 + 8bc^6x^2 + 4c^7x^4}$$

input
```
integrate(x**13*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
B*x**4/(4*c**3) + 3*b*(-A*c + 2*B*b)*log(b + c*x**2)/(2*c**5) + x**2*(A/(2
*c**3) - 3*B*b/(2*c**4)) + (-5*A*b**3*c + 7*B*b**4 + x**2*(-6*A*b**2*c**2
+ 8*B*b**3*c))/(4*b**2*c**5 + 8*b*c**6*x**2 + 4*c**7*x**4)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{7\,Bb^4-5\,Ab^3c+2\,(4\,Bb^3c-3\,Ab^2c^2)x^2}{4\,(c^7x^4+2\,bc^6x^2+b^2c^5)} + \frac{Bcx^4-2\,(3\,Bb-Ac)x^2}{4\,c^4} + \frac{3\,(2\,Bb^2-Abc)\log{(cx^2+b)}}{2\,c^5}$$

input
```
integrate(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/4*(7*B*b^4 - 5*A*b^3*c + 2*(4*B*b^3*c - 3*A*b^2*c^2)*x^2)/(c^7*x^4 + 2*b
*c^6*x^2 + b^2*c^5) + 1/4*(B*c*x^4 - 2*(3*B*b - A*c)*x^2)/c^4 + 3/2*(2*B*b
^2 - A*b*c)*log(c*x^2 + b)/c^5
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.19

$$\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3\,(2\,Bb^2-Abc)\log{(|cx^2+b|)}}{2\,c^5} + \frac{Bc^3x^4-6\,Bbc^2x^2+2\,Ac^3x^2}{4\,c^6} - \frac{18\,Bb^2c^2x^4-9\,Abc^3x^4+28\,Bb^3cx^2-12\,Ab^2c^2x^2+11\,Bb^4-4\,Ab^3c}{4\,(cx^2+b)^2c^5}$$

input
```
integrate(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
3/2*(2*B*b^2 - A*b*c)*log(abs(c*x^2 + b))/c^5 + 1/4*(B*c^3*x^4 - 6*B*b*c^2
*x^2 + 2*A*c^3*x^2)/c^6 - 1/4*(18*B*b^2*c^2*x^4 - 9*A*b*c^3*x^4 + 28*B*b^3
*c*x^2 - 12*A*b^2*c^2*x^2 + 11*B*b^4 - 4*A*b^3*c)/((c*x^2 + b)^2*c^5)
```

3.74.9 Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.06

$$\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{7\,B\,b^4-5\,A\,b^3\,c}{4\,c}+x^2\,\left(2\,B\,b^3-\frac{3\,A\,b^2\,c}{2}\right)}{b^2\,c^4+2\,b\,c^5\,x^2+c^6\,x^4} + x^2\,\left(\frac{A}{2\,c^3}-\frac{3\,B\,b}{2\,c^4}\right) + \frac{\ln\left(c\,x^2+b\right)\,\left(6\,B\,b^2-3\,A\,b\,c\right)}{2\,c^5} + \frac{B\,x^4}{4\,c^3}$$

input
```
int((x^13*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
((7*B*b^4 - 5*A*b^3*c)/(4*c) + x^2*(2*B*b^3 - (3*A*b^2*c)/2))/(b^2*c^4 + c
^6*x^4 + 2*b*c^5*x^2) + x^2*(A/(2*c^3) - (3*B*b)/(2*c^4)) + (log(b + c*x^2
)*(6*B*b^2 - 3*A*b*c))/(2*c^5) + (B*x^4)/(4*c^3)
```

3.74. $\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.75 $\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3} dx$

3.75.1 Optimal result

Integrand size = 24, antiderivative size = 118

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3} dx = -\frac{(3bB-Ac)x}{c^4}+\frac{Bx^3}{3c^3}+\frac{b^2(bB-Ac)x}{4c^4(b+cx^2)^2} - \frac{b(13bB-9Ac)x}{8c^4(b+cx^2)}+\frac{5\sqrt{b}(7bB-3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}$$

output
```
-(-A*c+3*B*b)*x/c^4+1/3*B*x^3/c^3+1/4*b^2*(-A*c+B*b)*x/c^4/(c*x^2+b)^2-1/8
*b*(-9*A*c+13*B*b)*x/c^4/(c*x^2+b)+5/8*(-3*A*c+7*B*b)*arctan(x*c^(1/2)/b^(
1/2))*b^(1/2)/c^(9/2)
```

3.75.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3} dx = \frac{-105b^3Bx+bc^2x^3(75A-56Bx^2)+5b^2cx(9A-35Bx^2)+8c^3x^5(3A+Bx^2)}{24c^4(b+cx^2)^2} + \frac{5\sqrt{b}(7bB-3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}$$

input
```
Integrate[(x^12*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
(-105*b^3*B*x + b*c^2*x^3*(75*A - 56*B*x^2) + 5*b^2*c*x*(9*A - 35*B*x^2) +
 8*c^3*x^5*(3*A + B*x^2))/(24*c^4*(b + c*x^2)^2) + (5*Sqrt[b]*(7*b*B - 3*A
*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/2))
```

3.75.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {9, 360, 2345, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^6(A+Bx^2)}{(b+cx^2)^3}dx$$

$$\Big\downarrow \text{ 360}$$

$$\frac{b^2x(bB-Ac)}{4c^4(b+cx^2)^2}-\frac{\int \frac{-4Bc^3x^6+4c^2(bB-Ac)x^4-4bc(bB-Ac)x^2+b^2(bB-Ac)}{(cx^2+b)^2}dx}{4c^4}$$

$$\Big\downarrow \text{ 2345}$$

$$\frac{b^2x(bB-Ac)}{4c^4(b+cx^2)^2}-\frac{\frac{bx(13bB-9Ac)}{2(b+cx^2)}-\frac{\int \frac{8bBc^2x^4-8bc(2bB-Ac)x^2+b^2(11bB-7Ac)}{cx^2+b}dx}{2b}}{4c^4}$$

$$\Big\downarrow \text{ 1467}$$

$$\frac{b^2x(bB-Ac)}{4c^4(b+cx^2)^2}-\frac{\frac{bx(13bB-9Ac)}{2(b+cx^2)}-\frac{\int \left(8bBcx^2-8b(3bB-Ac)+\frac{5(7b^3B-3Ab^2c)}{cx^2+b}\right)dx}{2b}}{4c^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^2x(bB-Ac)}{4c^4(b+cx^2)^2}-\frac{\frac{bx(13bB-9Ac)}{2(b+cx^2)}-\frac{\frac{5b^{3/2}(7bB-3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{c}}-8bx(3bB-Ac)+\frac{8}{3}bBcx^3}{2b}}{4c^4}$$

input
```
Int[(x^12*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
(b^2*(b*B - A*c)*x)/(4*c^4*(b + c*x^2)^2) - ((b*(13*b*B - 9*A*c)*x)/(2*(b
+ c*x^2)) - (-8*b*(3*b*B - A*c)*x + (8*b*B*c*x^3)/3 + (5*b^(3/2)*(7*b*B -
3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/Sqrt[c])/(2*b))/(4*c^4)
```

3.75.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.75.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

method	result
default	$\frac{\frac{1}{3}Bc\,x^3+Acx-3bBx}{c^4}-\frac{b\left(\frac{\left(-\frac{9}{8}A\,c^2+\frac{13}{8}Bbc\right)x^3-\frac{b(7Ac-11Bb)x}{8}}{\left(c\,x^2+b\right)^2}+\frac{5(3Ac-7Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{c^4}$
risch	$\frac{B\,x^3}{3c^3}+\frac{Ax}{c^3}-\frac{3bBx}{c^4}+\frac{\left(\frac{9}{8}Ab\,c^2-\frac{13}{8}B\,b^2c\right)x^3+\frac{b^2(7Ac-11Bb)x}{8}}{c^4(c\,x^2+b)^2}+\frac{15\sqrt{-bc}\,\ln(-\sqrt{-bc}\,x-b)A}{16c^4}-\frac{35\sqrt{-bc}\,\ln(-\sqrt{-bc}\,x-b)Bb}{16c^5}$

input
```
int(x^12*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(1/3*B*c*x^3+A*c*x-3*b*B*x)-b/c^4*(((-9/8*A*c^2+13/8*B*b*c)*x^3-1/8*
b*(7*A*c-11*B*b)*x)/(c*x^2+b)^2+5/8*(3*A*c-7*B*b)/(b*c)^(1/2)*arctan(c*x/(
b*c)^(1/2)))
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.03

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$

$$= \left[\frac{16\,Bc^3x^7-16\,(7\,Bbc^2-3\,Ac^3)x^5-50\,(7\,Bb^2c-3\,Abc^2)x^3-15\,((7\,Bbc^2-3\,Ac^3)x^4+7\,Bb^3-3\,Ab^2c}{48\,(c^6x^4+2\,bc^5x^2+b^2c^4)}\right.$$

input
```
integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
[1/48*(16*B*c^3*x^7 - 16*(7*B*b*c^2 - 3*A*c^3)*x^5 - 50*(7*B*b^2*c - 3*A*b
*c^2)*x^3 - 15*((7*B*b*c^2 - 3*A*c^3)*x^4 + 7*B*b^3 - 3*A*b^2*c + 2*(7*B*b
^2*c - 3*A*b*c^2)*x^2)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^
2 + b)) - 30*(7*B*b^3 - 3*A*b^2*c)*x)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4), 1
/24*(8*B*c^3*x^7 - 8*(7*B*b*c^2 - 3*A*c^3)*x^5 - 25*(7*B*b^2*c - 3*A*b*c^2
)*x^3 + 15*((7*B*b*c^2 - 3*A*c^3)*x^4 + 7*B*b^3 - 3*A*b^2*c + 2*(7*B*b^2*c
 - 3*A*b*c^2)*x^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - 15*(7*B*b^3 - 3*A*b
^2*c)*x)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)]
```

3.75.6 Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.81

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{Bx^3}{3c^3} + x\left(\frac{A}{c^3} - \frac{3Bb}{c^4}\right) - \frac{5\sqrt{-\frac{b}{c^9}}(-3Ac+7Bb)\log\left(-\frac{5c^4\sqrt{-\frac{b}{c^9}}(-3Ac+7Bb)}{-15Ac+35Bb}+x\right)}{16} + \frac{5\sqrt{-\frac{b}{c^9}}(-3Ac+7Bb)\log\left(\frac{5c^4\sqrt{-\frac{b}{c^9}}(-3Ac+7Bb)}{-15Ac+35Bb}+x\right)}{16} + \frac{x^3\cdot(9Abc^2-13Bb^2c)+x(7Ab^2c-11Bb^3)}{8b^2c^4+16bc^5x^2+8c^6x^4}$$

input
```
integrate(x**12*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
B*x**3/(3*c**3) + x*(A/c**3 - 3*B*b/c**4) - 5*sqrt(-b/c**9)*(-3*A*c + 7*B*
b)*log(-5*c**4*sqrt(-b/c**9)*(-3*A*c + 7*B*b)/(-15*A*c + 35*B*b) + x)/16 +
 5*sqrt(-b/c**9)*(-3*A*c + 7*B*b)*log(5*c**4*sqrt(-b/c**9)*(-3*A*c + 7*B*b
)/(-15*A*c + 35*B*b) + x)/16 + (x**3*(9*A*b*c**2 - 13*B*b**2*c) + x*(7*A*b
**2c - 11*B*b**3))/(8*b**2*c**4 + 16*b*c**5*x**2 + 8*c**6*x**4)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.02

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{(13\,Bb^2c-9\,Abc^2)x^3+(11\,Bb^3-7\,Ab^2c)x}{8\,(c^6x^4+2\,bc^5x^2+b^2c^4)} + \frac{5\,(7\,Bb^2-3\,Abc)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^4} + \frac{Bcx^3-3\,(3\,Bb-Ac)x}{3\,c^4}$$

input
```
integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((13*B*b^2*c - 9*A*b*c^2)*x^3 + (11*B*b^3 - 7*A*b^2*c)*x)/(c^6*x^4 +
2*b*c^5*x^2 + b^2*c^4) + 5/8*(7*B*b^2 - 3*A*b*c)*arctan(c*x/sqrt(b*c))/(sq
rt(b*c)*c^4) + 1/3*(B*c*x^3 - 3*(3*B*b - A*c)*x)/c^4
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.94

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{5\,(7\,Bb^2-3\,Abc)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^4} - \frac{13\,Bb^2cx^3-9\,Abc^2x^3+11\,Bb^3x-7\,Ab^2cx}{8\,(cx^2+b)^2c^4} + \frac{Bc^6x^3-9\,Bbc^5x+3\,Ac^6x}{3\,c^9}$$

input
```
integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
5/8*(7*B*b^2 - 3*A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) - 1/8*(13*B*
b^2*c*x^3 - 9*A*b*c^2*x^3 + 11*B*b^3*x - 7*A*b^2*c*x)/((c*x^2 + b)^2*c^4)
+ 1/3*(B*c^6*x^3 - 9*B*b*c^5*x + 3*A*c^6*x)/c^9
```

3.75.9 Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.17

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{x^3\left(\frac{9\,A\,b\,c^2}{8}-\frac{13\,B\,b^2\,c}{8}\right)-x\left(\frac{11\,B\,b^3}{8}-\frac{7\,A\,b^2\,c}{8}\right)}{b^2\,c^4+2\,b\,c^5\,x^2+c^6\,x^4} + x\left(\frac{A}{c^3}-\frac{3\,B\,b}{c^4}\right) + \frac{B\,x^3}{3\,c^3} + \frac{5\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c}\,x\,(3\,A\,c-7\,B\,b)}{7\,B\,b^2-3\,A\,b\,c}\right)\,(3\,A\,c-7\,B\,b)}{8\,c^{9/2}}$$

input
```
int((x^12*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
(x^3*((9*A*b*c^2)/8 - (13*B*b^2*c)/8) - x*((11*B*b^3)/8 - (7*A*b^2*c)/8))/
(b^2*c^4 + c^6*x^4 + 2*b*c^5*x^2) + x*(A/c^3 - (3*B*b)/c^4) + (B*x^3)/(3*c
^3) + (5*b^(1/2)*atan((b^(1/2)*c^(1/2)*x*(3*A*c - 7*B*b))/(7*B*b^2 - 3*A*b
*c))*(3*A*c - 7*B*b))/(8*c^(9/2))
```

3.76 $\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.76.1 Optimal result

Integrand size = 24, antiderivative size = 89

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{Bx^2}{2c^3}+\frac{b^2(bB-Ac)}{4c^4(b+cx^2)^2}-\frac{b(3bB-2Ac)}{2c^4(b+cx^2)}-\frac{(3bB-Ac)\log(b+cx^2)}{2c^4}$$

output
```
1/2*B*x^2/c^3+1/4*b^2*(-A*c+B*b)/c^4/(c*x^2+b)^2-1/2*b*(-2*A*c+3*B*b)/c^4/
(c*x^2+b)-1/2*(-A*c+3*B*b)*ln(c*x^2+b)/c^4
```

3.76.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{Bx^2}{2c^3}+\frac{b^3B-Ab^2c}{4c^4(b+cx^2)^2}+\frac{-3b^2B+2Abc}{2c^4(b+cx^2)}+\frac{(-3bB+Ac)\log(b+cx^2)}{2c^4}$$

input
```
Integrate[(x^11*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
(B*x^2)/(2*c^3) + (b^3*B - A*b^2*c)/(4*c^4*(b + c*x^2)^2) + (-3*b^2*B + 2*
A*b*c)/(2*c^4*(b + c*x^2)) + ((-3*b*B + A*c)*Log[b + c*x^2])/(2*c^4)
```

3.76.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^5(A+Bx^2)}{(b+cx^2)^3}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{x^4(Bx^2+A)}{(cx^2+b)^3}dx^2$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{2}\int \left(-\frac{(bB-Ac)b^2}{c^3(cx^2+b)^3}+\frac{(3bB-2Ac)b}{c^3(cx^2+b)^2}+\frac{Ac-3bB}{c^3(cx^2+b)}+\frac{B}{c^3}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{b^2(bB-Ac)}{2c^4(b+cx^2)^2}-\frac{b(3bB-2Ac)}{c^4(b+cx^2)}-\frac{(3bB-Ac)\log(b+cx^2)}{c^4}+\frac{Bx^2}{c^3}\right)$$

input
```
Int[(x^11*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
((B*x^2)/c^3 + (b^2*(b*B - A*c))/(2*c^4*(b + c*x^2)^2) - (b*(3*b*B - 2*A*c
))/(c^4*(b + c*x^2)) - ((3*b*B - A*c)*Log[b + c*x^2])/c^4)/2
```

3.76.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.76.4 Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

method	result
norman	$\frac{\frac{b(Ac-3Bb)x^7}{c^3}+\frac{B\,x^{11}}{2c}+\frac{b^2(3Ac-9Bb)x^5}{4c^4}}{x^5(c\,x^2+b)^2}+\frac{(Ac-3Bb)\ln\left(c\,x^2+b\right)}{2c^4}$
default	$\frac{B\,x^2}{2c^3}+\frac{\frac{(Ac-3Bb)\ln\left(c\,x^2+b\right)}{c}-\frac{b^2(Ac-Bb)}{2c\left(c\,x^2+b\right)^2}+\frac{b(2Ac-3Bb)}{c\left(c\,x^2+b\right)}}{2c^3}$
risch	$\frac{B\,x^2}{2c^3}+\frac{(Abc-\frac{3}{2}B\,b^2)x^2+\frac{b^2(3Ac-5Bb)}{4c}}{c^3(c\,x^2+b)^2}+\frac{\ln(c\,x^2+b)A}{2c^3}-\frac{3\ln(c\,x^2+b)Bb}{2c^4}$
parallelrisch	$\frac{2B\,c^3x^6+2A\ln(c\,x^2+b)x^4c^3-6B\ln(c\,x^2+b)x^4b\,c^2+4A\ln(c\,x^2+b)x^2b\,c^2-12B\ln(c\,x^2+b)x^2b^2c+4Ab\,c^2x^2-12B\,b^2c\,x^2+2A\ln}{4c^4(c\,x^2+b)^2}$

input
```
int(x^11*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(b*(A*c-3*B*b)/c^3*x^7+1/2*B/c*x^11+1/4*b^2*(3*A*c-9*B*b)/c^4*x^5)/x^5/(c*
x^2+b)^2+1/2*(A*c-3*B*b)/c^4*ln(c*x^2+b)
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.60

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$
$$= \frac{2\,Bc^3x^6+4\,Bbc^2x^4-5\,Bb^3+3\,Ab^2c-4\,(Bb^2c-Abc^2)x^2-2\,((3\,Bbc^2-Ac^3)x^4+3\,Bb^3-Ab^2c+2\,(3\,B}{4\,(c^6x^4+2\,bc^5x^2+b^2c^4)}$$

input
```
integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/4*(2*B*c^3*x^6 + 4*B*b*c^2*x^4 - 5*B*b^3 + 3*A*b^2*c - 4*(B*b^2*c - A*b*
c^2)*x^2 - 2*((3*B*b*c^2 - A*c^3)*x^4 + 3*B*b^3 - A*b^2*c + 2*(3*B*b^2*c -
 A*b*c^2)*x^2)*log(c*x^2 + b))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)
```

3.76.6 Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{Bx^2}{2c^3} + \frac{3Ab^2c-5Bb^3+x^2\cdot(4Abc^2-6Bb^2c)}{4b^2c^4+8bc^5x^2+4c^6x^4} - \frac{(-Ac+3Bb)\log{(b+cx^2)}}{2c^4}$$

input
```
integrate(x**11*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
B*x**2/(2*c**3) + (3*A*b**2*c - 5*B*b**3 + x**2*(4*A*b*c**2 - 6*B*b**2*c))
/(4*b**2*c**4 + 8*b*c**5*x**2 + 4*c**6*x**4) - (-A*c + 3*B*b)*log(b + c*x*
*2)/(2*c**4)
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{5\,Bb^3-3\,Ab^2c+2\,(3\,Bb^2c-2\,Abc^2)x^2}{4\,(c^6x^4+2\,bc^5x^2+b^2c^4)} + \frac{Bx^2}{2\,c^3} - \frac{(3\,Bb-Ac)\log(cx^2+b)}{2\,c^4}$$

input
```
integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(5*B*b^3 - 3*A*b^2*c + 2*(3*B*b^2*c - 2*A*b*c^2)*x^2)/(c^6*x^4 + 2*b*
c^5*x^2 + b^2*c^4) + 1/2*B*x^2/c^3 - 1/2*(3*B*b - A*c)*log(c*x^2 + b)/c^4
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{Bx^2}{2\,c^3} - \frac{(3\,Bb-Ac)\log(|cx^2+b|)}{2\,c^4} + \frac{9\,Bbc^2x^4-3\,Ac^3x^4+12\,Bb^2cx^2-2\,Abc^2x^2+4\,Bb^3}{4\,(cx^2+b)^2c^4}$$

input
```
integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
1/2*B*x^2/c^3 - 1/2*(3*B*b - A*c)*log(abs(c*x^2 + b))/c^4 + 1/4*(9*B*b*c^2
*x^4 - 3*A*c^3*x^4 + 12*B*b^2*c*x^2 - 2*A*b*c^2*x^2 + 4*B*b^3)/((c*x^2 + b
)^2*c^4)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{B\,x^2}{2\,c^3} - \frac{x^2\left(\frac{3\,B\,b^2}{2}-A\,b\,c\right)+\frac{5\,B\,b^3-3\,A\,b^2\,c}{4\,c}}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4} + \frac{\ln(c\,x^2+b)\,(A\,c-3\,B\,b)}{2\,c^4}$$

input
```
int((x^11*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
(B*x^2)/(2*c^3) - (x^2*((3*B*b^2)/2 - A*b*c) + (5*B*b^3 - 3*A*b^2*c)/(4*c)
)/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + (log(b + c*x^2)*(A*c - 3*B*b))/(2*c^
4)
```

3.76. $\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.77 $\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3} dx$

3.77.1 Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3} dx = \frac{Bx}{c^3} - \frac{b(bB-Ac)x}{4c^3(b+cx^2)^2} + \frac{(9bB-5Ac)x}{8c^3(b+cx^2)} - \frac{3(5bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{7/2}}$$

output
```
B*x/c^3-1/4*b*(-A*c+B*b)*x/c^3/(c*x^2+b)^2+1/8*(-5*A*c+9*B*b)*x/c^3/(c*x^2
+b)-3/8*(-A*c+5*B*b)*arctan(x*c^(1/2)/b^(1/2))/c^(7/2)/b^(1/2)
```

3.77.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3} dx = \frac{x(15b^2B + c^2x^2(-5A+8Bx^2) + b(-3Ac+25Bcx^2))}{8c^3(b+cx^2)^2} - \frac{3(5bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{7/2}}$$

input
```
Integrate[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
(x*(15*b^2*B + c^2*x^2*(-5*A + 8*B*x^2) + b*(-3*A*c + 25*B*c*x^2)))/(8*c^3
*(b + c*x^2)^2) - (3*(5*b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*Sqrt[b]
*c^(7/2))
```

3.77.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {9, 360, 25, 1471, 27, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^4\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}dx$$

$$\Big\downarrow \text{ 360}$$

$$-\frac{\int -\frac{4Bc^2x^4-4c(bB-Ac)x^2+b(bB-Ac)}{\left(cx^2+b\right)^2}dx}{4c^3}-\frac{bx(bB-Ac)}{4c^3\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{4Bc^2x^4-4c(bB-Ac)x^2+b(bB-Ac)}{\left(cx^2+b\right)^2}dx}{4c^3}-\frac{bx(bB-Ac)}{4c^3\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{ 1471}$$

$$\frac{\frac{x(9bB-5Ac)}{2\left(b+cx^2\right)}-\frac{\int \frac{b\left(-8Bcx^2+7bB-3Ac\right)}{cx^2+b}dx}{2b}}{4c^3}-\frac{bx(bB-Ac)}{4c^3\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{x(9bB-5Ac)}{2\left(b+cx^2\right)}-\frac{1}{2}\int \frac{-8Bcx^2+7bB-3Ac}{cx^2+b}dx}{4c^3}-\frac{bx(bB-Ac)}{4c^3\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{1}{2}\left(8Bx-3(5bB-Ac)\int \frac{1}{cx^2+b}dx\right)+\frac{x(9bB-5Ac)}{2\left(b+cx^2\right)}}{4c^3}-\frac{bx(bB-Ac)}{4c^3\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{1}{2}\left(8Bx-\frac{3(5bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{c}}\right)+\frac{x(9bB-5Ac)}{2\left(b+cx^2\right)}}{4c^3}-\frac{bx(bB-Ac)}{4c^3\left(b+cx^2\right)^2}$$

input
```
Int[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*(b*(b*B - A*c)*x)/(c^3*(b + c*x^2)^2) + (((9*b*B - 5*A*c)*x)/(2*(b +
c*x^2)) + (8*B*x - (3*(5*b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(Sqrt[b]*
Sqrt[c]))/2)/(4*c^3)
```

3.77.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.77.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

method	result
default	$\frac{Bx}{c^3}+\frac{\frac{\left(-\frac{5}{8}A\,c^2+\frac{9}{8}Bbc\right)x^3-\frac{b(3Ac-7Bb)x}{8}}{\left(c\,x^2+b\right)^2}+\frac{3(Ac-5Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}}{c^3}$
risch	$\frac{Bx}{c^3}+\frac{\left(-\frac{5}{8}A\,c^2+\frac{9}{8}Bbc\right)x^3-\frac{b(3Ac-7Bb)x}{8}}{c^3\left(c\,x^2+b\right)^2}-\frac{3\ln\left(cx+\sqrt{-bc}\right)A}{16c^2\sqrt{-bc}}+\frac{15\ln\left(cx+\sqrt{-bc}\right)Bb}{16c^3\sqrt{-bc}}+\frac{3\ln\left(-cx+\sqrt{-bc}\right)A}{16c^2\sqrt{-bc}}-\frac{15\ln\left(-cx+\sqrt{\right.}}{16c^3\sqrt{-}}$

input
```
int(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
B*x/c^3+1/c^3*(((-5/8*A*c^2+9/8*B*b*c)*x^3-1/8*b*(3*A*c-7*B*b)*x)/(c*x^2+b
)^2+3/8*(A*c-5*B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 328, normalized size of antiderivative = 3.45

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$
$$= \left[\frac{16\,Bbc^3x^5+10\,(5\,Bb^2c^2-Abc^3)x^3+3\,((5\,Bbc^2-Ac^3)x^4+5\,Bb^3-Ab^2c+2\,(5\,Bb^2c-Abc^2)x^2)\sqrt{-bc}}{16\,(bc^6x^4+2\,b^2c^5x^2+b^3c^4)}\right.$$

input
```
integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
[1/16*(16*B*b*c^3*x^5 + 10*(5*B*b^2*c^2 - A*b*c^3)*x^3 + 3*((5*B*b*c^2 - A
*c^3)*x^4 + 5*B*b^3 - A*b^2*c + 2*(5*B*b^2*c - A*b*c^2)*x^2)*sqrt(-b*c)*lo
g((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)) + 6*(5*B*b^3*c - A*b^2*c^2)*x)
/(b*c^6*x^4 + 2*b^2*c^5*x^2 + b^3*c^4), 1/8*(8*B*b*c^3*x^5 + 5*(5*B*b^2*c^
2 - A*b*c^3)*x^3 - 3*((5*B*b*c^2 - A*c^3)*x^4 + 5*B*b^3 - A*b^2*c + 2*(5*B
*b^2*c - A*b*c^2)*x^2)*sqrt(b*c)*arctan(sqrt(b*c)*x/b) + 3*(5*B*b^3*c - A*
b^2*c^2)*x)/(b*c^6*x^4 + 2*b^2*c^5*x^2 + b^3*c^4)]
```

3.77.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(92) = 184$.

Time = 0.58 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.04

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{Bx}{c^3} + \frac{3\sqrt{-\frac{1}{bc^7}}(-Ac+5Bb)\log\left(-\frac{3bc^3\sqrt{-\frac{1}{bc^7}}(-Ac+5Bb)}{-3Ac+15Bb}+x\right)}{16} - \frac{3\sqrt{-\frac{1}{bc^7}}(-Ac+5Bb)\log\left(\frac{3bc^3\sqrt{-\frac{1}{bc^7}}(-Ac+5Bb)}{-3Ac+15Bb}+x\right)}{16} + \frac{x^3(-5Ac^2+9Bbc)+x(-3Abc+7Bb^2)}{8b^2c^3+16bc^4x^2+8c^5x^4}$$

input
```
integrate(x**10*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
B*x/c**3 + 3*sqrt(-1/(b*c**7))*(-A*c + 5*B*b)*log(-3*b*c**3*sqrt(-1/(b*c**
7))*(-A*c + 5*B*b)/(-3*A*c + 15*B*b) + x)/16 - 3*sqrt(-1/(b*c**7))*(-A*c +
 5*B*b)*log(3*b*c**3*sqrt(-1/(b*c**7))*(-A*c + 5*B*b)/(-3*A*c + 15*B*b) +
x)/16 + (x**3*(-5*A*c**2 + 9*B*b*c) + x*(-3*A*b*c + 7*B*b**2))/(8*b**2*c**
3 + 16*b*c**4*x**2 + 8*c**5*x**4)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(9\,Bbc-5\,Ac^2)x^3+(7\,Bb^2-3\,Abc)x}{8\,(c^5x^4+2\,bc^4x^2+b^2c^3)} + \frac{Bx}{c^3} - \frac{3\,(5\,Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^3}$$

input
```
integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/8*((9*B*b*c - 5*A*c^2)*x^3 + (7*B*b^2 - 3*A*b*c)*x)/(c^5*x^4 + 2*b*c^4*x
^2 + b^2*c^3) + B*x/c^3 - 3/8*(5*B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*
c)*c^3)
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{Bx}{c^3} - \frac{3\,(5\,Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^3} + \frac{9\,Bbcx^3-5\,Ac^2x^3+7\,Bb^2x-3\,Abcx}{8\,(cx^2+b)^2c^3}$$

input
```
integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
B*x/c^3 - 3/8*(5*B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/8*(9
*B*b*c*x^3 - 5*A*c^2*x^3 + 7*B*b^2*x - 3*A*b*c*x)/((c*x^2 + b)^2*c^3)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{B\,x}{c^3} - \frac{x^3\left(\frac{5\,A\,c^2}{8}-\frac{9\,B\,b\,c}{8}\right)-x\left(\frac{7\,B\,b^2}{8}-\frac{3\,A\,b\,c}{8}\right)}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(A\,c-5\,B\,b)}{8\,\sqrt{b}\,c^{7/2}}$$

input
```
int((x^10*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
(B*x)/c^3 - (x^3*((5*A*c^2)/8 - (9*B*b*c)/8) - x*((7*B*b^2)/8 - (3*A*b*c)/
8))/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + (3*atan((c^(1/2)*x)/b^(1/2))*(A*c
- 5*B*b))/(8*b^(1/2)*c^(7/2))
```

3.78 $\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.78.1 Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{b(bB-Ac)}{4c^3(b+cx^2)^2}+\frac{2bB-Ac}{2c^3(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^3}$$

output
```
-1/4*b*(-A*c+B*b)/c^3/(c*x^2+b)^2+1/2*(-A*c+2*B*b)/c^3/(c*x^2+b)+1/2*B*ln(
c*x^2+b)/c^3
```

3.78.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3b^2B-2Ac^2x^2-bc(A-4Bx^2)+2B(b+cx^2)^2\log(b+cx^2)}{4c^3(b+cx^2)^2}$$

input
```
Integrate[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
(3*b^2*B - 2*A*c^2*x^2 - b*c*(A - 4*B*x^2) + 2*B*(b + c*x^2)^2*Log[b + c*x
^2])/(4*c^3*(b + c*x^2)^2)
```

3.78.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9 (A+Bx^2)}{(bx^2+cx^4)^3} dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^3 (A+Bx^2)}{(b+cx^2)^3} dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2} \int \frac{x^2 (Bx^2+A)}{(cx^2+b)^3} dx^2$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{2} \int \left(\frac{B}{c^2 (cx^2+b)} + \frac{Ac-2bB}{c^2 (cx^2+b)^2} + \frac{b(bB-Ac)}{c^2 (cx^2+b)^3}\right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(-\frac{b(bB-Ac)}{2c^3 (b+cx^2)^2} + \frac{2bB-Ac}{c^3 (b+cx^2)} + \frac{B\log(b+cx^2)}{c^3}\right)$$

input
```
Int[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
(-1/2*(b*(b*B - A*c))/(c^3*(b + c*x^2)^2) + (2*b*B - A*c)/(c^3*(b + c*x^2)
) + (B*Log[b + c*x^2])/c^3)/2
```

3.78.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.78.4 Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{-\frac{(Ac-2Bb)x^2}{2c^2}-\frac{b(Ac-3Bb)}{4c^3}}{(c\,x^2+b)^2}+\frac{B\ln(c\,x^2+b)}{2c^3}$	57
default	$\frac{B\ln(c\,x^2+b)}{2c^3}+\frac{b(Ac-Bb)}{4c^3(c\,x^2+b)^2}-\frac{Ac-2Bb}{2c^3(c\,x^2+b)}$	61
norman	$\frac{-\frac{(Ac-2Bb)x^7}{2c^2}-\frac{b(Ac-3Bb)x^5}{4c^3}}{x^5(c\,x^2+b)^2}+\frac{B\ln(c\,x^2+b)}{2c^3}$	63
parallelrisch	$-\frac{-2B\ln(c\,x^2+b)x^4c^2-4B\ln(c\,x^2+b)x^2bc+2A\,c^2x^2-4Bbc\,x^2-2B\ln(c\,x^2+b)b^2+Abc-3B\,b^2}{4c^3(c\,x^2+b)^2}$	90

input
```
int(x^9*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*(A*c-2*B*b)/c^2*x^2-1/4*b*(A*c-3*B*b)/c^3)/(c*x^2+b)^2+1/2*B*ln(c*x^
2+b)/c^3
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.33

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3\,Bb^2-Abc+2\,(2\,Bbc-Ac^2)x^2+2\,(Bc^2x^4+2\,Bbcx^2+Bb^2)\log{(cx^2+b)}}{4\,(c^5x^4+2\,bc^4x^2+b^2c^3)}$$

input
```
integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/4*(3*B*b^2 - A*b*c + 2*(2*B*b*c - A*c^2)*x^2 + 2*(B*c^2*x^4 + 2*B*b*c*x^
2 + B*b^2)*log(c*x^2 + b))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

3.78.6 Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{B\log{(b+cx^2)}}{2c^3}+\frac{-Abc+3Bb^2+x^2(-2Ac^2+4Bbc)}{4b^2c^3+8bc^4x^2+4c^5x^4}$$

input
```
integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
B*log(b + c*x**2)/(2*c**3) + (-A*b*c + 3*B*b**2 + x**2*(-2*A*c**2 + 4*B*b*
c))/(4*b**2*c**3 + 8*b*c**4*x**2 + 4*c**5*x**4)
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3\,Bb^2-Abc+2\,(2\,Bbc-Ac^2)x^2}{4\,(c^5x^4+2\,bc^4x^2+b^2c^3)} + \frac{B\log(cx^2+b)}{2\,c^3}$$

input
```
integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/4*(3*B*b^2 - A*b*c + 2*(2*B*b*c - A*c^2)*x^2)/(c^5*x^4 + 2*b*c^4*x^2 + b
^2*c^3) + 1/2*B*log(c*x^2 + b)/c^3
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{B\log(|cx^2+b|)}{2\,c^3} - \frac{3\,Bcx^4+2\,Bbx^2+2\,Acx^2+Ab}{4\,(cx^2+b)^2c^2}$$

input
```
integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
1/2*B*log(abs(c*x^2 + b))/c^3 - 1/4*(3*B*c*x^4 + 2*B*b*x^2 + 2*A*c*x^2 + A
*b)/((c*x^2 + b)^2*c^2)
```

3.78.9 Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{3\,B\,b^2-A\,b\,c}{4\,c^3}-\frac{x^2\,(A\,c-2\,B\,b)}{2\,c^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4} + \frac{B\,\ln(c\,x^2+b)}{2\,c^3}$$

input
```
int((x^9*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
((3*B*b^2 - A*b*c)/(4*c^3) - (x^2*(A*c - 2*B*b))/(2*c^2))/(b^2 + c^2*x^4 +
 2*b*c*x^2) + (B*log(b + c*x^2))/(2*c^3)
```

3.79 $\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.79.1 Optimal result

Integrand size = 24, antiderivative size = 90

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(bB-Ac)x}{4c^2(b+cx^2)^2} - \frac{(5bB-Ac)x}{8bc^2(b+cx^2)} + \frac{(3bB+Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{3/2}c^{5/2}}$$

output
```
1/4*(-A*c+B*b)*x/c^2/(c*x^2+b)^2-1/8*(-A*c+5*B*b)*x/b/c^2/(c*x^2+b)+1/8*(A
*c+3*B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(3/2)/c^(5/2)
```

3.79.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{\sqrt{c}x(-3b^2B+Ac^2x^2-bc(A+5Bx^2))}{b(b+cx^2)^2} + \frac{(3bB+Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}}{8c^{5/2}}$$

input
```
Integrate[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
((Sqrt[c]*x*(-3*b^2*B + A*c^2*x^2 - b*c*(A + 5*B*x^2)))/(b*(b + c*x^2)^2)
+ ((3*b*B + A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2))/(8*c^(5/2))
```

3.79.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 360, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^2(A+Bx^2)}{(b+cx^2)^3}dx$$

$$\Big\downarrow \text{ 360}$$

$$\frac{x(bB-Ac)}{4c^2(b+cx^2)^2}-\frac{\int \frac{-4Bcx^2+bB-Ac}{(cx^2+b)^2}dx}{4c^2}$$

$$\Big\downarrow \text{ 298}$$

$$\frac{x(bB-Ac)}{4c^2(b+cx^2)^2}-\frac{\frac{x(5bB-Ac)}{2b(b+cx^2)}-\frac{(Ac+3bB)\int \frac{1}{cx^2+b}dx}{2b}}{4c^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{x(bB-Ac)}{4c^2(b+cx^2)^2}-\frac{\frac{x(5bB-Ac)}{2b(b+cx^2)}-\frac{(Ac+3bB)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}}{4c^2}$$

input
```
Int[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
((b*B - A*c)*x)/(4*c^2*(b + c*x^2)^2) - (((5*b*B - A*c)*x)/(2*b*(b + c*x^2
)) - ((3*b*B + A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(3/2)*Sqrt[c]))/(4*c
^2)
```

3.79.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

3.79.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\frac{(Ac-5Bb)x^3}{8bc}-\frac{(Ac+3Bb)x}{8c^2}}{(c\,x^2+b)^2}+\frac{(Ac+3Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8c^2b\sqrt{bc}}$	76
risch	$\frac{\frac{(Ac-5Bb)x^3}{8bc}-\frac{(Ac+3Bb)x}{8c^2}}{(c\,x^2+b)^2}-\frac{\ln(cx+\sqrt{-bc})A}{16\sqrt{-bc}\,cb}-\frac{3\ln(cx+\sqrt{-bc})B}{16\sqrt{-bc}\,c^2}+\frac{\ln(-cx+\sqrt{-bc})A}{16\sqrt{-bc}\,cb}+\frac{3\ln(-cx+\sqrt{-bc})B}{16\sqrt{-bc}\,c^2}$	146

input
```
int(x^8*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(A*c-5*B*b)/b/c*x^3-1/8*(A*c+3*B*b)/c^2*x)/(c*x^2+b)^2+1/8*(A*c+3*B*b
)/c^2/b/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.34

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$
$$= \left[-\frac{2\,(5\,Bb^2c^2-Abc^3)x^3+((3\,Bbc^2+Ac^3)x^4+3\,Bb^3+Ab^2c+2\,(3\,Bb^2c+Abc^2)x^2)\sqrt{-bc}\log\left(\frac{cx^2-2\,\sqrt{-b}}{cx^2+b}\right.}{16\,(b^2c^5x^4+2\,b^3c^4x^2+b^4c^3)}\right.$$
$$-\frac{(5\,Bb^2c^2-Abc^3)x^3-((3\,Bbc^2+Ac^3)x^4+3\,Bb^3+Ab^2c+2\,(3\,Bb^2c+Abc^2)x^2)\sqrt{bc}\arctan\left(\frac{\sqrt{bc}x}{b}\right)+(}{8\,(b^2c^5x^4+2\,b^3c^4x^2+b^4c^3)}$$

input
```
integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*(5*B*b^2*c^2 - A*b*c^3)*x^3 + ((3*B*b*c^2 + A*c^3)*x^4 + 3*B*b^3
 + A*b^2*c + 2*(3*B*b^2*c + A*b*c^2)*x^2)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-
b*c)*x - b)/(c*x^2 + b)) + 2*(3*B*b^3*c + A*b^2*c^2)*x)/(b^2*c^5*x^4 + 2*b
^3*c^4*x^2 + b^4*c^3), -1/8*((5*B*b^2*c^2 - A*b*c^3)*x^3 - ((3*B*b*c^2 + A
*c^3)*x^4 + 3*B*b^3 + A*b^2*c + 2*(3*B*b^2*c + A*b*c^2)*x^2)*sqrt(b*c)*arc
tan(sqrt(b*c)*x/b) + (3*B*b^3*c + A*b^2*c^2)*x)/(b^2*c^5*x^4 + 2*b^3*c^4*x
^2 + b^4*c^3)]
```

3.79.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.72

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{\sqrt{-\frac{1}{b^3c^5}}(Ac+3Bb)\log\left(-b^2c^2\sqrt{-\frac{1}{b^3c^5}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{b^3c^5}}(Ac+3Bb)\log\left(b^2c^2\sqrt{-\frac{1}{b^3c^5}}+x\right)}{16} + \frac{x^3(Ac^2-5Bbc)+x(-Abc-3Bb^2)}{8b^3c^2+16b^2c^3x^2+8bc^4x^4}$$

input
```
integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
-sqrt(-1/(b**3*c**5))*(A*c + 3*B*b)*log(-b**2*c**2*sqrt(-1/(b**3*c**5)) +
x)/16 + sqrt(-1/(b**3*c**5))*(A*c + 3*B*b)*log(b**2*c**2*sqrt(-1/(b**3*c**
5)) + x)/16 + (x**3*(A*c**2 - 5*B*b*c) + x*(-A*b*c - 3*B*b**2))/(8*b**3*c*
*2 + 16*b**2*c**3*x**2 + 8*b*c**4*x**4)
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{(5\,Bbc-Ac^2)x^3+(3\,Bb^2+Abc)x}{8\,(bc^4x^4+2\,b^2c^3x^2+b^3c^2)}+\frac{(3\,Bb+Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}bc^2}$$

input
```
integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((5*B*b*c - A*c^2)*x^3 + (3*B*b^2 + A*b*c)*x)/(b*c^4*x^4 + 2*b^2*c^3*
x^2 + b^3*c^2) + 1/8*(3*B*b + A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b*c^2)
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(3\,Bb+Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}bc^2}-\frac{5\,Bbcx^3-Ac^2x^3+3\,Bb^2x+Abcx}{8\,(cx^2+b)^2bc^2}$$

input
```
integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
1/8*(3*B*b + A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b*c^2) - 1/8*(5*B*b*c*x
^3 - A*c^2*x^3 + 3*B*b^2*x + A*b*c*x)/((c*x^2 + b)^2*b*c^2)
```

3.79.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(A\,c+3\,B\,b)}{8\,b^{3/2}\,c^{5/2}} - \frac{\frac{x\,(A\,c+3\,B\,b)}{8\,c^2} - \frac{x^3\,(A\,c-5\,B\,b)}{8\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

input
```
int((x^8*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
(atan((c^(1/2)*x)/b^(1/2))*(A*c + 3*B*b))/(8*b^(3/2)*c^(5/2)) - ((x*(A*c + 
 3*B*b))/(8*c^2) - (x^3*(A*c - 5*B*b))/(8*b*c))/(b^2 + c^2*x^4 + 2*b*c*x^2
)
```

3.80 $\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.80.1 Optimal result

Integrand size = 24, antiderivative size = 32

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(A+Bx^2)^2}{4(bB-Ac)(b+cx^2)^2}$$

output
```
1/4*(B*x^2+A)^2/(-A*c+B*b)/(c*x^2+b)^2
```

3.80.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{bB+c(A+2Bx^2)}{4c^2(b+cx^2)^2}$$

input
```
Integrate[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*(b*B + c*(A + 2*B*x^2))/(c^2*(b + c*x^2)^2)
```

3.80.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 353, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 (A+Bx^2)}{(bx^2+cx^4)^3} \, dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{x (A+Bx^2)}{(b+cx^2)^3} dx$$
$$\Big\downarrow \text{ 353}$$
$$\frac{1}{2} \int \frac{Bx^2+A}{(cx^2+b)^3} dx^2$$
$$\Big\downarrow \text{ 48}$$
$$\frac{(A+Bx^2)^2}{4 (b+cx^2)^2 (bB-Ac)}$$

input
```
Int[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
(A + B*x^2)^2/(4*(b*B - A*c)*(b + c*x^2)^2)
```

3.80.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.80.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
gosper	$-\frac{2Bc\,x^{2}+Ac+Bb}{4\left(c\,x^{2}+b\right)^{2} c^{2}}$	29
parallelrisch	$-\frac{2Bc\,x^{2}+Ac+Bb}{4\left(c\,x^{2}+b\right)^{2} c^{2}}$	29
risch	$\frac{-\frac{B\,x^{2}}{2c}-\frac{Ac+Bb}{4c^{2}}}{\left(c\,x^{2}+b\right)^{2}}$	33
default	$-\frac{Ac-Bb}{4c^{2}\left(c\,x^{2}+b\right)^{2}}-\frac{B}{2c^{2}\left(c\,x^{2}+b\right)}$	39
norman	$\frac{-\frac{B\,x^{7}}{2c}-\frac{\left(Ac+Bb\right)x^{5}}{4c^{2}}}{x^{5}\left(c\,x^{2}+b\right)^{2}}$	39

input
```
int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*B*c*x^2+A*c+B*b)/(c*x^2+b)^2/c^2
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{x^7(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx=-\frac{2\,Bcx^2+Bb+Ac}{4\,\left(c^4x^4+2\,bc^3x^2+b^2c^2\right)}$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*B*c*x^2 + B*b + A*c)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

3.80.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{-Ac-Bb-2Bcx^2}{4b^2c^2+8bc^3x^2+4c^4x^4}$$

input
```
integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
(-A*c - B*b - 2*B*c*x**2)/(4*b**2*c**2 + 8*b*c**3*x**2 + 4*c**4*x**4)
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{2\,Bcx^2+Bb+Ac}{4\,(c^4x^4+2\,bc^3x^2+b^2c^2)}$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*B*c*x^2 + B*b + A*c)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{2\,Bcx^2+Bb+Ac}{4\,(cx^2+b)^2c^2}$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-1/4*(2*B*c*x^2 + B*b + A*c)/((c*x^2 + b)^2*c^2)
```

3.80.9 Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{\frac{A\,c+B\,b}{4\,c^2}+\frac{B\,x^2}{2\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

input
```
int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
-((A*c + B*b)/(4*c^2) + (B*x^2)/(2*c))/(b^2 + c^2*x^4 + 2*b*c*x^2)
```

3.81 $\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.81.1 Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{(bB-Ac)x}{4bc\,(b+cx^2)^2}+\frac{(bB+3Ac)x}{8b^2c\,(b+cx^2)}+\frac{(bB+3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}}$$

output
```
-1/4*(-A*c+B*b)*x/b/c/(c*x^2+b)^2+1/8*(3*A*c+B*b)*x/b^2/c/(c*x^2+b)+1/8*(3
*A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(5/2)/c^(3/2)
```

3.81.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{x(-b^2B+3Ac^2x^2+bc(5A+Bx^2))}{8b^2c\,(b+cx^2)^2}+\frac{(bB+3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}}$$

input
```
Integrate[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
(x*(-(b^2*B) + 3*A*c^2*x^2 + b*c*(5*A + B*x^2)))/(8*b^2*c*(b + c*x^2)^2) +
 ((b*B + 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(5/2)*c^(3/2))
```

3.81.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 298, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{A+Bx^2}{(b+cx^2)^3}dx$$

$$\Big\downarrow \; 298$$

$$\frac{(3Ac+bB)\int \frac{1}{(cx^2+b)^2}dx}{4bc}-\frac{x(bB-Ac)}{4bc(b+cx^2)^2}$$

$$\Big\downarrow \; 215$$

$$\frac{(3Ac+bB)\left(\frac{\int \frac{1}{cx^2+b}dx}{2b}+\frac{x}{2b(b+cx^2)}\right)}{4bc}-\frac{x(bB-Ac)}{4bc(b+cx^2)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{(3Ac+bB)\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}+\frac{x}{2b(b+cx^2)}\right)}{4bc}-\frac{x(bB-Ac)}{4bc(b+cx^2)^2}$$

input
```
Int[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*((b*B - A*c)*x)/(b*c*(b + c*x^2)^2) + ((b*B + 3*A*c)*(x/(2*b*(b + c*x
^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*b^(3/2)*Sqrt[c])))/(4*b*c)
```

3.81.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.81.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\frac{(3Ac+Bb)x^3}{8b^2}+\frac{(5Ac-Bb)x}{8bc}}{(c\,x^2+b)^2}+\frac{(3Ac+Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8b^2c\sqrt{bc}}$	77
risch	$\frac{\frac{(3Ac+Bb)x^3}{8b^2}+\frac{(5Ac-Bb)x}{8bc}}{(c\,x^2+b)^2}-\frac{3\ln\left(cx+\sqrt{-bc}\right)A}{16\sqrt{-bc}\,b^2}-\frac{\ln\left(cx+\sqrt{-bc}\right)B}{16\sqrt{-bc}\,cb}+\frac{3\ln\left(-cx+\sqrt{-bc}\right)A}{16\sqrt{-bc}\,b^2}+\frac{\ln\left(-cx+\sqrt{-bc}\right)B}{16\sqrt{-bc}\,cb}$	147

input
```
int(x^6*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*A*c+B*b)/b^2*x^3+1/8*(5*A*c-B*b)/b/c*x)/(c*x^2+b)^2+1/8*(3*A*c+B*b
)/b^2/c/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.26

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$
$$= \left[\frac{2\,(Bb^2c^2+3\,Abc^3)x^3-((Bbc^2+3\,Ac^3)x^4+Bb^3+3\,Ab^2c+2\,(Bb^2c+3\,Abc^2)x^2)\sqrt{-bc}\log\left(\frac{cx^2-2\,\sqrt{-bc}x}{cx^2+b}\right.}{16\,(b^3c^4x^4+2\,b^4c^3x^2+b^5c^2)}\right.$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
[1/16*(2*(B*b^2*c^2 + 3*A*b*c^3)*x^3 - ((B*b*c^2 + 3*A*c^3)*x^4 + B*b^3 +
3*A*b^2*c + 2*(B*b^2*c + 3*A*b*c^2)*x^2)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b
*c)*x - b)/(c*x^2 + b)) - 2*(B*b^3*c - 5*A*b^2*c^2)*x)/(b^3*c^4*x^4 + 2*b^
4*c^3*x^2 + b^5*c^2), 1/8*((B*b^2*c^2 + 3*A*b*c^3)*x^3 + ((B*b*c^2 + 3*A*c
^3)*x^4 + B*b^3 + 3*A*b^2*c + 2*(B*b^2*c + 3*A*b*c^2)*x^2)*sqrt(b*c)*arcta
n(sqrt(b*c)*x/b) - (B*b^3*c - 5*A*b^2*c^2)*x)/(b^3*c^4*x^4 + 2*b^4*c^3*x^2
 + b^5*c^2)]
```

3.81.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.63

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{\sqrt{-\frac{1}{b^5c^3}}\cdot(3Ac+Bb)\log\left(-b^3c\sqrt{-\frac{1}{b^5c^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{b^5c^3}}\cdot(3Ac+Bb)\log\left(b^3c\sqrt{-\frac{1}{b^5c^3}}+x\right)}{16} + \frac{x^3\cdot(3Ac^2+Bbc)+x(5Abc-Bb^2)}{8b^4c+16b^3c^2x^2+8b^2c^3x^4}$$

input
```
integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
-sqrt(-1/(b**5*c**3))*(3*A*c + B*b)*log(-b**3*c*sqrt(-1/(b**5*c**3)) + x)/
16 + sqrt(-1/(b**5*c**3))*(3*A*c + B*b)*log(b**3*c*sqrt(-1/(b**5*c**3)) +
x)/16 + (x**3*(3*A*c**2 + B*b*c) + x*(5*A*b*c - B*b**2))/(8*b**4*c + 16*b*
*3*c**2*x**2 + 8*b**2*c**3*x**4)
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(Bbc+3\,Ac^2)x^3-(Bb^2-5\,Abc)x}{8\,(b^2c^3x^4+2\,b^3c^2x^2+b^4c)}+\frac{(Bb+3\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^2c}$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/8*((B*b*c + 3*A*c^2)*x^3 - (B*b^2 - 5*A*b*c)*x)/(b^2*c^3*x^4 + 2*b^3*c^2
*x^2 + b^4*c) + 1/8*(B*b + 3*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2*c)
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.85

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(Bb+3\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^2c}+\frac{Bbcx^3+3\,Ac^2x^3-Bb^2x+5\,Abcx}{8\,(cx^2+b)^2b^2c}$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
1/8*(B*b + 3*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2*c) + 1/8*(B*b*c*x^3
 + 3*A*c^2*x^3 - B*b^2*x + 5*A*b*c*x)/((c*x^2 + b)^2*b^2*c)
```

3.81.9 Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{x^3\,(3\,A\,c+B\,b)}{8\,b^2}+\frac{x\,(5\,A\,c-B\,b)}{8\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(3\,A\,c+B\,b)}{8\,b^{5/2}\,c^{3/2}}$$

input
```
int((x^6*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
((x^3*(3*A*c + B*b))/(8*b^2) + (x*(5*A*c - B*b))/(8*b*c))/(b^2 + c^2*x^4 +
 2*b*c*x^2) + (atan((c^(1/2)*x)/b^(1/2))*(3*A*c + B*b))/(8*b^(5/2)*c^(3/2)
)
```

3.82 $\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.82.1 Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{bB-Ac}{4bc\,(b+cx^2)^2}+\frac{A}{2b^2\,(b+cx^2)}+\frac{A\log(x)}{b^3}-\frac{A\log\left(b+cx^2\right)}{2b^3}$$

output
```
1/4*(A*c-B*b)/b/c/(c*x^2+b)^2+1/2*A/b^2/(c*x^2+b)+A*ln(x)/b^3-1/2*A*ln(c*x
^2+b)/b^3
```

3.82.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{b(-b^2B+3Abc+2Ac^2x^2)}{c(b+cx^2)^2}+4A\log(x)-2A\log\left(b+cx^2\right)}{4b^3}$$

input
```
Integrate[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
((b*(-(b^2*B) + 3*A*b*c + 2*A*c^2*x^2))/(c*(b + c*x^2)^2) + 4*A*Log[x] - 2
*A*Log[b + c*x^2])/(4*b^3)
```

3.82.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx^2)}{(bx^2+cx^4)^3} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{A+Bx^2}{x (b+cx^2)^3}dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2} \int \frac{Bx^2+A}{x^2 (cx^2+b)^3}dx^2$$

$$\Big\downarrow \; 86$$

$$\frac{1}{2} \int \left(-\frac{cA}{b^3 (cx^2+b)}+\frac{A}{b^3x^2}-\frac{cA}{b^2 (cx^2+b)^2}+\frac{bB-Ac}{b (cx^2+b)^3}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2} \left(-\frac{A \log (b+cx^2)}{b^3}+\frac{A \log (x^2)}{b^3}+\frac{A}{b^2 (b+cx^2)}-\frac{bB-Ac}{2bc (b+cx^2)^2}\right)$$

input
```
Int[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
(-1/2*(b*B - A*c)/(b*c*(b + c*x^2)^2) + A/(b^2*(b + c*x^2)) + (A*Log[x^2])
/b^3 - (A*Log[b + c*x^2])/b^3)/2
```

3.82.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.82.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

method	result
risch	$\frac{\frac{A c\,x^{2}}{2 b^{2}}+\frac{3 A c -B b}{4 b c}}{\left(c\,x^{2}+b \right)^{2}}+\frac{A \ln \left(x \right)}{b^{3}}-\frac{A \ln \left(c\,x^{2}+b \right)}{2 b^{3}}$
default	$\frac{A \ln \left(x \right)}{b^{3}}-\frac{-\frac{A b}{c\,x^{2}+b}+A \ln \left(c\,x^{2}+b \right)-\frac{b^{2} \left(A c -B b \right)}{2 c \left(c\,x^{2}+b \right)^{2}}}{2 b^{3}}$
norman	$\frac{-\frac{\left(2 A c -B b \right) x^{7}}{2 b^{2}}-\frac{c \left(3 A c -B b \right) x^{9}}{4 b^{3}}}{x^{5} \left(c\,x^{2}+b \right)^{2}}+\frac{A \ln \left(x \right)}{b^{3}}-\frac{A \ln \left(c\,x^{2}+b \right)}{2 b^{3}}$
parallelrisch	$\frac{4 A \ln \left(x \right) x^{4} c^{2}-2 A \ln \left(c\,x^{2}+b \right) x^{4} c^{2}-3 A\,c^{2} x^{4}+x^{4} B b c +8 A \ln \left(x \right) x^{2} b c -4 A \ln \left(c\,x^{2}+b \right) x^{2} b c -4 A b c\,x^{2}+2 b^{2} B\,x^{2}+4 A\,b^{2} \ln \left(x \right)-2 A \text{l}}{4 b^{3} \left(c\,x^{2}+b \right)^{2}}$

input
```
int(x^5*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/2/b^2*A*c*x^2+1/4*(3*A*c-B*b)/b/c)/(c*x^2+b)^2+A*ln(x)/b^3-1/2*A*ln(c*x
^2+b)/b^3
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.75

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$
$$= \frac{2\,Abc^2x^2 - Bb^3 + 3\,Ab^2c - 2\,(Ac^3x^4 + 2\,Abc^2x^2 + Ab^2c)\log\left(cx^2+b\right) + 4\,(Ac^3x^4 + 2\,Abc^2x^2 + Ab^2c)\log (}{4\,(b^3c^3x^4 + 2\,b^4c^2x^2 + b^5c)}$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/4*(2*A*b*c^2*x^2 - B*b^3 + 3*A*b^2*c - 2*(A*c^3*x^4 + 2*A*b*c^2*x^2 + A*
b^2*c)*log(c*x^2 + b) + 4*(A*c^3*x^4 + 2*A*b*c^2*x^2 + A*b^2*c)*log(x))/(b
^3*c^3*x^4 + 2*b^4*c^2*x^2 + b^5*c)
```

3.82.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{A\log{(x)}}{b^3} - \frac{A\log{\left(\frac{b}{c}+x^2\right)}}{2b^3} + \frac{3Abc + 2Ac^2x^2 - Bb^2}{4b^4c + 8b^3c^2x^2 + 4b^2c^3x^4}$$

input
```
integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
A*log(x)/b**3 - A*log(b/c + x**2)/(2*b**3) + (3*A*b*c + 2*A*c**2*x**2 - B*
b**2)/(4*b**4*c + 8*b**3*c**2*x**2 + 4*b**2*c**3*x**4)
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.13

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{2\,Ac^2x^2 - Bb^2 + 3\,Abc}{4\,(b^2c^3x^4 + 2\,b^3c^2x^2 + b^4c)} - \frac{A\log\left(cx^2+b\right)}{2\,b^3} + \frac{A\log\left(x^2\right)}{2\,b^3}$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/4*(2*A*c^2*x^2 - B*b^2 + 3*A*b*c)/(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)
- 1/2*A*log(c*x^2 + b)/b^3 + 1/2*A*log(x^2)/b^3
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{A\log\left(x^2\right)}{2\,b^3} - \frac{A\log\left(|cx^2+b|\right)}{2\,b^3} + \frac{3\,Ac^3x^4 + 8\,Abc^2x^2 - Bb^3 + 6\,Ab^2c}{4\,(cx^2+b)^2b^3c}$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
1/2*A*log(x^2)/b^3 - 1/2*A*log(abs(c*x^2 + b))/b^3 + 1/4*(3*A*c^3*x^4 + 8*
A*b*c^2*x^2 - B*b^3 + 6*A*b^2*c)/((c*x^2 + b)^2*b^3*c)
```

3.82.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{3\,A\,c-B\,b}{4\,b\,c} + \frac{A\,c\,x^2}{2\,b^2}}{b^2 + 2\,b\,c\,x^2 + c^2\,x^4} - \frac{A\,\ln\left(c\,x^2+b\right)}{2\,b^3} + \frac{A\,\ln\left(x\right)}{b^3}$$

input
```
int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
((3*A*c - B*b)/(4*b*c) + (A*c*x^2)/(2*b^2))/(b^2 + c^2*x^4 + 2*b*c*x^2) -
(A*log(b + c*x^2))/(2*b^3) + (A*log(x))/b^3
```

3.83 $\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.83.1 Optimal result

Integrand size = 24, antiderivative size = 96

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{A}{b^3x}+\frac{(bB-Ac)x}{4b^2(b+cx^2)^2}+\frac{(3bB-7Ac)x}{8b^3(b+cx^2)}+\frac{3(bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{c}}$$

output
```
-A/b^3/x+1/4*(-A*c+B*b)*x/b^2/(c*x^2+b)^2+1/8*(-7*A*c+3*B*b)*x/b^3/(c*x^2+
b)+3/8*(-5*A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(7/2)/c^(1/2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{A}{b^3x}+\frac{(bB-Ac)x}{4b^2(b+cx^2)^2}+\frac{(3bB-7Ac)x}{8b^3(b+cx^2)}+\frac{3(bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{c}}$$

input
```
Integrate[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
-(A/(b^3*x)) + ((b*B - A*c)*x)/(4*b^2*(b + c*x^2)^2) + ((3*b*B - 7*A*c)*x)
/(8*b^3*(b + c*x^2)) + (3*(b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^
(7/2)*Sqrt[c])
```

3.83.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {9, 361, 25, 27, 361, 25, 359, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (A+Bx^2)}{(bx^2+cx^4)^3} \, dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{A+Bx^2}{x^2 (b+cx^2)^3}dx$$
$$\Big\downarrow \; 361$$
$$\frac{x(bB-Ac)}{4b^2 (b+cx^2)^2}-\frac{1}{4}\int -\frac{3(bB-Ac)x^2+4Ab}{b^2x^2 (cx^2+b)^2}dx$$
$$\Big\downarrow \; 25$$
$$\frac{1}{4}\int \frac{3(bB-Ac)x^2+4Ab}{b^2x^2 (cx^2+b)^2}dx+\frac{x(bB-Ac)}{4b^2 (b+cx^2)^2}$$
$$\Big\downarrow \; 27$$
$$\frac{\int \frac{3(bB-Ac)x^2+4Ab}{x^2 (cx^2+b)^2}dx}{4b^2}+\frac{x(bB-Ac)}{4b^2 (b+cx^2)^2}$$
$$\Big\downarrow \; 361$$
$$\frac{\frac{x(3bB-7Ac)}{2b(b+cx^2)}-\frac{1}{2}\int -\frac{\left(3B-\frac{7Ac}{b}\right)x^2+8A}{x^2 (cx^2+b)}dx}{4b^2}+\frac{x(bB-Ac)}{4b^2 (b+cx^2)^2}$$
$$\Big\downarrow \; 25$$
$$\frac{\frac{1}{2}\int \frac{\left(3B-\frac{7Ac}{b}\right)x^2+8A}{x^2 (cx^2+b)}dx+\frac{x(3bB-7Ac)}{2b(b+cx^2)}}{4b^2}+\frac{x(bB-Ac)}{4b^2 (b+cx^2)^2}$$
$$\Big\downarrow \; 359$$
$$\frac{\frac{1}{2}\left(\frac{3(bB-5Ac)\int \frac{1}{cx^2+b}dx}{b}-\frac{8A}{bx}\right)+\frac{x(3bB-7Ac)}{2b(b+cx^2)}}{4b^2}+\frac{x(bB-Ac)}{4b^2 (b+cx^2)^2}$$

$$\downarrow 218$$

$$\frac{\frac{1}{2}\left(\frac{3(bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}-\frac{8A}{bx}\right)+\frac{x(3bB-7Ac)}{2b(b+cx^2)}}{4b^2}+\frac{x(bB-Ac)}{4b^2\left(b+cx^2\right)^2}$$

input
```
Int[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
((b*B - A*c)*x)/(4*b^2*(b + c*x^2)^2) + (((3*b*B - 7*A*c)*x)/(2*b*(b + c*x
^2)) + ((-8*A)/(b*x) + (3*(b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(b^(3
/2)*Sqrt[c]))/2)/(4*b^2)
```

3.83.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

3.83.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

method	result
default	$-\frac{A}{b^3 x}-\frac{\frac{\left(\frac{7}{8}A\,c^2-\frac{3}{8}Bbc\right)x^3+\frac{b(9Ac-5Bb)x}{8}}{\left(c\,x^2+b\right)^2}+\frac{3(5Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}}{b^3}$
risch	$\frac{-\frac{3c(5Ac-Bb)x^4}{8b^3}-\frac{5(5Ac-Bb)x^2}{8b^2}-\frac{A}{b}}{x(c\,x^2+b)^2}-\frac{15\ln(-\sqrt{-bc}\,x-b)Ac}{16\sqrt{-bc}\,b^3}+\frac{3\ln(-\sqrt{-bc}\,x-b)B}{16\sqrt{-bc}\,b^2}+\frac{15\ln(-\sqrt{-bc}\,x+b)Ac}{16\sqrt{-bc}\,b^3}-\frac{3\ln(-\sqrt{-bc}\,x\cdots}{16\sqrt{-bc}\,b^{\cdots}}$

input
```
int(x^4*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-A/b^3/x-1/b^3*(((7/8*A*c^2-3/8*B*b*c)*x^3+1/8*b*(9*A*c-5*B*b)*x)/(c*x^2+b
)^2+3/8*(5*A*c-B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.38

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$
$$=\left[-\frac{16\,Ab^3c-6\,(Bb^2c^2-5\,Abc^3)x^4-10\,(Bb^3c-5\,Ab^2c^2)x^2-3\,((Bbc^2-5\,Ac^3)x^5+2\,(Bb^2c-5\,Abc^2)x^3\cdots}{16\,(b^4c^3x^5+2\,b^5c^2x^3+b^6cx)}\right.$$
$$-\frac{8\,Ab^3c-3\,(Bb^2c^2-5\,Abc^3)x^4-5\,(Bb^3c-5\,Ab^2c^2)x^2-3\,((Bbc^2-5\,Ac^3)x^5+2\,(Bb^2c-5\,Abc^2)x^3+\cdots}{8\,(b^4c^3x^5+2\,b^5c^2x^3+b^6cx)}$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
[-1/16*(16*A*b^3*c - 6*(B*b^2*c^2 - 5*A*b*c^3)*x^4 - 10*(B*b^3*c - 5*A*b^2
*c^2)*x^2 - 3*((B*b*c^2 - 5*A*c^3)*x^5 + 2*(B*b^2*c - 5*A*b*c^2)*x^3 + (B*
b^3 - 5*A*b^2*c)*x)*sqrt(-b*c)*log((c*x^2 + 2*sqrt(-b*c)*x - b)/(c*x^2 + b
)))/(b^4*c^3*x^5 + 2*b^5*c^2*x^3 + b^6*c*x), -1/8*(8*A*b^3*c - 3*(B*b^2*c^
2 - 5*A*b*c^3)*x^4 - 5*(B*b^3*c - 5*A*b^2*c^2)*x^2 - 3*((B*b*c^2 - 5*A*c^3
)*x^5 + 2*(B*b^2*c - 5*A*b*c^2)*x^3 + (B*b^3 - 5*A*b^2*c)*x)*sqrt(b*c)*arc
tan(sqrt(b*c)*x/b))/(b^4*c^3*x^5 + 2*b^5*c^2*x^3 + b^6*c*x)]
```

3.83.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(92) = 184$.

Time = 0.36 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.02

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{3\sqrt{-\frac{1}{b^7c}}(-5Ac+Bb)\log\left(-\frac{3b^4\sqrt{-\frac{1}{b^7c}}(-5Ac+Bb)}{-15Ac+3Bb}+x\right)}{16} + \frac{3\sqrt{-\frac{1}{b^7c}}(-5Ac+Bb)\log\left(\frac{3b^4\sqrt{-\frac{1}{b^7c}}(-5Ac+Bb)}{-15Ac+3Bb}+x\right)}{16} + \frac{-8Ab^2+x^4(-15Ac^2+3Bbc)+x^2(-25Abc+5Bb^2)}{8b^5x+16b^4cx^3+8b^3c^2x^5}$$

input
```
integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
-3*sqrt(-1/(b**7*c))*(-5*A*c + B*b)*log(-3*b**4*sqrt(-1/(b**7*c))*(-5*A*c
+ B*b)/(-15*A*c + 3*B*b) + x)/16 + 3*sqrt(-1/(b**7*c))*(-5*A*c + B*b)*log(
3*b**4*sqrt(-1/(b**7*c))*(-5*A*c + B*b)/(-15*A*c + 3*B*b) + x)/16 + (-8*A*
b**2 + x**4*(-15*A*c**2 + 3*B*b*c) + x**2*(-25*A*b*c + 5*B*b**2))/(8*b**5*
x + 16*b**4*c*x**3 + 8*b**3*c**2*x**5)
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3\,(Bbc-5\,Ac^2)x^4-8\,Ab^2+5\,(Bb^2-5\,Abc)x^2}{8\,(b^3c^2x^5+2\,b^4cx^3+b^5x)} + \frac{3\,(Bb-5\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^3}$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/8*(3*(B*b*c - 5*A*c^2)*x^4 - 8*A*b^2 + 5*(B*b^2 - 5*A*b*c)*x^2)/(b^3*c^2
*x^5 + 2*b^4*c*x^3 + b^5*x) + 3/8*(B*b - 5*A*c)*arctan(c*x/sqrt(b*c))/(sqr
t(b*c)*b^3)
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3\,(Bb-5\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^3} - \frac{A}{b^3x} + \frac{3\,Bbcx^3-7\,Ac^2x^3+5\,Bb^2x-9\,Abcx}{8\,(cx^2+b)^2b^3}$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
3/8*(B*b - 5*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - A/(b^3*x) + 1/8*
(3*B*b*c*x^3 - 7*A*c^2*x^3 + 5*B*b^2*x - 9*A*b*c*x)/((c*x^2 + b)^2*b^3)
```

3.83.9 Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.18

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{\frac{A}{b}+\frac{5\,x^2\,(5\,A\,c-B\,b)}{8\,b^2}+\frac{3\,c\,x^4\,(5\,A\,c-B\,b)}{8\,b^3}}{b^2\,x+2\,b\,c\,x^3+c^2\,x^5} - \frac{3\,\mathrm{atan}\left(\frac{3\,\sqrt{c}\,x\,(5\,A\,c-B\,b)}{\sqrt{b}\,(15\,A\,c-3\,B\,b)}\right)\,(5\,A\,c-B\,b)}{8\,b^{7/2}\,\sqrt{c}}$$

input
```
int((x^4*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
- (A/b + (5*x^2*(5*A*c - B*b))/(8*b^2) + (3*c*x^4*(5*A*c - B*b))/(8*b^3))/
(b^2*x + c^2*x^5 + 2*b*c*x^3) - (3*atan((3*c^(1/2)*x*(5*A*c - B*b))/(b^(1/
2)*(15*A*c - 3*B*b)))*(5*A*c - B*b))/(8*b^(7/2)*c^(1/2))
```

3.84 $\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.84.1 Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{A}{2b^3x^2}+\frac{bB-Ac}{4b^2(b+cx^2)^2}+\frac{bB-2Ac}{2b^3(b+cx^2)}+\frac{(bB-3Ac)\log(x)}{b^4}-\frac{(bB-3Ac)\log(b+cx^2)}{2b^4}$$

output
```
-1/2*A/b^3/x^2+1/4*(-A*c+B*b)/b^2/(c*x^2+b)^2+1/2*(-2*A*c+B*b)/b^3/(c*x^2+
b)+(-3*A*c+B*b)*ln(x)/b^4-1/2*(-3*A*c+B*b)*ln(c*x^2+b)/b^4
```

3.84.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{-\frac{2Ab}{x^2}+\frac{b^2(bB-Ac)}{(b+cx^2)^2}+\frac{2b(bB-2Ac)}{b+cx^2}+4(bB-3Ac)\log(x)-2(bB-3Ac)\log(b+cx^2)}{4b^4}$$

input
```
Integrate[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
((-2*A*b)/x^2 + (b^2*(b*B - A*c))/(b + c*x^2)^2 + (2*b*(b*B - 2*A*c))/(b +
 c*x^2) + 4*(b*B - 3*A*c)*Log[x] - 2*(b*B - 3*A*c)*Log[b + c*x^2])/(4*b^4)
```

3.84.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(A+B x^2\right)}{\left(b x^2+c x^4\right)^3} \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{A+B x^2}{x^3 \left(b+c x^2\right)^3}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2} \int \frac{B x^2+A}{x^4 \left(c x^2+b\right)^3}dx^2$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{2} \int \left(\frac{A}{b^3 x^4}-\frac{c (b B-3 A c)}{b^4 \left(c x^2+b\right)}+\frac{b B-3 A c}{b^4 x^2}-\frac{c (b B-2 A c)}{b^3 \left(c x^2+b\right)^2}-\frac{c (b B-A c)}{b^2 \left(c x^2+b\right)^3}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(\frac{\log \left(x^2\right) (b B-3 A c)}{b^4}-\frac{(b B-3 A c) \log \left(b+c x^2\right)}{b^4}+\frac{b B-2 A c}{b^3 \left(b+c x^2\right)}-\frac{A}{b^3 x^2}+\frac{b B-A c}{2 b^2 \left(b+c x^2\right)^2}\right)$$

input
```
Int[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
(-(A/(b^3*x^2)) + (b*B - A*c)/(2*b^2*(b + c*x^2)^2) + (b*B - 2*A*c)/(b^3*(
b + c*x^2)) + ((b*B - 3*A*c)*Log[x^2])/b^4 - ((b*B - 3*A*c)*Log[b + c*x^2]
)/b^4)/2
```

3.84.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.84.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

method	result
norman	$\frac{\frac{c(3Ac-Bb)x^7}{b^3}-\frac{A\,x^3}{2b}+\frac{c^2(9Ac-3Bb)x^9}{4b^4}}{x^5(c\,x^2+b)^2}-\frac{(3Ac-Bb)\ln(x)}{b^4}+\frac{(3Ac-Bb)\ln(c\,x^2+b)}{2b^4}$
default	$-\frac{A}{2b^3x^2}+\frac{(-3Ac+Bb)\ln(x)}{b^4}+\frac{c\left(\frac{(3Ac-Bb)\ln(c\,x^2+b)}{c}-\frac{b^2(Ac-Bb)}{2c(c\,x^2+b)^2}-\frac{b(2Ac-Bb)}{c(c\,x^2+b)}\right)}{2b^4}$
risch	$\frac{-\frac{c(3Ac-Bb)x^4}{2b^3}-\frac{3(3Ac-Bb)x^2}{4b^2}-\frac{A}{2b}}{x^2(c\,x^2+b)^2}-\frac{3\ln(x)Ac}{b^4}+\frac{\ln(x)B}{b^3}+\frac{3\ln(-c\,x^2-b)Ac}{2b^4}-\frac{\ln(-c\,x^2-b)B}{2b^3}$
parallelrisch	$-\frac{12A\ln(x)x^6c^3-6A\ln(c\,x^2+b)x^6c^3-4B\ln(x)x^6b\,c^2+2B\ln(c\,x^2+b)x^6b\,c^2-9A\,c^3x^6+3x^6Bb\,c^2+24A\ln(x)x^4b\,c^2-12A\ln(c}{}$

input
```
int(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(c*(3*A*c-B*b)/b^3*x^7-1/2*A/b*x^3+1/4*c^2*(9*A*c-3*B*b)/b^4*x^9)/x^5/(c*x
^2+b)^2-(3*A*c-B*b)/b^4*ln(x)+1/2*(3*A*c-B*b)/b^4*ln(c*x^2+b)
```

3.84.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(89) = 178$.

Time = 0.24 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.03

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{2\,(Bb^2c-3\,Abc^2)x^4-2\,Ab^3+3\,(Bb^3-3\,Ab^2c)x^2-2\,((Bbc^2-3\,Ac^3)x^6+2\,(Bb^2c-3\,Abc^2)x^4+(Bb^3-}{4\,(b^4c^2x^6+2\,b^5cx}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/4*(2*(B*b^2*c - 3*A*b*c^2)*x^4 - 2*A*b^3 + 3*(B*b^3 - 3*A*b^2*c)*x^2 - 2
*((B*b*c^2 - 3*A*c^3)*x^6 + 2*(B*b^2*c - 3*A*b*c^2)*x^4 + (B*b^3 - 3*A*b^2
*c)*x^2)*log(c*x^2 + b) + 4*((B*b*c^2 - 3*A*c^3)*x^6 + 2*(B*b^2*c - 3*A*b*
c^2)*x^4 + (B*b^3 - 3*A*b^2*c)*x^2)*log(x))/(b^4*c^2*x^6 + 2*b^5*c*x^4 + b
^6*x^2)
```

3.84.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{-2Ab^2+x^4(-6Ac^2+2Bbc)+x^2(-9Abc+3Bb^2)}{4b^5x^2+8b^4cx^4+4b^3c^2x^6} + \frac{(-3Ac+Bb)\log(x)}{b^4} - \frac{(-3Ac+Bb)\log\left(\frac{b}{c}+x^2\right)}{2b^4}$$

input
```
integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
(-2*A*b**2 + x**4*(-6*A*c**2 + 2*B*b*c) + x**2*(-9*A*b*c + 3*B*b**2))/(4*b
**5*x**2 + 8*b**4*c*x**4 + 4*b**3*c**2*x**6) + (-3*A*c + B*b)*log(x)/b**4
- (-3*A*c + B*b)*log(b/c + x**2)/(2*b**4)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.12

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{2\,(Bbc-3\,Ac^2)x^4-2\,Ab^2+3\,(Bb^2-3\,Abc)x^2}{4\,(b^3c^2x^6+2\,b^4cx^4+b^5x^2)} - \frac{(Bb-3\,Ac)\log(cx^2+b)}{2\,b^4} + \frac{(Bb-3\,Ac)\log(x^2)}{2\,b^4}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/4*(2*(B*b*c - 3*A*c^2)*x^4 - 2*A*b^2 + 3*(B*b^2 - 3*A*b*c)*x^2)/(b^3*c^2
*x^6 + 2*b^4*c*x^4 + b^5*x^2) - 1/2*(B*b - 3*A*c)*log(c*x^2 + b)/b^4 + 1/2
*(B*b - 3*A*c)*log(x^2)/b^4
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(Bb-3\,Ac)\log(|x|)}{b^4} - \frac{(Bbc-3\,Ac^2)\log(|cx^2+b|)}{2\,b^4c} + \frac{2\,(Bb^2c-3\,Abc^2)x^4-2\,Ab^3+3\,(Bb^3-3\,Ab^2c)x^2}{4\,(cx^2+b)^2b^4x^2}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
(B*b - 3*A*c)*log(abs(x))/b^4 - 1/2*(B*b*c - 3*A*c^2)*log(abs(c*x^2 + b))/
(b^4*c) + 1/4*(2*(B*b^2*c - 3*A*b*c^2)*x^4 - 2*A*b^3 + 3*(B*b^3 - 3*A*b^2*
c)*x^2)/((c*x^2 + b)^2*b^4*x^2)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\ln(c\,x^2+b)\,\left(3\,A\,c-B\,b\right)}{2\,b^4} - \frac{\frac{A}{2\,b}+\frac{3\,x^2\,(3\,A\,c-B\,b)}{4\,b^2}+\frac{c\,x^4\,(3\,A\,c-B\,b)}{2\,b^3}}{b^2\,x^2+2\,b\,c\,x^4+c^2\,x^6} - \frac{\ln(x)\,\left(3\,A\,c-B\,b\right)}{b^4}$$

input
```
int((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
(log(b + c*x^2)*(3*A*c - B*b))/(2*b^4) - (A/(2*b) + (3*x^2*(3*A*c - B*b))/
(4*b^2) + (c*x^4*(3*A*c - B*b))/(2*b^3))/(b^2*x^2 + c^2*x^6 + 2*b*c*x^4) -
 (log(x)*(3*A*c - B*b))/b^4
```

3.84. $\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.85 $\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3} dx$

3.85.1 Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3} dx = -\frac{A}{3b^3x^3} - \frac{bB-3Ac}{b^4x} - \frac{c(bB-Ac)x}{4b^3(b+cx^2)^2} - \frac{c(7bB-11Ac)x}{8b^4(b+cx^2)} - \frac{5\sqrt{c}(3bB-7Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}}$$

output
```
-1/3*A/b^3/x^3+(3*A*c-B*b)/b^4/x-1/4*c*(-A*c+B*b)*x/b^3/(c*x^2+b)^2-1/8*c*
(-11*A*c+7*B*b)*x/b^4/(c*x^2+b)-5/8*(-7*A*c+3*B*b)*arctan(x*c^(1/2)/b^(1/2
))*c^(1/2)/b^(9/2)
```

3.85.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3} dx = -\frac{A}{3b^3x^3} + \frac{-bB+3Ac}{b^4x} - \frac{c(bB-Ac)x}{4b^3(b+cx^2)^2} - \frac{(7bBc-11Ac^2)x}{8b^4(b+cx^2)} - \frac{5\sqrt{c}(3bB-7Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}}$$

input
```
Integrate[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/3*A/(b^3*x^3) + (-(b*B) + 3*A*c)/(b^4*x) - (c*(b*B - A*c)*x)/(4*b^3*(b
+ c*x^2)^2) - ((7*b*B*c - 11*A*c^2)*x)/(8*b^4*(b + c*x^2)) - (5*Sqrt[c]*(3
*b*B - 7*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(9/2))
```

3.85.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {9, 361, 25, 1582, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^2)}{(bx^2+cx^4)^3} \, dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{A+Bx^2}{x^4 (b+cx^2)^3} dx$$

$$\Big\downarrow \ 361$$

$$-\frac{1}{4}c\int -\frac{-\frac{3(bB-Ac)x^4}{b^3}+\frac{4(bB-Ac)x^2}{b^2c}+\frac{4A}{bc}}{x^4 (cx^2+b)^2}dx-\frac{cx(bB-Ac)}{4b^3 (b+cx^2)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{4}c\int \frac{-\frac{3(bB-Ac)x^4}{b^3}+\frac{4(bB-Ac)x^2}{b^2c}+\frac{4A}{bc}}{x^4 (cx^2+b)^2}dx-\frac{cx(bB-Ac)}{4b^3 (b+cx^2)^2}$$

$$\Big\downarrow \ 1582$$

$$\frac{1}{4}c\left(\frac{\int \frac{-\frac{c^2(7bB-11Ac)x^4}{b}+8c(bB-2Ac)x^2+8Abc}{x^4(cx^2+b)}dx}{2b^3c^2}-\frac{x(7bB-11Ac)}{2b^4 (b+cx^2)}\right)-\frac{cx(bB-Ac)}{4b^3 (b+cx^2)^2}$$

$$\Big\downarrow \ 1584$$

$$\frac{1}{4}c\left(\frac{\int \left(-\frac{5(3bB-7Ac)c^2}{b(cx^2+b)}+\frac{8(bB-3Ac)c}{bx^2}+\frac{8Ac}{x^4}\right)dx}{2b^3c^2}-\frac{x(7bB-11Ac)}{2b^4 (b+cx^2)}\right)-\frac{cx(bB-Ac)}{4b^3 (b+cx^2)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}c\left(-\frac{\frac{5c^{3/2}(3bB-7Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{8c(bB-3Ac)}{bx}-\frac{8Ac}{3x^3}}{2b^3c^2}-\frac{x(7bB-11Ac)}{2b^4\left(b+cx^2\right)}\right)-\frac{cx(bB-Ac)}{4b^3\left(b+cx^2\right)^2}$$

input
```
Int[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*(c*(b*B - A*c)*x)/(b^3*(b + c*x^2)^2) + (c*(-1/2*((7*b*B - 11*A*c)*x)
/(b^4*(b + c*x^2)) + ((-8*A*c)/(3*x^3) - (8*c*(b*B - 3*A*c))/(b*x) - (5*c^
(3/2)*(3*b*B - 7*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2))/(2*b^3*c^2)))/
4
```

3.85.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.85.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

method	result
default	$-\frac{A}{3b^3x^3}-\frac{-3Ac+Bb}{b^4x}+\frac{c\left(\frac{\left(\frac{11}{8}Ac^2-\frac{7}{8}Bbc\right)x^3+\frac{b(13Ac-9Bb)x}{8}}{(cx^2+b)^2}+\frac{5(7Ac-3Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{b^4}$
risch	$\frac{\frac{5c^2(7Ac-3Bb)x^6}{8b^4}+\frac{25c(7Ac-3Bb)x^4}{24b^3}+\frac{(7Ac-3Bb)x^2}{3b^2}-\frac{A}{3b}}{x^3(cx^2+b)^2}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(b^9\_Z^2+49A^2c^3-42ABbc^2+9B^2b^2c\right)}\_R\ln\left(\left(3\_R^2b^9+9\right.\right.\right.}{16}$

input
```
int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A/b^3/x^3-(-3*A*c+B*b)/b^4/x+1/b^4*c*(((11/8*A*c^2-7/8*B*b*c)*x^3+1/8
*b*(13*A*c-9*B*b)*x)/(c*x^2+b)^2+5/8*(7*A*c-3*B*b)/(b*c)^(1/2)*arctan(c*x/
(b*c)^(1/2)))
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 368, normalized size of antiderivative = 3.15

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$
$$= \left[-\frac{30\,(3\,Bbc^2-7\,Ac^3)x^6+50\,(3\,Bb^2c-7\,Abc^2)x^4+16\,Ab^3+16\,(3\,Bb^3-7\,Ab^2c)x^2+15\,((3\,Bbc^2-7\,A}{48\,(b^4c^2x^7+2\,b^5cx^5+b^6x}\right.$$
$$-\frac{15\,(3\,Bbc^2-7\,Ac^3)x^6+25\,(3\,Bb^2c-7\,Abc^2)x^4+8\,Ab^3+8\,(3\,Bb^3-7\,Ab^2c)x^2+15\,((3\,Bbc^2-7\,Ac^3)}{24\,(b^4c^2x^7+2\,b^5cx^5+b^6x^3)}$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
[-1/48*(30*(3*B*b*c^2 - 7*A*c^3)*x^6 + 50*(3*B*b^2*c - 7*A*b*c^2)*x^4 + 16
*A*b^3 + 16*(3*B*b^3 - 7*A*b^2*c)*x^2 + 15*((3*B*b*c^2 - 7*A*c^3)*x^7 + 2*
(3*B*b^2*c - 7*A*b*c^2)*x^5 + (3*B*b^3 - 7*A*b^2*c)*x^3)*sqrt(-c/b)*log((c
*x^2 + 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^
6*x^3), -1/24*(15*(3*B*b*c^2 - 7*A*c^3)*x^6 + 25*(3*B*b^2*c - 7*A*b*c^2)*x
^4 + 8*A*b^3 + 8*(3*B*b^3 - 7*A*b^2*c)*x^2 + 15*((3*B*b*c^2 - 7*A*c^3)*x^7
 + 2*(3*B*b^2*c - 7*A*b*c^2)*x^5 + (3*B*b^3 - 7*A*b^2*c)*x^3)*sqrt(c/b)*ar
ctan(x*sqrt(c/b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)]
```

3.85.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(109) = 218.

Time = 0.40 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.93

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$
$$= \frac{5\sqrt{-\frac{c}{b^9}}(-7Ac+3Bb)\log\left(-\frac{5b^5\sqrt{-\frac{c}{b^9}}(-7Ac+3Bb)}{-35Ac^2+15Bbc}+x\right)}{16} - \frac{5\sqrt{-\frac{c}{b^9}}(-7Ac+3Bb)\log\left(\frac{5b^5\sqrt{-\frac{c}{b^9}}(-7Ac+3Bb)}{-35Ac^2+15Bbc}+x\right)}{16} + \frac{-8Ab^3+x^6\cdot(105Ac^3-45Bbc^2)+x^4\cdot(175Abc^2-75Bb^2c)+x^2\cdot(56Ab^2c-24Bb^3)}{24b^6x^3+48b^5cx^5+24b^4c^2x^7}$$

input
```
integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
5*sqrt(-c/b**9)*(-7*A*c + 3*B*b)*log(-5*b**5*sqrt(-c/b**9)*(-7*A*c + 3*B*b
)/(-35*A*c**2 + 15*B*b*c) + x)/16 - 5*sqrt(-c/b**9)*(-7*A*c + 3*B*b)*log(5
*b**5*sqrt(-c/b**9)*(-7*A*c + 3*B*b)/(-35*A*c**2 + 15*B*b*c) + x)/16 + (-8
*A*b**3 + x**6*(105*A*c**3 - 45*B*b*c**2) + x**4*(175*A*b*c**2 - 75*B*b**2
*c) + x**2*(56*A*b**2*c - 24*B*b**3))/(24*b**6*x**3 + 48*b**5*c*x**5 + 24*
b**4*c**2*x**7)
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.09

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{15\,(3\,Bbc^2-7\,Ac^3)x^6+25\,(3\,Bb^2c-7\,Abc^2)x^4+8\,Ab^3+8\,(3\,Bb^3-7\,Ab^2c)x^2}{24\,(b^4c^2x^7+2\,b^5cx^5+b^6x^3)} - \frac{5\,(3\,Bbc-7\,Ac^2)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^4}$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/24*(15*(3*B*b*c^2 - 7*A*c^3)*x^6 + 25*(3*B*b^2*c - 7*A*b*c^2)*x^4 + 8*A
*b^3 + 8*(3*B*b^3 - 7*A*b^2*c)*x^2)/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)
- 5/8*(3*B*b*c - 7*A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4)
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{5\,(3\,Bbc-7\,Ac^2)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^4} - \frac{7\,Bbc^2x^3-11\,Ac^3x^3+9\,Bb^2cx-13\,Abc^2x}{8\,(cx^2+b)^2b^4} - \frac{3\,Bbx^2-9\,Acx^2+Ab}{3\,b^4x^3}$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-5/8*(3*B*b*c - 7*A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) - 1/8*(7*B*
b*c^2*x^3 - 11*A*c^3*x^3 + 9*B*b^2*c*x - 13*A*b*c^2*x)/((c*x^2 + b)^2*b^4)
 - 1/3*(3*B*b*x^2 - 9*A*c*x^2 + A*b)/(b^4*x^3)
```

3.85.9 Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{x^2\,(7\,A\,c-3\,B\,b)}{3\,b^2}-\frac{A}{3\,b}+\frac{5\,c^2\,x^6\,(7\,A\,c-3\,B\,b)}{8\,b^4}+\frac{25\,c\,x^4\,(7\,A\,c-3\,B\,b)}{24\,b^3}}{b^2\,x^3+2\,b\,c\,x^5+c^2\,x^7} + \frac{5\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(7\,A\,c-3\,B\,b)}{8\,b^{9/2}}$$

input
```
int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
((x^2*(7*A*c - 3*B*b))/(3*b^2) - A/(3*b) + (5*c^2*x^6*(7*A*c - 3*B*b))/(8*
b^4) + (25*c*x^4*(7*A*c - 3*B*b))/(24*b^3))/(b^2*x^3 + c^2*x^7 + 2*b*c*x^5
) + (5*c^(1/2)*atan((c^(1/2)*x)/b^(1/2))*(7*A*c - 3*B*b))/(8*b^(9/2))
```

3.86 $\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.86.1 Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{A}{4b^3x^4} - \frac{bB-3Ac}{2b^4x^2} - \frac{c(bB-Ac)}{4b^3(b+cx^2)^2} - \frac{c(2bB-3Ac)}{2b^4(b+cx^2)} - \frac{3c(bB-2Ac)\log(x)}{b^5} + \frac{3c(bB-2Ac)\log(b+cx^2)}{2b^5}$$

output
```
-1/4*A/b^3/x^4+1/2*(3*A*c-B*b)/b^4/x^2-1/4*c*(-A*c+B*b)/b^3/(c*x^2+b)^2-1/
2*c*(-3*A*c+2*B*b)/b^4/(c*x^2+b)-3*c*(-2*A*c+B*b)*ln(x)/b^5+3/2*c*(-2*A*c+
B*b)*ln(c*x^2+b)/b^5
```

3.86.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{-\frac{Ab^2}{x^4} - \frac{2b(bB-3Ac)}{x^2} + \frac{b^2c(-bB+Ac)}{(b+cx^2)^2} + \frac{2bc(-2bB+3Ac)}{b+cx^2} + 12c(-bB+2Ac)\log(x) + 6c(bB-2Ac)\log(b+cx^2)}{4b^5}$$

input
```
Integrate[(x*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
(-((A*b^2)/x^4) - (2*b*(b*B - 3*A*c))/x^2 + (b^2*c*(-(b*B) + A*c))/(b + c*
x^2)^2 + (2*b*c*(-2*b*B + 3*A*c))/(b + c*x^2) + 12*c*(-(b*B) + 2*A*c)*Log[
x] + 6*c*(b*B - 2*A*c)*Log[b + c*x^2])/(4*b^5)
```

3.86.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow 9$$

$$\int \frac{A+Bx^2}{x^5(b+cx^2)^3}dx$$

$$\Big\downarrow 354$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^6(cx^2+b)^3}dx^2$$

$$\Big\downarrow 86$$

$$\frac{1}{2}\int \left(\frac{3(bB-2Ac)c^2}{b^5(cx^2+b)}+\frac{(2bB-3Ac)c^2}{b^4(cx^2+b)^2}+\frac{(bB-Ac)c^2}{b^3(cx^2+b)^3}-\frac{3(bB-2Ac)c}{b^5x^2}+\frac{bB-3Ac}{b^4x^4}+\frac{A}{b^3x^6}\right)dx^2$$

$$\Big\downarrow 2009$$

$$\frac{1}{2}\left(-\frac{3c\log(x^2)(bB-2Ac)}{b^5}+\frac{3c(bB-2Ac)\log(b+cx^2)}{b^5}-\frac{c(2bB-3Ac)}{b^4(b+cx^2)}-\frac{bB-3Ac}{b^4x^2}-\frac{c(bB-Ac)}{2b^3(b+cx^2)^2}-\frac{A}{2b^3x}\right.$$

input
```
Int[(x*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
(-1/2*A/(b^3*x^4) - (b*B - 3*A*c)/(b^4*x^2) - (c*(b*B - A*c))/(2*b^3*(b +
c*x^2)^2) - (c*(2*b*B - 3*A*c))/(b^4*(b + c*x^2)) - (3*c*(b*B - 2*A*c)*Log
[x^2])/b^5 + (3*c*(b*B - 2*A*c)*Log[b + c*x^2])/b^5)/2
```

3.86.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.86.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02

method	result
default	$-\frac{A}{4b^3x^4}-\frac{-3Ac+Bb}{2x^2b^4}+\frac{3c(2Ac-Bb)\ln(x)}{b^5}-\frac{c^2\left(-\frac{b(3Ac-2Bb)}{c(cx^2+b)}+\frac{(6Ac-3Bb)\ln(cx^2+b)}{c}-\frac{b^2(Ac-Bb)}{2c(cx^2+b)^2}\right)}{2b^5}$
norman	$\frac{-\frac{Ax}{4b}+\frac{(2Ac-Bb)x^3}{2b^2}-\frac{c(6Ac^2-3Bbc)x^7}{b^4}-\frac{c^2(18Ac^2-9Bbc)x^9}{4b^5}}{x^5(cx^2+b)^2}+\frac{3c(2Ac-Bb)\ln(x)}{b^5}-\frac{3c(2Ac-Bb)\ln(cx^2+b)}{2b^5}$
risch	$\frac{\frac{3c^2(2Ac-Bb)x^6}{2b^4}+\frac{9c(2Ac-Bb)x^4}{4b^3}+\frac{(2Ac-Bb)x^2}{2b^2}-\frac{A}{4b}}{x^4(cx^2+b)^2}+\frac{6c^2\ln(x)A}{b^5}-\frac{3c\ln(x)B}{b^4}-\frac{3c^2\ln(cx^2+b)A}{b^5}+\frac{3c\ln(cx^2+b)B}{2b^4}$
parallelrisch	$24A\ln(x)x^8c^4-12A\ln(cx^2+b)x^8c^4-12B\ln(x)x^8bc^3+6B\ln(cx^2+b)x^8bc^3-18Ax^8c^4+9Bx^8bc^3+48A\ln(x)x^6bc^3-24A\ln($

input
```
int(x*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*A/b^3/x^4-1/2*(-3*A*c+B*b)/x^2/b^4+3*c*(2*A*c-B*b)/b^5*ln(x)-1/2/b^5*
c^2*(-b*(3*A*c-2*B*b)/c/(c*x^2+b)+(6*A*c-3*B*b)/c*ln(c*x^2+b)-1/2*b^2*(A*c
-B*b)/c/(c*x^2+b)^2)
```

3.86.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(111) = 222$.

Time = 0.25 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.89

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3}\,dx =$$
$$-\frac{6\,(Bb^2c^2-2\,Abc^3)x^6+Ab^4+9\,(Bb^3c-2\,Ab^2c^2)x^4+2\,(Bb^4-2\,Ab^3c)x^2-6\,((Bbc^3-2\,Ac^4)x^8+2\,(B}{}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
-1/4*(6*(B*b^2*c^2 - 2*A*b*c^3)*x^6 + A*b^4 + 9*(B*b^3*c - 2*A*b^2*c^2)*x^
4 + 2*(B*b^4 - 2*A*b^3*c)*x^2 - 6*((B*b*c^3 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2
- 2*A*b*c^3)*x^6 + (B*b^3*c - 2*A*b^2*c^2)*x^4)*log(c*x^2 + b) + 12*((B*b*
c^3 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2 - 2*A*b*c^3)*x^6 + (B*b^3*c - 2*A*b^2*c^
2)*x^4)*log(x))/(b^5*c^2*x^8 + 2*b^6*c*x^6 + b^7*x^4)
```

3.86.6 Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.12

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{-Ab^3+x^6\cdot(12Ac^3-6Bbc^2)+x^4\cdot(18Abc^2-9Bb^2c)+x^2\cdot(4Ab^2c-2Bb^3)}{4b^6x^4+8b^5cx^6+4b^4c^2x^8} - \frac{3c(-2Ac+Bb)\log(x)}{b^5} + \frac{3c(-2Ac+Bb)\log\left(\frac{b}{c}+x^2\right)}{2b^5}$$

input
```
integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
(-A*b**3 + x**6*(12*A*c**3 - 6*B*b*c**2) + x**4*(18*A*b*c**2 - 9*B*b**2*c)
 + x**2*(4*A*b**2*c - 2*B*b**3))/(4*b**6*x**4 + 8*b**5*c*x**6 + 4*b**4*c**
2*x**8) - 3*c*(-2*A*c + B*b)*log(x)/b**5 + 3*c*(-2*A*c + B*b)*log(b/c + x*
*2)/(2*b**5)
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.13

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{6\,(Bbc^2-2\,Ac^3)x^6+9\,(Bb^2c-2\,Abc^2)x^4+Ab^3+2\,(Bb^3-2\,Ab^2c)x^2}{4\,(b^4c^2x^8+2\,b^5cx^6+b^6x^4)} + \frac{3\,(Bbc-2\,Ac^2)\log{(cx^2+b)}}{2\,b^5} - \frac{3\,(Bbc-2\,Ac^2)\log{(x^2)}}{2\,b^5}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(6*(B*b*c^2 - 2*A*c^3)*x^6 + 9*(B*b^2*c - 2*A*b*c^2)*x^4 + A*b^3 + 2*
(B*b^3 - 2*A*b^2*c)*x^2)/(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4) + 3/2*(B*b*
c - 2*A*c^2)*log(c*x^2 + b)/b^5 - 3/2*(B*b*c - 2*A*c^2)*log(x^2)/b^5
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.09

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{3\,(Bbc-2\,Ac^2)\log{(|x|)}}{b^5} + \frac{3\,(Bbc^2-2\,Ac^3)\log{(|cx^2+b|)}}{2\,b^5c} - \frac{6\,Bbc^2x^6-12\,Ac^3x^6+9\,Bb^2cx^4-18\,Abc^2x^4+2\,Bb^3x^2-4\,Ab^2cx^2+Ab^3}{4\,(cx^4+bx^2)^2b^4}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-3*(B*b*c - 2*A*c^2)*log(abs(x))/b^5 + 3/2*(B*b*c^2 - 2*A*c^3)*log(abs(c*x
^2 + b))/(b^5*c) - 1/4*(6*B*b*c^2*x^6 - 12*A*c^3*x^6 + 9*B*b^2*c*x^4 - 18*
A*b*c^2*x^4 + 2*B*b^3*x^2 - 4*A*b^2*c*x^2 + A*b^3)/((c*x^4 + b*x^2)^2*b^4)
```

3.86.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.08

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{x^2\,(2\,A\,c-B\,b)}{2\,b^2}-\frac{A}{4\,b}+\frac{3\,c^2\,x^6\,(2\,A\,c-B\,b)}{2\,b^4}+\frac{9\,c\,x^4\,(2\,A\,c-B\,b)}{4\,b^3}}{b^2\,x^4+2\,b\,c\,x^6+c^2\,x^8} - \frac{\ln\left(c\,x^2+b\right)\,\left(6\,A\,c^2-3\,B\,b\,c\right)}{2\,b^5}+\frac{\ln\left(x\right)\,\left(6\,A\,c^2-3\,B\,b\,c\right)}{b^5}$$

input
```
int((x*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
((x^2*(2*A*c - B*b))/(2*b^2) - A/(4*b) + (3*c^2*x^6*(2*A*c - B*b))/(2*b^4)
 + (9*c*x^4*(2*A*c - B*b))/(4*b^3))/(b^2*x^4 + c^2*x^8 + 2*b*c*x^6) - (log
(b + c*x^2)*(6*A*c^2 - 3*B*b*c))/(2*b^5) + (log(x)*(6*A*c^2 - 3*B*b*c))/b^
5
```

3.87 $\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^3} dx$

3.87.1 Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^3} dx = -\frac{A}{5b^3x^5} - \frac{bB-3Ac}{3b^4x^3} + \frac{3c(bB-2Ac)}{b^5x} + \frac{c^2(bB-Ac)x}{4b^4\left(b+cx^2\right)^2} + \frac{c^2(11bB-15Ac)x}{8b^5\left(b+cx^2\right)} + \frac{7c^{3/2}(5bB-9Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}}$$

output
```
-1/5*A/b^3/x^5+1/3*(3*A*c-B*b)/b^4/x^3+3*c*(-2*A*c+B*b)/b^5/x+1/4*c^2*(-A*
c+B*b)*x/b^4/(c*x^2+b)^2+1/8*c^2*(-15*A*c+11*B*b)*x/b^5/(c*x^2+b)+7/8*c^(3
/2)*(-9*A*c+5*B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(11/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^3} dx = -\frac{A}{5b^3x^5} - \frac{bB-3Ac}{3b^4x^3} + \frac{3c(bB-2Ac)}{b^5x} + \frac{c^2(bB-Ac)x}{4b^4\left(b+cx^2\right)^2} + \frac{c^2(11bB-15Ac)x}{8b^5\left(b+cx^2\right)} + \frac{7c^{3/2}(5bB-9Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}}$$

input
```
Integrate[(A + B*x^2)/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/5*A/(b^3*x^5) - (b*B - 3*A*c)/(3*b^4*x^3) + (3*c*(b*B - 2*A*c))/(b^5*x)
 + (c^2*(b*B - A*c)*x)/(4*b^4*(b + c*x^2)^2) + (c^2*(11*b*B - 15*A*c)*x)/(
8*b^5*(b + c*x^2)) + (7*c^(3/2)*(5*b*B - 9*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]
])/(8*b^(11/2))
```

3.87.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2026, 361, 25, 2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{A+Bx^2}{x^6(b+cx^2)^3}dx$$

$$\Big\downarrow \text{361}$$

$$\frac{c^2x(bB-Ac)}{4b^4(b+cx^2)^2}-\frac{1}{4}c^2\int -\frac{\frac{3(bB-Ac)x^6}{b^4}-\frac{4(bB-Ac)x^4}{b^3c}+\frac{4(bB-Ac)x^2}{b^2c^2}+\frac{4A}{bc^2}}{x^6(cx^2+b)^2}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}c^2\int \frac{\frac{3(bB-Ac)x^6}{b^4}-\frac{4(bB-Ac)x^4}{b^3c}+\frac{4(bB-Ac)x^2}{b^2c^2}+\frac{4A}{bc^2}}{x^6(cx^2+b)^2}dx+\frac{c^2x(bB-Ac)}{4b^4(b+cx^2)^2}$$

$$\Big\downarrow \text{2336}$$

$$\frac{1}{4}c^2\left(\frac{x(11bB-15Ac)}{2b^5(b+cx^2)}-\frac{\int -\frac{\frac{(11bB-15Ac)x^6}{b^4}-\frac{8(2bB-3Ac)x^4}{b^3c}+\frac{8(bB-2Ac)x^2}{b^2c^2}+\frac{8A}{bc^2}}{x^6(cx^2+b)}dx}{2b}\right)+\frac{c^2x(bB-Ac)}{4b^4(b+cx^2)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}c^2\left(\frac{\int \frac{\frac{(11bB-15Ac)x^6}{b^4}-\frac{8(2bB-3Ac)x^4}{b^3c}+\frac{8(bB-2Ac)x^2}{b^2c^2}+\frac{8A}{bc^2}}{x^6(cx^2+b)}dx}{2b}+\frac{x(11bB-15Ac)}{2b^5(b+cx^2)}\right)+\frac{c^2x(bB-Ac)}{4b^4(b+cx^2)^2}$$

$$\Big\downarrow \text{2333}$$

$$\frac{1}{4}c^2\left(\frac{\int\left(\frac{8A}{b^2c^2x^6}+\frac{7(5bB-9Ac)}{b^4(cx^2+b)}-\frac{24(bB-2Ac)}{b^4cx^2}+\frac{8(bB-3Ac)}{b^3c^2x^4}\right)dx}{2b}+\frac{x(11bB-15Ac)}{2b^5\,(b+cx^2)}\right)+\frac{c^2x(bB-Ac)}{4b^4\,(b+cx^2)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}c^2\left(\frac{\frac{7(5bB-9Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{9/2}\sqrt{c}}+\frac{24(bB-2Ac)}{b^4cx}-\frac{8(bB-3Ac)}{3b^3c^2x^3}-\frac{8A}{5b^2c^2x^5}}{2b}+\frac{x(11bB-15Ac)}{2b^5\,(b+cx^2)}\right)+\frac{c^2x(bB-Ac)}{4b^4\,(b+cx^2)^2}$$

input
```
Int[(A + B*x^2)/(b*x^2 + c*x^4)^3,x]
```

output
```
(c^2*(b*B - A*c)*x)/(4*b^4*(b + c*x^2)^2) + (c^2*(((11*b*B - 15*A*c)*x)/(2
*b^5*(b + c*x^2)) + ((-8*A)/(5*b^2*c^2*x^5) - (8*(b*B - 3*A*c))/(3*b^3*c^2
*x^3) + (24*(b*B - 2*A*c))/(b^4*c*x) + (7*(5*b*B - 9*A*c)*ArcTan[(Sqrt[c]*
x)/Sqrt[b]])/(b^(9/2)*Sqrt[c]))/(2*b)))/4
```

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.87.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.85

method	result
default	$-\frac{A}{5b^3x^5}-\frac{-3Ac+Bb}{3x^3b^4}-\frac{3c(2Ac-Bb)}{b^5x}-\frac{c^2\left(\frac{\left(\frac{15}{8}Ac^2-\frac{11}{8}Bbc\right)x^3+\frac{b(17Ac-13Bb)x}{8}}{\left(cx^2+b\right)^2}+\frac{7(9Ac-5Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{b^5}$
risch	$\frac{-\frac{7c^3(9Ac-5Bb)x^8}{8b^5}-\frac{35c^2(9Ac-5Bb)x^6}{24b^4}-\frac{7c(9Ac-5Bb)x^4}{15b^3}+\frac{(9Ac-5Bb)x^2}{15b^2}-\frac{A}{5b}}{x^5(cx^2+b)^2}+\frac{7\left(\sum_{\_R=\mathrm{RootOf}\left(b^{11}\_Z^2+81A^2c^5-90ABbc^4+25B^2b^2c^3\right)}\right.}{}$

input
```
int((B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/5*A/b^3/x^5-1/3*(-3*A*c+B*b)/x^3/b^4-3*c*(2*A*c-B*b)/b^5/x-1/b^5*c^2*((
(15/8*A*c^2-11/8*B*b*c)*x^3+1/8*b*(17*A*c-13*B*b)*x)/(c*x^2+b)^2+7/8*(9*A*
c-5*B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.04

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^3}\,dx$$
$$= \left[\frac{210\,(5\,Bbc^3-9\,Ac^4)x^8+350\,(5\,Bb^2c^2-9\,Abc^3)x^6-48\,Ab^4+112\,(5\,Bb^3c-9\,Ab^2c^2)x^4-16\,(5\,Bb^4-9}{240\,(b^5c^2x}\right.$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
[1/240*(210*(5*B*b*c^3 - 9*A*c^4)*x^8 + 350*(5*B*b^2*c^2 - 9*A*b*c^3)*x^6
- 48*A*b^4 + 112*(5*B*b^3*c - 9*A*b^2*c^2)*x^4 - 16*(5*B*b^4 - 9*A*b^3*c)*
x^2 - 105*((5*B*b*c^3 - 9*A*c^4)*x^9 + 2*(5*B*b^2*c^2 - 9*A*b*c^3)*x^7 + (
5*B*b^3*c - 9*A*b^2*c^2)*x^5)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b
)/(c*x^2 + b)))/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5), 1/120*(105*(5*B*b*c
^3 - 9*A*c^4)*x^8 + 175*(5*B*b^2*c^2 - 9*A*b*c^3)*x^6 - 24*A*b^4 + 56*(5*B
*b^3*c - 9*A*b^2*c^2)*x^4 - 8*(5*B*b^4 - 9*A*b^3*c)*x^2 + 105*((5*B*b*c^3
- 9*A*c^4)*x^9 + 2*(5*B*b^2*c^2 - 9*A*b*c^3)*x^7 + (5*B*b^3*c - 9*A*b^2*c^
2)*x^5)*sqrt(c/b)*arctan(x*sqrt(c/b)))/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^
5)]
```

3.87.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.86

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^3}\,dx = -\frac{7\sqrt{-\frac{c^3}{b^{11}}}(-9Ac+5Bb)\log\left(-\frac{7b^6\sqrt{-\frac{c^3}{b^{11}}}(-9Ac+5Bb)}{-63Ac^3+35Bbc^2}+x\right)}{16}$$
$$+\frac{7\sqrt{-\frac{c^3}{b^{11}}}(-9Ac+5Bb)\log\left(\frac{7b^6\sqrt{-\frac{c^3}{b^{11}}}(-9Ac+5Bb)}{-63Ac^3+35Bbc^2}+x\right)}{16}$$
$$+\frac{-24Ab^4+x^8(-945Ac^4+525Bbc^3)+x^6(-1575Abc^3+875Bb^2c^2)+x^4(-504Ab^2c^2+280Bb^3c)+x^2\cdot}{120b^7x^5+240b^6cx^7+120b^5c^2x^9}$$

input
```
integrate((B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
-7*sqrt(-c**3/b**11)*(-9*A*c + 5*B*b)*log(-7*b**6*sqrt(-c**3/b**11)*(-9*A*
c + 5*B*b)/(-63*A*c**3 + 35*B*b*c**2) + x)/16 + 7*sqrt(-c**3/b**11)*(-9*A*
c + 5*B*b)*log(7*b**6*sqrt(-c**3/b**11)*(-9*A*c + 5*B*b)/(-63*A*c**3 + 35*
B*b*c**2) + x)/16 + (-24*A*b**4 + x**8*(-945*A*c**4 + 525*B*b*c**3) + x**6
*(-1575*A*b*c**3 + 875*B*b**2*c**2) + x**4*(-504*A*b**2*c**2 + 280*B*b**3*
c) + x**2*(72*A*b**3*c - 40*B*b**4))/(120*b**7*x**5 + 240*b**6*c*x**7 + 12
0*b**5*c**2*x**9)
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^3}\,dx = \frac{105\,(5\,Bbc^3-9\,Ac^4)x^8+175\,(5\,Bb^2c^2-9\,Abc^3)x^6-24\,Ab^4+56\,(5\,Bb^3c-9\,Ab^2c^2)x^4-8\,(5\,Bb^4-9\,Ab^3}{120\,(b^5c^2x^9+2\,b^6cx^7+b^7x^5)} + \frac{7\,(5\,Bbc^2-9\,Ac^3)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^5}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/120*(105*(5*B*b*c^3 - 9*A*c^4)*x^8 + 175*(5*B*b^2*c^2 - 9*A*b*c^3)*x^6 -
 24*A*b^4 + 56*(5*B*b^3*c - 9*A*b^2*c^2)*x^4 - 8*(5*B*b^4 - 9*A*b^3*c)*x^2
)/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5) + 7/8*(5*B*b*c^2 - 9*A*c^3)*arctan
(c*x/sqrt(b*c))/(sqrt(b*c)*b^5)
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^3}\,dx = \frac{7\,(5\,Bbc^2-9\,Ac^3)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^5} + \frac{11\,Bbc^3x^3-15\,Ac^4x^3+13\,Bb^2c^2x-17\,Abc^3x}{8\,(cx^2+b)^2b^5} + \frac{45\,Bbcx^4-90\,Ac^2x^4-5\,Bb^2x^2+15\,Abcx^2-3\,Ab^2}{15\,b^5x^5}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
7/8*(5*B*b*c^2 - 9*A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^5) + 1/8*(11*
B*b*c^3*x^3 - 15*A*c^4*x^3 + 13*B*b^2*c^2*x - 17*A*b*c^3*x)/((c*x^2 + b)^2
*b^5) + 1/15*(45*B*b*c*x^4 - 90*A*c^2*x^4 - 5*B*b^2*x^2 + 15*A*b*c*x^2 - 3
*A*b^2)/(b^5*x^5)
```

3.87.9 Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^3}\,dx = -\frac{\frac{A}{5\,b}-\frac{x^2\,(9\,A\,c-5\,B\,b)}{15\,b^2}+\frac{35\,c^2\,x^6\,(9\,A\,c-5\,B\,b)}{24\,b^4}+\frac{7\,c^3\,x^8\,(9\,A\,c-5\,B\,b)}{8\,b^5}+\frac{7\,c\,x^4\,(9\,A\,c-5\,B\,b)}{15\,b^3}}{b^2\,x^5+2\,b\,c\,x^7+c^2\,x^9} - \frac{7\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(9\,A\,c-5\,B\,b)}{8\,b^{11/2}}$$

input
```
int((A + B*x^2)/(b*x^2 + c*x^4)^3,x)
```

output
```
- (A/(5*b) - (x^2*(9*A*c - 5*B*b))/(15*b^2) + (35*c^2*x^6*(9*A*c - 5*B*b))
/(24*b^4) + (7*c^3*x^8*(9*A*c - 5*B*b))/(8*b^5) + (7*c*x^4*(9*A*c - 5*B*b)
)/(15*b^3))/(b^2*x^5 + c^2*x^9 + 2*b*c*x^7) - (7*c^(3/2)*atan((c^(1/2)*x)/
b^(1/2))*(9*A*c - 5*B*b))/(8*b^(11/2))
```

3.88 $\int \frac{A+Bx^2}{x(bx^2+cx^4)^3} dx$

3.88.1 Optimal result

Integrand size = 24, antiderivative size = 148

$$\int \frac{A+Bx^2}{x(bx^2+cx^4)^3} dx = -\frac{A}{6b^3x^6} - \frac{bB-3Ac}{4b^4x^4} + \frac{3c(bB-2Ac)}{2b^5x^2} + \frac{c^2(bB-Ac)}{4b^4(b+cx^2)^2} + \frac{c^2(3bB-4Ac)}{2b^5(b+cx^2)} + \frac{2c^2(3bB-5Ac)\log(x)}{b^6} - \frac{c^2(3bB-5Ac)\log(b+cx^2)}{b^6}$$

output
```
-1/6*A/b^3/x^6+1/4*(3*A*c-B*b)/b^4/x^4+3/2*c*(-2*A*c+B*b)/b^5/x^2+1/4*c^2*
(-A*c+B*b)/b^4/(c*x^2+b)^2+1/2*c^2*(-4*A*c+3*B*b)/b^5/(c*x^2+b)+2*c^2*(-5*
A*c+3*B*b)*ln(x)/b^6-c^2*(-5*A*c+3*B*b)*ln(c*x^2+b)/b^6
```

3.88.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^2}{x(bx^2+cx^4)^3} dx = \frac{-\frac{2Ab^3}{x^6} - \frac{3b^2(bB-3Ac)}{x^4} + \frac{18bc(bB-2Ac)}{x^2} + \frac{3b^2c^2(bB-Ac)}{(b+cx^2)^2} + \frac{6bc^2(3bB-4Ac)}{b+cx^2} + 24c^2(3bB-5Ac)\log(x) + 12c^2(-3bB-}{12b^6}$$

input
```
Integrate[(A + B*x^2)/(x*(b*x^2 + c*x^4)^3),x]
```

output
```
((-2*A*b^3)/x^6 - (3*b^2*(b*B - 3*A*c))/x^4 + (18*b*c*(b*B - 2*A*c))/x^2 +
 (3*b^2*c^2*(b*B - A*c))/(b + c*x^2)^2 + (6*b*c^2*(3*b*B - 4*A*c))/(b + c*
x^2) + 24*c^2*(3*b*B - 5*A*c)*Log[x] + 12*c^2*(-3*b*B + 5*A*c)*Log[b + c*x
^2])/(12*b^6)
```

3.88.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^3}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{A+Bx^2}{x^7\left(b+cx^2\right)^3}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^8\left(cx^2+b\right)^3}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(-\frac{2(3bB-5Ac)c^3}{b^6\left(cx^2+b\right)}-\frac{(3bB-4Ac)c^3}{b^5\left(cx^2+b\right)^2}-\frac{(bB-Ac)c^3}{b^4\left(cx^2+b\right)^3}+\frac{2(3bB-5Ac)c^2}{b^6x^2}-\frac{3(bB-2Ac)c}{b^5x^4}+\frac{bB-3Ac}{b^4x^6}+\frac{}{b^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2c^2\log\left(x^2\right)(3bB-5Ac)}{b^6}-\frac{2c^2(3bB-5Ac)\log\left(b+cx^2\right)}{b^6}+\frac{c^2(3bB-4Ac)}{b^5\left(b+cx^2\right)}+\frac{3c(bB-2Ac)}{b^5x^2}+\frac{c^2(bB-Ac)}{2b^4\left(b+cx^2\right)^2}\right.$$

input
```
Int[(A + B*x^2)/(x*(b*x^2 + c*x^4)^3),x]
```

output
```
(-1/3*A/(b^3*x^6) - (b*B - 3*A*c)/(2*b^4*x^4) + (3*c*(b*B - 2*A*c))/(b^5*x
^2) + (c^2*(b*B - A*c))/(2*b^4*(b + c*x^2)^2) + (c^2*(3*b*B - 4*A*c))/(b^5
*(b + c*x^2)) + (2*c^2*(3*b*B - 5*A*c)*Log[x^2])/b^6 - (2*c^2*(3*b*B - 5*A
*c)*Log[b + c*x^2])/b^6)/2
```

3.88.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.88. $\int \frac{A+Bx^2}{x(bx^2+cx^4)^3}\,dx$

3.88.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.97

method	result
default	$-\frac{A}{6b^3x^6}-\frac{-3Ac+Bb}{4x^4b^4}-\frac{3c(2Ac-Bb)}{2b^5x^2}-\frac{2c^2(5Ac-3Bb)\ln(x)}{b^6}+\frac{c^3\left(\frac{(10Ac-6Bb)\ln\left(c\,x^2+b\right)}{c}-\frac{b^2(Ac-Bb)}{2c\left(c\,x^2+b\right)^2}-\frac{b(4Ac-3Bb)}{c\left(c\,x^2+b\right)}\right.}{2b^6}$
norman	$\frac{-\frac{A}{6b}+\frac{(5Ac-3Bb)x^2}{12b^2}-\frac{c(5Ac-3Bb)x^4}{3b^3}+\frac{2c\left(5A\,c^3-3Bb\,c^2\right)x^8}{b^5}+\frac{c^2\left(15A\,c^3-9Bb\,c^2\right)x^{10}}{2b^6}}{x^6(c\,x^2+b)^2}+\frac{c^2(5Ac-3Bb)\ln(c\,x^2+b)}{b^6}-\frac{2c^2(5Ac-}{b}$
risch	$\frac{-\frac{c^3(5Ac-3Bb)x^8}{b^5}-\frac{3c^2(5Ac-3Bb)x^6}{2b^4}-\frac{c(5Ac-3Bb)x^4}{3b^3}+\frac{(5Ac-3Bb)x^2}{12b^2}-\frac{A}{6b}}{x^6(c\,x^2+b)^2}-\frac{10c^3\ln(x)A}{b^6}+\frac{6c^2\ln(x)B}{b^5}+\frac{5c^3\ln(-c\,x^2-b)A}{b^6}$
parallelrisch	$-\frac{120A\ln(x)x^{10}c^5-60A\ln(c\,x^2+b)x^{10}c^5-72B\ln(x)x^{10}b\,c^4+36B\ln(c\,x^2+b)x^{10}b\,c^4-90A\,x^{10}c^5+54B\,x^{10}b\,c^4+240A\ln(x)x^8}{}$

input
```
int((B*x^2+A)/x/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6*A/b^3/x^6-1/4*(-3*A*c+B*b)/x^4/b^4-3/2*c*(2*A*c-B*b)/b^5/x^2-2*c^2*(5
*A*c-3*B*b)/b^6*ln(x)+1/2/b^6*c^3*((10*A*c-6*B*b)/c*ln(c*x^2+b)-1/2*b^2*(A
*c-B*b)/c/(c*x^2+b)^2-b*(4*A*c-3*B*b)/c/(c*x^2+b))
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.80

$$\int\frac{A+Bx^2}{x\left(bx^2+cx^4\right)^3}\,dx$$
$$=\frac{12\left(3\,Bb^2c^3-5\,Abc^4\right)x^8+18\left(3\,Bb^3c^2-5\,Ab^2c^3\right)x^6-2\,Ab^5+4\left(3\,Bb^4c-5\,Ab^3c^2\right)x^4-\left(3\,Bb^5-5\,Ab^4c\right)x}{}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/12*(12*(3*B*b^2*c^3 - 5*A*b*c^4)*x^8 + 18*(3*B*b^3*c^2 - 5*A*b^2*c^3)*x^
6 - 2*A*b^5 + 4*(3*B*b^4*c - 5*A*b^3*c^2)*x^4 - (3*B*b^5 - 5*A*b^4*c)*x^2
 - 12*((3*B*b*c^4 - 5*A*c^5)*x^10 + 2*(3*B*b^2*c^3 - 5*A*b*c^4)*x^8 + (3*B*
b^3*c^2 - 5*A*b^2*c^3)*x^6)*log(c*x^2 + b) + 24*((3*B*b*c^4 - 5*A*c^5)*x^1
0 + 2*(3*B*b^2*c^3 - 5*A*b*c^4)*x^8 + (3*B*b^3*c^2 - 5*A*b^2*c^3)*x^6)*log
(x))/(b^6*c^2*x^10 + 2*b^7*c*x^8 + b^8*x^6)
```

3.88.6 Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^3}\,dx = \frac{-2Ab^4 + x^8(-60Ac^4 + 36Bbc^3) + x^6(-90Abc^3 + 54Bb^2c^2) + x^4(-20Ab^2c^2 + 12Bb^3c) + x^2\cdot(5Ab^3c - 3B}{12b^7x^6 + 24b^6cx^8 + 12b^5c^2x^{10}} + \frac{2c^2(-5Ac + 3Bb)\log{(x)}}{b^6} - \frac{c^2(-5Ac + 3Bb)\log{\left(\frac{b}{c} + x^2\right)}}{b^6}$$

input
```
integrate((B*x**2+A)/x/(c*x**4+b*x**2)**3,x)
```

output
```
(-2*A*b**4 + x**8*(-60*A*c**4 + 36*B*b*c**3) + x**6*(-90*A*b*c**3 + 54*B*b
**2*c**2) + x**4*(-20*A*b**2*c**2 + 12*B*b**3*c) + x**2*(5*A*b**3*c - 3*B*
b**4))/(12*b**7*x**6 + 24*b**6*c*x**8 + 12*b**5*c**2*x**10) + 2*c**2*(-5*A
*c + 3*B*b)*log(x)/b**6 - c**2*(-5*A*c + 3*B*b)*log(b/c + x**2)/b**6
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^3}\,dx = \frac{12\,(3\,Bbc^3 - 5\,Ac^4)x^8 + 18\,(3\,Bb^2c^2 - 5\,Abc^3)x^6 - 2\,Ab^4 + 4\,(3\,Bb^3c - 5\,Ab^2c^2)x^4 - (3\,Bb^4 - 5\,Ab^3c)x^2}{12\,(b^5c^2x^{10} + 2\,b^6cx^8 + b^7x^6)} - \frac{(3\,Bbc^2 - 5\,Ac^3)\log\left(cx^2+b\right)}{b^6} + \frac{(3\,Bbc^2 - 5\,Ac^3)\log\left(x^2\right)}{b^6}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/12*(12*(3*B*b*c^3 - 5*A*c^4)*x^8 + 18*(3*B*b^2*c^2 - 5*A*b*c^3)*x^6 - 2*
A*b^4 + 4*(3*B*b^3*c - 5*A*b^2*c^2)*x^4 - (3*B*b^4 - 5*A*b^3*c)*x^2)/(b^5*
c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6) - (3*B*b*c^2 - 5*A*c^3)*log(c*x^2 + b)/b
^6 + (3*B*b*c^2 - 5*A*c^3)*log(x^2)/b^6
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.36

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^3}\,dx = \frac{\left(3\,Bbc^2-5\,Ac^3\right)\log\left(x^2\right)}{b^6}-\frac{\left(3\,Bbc^3-5\,Ac^4\right)\log\left(|cx^2+b|\right)}{b^6c} + \frac{18\,Bbc^4x^4-30\,Ac^5x^4+42\,Bb^2c^3x^2-68\,Abc^4x^2+25\,Bb^3c^2-39\,Ab^2c^3}{4\left(cx^2+b\right)^2b^6} - \frac{66\,Bbc^2x^6-110\,Ac^3x^6-18\,Bb^2cx^4+36\,Abc^2x^4+3\,Bb^3x^2-9\,Ab^2cx^2+2\,Ab^3}{12\,b^6x^6}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
(3*B*b*c^2 - 5*A*c^3)*log(x^2)/b^6 - (3*B*b*c^3 - 5*A*c^4)*log(abs(c*x^2 +
 b))/(b^6*c) + 1/4*(18*B*b*c^4*x^4 - 30*A*c^5*x^4 + 42*B*b^2*c^3*x^2 - 68*
A*b*c^4*x^2 + 25*B*b^3*c^2 - 39*A*b^2*c^3)/((c*x^2 + b)^2*b^6) - 1/12*(66*
B*b*c^2*x^6 - 110*A*c^3*x^6 - 18*B*b^2*c*x^4 + 36*A*b*c^2*x^4 + 3*B*b^3*x^
2 - 9*A*b^2*c*x^2 + 2*A*b^3)/(b^6*x^6)
```

3.88.9 Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^3}\,dx = \frac{\ln\left(c\,x^2+b\right)\,\left(5\,A\,c^3-3\,B\,b\,c^2\right)}{b^6} - \frac{\frac{A}{6\,b}-\frac{x^2\,\left(5\,A\,c-3\,B\,b\right)}{12\,b^2}+\frac{3\,c^2\,x^6\,\left(5\,A\,c-3\,B\,b\right)}{2\,b^4}+\frac{c^3\,x^8\,\left(5\,A\,c-3\,B\,b\right)}{b^5}+\frac{c\,x^4\,\left(5\,A\,c-3\,B\,b\right)}{3\,b^3}}{b^2\,x^6+2\,b\,c\,x^8+c^2\,x^{10}} - \frac{\ln\left(x\right)\,\left(10\,A\,c^3-6\,B\,b\,c^2\right)}{b^6}$$

input
```
int((A + B*x^2)/(x*(b*x^2 + c*x^4)^3),x)
```

output
```
(log(b + c*x^2)*(5*A*c^3 - 3*B*b*c^2))/b^6 - (A/(6*b) - (x^2*(5*A*c - 3*B*
b))/(12*b^2) + (3*c^2*x^6*(5*A*c - 3*B*b))/(2*b^4) + (c^3*x^8*(5*A*c - 3*B
*b))/b^5 + (c*x^4*(5*A*c - 3*B*b))/(3*b^3))/(b^2*x^6 + c^2*x^10 + 2*b*c*x^
8) - (log(x)*(10*A*c^3 - 6*B*b*c^2))/b^6
```

3.89 $\int x^7(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

3.89.1 Optimal result

Integrand size = 26, antiderivative size = 218

$$\begin{aligned}\int x^7(A+Bx^2)\sqrt{bx^2+cx^4}\,dx &= \frac{7b^3(3bB-4Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{1024c^5} \\ &- \frac{7b^2(3bB-4Ac)(bx^2+cx^4)^{3/2}}{384c^4} \\ &+ \frac{7b(3bB-4Ac)x^2(bx^2+cx^4)^{3/2}}{320c^3} \\ &- \frac{(3bB-4Ac)x^4(bx^2+cx^4)^{3/2}}{40c^2} + \frac{Bx^6(bx^2+cx^4)^{3/2}}{12c} \\ &- \frac{7b^5(3bB-4Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{11/2}}\end{aligned}$$

output
```
-7/384*b^2*(-4*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/c^4+7/320*b*(-4*A*c+3*B*b)*x
^2*(c*x^4+b*x^2)^(3/2)/c^3-1/40*(-4*A*c+3*B*b)*x^4*(c*x^4+b*x^2)^(3/2)/c^2
+1/12*B*x^6*(c*x^4+b*x^2)^(3/2)/c-7/1024*b^5*(-4*A*c+3*B*b)*arctanh(x^2*c^
(1/2)/(c*x^4+b*x^2)^(1/2))/c^(11/2)+7/1024*b^3*(-4*A*c+3*B*b)*(2*c*x^2+b)*
(c*x^4+b*x^2)^(1/2)/c^5
```

3.89.2 Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.87

$$\int x^7 \left(A+Bx^2\right) \sqrt{bx^2+cx^4}\, dx$$
$$= \frac{\sqrt{x^2\left(b+cx^2\right)}\Big(\sqrt{c}(315b^5B-210b^4c(2A+Bx^2)+64bc^4x^6(3A+2Bx^2)+56b^3c^2x^2(5A+3Bx^2)+256c^5}{15360c^{11/2}}$$

input
```
Integrate[x^7*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*(315*b^5*B - 210*b^4*c*(2*A + B*x^2) + 64*
b*c^4*x^6*(3*A + 2*B*x^2) + 56*b^3*c^2*x^2*(5*A + 3*B*x^2) + 256*c^5*x^8*(
6*A + 5*B*x^2) - 16*b^2*c^3*x^4*(14*A + 9*B*x^2)) + (210*b^5*(3*b*B - 4*A*
c)*ArcTanh[(Sqrt[c]*x)/(Sqrt[b] - Sqrt[b + c*x^2])])/(x*Sqrt[b + c*x^2])))
/(15360*c^(11/2))
```

3.89.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1940, 1221, 1134, 1134, 1160, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(A+Bx^2\right) \sqrt{bx^2+cx^4}\, dx$$

$$\Big\downarrow \text{ 1940}$$

$$\frac{1}{2}\int x^6 \left(Bx^2+A\right) \sqrt{cx^4+bx^2}dx^2$$

$$\Big\downarrow \text{ 1221}$$

$$\frac{1}{2}\left(\frac{Bx^6\left(bx^2+cx^4\right)^{3/2}}{6c}-\frac{(3bB-4Ac)\int x^6\sqrt{cx^4+bx^2}dx^2}{4c}\right)$$

$$\Big\downarrow \text{ 1134}$$

$$\frac{1}{2}\left(\frac{Bx^6\left(bx^2+cx^4\right)^{3/2}}{6c}-\frac{(3bB-4Ac)\left(\frac{x^4\left(bx^2+cx^4\right)^{3/2}}{5c}-\frac{7b\int x^4\sqrt{cx^4+bx^2}dx^2}{10c}\right)}{4c}\right)$$

$$\Big\downarrow \text{ 1134}$$

$$\frac{1}{2}\left(\frac{Bx^6\left(bx^2+cx^4\right)^{3/2}}{6c}-\frac{(3bB-4Ac)\left(\frac{x^4\left(bx^2+cx^4\right)^{3/2}}{5c}-\frac{7b\left(\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{4c}-\frac{5b\int x^2\sqrt{cx^4+bx^2}dx^2}{8c}\right)}{10c}\right)}{4c}\right)$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{1}{2}\left(\frac{Bx^6\left(bx^2+cx^4\right)^{3/2}}{6c}-\frac{(3bB-4Ac)\left(\frac{x^4\left(bx^2+cx^4\right)^{3/2}}{5c}-\frac{7b\left(\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{4c}-\frac{5b\left(\frac{\left(bx^2+cx^4\right)^{3/2}}{3c}-\frac{b\int\sqrt{cx^4+bx^2}dx^2}{2c}\right)}{8c}\right)}{10c}\right)}{4c}\right)$$

$$\Big\downarrow \text{ 1087}$$

3.89. $\int x^7(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

$$\frac{1}{2}\left(\frac{Bx^6\left(bx^2+cx^4\right)^{3/2}}{6c}-\frac{(3bB-4Ac)\left(\frac{x^4\left(bx^2+cx^4\right)^{3/2}}{5c}-\frac{7b\left(\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{4c}-\frac{5b\left(\frac{\left(bx^2+cx^4\right)^{3/2}}{3c}-\frac{b\left(\frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int \right.}{2c}\right.}{8c}\right.}{10c}\right.}{4c}\right.$$

$\downarrow$ 1091

3.89. $\int x^7(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

$$\frac{1}{2}\left(\frac{Bx^6\left(bx^2+cx^4\right)^{3/2}}{6c}-\frac{(3bB-4Ac)\left(\frac{x^4\left(bx^2+cx^4\right)^{3/2}}{5c}-\frac{7b\left(\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{4c}-\frac{5b\left(\frac{\left(bx^2+cx^4\right)^{3/2}}{3c}-\frac{b\left(\frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int \cdots}{\cdots}\right)}{2c}\right)}{8c}\right)}{10c}\right)}{4c}\right)$$

$\downarrow$ 219

3.89. $\int x^7(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

$$\frac{1}{2}\left(\frac{Bx^6\left(bx^2+cx^4\right)^{3/2}}{6c}-\frac{(3bB-4Ac)\left(\frac{x^4\left(bx^2+cx^4\right)^{3/2}}{5c}-\frac{7b\left(\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{4c}-\frac{5b\left(\frac{\left(bx^2+cx^4\right)^{3/2}}{3c}-\frac{b\left(\frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\mathrm{ar}}{}\right.}{2c}\right.}{8c}\right.}{10c}\right.}{4c}\right.$$

3.89. $\int x^7(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

input
```
Int[x^7*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
((B*x^6*(b*x^2 + c*x^4)^(3/2))/(6*c) - ((3*b*B - 4*A*c)*((x^4*(b*x^2 + c*x
^4)^(3/2))/(5*c) - (7*b*((x^2*(b*x^2 + c*x^4)^(3/2))/(4*c) - (5*b*((b*x^2
+ c*x^4)^(3/2)/(3*c) - (b*(((b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(4*c) - (b^
2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(4*c^(3/2))))/(2*c)))/(8*c))
)/(10*c)))/(4*c))/2
```

3.89.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.89.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{\frac{7\left(A\,b^{5} c-\frac{3}{4} B\,b^{6}\right) \ln\left(\frac{2 c\,x^{2}+2 \sqrt{x^{2} \left(c\,x^{2}+b\right)}\, \sqrt{c}+b}{\sqrt{c}}\right)}{512}+\frac{7\left(\frac{192 x^{8} \left(\frac{5 x^{2} B}{6}+A\right) c^{\frac{11}{2}}}{35}+\left(-\frac{3 \left(\frac{x^{2} B}{2}+A\right) b^{3} c^{\frac{3}{2}}}{2}+b^{2} x^{2} \left(\frac{3 x^{2} B}{5}+A\right) c^{\frac{5}{2}}-\frac{4 x^{4} \left(9\right.}{}\right.\right.}{384}}{c^{\frac{11}{2}}}$
risch	$-\frac{\left(-1280 B\,c^{5} x^{10}-1536 A\,c^{5} x^{8}-128 B b\,c^{4} x^{8}-192 A b\,c^{4} x^{6}+144 B\,b^{2} c^{3} x^{6}+224 A\,b^{2} c^{3} x^{4}-168 B\,b^{3} c^{2} x^{4}-280 A\,b^{3} c^{2} x^{2}+210 B\right.}{15360 c^{5}}$
default	$\frac{\sqrt{x^{4} c+b\,x^{2}}\, \left(1280 B\,c^{\frac{9}{2}} \left(c\,x^{2}+b\right)^{\frac{3}{2}} x^{9}+1536 A\,c^{\frac{9}{2}} \left(c\,x^{2}+b\right)^{\frac{3}{2}} x^{7}-1152 B\,c^{\frac{7}{2}} \left(c\,x^{2}+b\right)^{\frac{3}{2}} b\,x^{7}-1344 A\,c^{\frac{7}{2}} \left(c\,x^{2}+b\right)^{\frac{3}{2}} b\,x^{5}+1008 B\,c\right.}{}$

input
```
int(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
7/384*(3/4*(A*b^5*c-3/4*B*b^6)*ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2)*c^(1/2)
+b)/c^(1/2))+(192/35*x^8*(5/6*x^2*B+A)*c^(11/2)+(-3/2*(1/2*x^2*B+A)*b^3*c^
(3/2)+b^2*x^2*(3/5*x^2*B+A)*c^(5/2)-4/5*x^4*(9/14*x^2*B+A)*b*c^(7/2)+24/35
*x^6*(2/3*x^2*B+A)*c^(9/2)+9/8*B*c^(1/2)*b^4)*b)*(x^2*(c*x^2+b))^(1/2)-3/4
*(A*c-3/4*B*b)*ln(2)*b^5)/c^(11/2)
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.69

$$\int x^7 (A + Bx^2) \sqrt{bx^2 + cx^4} \, dx$$

$$= \left[-\frac{105\,(3\,Bb^6 - 4\,Ab^5c)\sqrt{c}\log\left(-2\,cx^2 - b - 2\,\sqrt{cx^4 + bx^2}\sqrt{c}\right) - 2\,(1280\,Bc^6x^{10} + 128\,(Bbc^5 + 12\,Ac^6)x^8\right.$$

input
```
integrate(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/30720*(105*(3*B*b^6 - 4*A*b^5*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x
^4 + b*x^2)*sqrt(c)) - 2*(1280*B*c^6*x^10 + 128*(B*b*c^5 + 12*A*c^6)*x^8 +
 315*B*b^5*c - 420*A*b^4*c^2 - 48*(3*B*b^2*c^4 - 4*A*b*c^5)*x^6 + 56*(3*B*
b^3*c^3 - 4*A*b^2*c^4)*x^4 - 70*(3*B*b^4*c^2 - 4*A*b^3*c^3)*x^2)*sqrt(c*x^
4 + b*x^2))/c^6, 1/15360*(105*(3*B*b^6 - 4*A*b^5*c)*sqrt(-c)*arctan(sqrt(c
*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (1280*B*c^6*x^10 + 128*(B*b*c^5 + 12
*A*c^6)*x^8 + 315*B*b^5*c - 420*A*b^4*c^2 - 48*(3*B*b^2*c^4 - 4*A*b*c^5)*x
^6 + 56*(3*B*b^3*c^3 - 4*A*b^2*c^4)*x^4 - 70*(3*B*b^4*c^2 - 4*A*b^3*c^3)*x
^2)*sqrt(c*x^4 + b*x^2))/c^6]
```

3.89.6 Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.58

$$\int x^7(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$

$$= \frac{A\left(\begin{cases} \frac{7b^5\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{256c^4} + \sqrt{bx^2+cx^4}\left(-\frac{7b^4}{128c^4}+\frac{7b^3x^2}{192c^3}-\frac{7b^2x^4}{240c^2}+\frac{bx^6}{40c}+\frac{x^8}{5}\right) & \text{for } c \\ \frac{2(bx^2)^{\frac{9}{2}}}{9b^4} & \text{for } b \\ 0 & \text{othe}\end{cases}\right)}{2}$$

$$+\frac{B\left(\begin{cases} -\frac{21b^6\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{1024c^5} + \sqrt{bx^2+cx^4}\cdot\left(\frac{21b^5}{512c^5}-\frac{7b^4x^2}{256c^4}+\frac{7b^3x^4}{320c^3}-\frac{3b^2x^6}{160c^2}+\frac{bx^8}{60c}\right. \\ \frac{2(bx^2)^{\frac{11}{2}}}{11b^5} \\ 0 \end{cases}\right)}{2}$$

input
```
integrate(x**7*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

output
```
A*Piecewise((7*b**5*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2
*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sq
rt(c*(b/(2*c) + x**2)**2), True))/(256*c**4) + sqrt(b*x**2 + c*x**4)*(-7*b
**4/(128*c**4) + 7*b**3*x**2/(192*c**3) - 7*b**2*x**4/(240*c**2) + b*x**6/
(40*c) + x**8/5), Ne(c, 0)), (2*(b*x**2)**(9/2)/(9*b**4), Ne(b, 0)), (0, T
rue))/2 + B*Piecewise((-21*b**6*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 +
 c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c
) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(1024*c**5) + sqrt(b*x**2 +
c*x**4)*(21*b**5/(512*c**5) - 7*b**4*x**2/(256*c**4) + 7*b**3*x**4/(320*c*
*3) - 3*b**2*x**6/(160*c**2) + b*x**8/(60*c) + x**10/6), Ne(c, 0)), (2*(b*
x**2)**(11/2)/(11*b**5), Ne(b, 0)), (0, True))/2
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.47

$$\int x^7(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$
$$=\frac{1}{7680}\left(\frac{768\,(cx^4+bx^2)^{\frac{3}{2}}x^4}{c}-\frac{420\,\sqrt{cx^4+bx^2}b^3x^2}{c^3}-\frac{672\,(cx^4+bx^2)^{\frac{3}{2}}bx^2}{c^2}+\frac{105\,b^5\log\left(2\,cx^2+b+2\,\sqrt{cx\right.}}{c^{\frac{9}{2}}}\right.$$
$$+\frac{1}{30720}\left(\frac{2560\,(cx^4+bx^2)^{\frac{3}{2}}x^6}{c}-\frac{2304\,(cx^4+bx^2)^{\frac{3}{2}}bx^4}{c^2}+\frac{1260\,\sqrt{cx^4+bx^2}b^4x^2}{c^4}+\frac{2016\,(cx^4+bx^2)^{\frac{3}{2}}b^2x^2}{c^3}\right.$$

input
```
integrate(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/7680*(768*(c*x^4 + b*x^2)^(3/2)*x^4/c - 420*sqrt(c*x^4 + b*x^2)*b^3*x^2/
c^3 - 672*(c*x^4 + b*x^2)^(3/2)*b*x^2/c^2 + 105*b^5*log(2*c*x^2 + b + 2*sq
rt(c*x^4 + b*x^2)*sqrt(c))/c^(9/2) - 210*sqrt(c*x^4 + b*x^2)*b^4/c^4 + 560
*(c*x^4 + b*x^2)^(3/2)*b^2/c^3)*A + 1/30720*(2560*(c*x^4 + b*x^2)^(3/2)*x^
6/c - 2304*(c*x^4 + b*x^2)^(3/2)*b*x^4/c^2 + 1260*sqrt(c*x^4 + b*x^2)*b^4*
x^2/c^4 + 2016*(c*x^4 + b*x^2)^(3/2)*b^2*x^2/c^3 - 315*b^6*log(2*c*x^2 + b
 + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(11/2) + 630*sqrt(c*x^4 + b*x^2)*b^5/c
^5 - 1680*(c*x^4 + b*x^2)^(3/2)*b^3/c^4)*B
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.12

$$\int x^7(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$
$$=\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,Bx^2\mathrm{sgn}(x)+\frac{Bbc^9\mathrm{sgn}(x)+12\,Ac^{10}\mathrm{sgn}(x)}{c^{10}}\right)x^2-\frac{3\,(3\,Bb^2c^8\mathrm{sgn}(x)-4\,Abc^9\mathrm{sgn}(x}{c^{10}}\right.\right.\right.\right.$$
$$+\frac{7\,(3\,Bb^6\mathrm{sgn}(x)-4\,Ab^5c\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{1024\,c^{\frac{11}{2}}}$$
$$-\frac{7\,(3\,Bb^6\log(|b|)-4\,Ab^5c\log(|b|))\mathrm{sgn}(x)}{2048\,c^{\frac{11}{2}}}$$

input
```
integrate(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/15360*(2*(4*(2*(8*(10*B*x^2*sgn(x) + (B*b*c^9*sgn(x) + 12*A*c^10*sgn(x))
/c^10)*x^2 - 3*(3*B*b^2*c^8*sgn(x) - 4*A*b*c^9*sgn(x))/c^10)*x^2 + 7*(3*B*
b^3*c^7*sgn(x) - 4*A*b^2*c^8*sgn(x))/c^10)*x^2 - 35*(3*B*b^4*c^6*sgn(x) -
4*A*b^3*c^7*sgn(x))/c^10)*x^2 + 105*(3*B*b^5*c^5*sgn(x) - 4*A*b^4*c^6*sgn(
x))/c^10)*sqrt(c*x^2 + b)*x + 7/1024*(3*B*b^6*sgn(x) - 4*A*b^5*c*sgn(x))*l
og(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(11/2) - 7/2048*(3*B*b^6*log(abs(b
)) - 4*A*b^5*c*log(abs(b)))*sgn(x)/c^(11/2)
```

3.89.9 Mupad [B] (verification not implemented)

Time = 10.30 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.33

$$\int x^7(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{A\,x^4\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{10\,c}+\frac{B\,x^6\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{12\,c} - \frac{3\,B\,b\left(\frac{7\,b\left(\frac{5\,b\left(\frac{b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,|x|\,\sqrt{c\,x^2+b}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{24\,c^2}\right)}{8\,c}-\frac{x^2\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{4\,c}\right)}{10\,c}+\frac{x^4\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{5\,c}\right)}{8\,c} + \frac{7\,A\,b\left(\frac{5\,b\left(\frac{b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,|x|\,\sqrt{c\,x^2+b}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{24\,c^2}\right)}{8\,c}-\frac{x^2\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{4\,c}\right)}{20\,c}$$

input
```
int(x^7*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

output
```
(A*x^4*(b*x^2 + c*x^4)^(3/2))/(10*c) + (B*x^6*(b*x^2 + c*x^4)^(3/2))/(12*c
) - (3*B*b*((7*b*((5*b*((b^3*log(b + 2*c*x^2 + 2*c^(1/2)*abs(x)*(b + c*x^2
)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4 - 3*b^2 + 2*b*c
*x^2))/(24*c^2)))/(8*c) - (x^2*(b*x^2 + c*x^4)^(3/2))/(4*c)))/(10*c) + (x^
4*(b*x^2 + c*x^4)^(3/2))/(5*c)))/(8*c) + (7*A*b*((5*b*((b^3*log(b + 2*c*x^
2 + 2*c^(1/2)*abs(x)*(b + c*x^2)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(
1/2)*(8*c^2*x^4 - 3*b^2 + 2*b*c*x^2))/(24*c^2)))/(8*c) - (x^2*(b*x^2 + c*x
^4)^(3/2))/(4*c)))/(20*c)
```

3.90 $\int x^5(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

3.90.1 Optimal result

Integrand size = 26, antiderivative size = 181

$$\int x^5(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = -\frac{b^2(7bB-10Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{256c^4} + \frac{b(7bB-10Ac)(bx^2+cx^4)^{3/2}}{96c^3} - \frac{(7bB-10Ac)x^2(bx^2+cx^4)^{3/2}}{80c^2} + \frac{Bx^4(bx^2+cx^4)^{3/2}}{10c} + \frac{b^4(7bB-10Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{9/2}}$$

output
```
1/96*b*(-10*A*c+7*B*b)*(c*x^4+b*x^2)^(3/2)/c^3-1/80*(-10*A*c+7*B*b)*x^2*(c
*x^4+b*x^2)^(3/2)/c^2+1/10*B*x^4*(c*x^4+b*x^2)^(3/2)/c+1/256*b^4*(-10*A*c+
7*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(9/2)-1/256*b^2*(-10*A*c
+7*B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^4
```

3.90.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.94

$$\int x^5 \left(A+Bx^2\right) \sqrt{bx^2+cx^4}\,dx$$
$$= \frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}x(-105b^4B+16bc^3x^4(5A+3Bx^2)+96c^4x^6(5A+4Bx^2)+10b^3c(15A+7Bx^2)-4b^2\right.}{3840c^{9/2}x}$$

input
```
Integrate[x^5*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*(-105*b^4*B + 16*b*c^3*x^4*(5*A + 3*B*x^
2) + 96*c^4*x^6*(5*A + 4*B*x^2) + 10*b^3*c*(15*A + 7*B*x^2) - 4*b^2*c^2*x^
2*(25*A + 14*B*x^2)) + (30*b^4*(7*b*B - 10*A*c)*ArcTanh[(Sqrt[c]*x)/(-Sqrt
[b] + Sqrt[b + c*x^2])])/Sqrt[b + c*x^2]))/(3840*c^(9/2)*x)
```

3.90.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1940, 1221, 1134, 1160, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(A+Bx^2\right) \sqrt{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int x^4 \left(Bx^2+A\right) \sqrt{cx^4+bx^2}dx^2$$

$$\Big\downarrow \text{1221}$$

$$\frac{1}{2}\left(\frac{Bx^4\left(bx^2+cx^4\right)^{3/2}}{5c}-\frac{(7bB-10Ac)\int x^4\sqrt{cx^4+bx^2}dx^2}{10c}\right)$$

$$\Big\downarrow \text{1134}$$

$$\frac{1}{2}\left(\frac{Bx^4\left(bx^2+cx^4\right)^{3/2}}{5c}-\frac{(7bB-10Ac)\left(\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{4c}-\frac{5b\int x^2\sqrt{cx^4+bx^2}dx^2}{8c}\right)}{10c}\right)$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{1}{2}\left(\frac{Bx^4\left(bx^2+cx^4\right)^{3/2}}{5c}-\frac{(7bB-10Ac)\left(\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{4c}-\frac{5b\left(\frac{\left(bx^2+cx^4\right)^{3/2}}{3c}-\frac{b\int\sqrt{cx^4+bx^2}dx^2}{2c}\right)}{8c}\right)}{10c}\right)$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{1}{2}\left(\frac{Bx^4\left(bx^2+cx^4\right)^{3/2}}{5c}-\frac{(7bB-10Ac)\left(\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{4c}-\frac{5b\left(\frac{\left(bx^2+cx^4\right)^{3/2}}{3c}-\frac{b\left(\frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^4+bx^2}}dx^2}{8c}\right)}{2c}\right)}{8c}\right)}{10c}\right)$$

$$\Big\downarrow \text{ 1091}$$

3.90. $\int x^5(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

$$\frac{1}{2}\left(\frac{Bx^4(bx^2+cx^4)^{3/2}}{5c}-\frac{(7bB-10Ac)\left(\frac{x^2(bx^2+cx^4)^{3/2}}{4c}-\frac{5b\left(\frac{(bx^2+cx^4)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int\frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{4c}\right)}{2c}\right)}{8c}\right)}{10c}\right)$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{Bx^4(bx^2+cx^4)^{3/2}}{5c}-\frac{(7bB-10Ac)\left(\frac{x^2(bx^2+cx^4)^{3/2}}{4c}-\frac{5b\left(\frac{(bx^2+cx^4)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{4c^{3/2}}\right)}{2c}\right)}{8c}\right)}{10c}\right)$$

input
```
Int[x^5*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

3.90. $\int x^5(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

output
```
((B*x^4*(b*x^2 + c*x^4)^(3/2))/(5*c) - ((7*b*B - 10*A*c)*((x^2*(b*x^2 + c*
x^4)^(3/2))/(4*c) - (5*b*((b*x^2 + c*x^4)^(3/2)/(3*c) - (b*(((b + 2*c*x^2)
*Sqrt[b*x^2 + c*x^4])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^
4]])/(4*c^(3/2))))/(2*c)))/(8*c)))/(10*c))/2
```

3.90.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.90.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{\frac{5\left(-\frac{1}{2}A\,b^4c+\frac{7}{20}b^5B\right)\ln\left(\frac{2c\,x^2+2\sqrt{x^2\left(c\,x^2+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)}{128}+\frac{5\left(b^3\left(\frac{7x^2B}{15}+A\right)c^{\frac{3}{2}}-\frac{2x^2\left(\frac{14x^2B}{25}+A\right)b^2c^{\frac{5}{2}}}{3}+\frac{8x^4b\left(\frac{3x^2B}{5}+A\right)c^{\frac{7}{2}}}{15}+\frac{16x^6(}{}\right.}{128}}{c^{\frac{9}{2}}}$
risch	$\frac{\left(384B\,x^8c^4+480A\,x^6c^4+48B\,x^6b\,c^3+80A\,x^4b\,c^3-56B\,x^4b^2c^2-100A\,x^2b^2c^2+70B\,x^2b^3c+150A\,b^3c-105B\,b^4\right)\sqrt{x^2(c\,x^2+b)}}{3840c^4}$
default	$\sqrt{x^4c+b\,x^2}\,\left(384B\left(c\,x^2+b\right)^{\frac{3}{2}}c^{\frac{7}{2}}x^7+480A\left(c\,x^2+b\right)^{\frac{3}{2}}c^{\frac{7}{2}}x^5-336B\left(c\,x^2+b\right)^{\frac{3}{2}}c^{\frac{5}{2}}b\,x^5-400A\left(c\,x^2+b\right)^{\frac{3}{2}}c^{\frac{5}{2}}b\,x^3+280B\left(c\,x^2+b\right)\right.$

input
```
int(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
5/128/c^(9/2)*((-1/2*A*b^4*c+7/20*b^5*B)*ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/
2)*c^(1/2)+b)/c^(1/2))+(b^3*(7/15*x^2*B+A)*c^(3/2)-2/3*x^2*(14/25*x^2*B+A)
*b^2*c^(5/2)+8/15*x^4*b*(3/5*x^2*B+A)*c^(7/2)+16/5*x^6*(4/5*x^2*B+A)*c^(9/
2)-7/10*B*c^(1/2)*b^4)*(x^2*(c*x^2+b))^(1/2)+1/2*ln(2)*(A*c-7/10*B*b)*b^4)
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.77

$$\int x^5 (A + Bx^2) \sqrt{bx^2 + cx^4}\, dx$$

$$= \left[-\frac{15\,(7\,Bb^5 - 10\,Ab^4c)\sqrt{c}\log\left(-2\,cx^2 - b + 2\,\sqrt{cx^4 + bx^2}\sqrt{c}\right) - 2\,(384\,Bc^5x^8 + 48\,(Bbc^4 + 10\,Ac^5)x^6 - }{7680\,c^5} \right.$$

$$-\frac{15\,(7\,Bb^5 - 10\,Ab^4c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right) - (384\,Bc^5x^8 + 48\,(Bbc^4 + 10\,Ac^5)x^6 - 105\,Bb^4c + 15}{3840\,c^5}$$

input
```
integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/7680*(15*(7*B*b^5 - 10*A*b^4*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^
4 + b*x^2)*sqrt(c)) - 2*(384*B*c^5*x^8 + 48*(B*b*c^4 + 10*A*c^5)*x^6 - 105
*B*b^4*c + 150*A*b^3*c^2 - 8*(7*B*b^2*c^3 - 10*A*b*c^4)*x^4 + 10*(7*B*b^3*
c^2 - 10*A*b^2*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^5, -1/3840*(15*(7*B*b^5 -
10*A*b^4*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (3
84*B*c^5*x^8 + 48*(B*b*c^4 + 10*A*c^5)*x^6 - 105*B*b^4*c + 150*A*b^3*c^2 -
 8*(7*B*b^2*c^3 - 10*A*b*c^4)*x^4 + 10*(7*B*b^3*c^2 - 10*A*b^2*c^3)*x^2)*s
qrt(c*x^4 + b*x^2))/c^5]
```

3.90.6 Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.76

$$\int x^5(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$

$$= \frac{A\left(\begin{cases} -\frac{5b^4\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{128c^3} + \sqrt{bx^2+cx^4}\cdot\left(\frac{5b^3}{64c^3}-\frac{5b^2x^2}{96c^2}+\frac{bx^4}{24c}+\frac{x^6}{4}\right) & \text{for } c\neq 0\\ \frac{2(bx^2)^{\frac{7}{2}}}{7b^3} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{2}$$

$$+\frac{B\left(\begin{cases} \frac{7b^5\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{256c^4} + \sqrt{bx^2+cx^4}\left(-\frac{7b^4}{128c^4}+\frac{7b^3x^2}{192c^3}-\frac{7b^2x^4}{240c^2}+\frac{bx^6}{40c}+\frac{x^8}{5}\right) & \text{fo}\\ \frac{2(bx^2)^{\frac{9}{2}}}{9b^4} & \text{fo}\\ 0 & \text{ot}\end{cases}\right.}{2}$$

input
```
integrate(x**5*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

output
```
A*Piecewise((-5*b**4*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) +
2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/s
qrt(c*(b/(2*c) + x**2)**2), True))/(128*c**3) + sqrt(b*x**2 + c*x**4)*(5*b
**3/(64*c**3) - 5*b**2*x**2/(96*c**2) + b*x**4/(24*c) + x**6/4), Ne(c, 0))
, (2*(b*x**2)**(7/2)/(7*b**3), Ne(b, 0)), (0, True))/2 + B*Piecewise((7*b*
*5*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*c*x**2)/sqrt(c),
 Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c) + x
**2)**2), True))/(256*c**4) + sqrt(b*x**2 + c*x**4)*(-7*b**4/(128*c**4) +
7*b**3*x**2/(192*c**3) - 7*b**2*x**4/(240*c**2) + b*x**6/(40*c) + x**8/5),
 Ne(c, 0)), (2*(b*x**2)**(9/2)/(9*b**4), Ne(b, 0)), (0, True))/2
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.51

$$\int x^5 (A + Bx^2) \sqrt{bx^2 + cx^4} \, dx$$
$$= \frac{1}{768} \left(\frac{60 \sqrt{cx^4 + bx^2} b^2 x^2}{c^2} + \frac{96 (cx^4 + bx^2)^{\frac{3}{2}} x^2}{c} - \frac{15 b^4 \log \left(2 cx^2 + b + 2 \sqrt{cx^4 + bx^2} \sqrt{c}\right)}{c^{\frac{7}{2}}} + \frac{30 \sqrt{cx^4 + bx}}{c^3}\right.$$
$$+ \frac{1}{7680} \left(\frac{768 (cx^4 + bx^2)^{\frac{3}{2}} x^4}{c} - \frac{420 \sqrt{cx^4 + bx^2} b^3 x^2}{c^3} - \frac{672 (cx^4 + bx^2)^{\frac{3}{2}} bx^2}{c^2} + \frac{105 b^5 \log \left(2 cx^2 + b + 2 \sqrt{}\right.}{c^{\frac{9}{2}}}\right.$$

input
```
integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/768*(60*sqrt(c*x^4 + b*x^2)*b^2*x^2/c^2 + 96*(c*x^4 + b*x^2)^(3/2)*x^2/c
 - 15*b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 30*sq
rt(c*x^4 + b*x^2)*b^3/c^3 - 80*(c*x^4 + b*x^2)^(3/2)*b/c^2)*A + 1/7680*(76
8*(c*x^4 + b*x^2)^(3/2)*x^4/c - 420*sqrt(c*x^4 + b*x^2)*b^3*x^2/c^3 - 672*
(c*x^4 + b*x^2)^(3/2)*b*x^2/c^2 + 105*b^5*log(2*c*x^2 + b + 2*sqrt(c*x^4 +
 b*x^2)*sqrt(c))/c^(9/2) - 210*sqrt(c*x^4 + b*x^2)*b^4/c^4 + 560*(c*x^4 +
b*x^2)^(3/2)*b^2/c^3)*B
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.17

$$\int x^5 (A + Bx^2) \sqrt{bx^2 + cx^4} \, dx$$
$$= \frac{1}{3840} \left(2 \left(4 \left(6 \left(8 Bx^2 \mathrm{sgn}(x) + \frac{Bbc^7 \mathrm{sgn}(x) + 10 Ac^8 \mathrm{sgn}(x)}{c^8}\right) x^2 - \frac{7 Bb^2 c^6 \mathrm{sgn}(x) - 10 Abc^7 \mathrm{sgn}(x)}{c^8}\right) x^2 +\right.\right.$$
$$- \frac{(7 Bb^5 \mathrm{sgn}(x) - 10 Ab^4 c \mathrm{sgn}(x)) \log \left(\left|-\sqrt{c}x + \sqrt{cx^2 + b}\right|\right)}{256 c^{\frac{9}{2}}}$$
$$+ \frac{(7 Bb^5 \log (|b|) - 10 Ab^4 c \log (|b|)) \mathrm{sgn}(x)}{512 c^{\frac{9}{2}}}$$

input
```
integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/3840*(2*(4*(6*(8*B*x^2*sgn(x) + (B*b*c^7*sgn(x) + 10*A*c^8*sgn(x))/c^8)*
x^2 - (7*B*b^2*c^6*sgn(x) - 10*A*b*c^7*sgn(x))/c^8)*x^2 + 5*(7*B*b^3*c^5*s
gn(x) - 10*A*b^2*c^6*sgn(x))/c^8)*x^2 - 15*(7*B*b^4*c^4*sgn(x) - 10*A*b^3*
c^5*sgn(x))/c^8)*sqrt(c*x^2 + b)*x - 1/256*(7*B*b^5*sgn(x) - 10*A*b^4*c*sg
n(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(9/2) + 1/512*(7*B*b^5*log(
abs(b)) - 10*A*b^4*c*log(abs(b)))*sgn(x)/c^(9/2)
```

3.90.9 Mupad [B] (verification not implemented)

Time = 9.82 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.29

$$\int x^5(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$

$$= \frac{A\,x^2\,(c\,x^4+b\,x^2)^{3/2}}{8\,c} - \frac{5\,A\,b\left(\frac{b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,|x|\,\sqrt{c\,x^2+b}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4)}{24\,c^2}\right)}{16\,c}$$
$$+ \frac{B\,x^4\,(c\,x^4+b\,x^2)^{3/2}}{10\,c}$$
$$+ \frac{7\,B\,b\left(\frac{5\,b\left(\frac{b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,|x|\,\sqrt{c\,x^2+b}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{24\,c^2}\right)}{8\,c}-\frac{x^2\,(c\,x^4+b\,x^2)^{3/2}}{4\,c}\right)}{20\,c}$$

input
```
int(x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

output
```
(A*x^2*(b*x^2 + c*x^4)^(3/2))/(8*c) - (5*A*b*((b^3*log(b + 2*c*x^2 + 2*c^(
1/2)*abs(x)*(b + c*x^2)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c
^2*x^4 - 3*b^2 + 2*b*c*x^2))/(24*c^2)))/(16*c) + (B*x^4*(b*x^2 + c*x^4)^(3
/2))/(10*c) + (7*B*b*((5*b*((b^3*log(b + 2*c*x^2 + 2*c^(1/2)*abs(x)*(b + c
*x^2)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4 - 3*b^2 + 2
*b*c*x^2))/(24*c^2)))/(8*c) - (x^2*(b*x^2 + c*x^4)^(3/2))/(4*c)))/(20*c)
```

3.91 $\int x^3(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

3.91.1 Optimal result

Integrand size = 26, antiderivative size = 125

$$\int x^3(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{b(5bB-8Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^3} - \frac{(5bB-8Ac-6Bcx^2)(bx^2+cx^4)^{3/2}}{48c^2} - \frac{b^3(5bB-8Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}$$

output
```
-1/48*(-6*B*c*x^2-8*A*c+5*B*b)*(c*x^4+b*x^2)^(3/2)/c^2-1/128*b^3*(-8*A*c+5
*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(7/2)+1/128*b*(-8*A*c+5*B
*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^3
```

3.91.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.18

$$\int x^3(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$

$$= \frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{c}(15b^3B+8bc^2x^2(2A+Bx^2)+16c^3x^4(4A+3Bx^2)-2b^2c(12A+5Bx^2)) + \frac{6b^3(5bB-8A}{}\right.}{384c^{7/2}}$$

input
```
Integrate[x^3*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*(15*b^3*B + 8*b*c^2*x^2*(2*A + B*x^2) + 16
*c^3*x^4*(4*A + 3*B*x^2) - 2*b^2*c*(12*A + 5*B*x^2)) + (6*b^3*(5*b*B - 8*A
*c)*ArcTanh[(Sqrt[c]*x)/(Sqrt[b] - Sqrt[b + c*x^2])])/(x*Sqrt[b + c*x^2]))
)/(384*c^(7/2))
```

3.91.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1940, 1225, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int x^2\left(Bx^2+A\right)\sqrt{cx^4+bx^2}dx^2$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{2}\left(\frac{b(5bB-8Ac)\int\sqrt{cx^4+bx^2}dx^2}{16c^2}-\frac{\left(bx^2+cx^4\right)^{3/2}\left(-8Ac+5bB-6Bcx^2\right)}{24c^2}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{b(5bB-8Ac)\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^4+bx^2}}dx^2}{8c}\right)}{16c^2}-\frac{\left(bx^2+cx^4\right)^{3/2}\left(-8Ac+5bB-6Bcx^2\right)}{24c^2}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{b(5bB-8Ac)\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int\frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{4c}\right)}{16c^2}-\frac{\left(bx^2+cx^4\right)^{3/2}\left(-8Ac+5bB-6Bcx^2\right)}{24c^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{b(5bB-8Ac)\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{4c^{3/2}}\right)}{16c^2}-\frac{(bx^2+cx^4)^{3/2}\left(-8Ac+5bB-6Bcx^2\right)}{24c^2}\right)$$

input
```
Int[x^3*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(-1/24*((5*b*B - 8*A*c - 6*B*c*x^2)*(b*x^2 + c*x^4)^(3/2))/c^2 + (b*(5*b*B
 - 8*A*c)*(((b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(4*c) - (b^2*ArcTanh[(Sqrt[
c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(4*c^(3/2))))/(16*c^2))/2
```

3.91.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] && !LeQ[p, -1]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.91.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{\left(-48 B c^{3} x^{6}-64 A c^{3} x^{4}-8 B b c^{2} x^{4}-16 A b c^{2} x^{2}+10 B b^{2} c x^{2}+24 b^{2} A c-15 B b^{3}\right) \sqrt{x^{2} \left(c x^{2}+b\right)}}{384 c^{3}}+\frac{b^{3} (8 A c-5 B b) \ln \left(\sqrt{c}\, x+\sqrt{}\right.}{128 c^{\frac{7}{2}} x \sqrt{}}$
pseudoelliptic	$-\frac{\left(-\frac{1}{2} A b^{3} c+\frac{5}{16} B b^{4}\right) \ln \left(\frac{2 c x^{2}+2 \sqrt{x^{2} \left(c x^{2}+b\right)}\, \sqrt{c}+b}{\sqrt{c}}\right)+\left(b^{2} \left(\frac{5 x^{2} B}{12}+A\right) c^{\frac{3}{2}}-\frac{2 x^{2} \left(\frac{x^{2} B}{2}+A\right) b\, c^{\frac{5}{2}}}{3}+\left(-2 B x^{6}-\frac{8}{3} A x^{4}\right) c^{\frac{7}{2}}-\frac{5 B}{}\right.}{16 c^{\frac{7}{2}}}$
default	$\frac{\sqrt{x^{4} c+b x^{2}}\, \left(48 B \left(c x^{2}+b\right)^{\frac{3}{2}} c^{\frac{5}{2}} x^{5}+64 A \left(c x^{2}+b\right)^{\frac{3}{2}} c^{\frac{5}{2}} x^{3}-40 B \left(c x^{2}+b\right)^{\frac{3}{2}} c^{\frac{3}{2}} b\, x^{3}-48 A \left(c x^{2}+b\right)^{\frac{3}{2}} c^{\frac{3}{2}} b x+24 A \sqrt{c x^{2}+b}\, c^{\frac{3}{2}} b^{2} x+\right.}{384 x \sqrt{c x^{2}+b}\, c^{\frac{7}{2}}}$

input
```
int(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384*(-48*B*c^3*x^6-64*A*c^3*x^4-8*B*b*c^2*x^4-16*A*b*c^2*x^2+10*B*b^2*c
*x^2+24*A*b^2*c-15*B*b^3)/c^3*(x^2*(c*x^2+b))^(1/2)+1/128*b^3*(8*A*c-5*B*b
)/c^(7/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^
(1/2)
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.18

$$\int x^3 (A+Bx^2) \sqrt{bx^2+cx^4}\, dx$$
$$= \left[-\frac{3\,(5\,Bb^4-8\,Ab^3c)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,(48\,Bc^4x^6+15\,Bb^3c-24\,Ab^2c^2+8\,(Bb}{768\,c^4}\right.$$

input
```
integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/768*(3*(5*B*b^4 - 8*A*b^3*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 +
 b*x^2)*sqrt(c)) - 2*(48*B*c^4*x^6 + 15*B*b^3*c - 24*A*b^2*c^2 + 8*(B*b*c^
3 + 8*A*c^4)*x^4 - 2*(5*B*b^2*c^2 - 8*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c
^4, 1/384*(3*(5*B*b^4 - 8*A*b^3*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqr
t(-c)/(c*x^2 + b)) + (48*B*c^4*x^6 + 15*B*b^3*c - 24*A*b^2*c^2 + 8*(B*b*c^
3 + 8*A*c^4)*x^4 - 2*(5*B*b^2*c^2 - 8*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c
^4]
```

3.91.6 Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.30

$$\int x^3(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$
$$= \frac{A\left(\begin{cases} \frac{b^3\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for}\ \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise} \end{cases}\right)}{16c^2} + \sqrt{bx^2+cx^4}\left(-\frac{b^2}{8c^2}+\frac{bx^2}{12c}+\frac{x^4}{3}\right) & \text{for}\ c\neq 0 \\ \frac{2(bx^2)^{\frac{5}{2}}}{5b^2} & \text{for}\ b\neq 0 \\ 0 & \text{otherwise} \end{cases}\right)}{2}$$
$$+ \frac{B\left(\begin{cases} -\frac{5b^4\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for}\ \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise} \end{cases}\right)}{128c^3} + \sqrt{bx^2+cx^4}\cdot\left(\frac{5b^3}{64c^3}-\frac{5b^2x^2}{96c^2}+\frac{bx^4}{24c}+\frac{x^6}{4}\right) & \text{for}\ c\neq 0 \\ \frac{2(bx^2)^{\frac{7}{2}}}{7b^3} & \text{for}\ b\neq 0 \\ 0 & \text{otherwise} \end{cases}\right)}{2}$$

input
```
integrate(x**3*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

output
```
A*Piecewise((b**3*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*c
*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt
(c*(b/(2*c) + x**2)**2), True))/(16*c**2) + sqrt(b*x**2 + c*x**4)*(-b**2/(
8*c**2) + b*x**2/(12*c) + x**4/3), Ne(c, 0)), (2*(b*x**2)**(5/2)/(5*b**2),
 Ne(b, 0)), (0, True))/2 + B*Piecewise((-5*b**4*Piecewise((log(b + 2*sqrt(
c)*sqrt(b*x**2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) +
x**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(128*c**3) +
 sqrt(b*x**2 + c*x**4)*(5*b**3/(64*c**3) - 5*b**2*x**2/(96*c**2) + b*x**4/
(24*c) + x**6/4), Ne(c, 0)), (2*(b*x**2)**(7/2)/(7*b**3), Ne(b, 0)), (0, T
rue))/2
```

3.91.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(109) = 218$.

Time = 0.21 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.80

$$\int x^3(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = -\frac{1}{96}\left(\frac{12\sqrt{cx^4+bx^2}bx^2}{c} - \frac{3\,b^3\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}} + \frac{6\sqrt{cx^4+bx^2}b^2}{c^2} - \frac{16\,(cx^4+bx^2)^{\frac{3}{2}}}{c}\right) + \frac{1}{768}\left(\frac{60\sqrt{cx^4+bx^2}b^2x^2}{c^2} + \frac{96\,(cx^4+bx^2)^{\frac{3}{2}}x^2}{c} - \frac{15\,b^4\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{7}{2}}} + \frac{30\sqrt{cx^4+bx^2}\right.$$

input
```
integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/96*(12*sqrt(c*x^4 + b*x^2)*b*x^2/c - 3*b^3*log(2*c*x^2 + b + 2*sqrt(c*x
^4 + b*x^2)*sqrt(c))/c^(5/2) + 6*sqrt(c*x^4 + b*x^2)*b^2/c^2 - 16*(c*x^4 +
 b*x^2)^(3/2)/c)*A + 1/768*(60*sqrt(c*x^4 + b*x^2)*b^2*x^2/c^2 + 96*(c*x^4
 + b*x^2)^(3/2)*x^2/c - 15*b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqr
t(c))/c^(7/2) + 30*sqrt(c*x^4 + b*x^2)*b^3/c^3 - 80*(c*x^4 + b*x^2)^(3/2)*
b/c^2)*B
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.42

$$\int x^3(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$
$$=\frac{1}{384}\left(2\left(4\left(6\,Bx^2\mathrm{sgn}(x)+\frac{Bbc^5\mathrm{sgn}(x)+8\,Ac^6\mathrm{sgn}(x)}{c^6}\right)x^2-\frac{5\,Bb^2c^4\mathrm{sgn}(x)-8\,Abc^5\mathrm{sgn}(x)}{c^6}\right)x^2+\frac{3\,(5\,B\right.$$
$$+\frac{(5\,Bb^4\mathrm{sgn}(x)-8\,Ab^3c\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{128\,c^{\frac{7}{2}}}$$
$$-\frac{(5\,Bb^4\log(|b|)-8\,Ab^3c\log(|b|))\mathrm{sgn}(x)}{256\,c^{\frac{7}{2}}}$$

input
```
integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*B*x^2*sgn(x) + (B*b*c^5*sgn(x) + 8*A*c^6*sgn(x))/c^6)*x^2 -
 (5*B*b^2*c^4*sgn(x) - 8*A*b*c^5*sgn(x))/c^6)*x^2 + 3*(5*B*b^3*c^3*sgn(x)
- 8*A*b^2*c^4*sgn(x))/c^6)*sqrt(c*x^2 + b)*x + 1/128*(5*B*b^4*sgn(x) - 8*A
*b^3*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(7/2) - 1/256*(5*B
*b^4*log(abs(b)) - 8*A*b^3*c*log(abs(b)))*sgn(x)/c^(7/2)
```

3.91.9 Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.42

$$\int x^3(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$
$$=\frac{B\,x^2\,(c\,x^4+b\,x^2)^{3/2}}{8\,c}-\frac{5\,B\,b\left(\frac{b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,|x|\,\sqrt{c\,x^2+b}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4)}{24\,c^2}\right)}{16\,c}$$
$$+\frac{A\,b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,|x|\,\sqrt{c\,x^2+b}\right)}{32\,c^{5/2}}+\frac{A\,\sqrt{c\,x^4+b\,x^2}\,(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4)}{48\,c^2}$$

input
```
int(x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

output
```
(B*x^2*(b*x^2 + c*x^4)^(3/2))/(8*c) - (5*B*b*((b^3*log(b + 2*c*x^2 + 2*c^(
1/2)*abs(x)*(b + c*x^2)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c
^2*x^4 - 3*b^2 + 2*b*c*x^2))/(24*c^2)))/(16*c) + (A*b^3*log(b + 2*c*x^2 +
2*c^(1/2)*abs(x)*(b + c*x^2)^(1/2)))/(32*c^(5/2)) + (A*(b*x^2 + c*x^4)^(1/
2)*(8*c^2*x^4 - 3*b^2 + 2*b*c*x^2))/(48*c^2)
```

3.92 $\int x(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

3.92.1 Optimal result

Integrand size = 24, antiderivative size = 107

$$\int x\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=-\frac{(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{6c}+\frac{b^2(bB-2Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}$$

output
```
1/6*B*(c*x^4+b*x^2)^(3/2)/c+1/16*b^2*(-2*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x
^4+b*x^2)^(1/2))/c^(5/2)-1/16*(-2*A*c+B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)
/c^2
```

3.92.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.18

$$\int x\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}x(-3b^2B+2bc(3A+Bx^2)+4c^2x^2(3A+2Bx^2))+\frac{6b^2(bB-2Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{b}+\sqrt{b+cx^2}}\right)}{\sqrt{b+cx^2}}\right)}{48c^{5/2}x}$$

input
```
Integrate[x*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*(-3*b^2*B + 2*b*c*(3*A + B*x^2) + 4*c^2*
x^2*(3*A + 2*B*x^2)) + (6*b^2*(b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b]
+ Sqrt[b + c*x^2])])/Sqrt[b + c*x^2]))/(48*c^(5/2)*x)
```

3.92.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1940, 1160, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int (Bx^2+A)\sqrt{cx^4+bx^2}dx^2$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{B(bx^2+cx^4)^{3/2}}{3c}-\frac{(bB-2Ac)\int\sqrt{cx^4+bx^2}dx^2}{2c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{B(bx^2+cx^4)^{3/2}}{3c}-\frac{(bB-2Ac)\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^4+bx^2}}dx^2}{8c}\right)}{2c}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{B(bx^2+cx^4)^{3/2}}{3c}-\frac{(bB-2Ac)\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int\frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{4c}\right)}{2c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{B\left(bx^2+cx^4\right)^{3/2}}{3c}-\frac{(bB-2Ac)\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{4c^{3/2}}\right)}{2c}\right)$$

input
```
Int[x*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
((B*(b*x^2 + c*x^4)^(3/2))/(3*c) - ((b*B - 2*A*c)*(((b + 2*c*x^2)*Sqrt[b*x
^2 + c*x^4])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(4*c
^(3/2))))/(2*c))/2
```

3.92.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.92.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08

method	result
risch	$\frac{\left(8B\,c^{2}x^{4}+12A\,c^{2}x^{2}+2Bbc\,x^{2}+6Abc-3B\,b^{2}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{48c^{2}}-\frac{b^{2}\left(2Ac-Bb\right)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{16c^{\frac{5}{2}}\,x\sqrt{c\,x^{2}+b}}$
pseudoelliptic	$\frac{\left(-\frac{1}{2}b^{2}Ac+\frac{1}{4}B\,b^{3}\right)\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)+\left(b\left(\frac{x^{2}B}{3}+A\right)c^{\frac{3}{2}}+\left(\frac{4}{3}x^{4}B+2A\,x^{2}\right)c^{\frac{5}{2}}-\frac{B\sqrt{c}\,b^{2}}{2}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}+\frac{\ln(2)\left(A\right.}{}}{8c^{\frac{5}{2}}}$
default	$\frac{\sqrt{x^{4}c+b\,x^{2}}\,\left(8B\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}x^{3}+12A\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}x-6B\left(c\,x^{2}+b\right)^{\frac{3}{2}}\sqrt{c}\,bx-6A\sqrt{c\,x^{2}+b}\,c^{\frac{3}{2}}bx+3B\sqrt{c\,x^{2}+b}\,\sqrt{c}\,b^{2}x-6A\ln\left(\right.\right.}{48x\sqrt{c\,x^{2}+b}\,c^{\frac{5}{2}}}$

input
```
int(x*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*(8*B*c^2*x^4+12*A*c^2*x^2+2*B*b*c*x^2+6*A*b*c-3*B*b^2)/c^2*(x^2*(c*x^
2+b))^(1/2)-1/16*b^2*(2*A*c-B*b)/c^(5/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*(x^
2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.08

$$\int x\left(A+Bx^{2}\right)\sqrt{bx^{2}+cx^{4}}\,dx$$

$$=\left[-\frac{3\,(Bb^{3}-2\,Ab^{2}c)\sqrt{c}\log\left(-2\,cx^{2}-b+2\,\sqrt{cx^{4}+bx^{2}}\sqrt{c}\right)-2\,(8\,Bc^{3}x^{4}-3\,Bb^{2}c+6\,Abc^{2}+2\,(Bbc^{2}+6}{96\,c^{3}}\right.$$

$$-\frac{3\,(Bb^{3}-2\,Ab^{2}c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\sqrt{-c}}{cx^{2}+b}\right)-(8\,Bc^{3}x^{4}-3\,Bb^{2}c+6\,Abc^{2}+2\,(Bbc^{2}+6\,Ac^{3})x^{2})\sqrt{cx^{4}}}{48\,c^{3}}$$

input
```
integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(B*b^3 - 2*A*b^2*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*
x^2)*sqrt(c)) - 2*(8*B*c^3*x^4 - 3*B*b^2*c + 6*A*b*c^2 + 2*(B*b*c^2 + 6*A*
c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^3, -1/48*(3*(B*b^3 - 2*A*b^2*c)*sqrt(-c)*
arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (8*B*c^3*x^4 - 3*B*b^2*
c + 6*A*b*c^2 + 2*(B*b*c^2 + 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^3]
```

3.92.6 Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.38

$$\int x(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$
$$= \frac{A\left(\begin{cases} -\frac{b^2\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{8c}+\left(\frac{b}{4c}+\frac{x^2}{2}\right)\sqrt{bx^2+cx^4} & \text{for } c\neq 0\\ \frac{2(bx^2)^{\frac{3}{2}}}{3b} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{2}$$
$$+\frac{B\left(\begin{cases} \frac{b^3\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{16c^2}+\sqrt{bx^2+cx^4}\left(-\frac{b^2}{8c^2}+\frac{bx^2}{12c}+\frac{x^4}{3}\right) & \text{for } c\neq 0\\ \frac{2(bx^2)^{\frac{5}{2}}}{5b^2} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{2}$$

input
```
integrate(x*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

output
```
A*Piecewise((-b**2*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*
c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqr
t(c*(b/(2*c) + x**2)**2), True))/(8*c) + (b/(4*c) + x**2/2)*sqrt(b*x**2 +
c*x**4), Ne(c, 0)), (2*(b*x**2)**(3/2)/(3*b), Ne(b, 0)), (0, True))/2 + B*
Piecewise((b**3*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*c*x
**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c
*(b/(2*c) + x**2)**2), True))/(16*c**2) + sqrt(b*x**2 + c*x**4)*(-b**2/(8*
c**2) + b*x**2/(12*c) + x**4/3), Ne(c, 0)), (2*(b*x**2)**(5/2)/(5*b**2), N
e(b, 0)), (0, True))/2
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.65

$$\int x(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$
$$= \frac{1}{16}\left(4\sqrt{cx^4+bx^2}x^2 - \frac{b^2\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{3}{2}}} + \frac{2\sqrt{cx^4+bx^2}b}{c}\right)A$$
$$- \frac{1}{96}\left(\frac{12\sqrt{cx^4+bx^2}bx^2}{c} - \frac{3\,b^3\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}} + \frac{6\sqrt{cx^4+bx^2}b^2}{c^2} - \frac{16\,(cx^4+bx^2)^{\frac{3}{2}}}{c}\right)$$

input
```
integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/16*(4*sqrt(c*x^4 + b*x^2)*x^2 - b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x
^2)*sqrt(c))/c^(3/2) + 2*sqrt(c*x^4 + b*x^2)*b/c)*A - 1/96*(12*sqrt(c*x^4
+ b*x^2)*b*x^2/c - 3*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/
c^(5/2) + 6*sqrt(c*x^4 + b*x^2)*b^2/c^2 - 16*(c*x^4 + b*x^2)^(3/2)/c)*B
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.31

$$\int x(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$
$$= \frac{1}{48}\left(2\left(4\,Bx^2\mathrm{sgn}(x) + \frac{Bbc^3\mathrm{sgn}(x) + 6\,Ac^4\mathrm{sgn}(x)}{c^4}\right)x^2 - \frac{3\,(Bb^2c^2\mathrm{sgn}(x) - 2\,Abc^3\mathrm{sgn}(x))}{c^4}\right)\sqrt{cx^2+b}x$$
$$- \frac{(Bb^3\mathrm{sgn}(x) - 2\,Ab^2c\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{16\,c^{\frac{5}{2}}}$$
$$+ \frac{(Bb^3\log(|b|) - 2\,Ab^2c\log(|b|))\mathrm{sgn}(x)}{32\,c^{\frac{5}{2}}}$$

input
```
integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/48*(2*(4*B*x^2*sgn(x) + (B*b*c^3*sgn(x) + 6*A*c^4*sgn(x))/c^4)*x^2 - 3*(
B*b^2*c^2*sgn(x) - 2*A*b*c^3*sgn(x))/c^4)*sqrt(c*x^2 + b)*x - 1/16*(B*b^3*
sgn(x) - 2*A*b^2*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(5/2)
+ 1/32*(B*b^3*log(abs(b)) - 2*A*b^2*c*log(abs(b)))*sgn(x)/c^(5/2)
```

3.92.9 Mupad [B] (verification not implemented)

Time = 9.67 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.31

$$\int x(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{A\left(\frac{b}{4c}+\frac{x^2}{2}\right)\sqrt{c\,x^4+b\,x^2}}{2} + \frac{B\,b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,|x|\,\sqrt{c\,x^2+b}\right)}{32\,c^{5/2}} - \frac{A\,b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{16\,c^{3/2}} + \frac{B\,\sqrt{c\,x^4+b\,x^2}\,(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4)}{48\,c^2}$$

input
```
int(x*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

output
```
(A*(b/(4*c) + x^2/2)*(b*x^2 + c*x^4)^(1/2))/2 + (B*b^3*log(b + 2*c*x^2 + 2
*c^(1/2)*abs(x)*(b + c*x^2)^(1/2)))/(32*c^(5/2)) - (A*b^2*log((b/2 + c*x^2
)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(16*c^(3/2)) + (B*(b*x^2 + c*x^4)^(1/2
)*(8*c^2*x^4 - 3*b^2 + 2*b*c*x^2))/(48*c^2)
```

3.93 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx$

3.93.1 Optimal result

Integrand size = 26, antiderivative size = 100

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx = -\frac{(bB-4Ac)\sqrt{bx^2+cx^4}}{8c}+\frac{B(bx^2+cx^4)^{3/2}}{4cx^2} - \frac{b(bB-4Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}$$

output
```
1/4*B*(c*x^4+b*x^2)^(3/2)/c/x^2-1/8*b*(-4*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*
x^4+b*x^2)^(1/2))/c^(3/2)-1/8*(-4*A*c+B*b)*(c*x^4+b*x^2)^(1/2)/c
```

3.93.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx = \frac{x\left(\sqrt{c}x(b+cx^2)(bB+4Ac+2Bcx^2)+b(bB-4Ac)\sqrt{b+cx^2}\log\left(-\sqrt{c}x+\sqrt{b+cx^2}\right)\right)}{8c^{3/2}\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x,x]
```

output
```
(x*(Sqrt[c]*x*(b + c*x^2)*(b*B + 4*A*c + 2*B*c*x^2) + b*(b*B - 4*A*c)*Sqrt
[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(8*c^(3/2)*Sqrt[x^2*(b +
 c*x^2)])
```

3.93.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1940, 1221, 1131, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)\sqrt{cx^4+bx^2}}{x^2}dx^2$$

$$\Big\downarrow \text{1221}$$

$$\frac{1}{2}\left(\frac{B(bx^2+cx^4)^{3/2}}{2cx^2}-\frac{(bB-4Ac)\int \frac{\sqrt{cx^4+bx^2}}{x^2}dx^2}{4c}\right)$$

$$\Big\downarrow \text{1131}$$

$$\frac{1}{2}\left(\frac{B(bx^2+cx^4)^{3/2}}{2cx^2}-\frac{(bB-4Ac)\left(\frac{1}{2}b\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2+\sqrt{bx^2+cx^4}\right)}{4c}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{B(bx^2+cx^4)^{3/2}}{2cx^2}-\frac{(bB-4Ac)\left(b\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}+\sqrt{bx^2+cx^4}\right)}{4c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{B(bx^2+cx^4)^{3/2}}{2cx^2}-\frac{(bB-4Ac)\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}+\sqrt{bx^2+cx^4}\right)}{4c}\right)$$

input
```
Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x,x]
```

output
```
((B*(b*x^2 + c*x^4)^(3/2))/(2*c*x^2) - ((b*B - 4*A*c)*(Sqrt[b*x^2 + c*x^4]
 + (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/Sqrt[c]))/(4*c))/2
```

3.93.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.93. $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx$

3.93.4 Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

method	result
risch	$\frac{(2Bc\,x^2+4Ac+Bb)\sqrt{x^2(c\,x^2+b)}}{8c}+\frac{b(4Ac-Bb)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)\sqrt{x^2(c\,x^2+b)}}{8c^{\frac{3}{2}}x\sqrt{c\,x^2+b}}$
default	$\frac{\sqrt{x^4c+b\,x^2}\left(2B\sqrt{c}\,(c\,x^2+b)^{\frac{3}{2}}x+4A\,c^{\frac{3}{2}}\sqrt{c\,x^2+b}\,x-B\sqrt{c}\,\sqrt{c\,x^2+b}\,bx+4A\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)bc-B\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)b^2\right.}{8c^{\frac{3}{2}}\sqrt{c\,x^2+b}\,x}$
pseudoelliptic	$\frac{4B\,c^{\frac{3}{2}}x^2\sqrt{x^2(c\,x^2+b)}+4A\ln\left(\frac{2c\,x^2+2\sqrt{x^2(c\,x^2+b)}\,\sqrt{c}+b}{\sqrt{c}}\right)bc-4A\ln(2)bc+8A\,c^{\frac{3}{2}}\sqrt{x^2(c\,x^2+b)}-B\ln\left(\frac{2c\,x^2+2\sqrt{x^2(c\,x^2+b)}}{\sqrt{c}}\right.}{16c^{\frac{3}{2}}}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*B*c*x^2+4*A*c+B*b)/c*(x^2*(c*x^2+b))^(1/2)+1/8*b*(4*A*c-B*b)/c^(3/2
)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.72

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx$$
$$=\left[-\frac{(Bb^2-4\,Abc)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,(2\,Bc^2x^2+Bbc+4\,Ac^2)\sqrt{cx^4+bx^2}}{16\,c^2},\frac{(Bb^2-\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
[-1/16*((B*b^2 - 4*A*b*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)
*sqrt(c)) - 2*(2*B*c^2*x^2 + B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2))/c^2, 1/
8*((B*b^2 - 4*A*b*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 +
 b)) + (2*B*c^2*x^2 + B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2))/c^2]
```

3.93.6 Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x, x)
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.28

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx = \frac{1}{4}\left(\frac{b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{\sqrt{c}}+2\,\sqrt{cx^4+bx^2}\right)A + \frac{1}{16}\left(4\,\sqrt{cx^4+bx^2}x^2-\frac{b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{3}{2}}}+\frac{2\,\sqrt{cx^4+bx^2}b}{c}\right)B$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
1/4*(b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) + 2*sqrt(c
*x^4 + b*x^2))*A + 1/16*(4*sqrt(c*x^4 + b*x^2)*x^2 - b^2*log(2*c*x^2 + b +
 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(3/2) + 2*sqrt(c*x^4 + b*x^2)*b/c)*B
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx = \frac{1}{8}\left(2\,Bx^2\mathrm{sgn}(x)+\frac{Bbc\mathrm{sgn}(x)+4\,Ac^2\mathrm{sgn}(x)}{c^2}\right)\sqrt{cx^2+b}x + \frac{(Bb^2\mathrm{sgn}(x)-4\,Abc\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{8\,c^{\frac{3}{2}}} - \frac{(Bb^2\log\left(|b|\right)-4\,Abc\log\left(|b|\right))\mathrm{sgn}(x)}{16\,c^{\frac{3}{2}}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x,x, algorithm="giac")
```

output
```
1/8*(2*B*x^2*sgn(x) + (B*b*c*sgn(x) + 4*A*c^2*sgn(x))/c^2)*sqrt(c*x^2 + b)
*x + 1/8*(B*b^2*sgn(x) - 4*A*b*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 +
 b)))/c^(3/2) - 1/16*(B*b^2*log(abs(b)) - 4*A*b*c*log(abs(b)))*sgn(x)/c^(3
/2)
```

3.93.9 Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx = \frac{A\sqrt{c\,x^4+b\,x^2}}{2}+\frac{B\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\sqrt{c\,x^4+b\,x^2}}{2} + \frac{A\,b\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{4\,\sqrt{c}} - \frac{B\,b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{16\,c^{3/2}}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x,x)
```

output
```
(A*(b*x^2 + c*x^4)^(1/2))/2 + (B*(b/(4*c) + x^2/2)*(b*x^2 + c*x^4)^(1/2))/
2 + (A*b*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(4*c^(1/2)) -
 (B*b^2*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(16*c^(3/2))
```

3.94 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3} \, dx$

3.94.1 Optimal result

Integrand size = 26, antiderivative size = 97

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3} \, dx = \frac{(bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{A(bx^2+cx^4)^{3/2}}{bx^4} + \frac{(bB+2Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}$$

output
```
-A*(c*x^4+b*x^2)^(3/2)/b/x^4+1/2*(2*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*
x^2)^(1/2))/c^(1/2)+1/2*(2*A*c+B*b)*(c*x^4+b*x^2)^(1/2)/b
```

3.94.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3} \, dx = \frac{\sqrt{x^2(b+cx^2)}\left(-2A+Bx^2+\frac{2(bB+2Ac)x\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{b}+\sqrt{b+cx^2}}\right)}{\sqrt{c}\sqrt{b+cx^2}}\right)}{2x^2}$$

input
```
Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^3,x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(-2*A + B*x^2 + (2*(b*B + 2*A*c)*x*ArcTanh[(Sqrt[c]
*x)/(-Sqrt[b] + Sqrt[b + c*x^2])])/(Sqrt[c]*Sqrt[b + c*x^2])))/(2*x^2)
```

3.94.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1940, 1220, 1131, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3}dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)\sqrt{cx^4+bx^2}}{x^4}dx^2$$

$$\Big\downarrow \text{1220}$$

$$\frac{1}{2}\left(\frac{(2Ac+bB)\int \frac{\sqrt{cx^4+bx^2}}{x^2}dx^2}{b}-\frac{2A(bx^2+cx^4)^{3/2}}{bx^4}\right)$$

$$\Big\downarrow \text{1131}$$

$$\frac{1}{2}\left(\frac{(2Ac+bB)\left(\frac{1}{2}b\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2+\sqrt{bx^2+cx^4}\right)}{b}-\frac{2A(bx^2+cx^4)^{3/2}}{bx^4}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{(2Ac+bB)\left(b\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}+\sqrt{bx^2+cx^4}\right)}{b}-\frac{2A(bx^2+cx^4)^{3/2}}{bx^4}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{(2Ac+bB)\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}+\sqrt{bx^2+cx^4}\right)}{b}-\frac{2A(bx^2+cx^4)^{3/2}}{bx^4}\right)$$

input
```
Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^3,x]
```

output
```
((-2*A*(b*x^2 + c*x^4)^(3/2))/(b*x^4) + ((b*B + 2*A*c)*(Sqrt[b*x^2 + c*x^4
] + (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/Sqrt[c]))/b)/2
```

3.94.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.94.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\left(-x^{2} B+2 A\right) \sqrt{x^{2}\left(c\,x^{2}+b\right)}}{2 x^{2}}+\frac{\left(A c+\frac{B b}{2}\right) \ln \left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right) \sqrt{x^{2}\left(c\,x^{2}+b\right)}}{\sqrt{c}\,x \sqrt{c\,x^{2}+b}}$
pseudoelliptic	$\frac{-2\left(-\frac{x^{2} B}{2}+A\right) \sqrt{c}\,\sqrt{x^{2}\left(c\,x^{2}+b\right)}+x^{2}\left(-\ln (2)+\ln \left(\frac{2 c\,x^{2}+2 \sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)\right)\left(A c+\frac{B b}{2}\right)}{2 \sqrt{c}\,x^{2}}$
default	$-\frac{\sqrt{x^{4} c+b\,x^{2}}\,\left(-2 A \sqrt{c\,x^{2}+b}\,c^{\frac{3}{2}} x^{2}-B \sqrt{c\,x^{2}+b}\,\sqrt{c}\,b\,x^{2}+2 A\left(c\,x^{2}+b\right)^{\frac{3}{2}} \sqrt{c}-2 A \ln \left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right) b c x-B \ln \left(\sqrt{c}\,x+\sqrt{c\,x^{2}+\cdots\right.\right.}{2 x^{2} \sqrt{c\,x^{2}+b}\,b \sqrt{c}}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-B*x^2+2*A)/x^2*(x^2*(c*x^2+b))^(1/2)+(A*c+1/2*B*b)*ln(c^(1/2)*x+(c*
x^2+b)^(1/2))/c^(1/2)*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.66

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3}\,dx$$
$$= \left[\frac{(Bb+2\,Ac)\sqrt{c}x^2\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}(Bcx^2-2\,Ac)}{4\,cx^2},\right.$$
$$\left.-\frac{(Bb+2\,Ac)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-\sqrt{cx^4+bx^2}(Bcx^2-2\,Ac)}{2\,cx^2}\right]$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/4*((B*b + 2*A*c)*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*s
qrt(c)) + 2*sqrt(c*x^4 + b*x^2)*(B*c*x^2 - 2*A*c))/(c*x^2), -1/2*((B*b + 2
*A*c)*sqrt(-c)*x^2*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - sqrt
(c*x^4 + b*x^2)*(B*c*x^2 - 2*A*c))/(c*x^2)]
```

3.94.6 Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3}\,dx = \int \frac{\sqrt{x^2(b+cx^2)}(A+Bx^2)}{x^3}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**3, x)
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3}\,dx = \frac{1}{2}\left(\sqrt{c}\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-\frac{2\,\sqrt{cx^4+bx^2}}{x^2}\right)A + \frac{1}{4}\left(\frac{b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{\sqrt{c}}+2\,\sqrt{cx^4+bx^2}\right)B$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
1/2*(sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*sqrt(c*x
^4 + b*x^2)/x^2)*A + 1/4*(b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c
))/sqrt(c) + 2*sqrt(c*x^4 + b*x^2))*B
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3}\,dx = \frac{1}{2}\,\sqrt{cx^2+b}Bx\mathrm{sgn}(x) + \frac{2\,Ab\sqrt{c}\mathrm{sgn}(x)}{\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b} - \frac{(Bb\mathrm{sgn}(x)+2\,Ac\mathrm{sgn}(x))\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)}{4\,\sqrt{c}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/2*sqrt(c*x^2 + b)*B*x*sgn(x) + 2*A*b*sqrt(c)*sgn(x)/((sqrt(c)*x - sqrt(c
*x^2 + b))^2 - b) - 1/4*(B*b*sgn(x) + 2*A*c*sgn(x))*log((sqrt(c)*x - sqrt(
c*x^2 + b))^2)/sqrt(c)
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^3}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^3,x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^3, x)
```

3.95 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx$

3.95.1 Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx = -\frac{B\sqrt{bx^2+cx^4}}{x^2}-\frac{A(bx^2+cx^4)^{3/2}}{3bx^6}+B\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

output
```
-1/3*A*(c*x^4+b*x^2)^(3/2)/b/x^6+B*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2)
)*c^(1/2)-B*(c*x^4+b*x^2)^(1/2)/x^2
```

3.95.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.24

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx = -\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b+cx^2}(3bBx^2+A(b+cx^2))+3bB\sqrt{c}x^3\log\left(-\sqrt{c}x+\sqrt{b+cx^2}\right)\right)}{3bx^4\sqrt{b+cx^2}}$$

input
```
Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^5,x]
```

output
```
-1/3*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b + c*x^2]*(3*b*B*x^2 + A*(b + c*x^2)) +
 3*b*B*Sqrt[c]*x^3*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(b*x^4*Sqrt[b + c
*x^2])
```

3.95.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1940, 1220, 1125, 25, 27, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)\sqrt{cx^4+bx^2}}{x^6}dx^2$$

$$\Big\downarrow \text{1220}$$

$$\frac{1}{2}\left(B\int \frac{\sqrt{cx^4+bx^2}}{x^4}dx^2-\frac{2A(bx^2+cx^4)^{3/2}}{3bx^6}\right)$$

$$\Big\downarrow \text{1125}$$

$$\frac{1}{2}\left(B\left(-\int -\frac{c}{\sqrt{cx^4+bx^2}}dx^2-\frac{2\sqrt{bx^2+cx^4}}{x^2}\right)-\frac{2A(bx^2+cx^4)^{3/2}}{3bx^6}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(B\left(\int \frac{c}{\sqrt{cx^4+bx^2}}dx^2-\frac{2\sqrt{bx^2+cx^4}}{x^2}\right)-\frac{2A(bx^2+cx^4)^{3/2}}{3bx^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(B\left(c\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2-\frac{2\sqrt{bx^2+cx^4}}{x^2}\right)-\frac{2A(bx^2+cx^4)^{3/2}}{3bx^6}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(B\left(2c\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}-\frac{2\sqrt{bx^2+cx^4}}{x^2}\right)-\frac{2A(bx^2+cx^4)^{3/2}}{3bx^6}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(B\left(2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{2\sqrt{bx^2+cx^4}}{x^2}\right)-\frac{2A(bx^2+cx^4)^{3/2}}{3bx^6}\right)$$

input
```
Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^5,x]
```

output
```
((-2*A*(b*x^2 + c*x^4)^(3/2))/(3*b*x^6) + B*((-2*Sqrt[b*x^2 + c*x^4])/x^2
+ 2*Sqrt[c]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]))/2
```

3.95.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.95.4 Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08

method	result	size
risch	$-\frac{\left(A c \,x^{2}+3 b B \,x^{2}+A b \right) \sqrt{x^{2} \left(c \,x^{2}+b \right)}}{3 x^{4} b}+\frac{B \sqrt{c}\, \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+b}\right) \sqrt{x^{2} \left(c \,x^{2}+b \right)}}{x \sqrt{c \,x^{2}+b}}$	86
pseudoelliptic	$\frac{3 x^{4} B \left(-\ln \left(2\right)+\ln \left(\frac{2 c \,x^{2}+2 \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, \sqrt{c}+b}{\sqrt{c}}\right)\right) b \sqrt{c}-2 \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, \left(\left(A c +3 B b \right) x^{2}+A b \right)}{6 b \,x^{4}}$	87
default	$-\frac{\sqrt{x^{4} c +b \,x^{2}}\, \left(-3 B \sqrt{c \,x^{2}+b}\, c^{\frac{3}{2}} x^{4}+3 B \left(c \,x^{2}+b \right)^{\frac{3}{2}} \sqrt{c}\, x^{2}-3 B \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+b}\right) b c \,x^{3}+A \left(c \,x^{2}+b \right)^{\frac{3}{2}} \sqrt{c}\right)}{3 x^{4} \sqrt{c \,x^{2}+b}\, b \sqrt{c}}$	109

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(A*c*x^2+3*B*b*x^2+A*b)/x^4/b*(x^2*(c*x^2+b))^(1/2)+B*c^(1/2)*ln(c^(1
/2)*x+(c*x^2+b)^(1/2))*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.00

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx = \left[\frac{3\,Bb\sqrt{c}x^4\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}((3\,Bb+Ac)x^2+Ab)}{6\,bx^4}, -\frac{3\,Bb\sqrt{-c}x^4\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}((3\,Bb+Ac)x^2+Ab)}{3\,bx^4}\right]$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[1/6*(3*B*b*sqrt(c)*x^4*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c))
- 2*sqrt(c*x^4 + b*x^2)*((3*B*b + A*c)*x^2 + A*b))/(b*x^4), -1/3*(3*B*b*sq
rt(-c)*x^4*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 +
 b*x^2)*((3*B*b + A*c)*x^2 + A*b))/(b*x^4)]
```

3.95.6 Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx=\int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x^5}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**5, x)
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.20

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx =\frac{1}{2}\left(\sqrt{c}\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-\frac{2\,\sqrt{cx^4+bx^2}}{x^2}\right)B-\frac{1}{3}\,A\left(\frac{\sqrt{cx^4+bx^2}c}{bx^2}+\frac{\sqrt{cx^4+bx^2}}{x^4}\right)$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
1/2*(sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*sqrt(c*x
^4 + b*x^2)/x^2)*B - 1/3*A*(sqrt(c*x^4 + b*x^2)*c/(b*x^2) + sqrt(c*x^4 + b
*x^2)/x^4)
```

3.95.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(68) = 136$.

Time = 0.44 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.04

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx = -\frac{1}{2}B\sqrt{c}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)\operatorname{sgn}(x) + \frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Bb\sqrt{c}\operatorname{sgn}(x)+3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Ac^{\frac{3}{2}}\operatorname{sgn}(x)-6\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2Bb^2\sqrt{c}\right.}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^3}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="giac")
```

output
```
-1/2*B*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sgn(x) + 2/3*(3*(sqrt(
c)*x - sqrt(c*x^2 + b))^4*B*b*sqrt(c)*sgn(x) + 3*(sqrt(c)*x - sqrt(c*x^2 +
 b))^4*A*c^(3/2)*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^2*sqrt(c)*
sgn(x) + 3*B*b^3*sqrt(c)*sgn(x) + A*b^2*c^(3/2)*sgn(x))/((sqrt(c)*x - sqrt
(c*x^2 + b))^2 - b)^3
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^5}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^5,x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^5, x)
```

3.96 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx$

3.96.1 Optimal result

Integrand size = 26, antiderivative size = 61

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx = -\frac{A(bx^2+cx^4)^{3/2}}{5bx^8} - \frac{(5bB-2Ac)\,(bx^2+cx^4)^{3/2}}{15b^2x^6}$$

output
```
-1/5*A*(c*x^4+b*x^2)^(3/2)/b/x^8-1/15*(-2*A*c+5*B*b)*(c*x^4+b*x^2)^(3/2)/b
^2/x^6
```

3.96.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx = -\frac{(x^2(b+cx^2))^{3/2}\,(3Ab+5bBx^2-2Acx^2)}{15b^2x^8}$$

input
```
Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^7,x]
```

output
```
-1/15*((x^2*(b + c*x^2))^(3/2)*(3*A*b + 5*b*B*x^2 - 2*A*c*x^2))/(b^2*x^8)
```

3.96.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1940, 1220, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)\sqrt{cx^4+bx^2}}{x^8}dx^2$$

$$\Big\downarrow \text{1220}$$

$$\frac{1}{2}\left(\frac{(5bB-2Ac)\int \frac{\sqrt{cx^4+bx^2}}{x^6}dx^2}{5b}-\frac{2A(bx^2+cx^4)^{3/2}}{5bx^8}\right)$$

$$\Big\downarrow \text{1123}$$

$$\frac{1}{2}\left(-\frac{2(bx^2+cx^4)^{3/2}(5bB-2Ac)}{15b^2x^6}-\frac{2A(bx^2+cx^4)^{3/2}}{5bx^8}\right)$$

input
```
Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^7,x]
```

output
```
((-2*A*(b*x^2 + c*x^4)^(3/2))/(5*b*x^8) - (2*(5*b*B - 2*A*c)*(b*x^2 + c*x^
4)^(3/2))/(15*b^2*x^6))/2
```

3.96.3.1 Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.96.4 Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

method	result	size
pseudoelliptic	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(c\,x^{2}+b\right)\left(\left(\frac{5x^{2}B}{3}+A\right)b-\frac{2Ac\,x^{2}}{3}\right)}{5b^{2}x^{6}}$	47
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-2Ac\,x^{2}+5bB\,x^{2}+3Ab\right)\sqrt{x^{4}c+b\,x^{2}}}{15b^{2}x^{6}}$	48
default	$-\frac{\left(c\,x^{2}+b\right)\left(-2Ac\,x^{2}+5bB\,x^{2}+3Ab\right)\sqrt{x^{4}c+b\,x^{2}}}{15b^{2}x^{6}}$	48
trager	$-\frac{\left(-2A\,c^{2}x^{4}+5x^{4}Bbc+Abc\,x^{2}+5b^{2}B\,x^{2}+3b^{2}A\right)\sqrt{x^{4}c+b\,x^{2}}}{15b^{2}x^{6}}$	62
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(-2A\,c^{2}x^{4}+5x^{4}Bbc+Abc\,x^{2}+5b^{2}B\,x^{2}+3b^{2}A\right)}{15x^{6}b^{2}}$	62

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(x^2*(c*x^2+b))^(1/2)*(c*x^2+b)*((5/3*x^2*B+A)*b-2/3*A*c*x^2)/b^2/x^6
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx = -\frac{((5\,Bbc-2\,Ac^2)x^4+3\,Ab^2+(5\,Bb^2+Abc)x^2)\sqrt{cx^4+bx^2}}{15\,b^2x^6}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
-1/15*((5*B*b*c - 2*A*c^2)*x^4 + 3*A*b^2 + (5*B*b^2 + A*b*c)*x^2)*sqrt(c*x
^4 + b*x^2)/(b^2*x^6)
```

3.96.6 Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x^7}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**7,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**7, x)
```

3.96.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(53) = 106.

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.82

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx = -\frac{1}{3}\,B\left(\frac{\sqrt{cx^4+bx^2}c}{bx^2}+\frac{\sqrt{cx^4+bx^2}}{x^4}\right) + \frac{1}{15}\,A\left(\frac{2\,\sqrt{cx^4+bx^2}c^2}{b^2x^2}-\frac{\sqrt{cx^4+bx^2}c}{bx^4}-\frac{3\,\sqrt{cx^4+bx^2}}{x^6}\right)$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-1/3*B*(sqrt(c*x^4 + b*x^2)*c/(b*x^2) + sqrt(c*x^4 + b*x^2)/x^4) + 1/15*A*
(2*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^2) - sqrt(c*x^4 + b*x^2)*c/(b*x^4) - 3*s
qrt(c*x^4 + b*x^2)/x^6)
```

3.96.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(53) = 106$.

Time = 0.70 (sec) , antiderivative size = 250, normalized size of antiderivative = 4.10

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx = \frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8Bc^{\frac{3}{2}}\mathrm{sgn}(x)-30\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6Bbc^{\frac{3}{2}}\mathrm{sgn}(x)+30\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6Ac^{\frac{5}{2}}\mathrm{sg}\right.}{}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="giac")
```

output
```
2/15*(15*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*c^(3/2)*sgn(x) - 30*(sqrt(c)*x
- sqrt(c*x^2 + b))^6*B*b*c^(3/2)*sgn(x) + 30*(sqrt(c)*x - sqrt(c*x^2 + b))
^6*A*c^(5/2)*sgn(x) + 20*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^2*c^(3/2)*sgn
(x) + 10*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b*c^(5/2)*sgn(x) - 10*(sqrt(c)*
x - sqrt(c*x^2 + b))^2*B*b^3*c^(3/2)*sgn(x) + 10*(sqrt(c)*x - sqrt(c*x^2 +
 b))^2*A*b^2*c^(5/2)*sgn(x) + 5*B*b^4*c^(3/2)*sgn(x) - 2*A*b^3*c^(5/2)*sgn
(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^5
```

3.96.9 Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.85

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx = \frac{(A\,c^2+B\,b\,c)\,\sqrt{c\,x^4+b\,x^2}}{5\,b^2\,x^2} - \frac{(5\,B\,b^2+A\,c\,b)\,\sqrt{c\,x^4+b\,x^2}}{15\,b^2\,x^4} - \frac{A\,\sqrt{c\,x^4+b\,x^2}}{5\,x^6} - \frac{c\,(A\,c+8\,B\,b)\,\sqrt{c\,x^4+b\,x^2}}{15\,b^2\,x^2}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^7,x)
```

output
```
((A*c^2 + B*b*c)*(b*x^2 + c*x^4)^(1/2))/(5*b^2*x^2) - ((5*B*b^2 + A*b*c)*(
b*x^2 + c*x^4)^(1/2))/(15*b^2*x^4) - (A*(b*x^2 + c*x^4)^(1/2))/(5*x^6) - (
c*(A*c + 8*B*b)*(b*x^2 + c*x^4)^(1/2))/(15*b^2*x^2)
```

3.97 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9}\,dx$

3.97.1 Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9}\,dx = -\frac{A(bx^2+cx^4)^{3/2}}{7bx^{10}} - \frac{(7bB-4Ac)(bx^2+cx^4)^{3/2}}{35b^2x^8} + \frac{2c(7bB-4Ac)(bx^2+cx^4)^{3/2}}{105b^3x^6}$$

output
```
-1/7*A*(c*x^4+b*x^2)^(3/2)/b/x^10-1/35*(-4*A*c+7*B*b)*(c*x^4+b*x^2)^(3/2)/
b^2/x^8+2/105*c*(-4*A*c+7*B*b)*(c*x^4+b*x^2)^(3/2)/b^3/x^6
```

3.97.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9}\,dx = \frac{(x^2(b+cx^2))^{3/2}(7bBx^2(-3b+2cx^2)+A(-15b^2+12bcx^2-8c^2x^4))}{105b^3x^{10}}$$

input
```
Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^9,x]
```

output
```
((x^2*(b + c*x^2))^(3/2)*(7*b*B*x^2*(-3*b + 2*c*x^2) + A*(-15*b^2 + 12*b*c
*x^2 - 8*c^2*x^4)))/(105*b^3*x^10)
```

3.97.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1940, 1220, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9}dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)\sqrt{cx^4+bx^2}}{x^{10}}dx^2$$

$$\Big\downarrow \text{1220}$$

$$\frac{1}{2}\left(\frac{(7bB-4Ac)\int \frac{\sqrt{cx^4+bx^2}}{x^8}dx^2}{7b}-\frac{2A(bx^2+cx^4)^{3/2}}{7bx^{10}}\right)$$

$$\Big\downarrow \text{1129}$$

$$\frac{1}{2}\left(\frac{(7bB-4Ac)\left(-\frac{2c\int \frac{\sqrt{cx^4+bx^2}}{x^6}dx^2}{5b}-\frac{2(bx^2+cx^4)^{3/2}}{5bx^8}\right)}{7b}-\frac{2A(bx^2+cx^4)^{3/2}}{7bx^{10}}\right)$$

$$\Big\downarrow \text{1123}$$

$$\frac{1}{2}\left(\frac{\left(\frac{4c(bx^2+cx^4)^{3/2}}{15b^2x^6}-\frac{2(bx^2+cx^4)^{3/2}}{5bx^8}\right)(7bB-4Ac)}{7b}-\frac{2A(bx^2+cx^4)^{3/2}}{7bx^{10}}\right)$$

input
```
Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^9,x]
```

output
```
((-2*A*(b*x^2 + c*x^4)^(3/2))/(7*b*x^10) + ((7*b*B - 4*A*c)*((-2*(b*x^2 +
c*x^4)^(3/2))/(5*b*x^8) + (4*c*(b*x^2 + c*x^4)^(3/2))/(15*b^2*x^6)))/(7*b)
)/2
```

3.97.3.1 Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.97. $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9}\,dx$

3.97.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

method	result	size
pseudoelliptic	$-\frac{\left(\left(\frac{7x^2B}{5}+A\right)b^2-\frac{4\left(\frac{7x^2B}{6}+A\right)x^2cb}{5}+\frac{8Ac^2x^4}{15}\right)\sqrt{x^2(c\,x^2+b)}\,(c\,x^2+b)}{7b^3x^8}$	66
gosper	$-\frac{(c\,x^2+b)(8A\,c^2x^4-14x^4Bbc-12Abc\,x^2+21b^2B\,x^2+15b^2A)\sqrt{x^4c+b\,x^2}}{105b^3x^8}$	70
default	$-\frac{(c\,x^2+b)(8A\,c^2x^4-14x^4Bbc-12Abc\,x^2+21b^2B\,x^2+15b^2A)\sqrt{x^4c+b\,x^2}}{105b^3x^8}$	70
trager	$-\frac{(8A\,c^3x^6-14x^6Bb\,c^2-4Ab\,c^2x^4+7x^4B\,b^2c+3A\,b^2c\,x^2+21b^3B\,x^2+15b^3A)\sqrt{x^4c+b\,x^2}}{105b^3x^8}$	87
risch	$-\frac{\sqrt{x^2(c\,x^2+b)}\,(8A\,c^3x^6-14x^6Bb\,c^2-4Ab\,c^2x^4+7x^4B\,b^2c+3A\,b^2c\,x^2+21b^3B\,x^2+15b^3A)}{105x^8b^3}$	87

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/7*((7/5*x^2*B+A)*b^2-4/5*(7/6*x^2*B+A)*x^2*c*b+8/15*A*c^2*x^4)*(x^2*(c*
x^2+b))^(1/2)*(c*x^2+b)/b^3/x^8
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9}\,dx = \frac{(2\,(7\,Bbc^2-4\,Ac^3)x^6-(7\,Bb^2c-4\,Abc^2)x^4-15\,Ab^3-3\,(7\,Bb^3+Ab^2c)x^2)\sqrt{cx^4+bx^2}}{105\,b^3x^8}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="fricas")
```

output
```
1/105*(2*(7*B*b*c^2 - 4*A*c^3)*x^6 - (7*B*b^2*c - 4*A*b*c^2)*x^4 - 15*A*b^
3 - 3*(7*B*b^3 + A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^3*x^8)
```

3.97.6 Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9}\,dx = \int \frac{\sqrt{x^2(b+cx^2)}(A+Bx^2)}{x^9}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**9,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**9, x)
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.68

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9}\,dx = \frac{1}{15}B\left(\frac{2\sqrt{cx^4+bx^2}c^2}{b^2x^2} - \frac{\sqrt{cx^4+bx^2}c}{bx^4} - \frac{3\sqrt{cx^4+bx^2}}{x^6}\right) - \frac{1}{105}A\left(\frac{8\sqrt{cx^4+bx^2}c^3}{b^3x^2} - \frac{4\sqrt{cx^4+bx^2}c^2}{b^2x^4} + \frac{3\sqrt{cx^4+bx^2}c}{bx^6} + \frac{15\sqrt{cx^4+bx^2}}{x^8}\right)$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="maxima")
```

output
```
1/15*B*(2*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^2) - sqrt(c*x^4 + b*x^2)*c/(b*x^4
) - 3*sqrt(c*x^4 + b*x^2)/x^6) - 1/105*A*(8*sqrt(c*x^4 + b*x^2)*c^3/(b^3*x
^2) - 4*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^4) + 3*sqrt(c*x^4 + b*x^2)*c/(b*x^6
) + 15*sqrt(c*x^4 + b*x^2)/x^8)
```

3.97.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(84) = 168.

Time = 0.89 (sec) , antiderivative size = 310, normalized size of antiderivative = 3.23

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9}\,dx = \frac{4\left(105\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bc^{\frac{5}{2}}\operatorname{sgn}(x) - 175\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Bbc^{\frac{5}{2}}\operatorname{sgn}(x) + 280\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}A\right.}{}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="giac")
```

output
```
4/105*(105*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*c^(5/2)*sgn(x) - 175*(sqrt(c
)*x - sqrt(c*x^2 + b))^8*B*b*c^(5/2)*sgn(x) + 280*(sqrt(c)*x - sqrt(c*x^2
+ b))^8*A*c^(7/2)*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^2*c^(5/2
)*sgn(x) + 140*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b*c^(7/2)*sgn(x) - 42*(sq
rt(c)*x - sqrt(c*x^2 + b))^4*B*b^3*c^(5/2)*sgn(x) + 84*(sqrt(c)*x - sqrt(c
*x^2 + b))^4*A*b^2*c^(7/2)*sgn(x) + 49*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b
^4*c^(5/2)*sgn(x) - 28*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^3*c^(7/2)*sgn(x
) - 7*B*b^5*c^(5/2)*sgn(x) + 4*A*b^4*c^(7/2)*sgn(x))/((sqrt(c)*x - sqrt(c*
x^2 + b))^2 - b)^7
```

3.97.9 Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.67

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9}\,dx = \frac{4\,A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b^2\,x^4} - \frac{B\,\sqrt{c\,x^4+b\,x^2}}{5\,x^6} - \frac{A\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,b\,x^6} - \frac{B\,c\,\sqrt{c\,x^4+b\,x^2}}{15\,b\,x^4} - \frac{A\,\sqrt{c\,x^4+b\,x^2}}{7\,x^8} - \frac{8\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{105\,b^3\,x^2} + \frac{2\,B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{15\,b^2\,x^2}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^9,x)
```

output
```
(4*A*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^4) - (B*(b*x^2 + c*x^4)^(1/2))/
(5*x^6) - (A*c*(b*x^2 + c*x^4)^(1/2))/(35*b*x^6) - (B*c*(b*x^2 + c*x^4)^(1
/2))/(15*b*x^4) - (A*(b*x^2 + c*x^4)^(1/2))/(7*x^8) - (8*A*c^3*(b*x^2 + c*
x^4)^(1/2))/(105*b^3*x^2) + (2*B*c^2*(b*x^2 + c*x^4)^(1/2))/(15*b^2*x^2)
```

3.98 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx$

3.98.1 Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx = -\frac{A(bx^2+cx^4)^{3/2}}{9bx^{12}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{3/2}}{21b^2x^{10}} + \frac{4c(3bB-2Ac)(bx^2+cx^4)^{3/2}}{105b^3x^8} - \frac{8c^2(3bB-2Ac)(bx^2+cx^4)^{3/2}}{315b^4x^6}$$

output
```
-1/9*A*(c*x^4+b*x^2)^(3/2)/b/x^12-1/21*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/
b^2/x^10+4/105*c*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/b^3/x^8-8/315*c^2*(-2*
A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/b^4/x^6
```

3.98.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.66

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx = -\frac{(x^2(b+cx^2))^{3/2}(3bBx^2(15b^2-12bcx^2+8c^2x^4)+A(35b^3-30b^2cx^2+24bc^2x^4-16c^3x^6))}{315b^4x^{12}}$$

input
```
Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^11,x]
```

output
```
-1/315*((x^2*(b + c*x^2))^(3/2)*(3*b*B*x^2*(15*b^2 - 12*b*c*x^2 + 8*c^2*x^
4) + A*(35*b^3 - 30*b^2*c*x^2 + 24*b*c^2*x^4 - 16*c^3*x^6)))/(b^4*x^12)
```

3.98.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1940, 1220, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)\sqrt{cx^4+bx^2}}{x^{12}}dx^2$$

$$\Big\downarrow \text{1220}$$

$$\frac{1}{2}\left(\frac{(3bB-2Ac)\int \frac{\sqrt{cx^4+bx^2}}{x^{10}}dx^2}{3b}-\frac{2A(bx^2+cx^4)^{3/2}}{9bx^{12}}\right)$$

$$\Big\downarrow \text{1129}$$

$$\frac{1}{2}\left(\frac{(3bB-2Ac)\left(-\frac{4c\int \frac{\sqrt{cx^4+bx^2}}{x^8}dx^2}{7b}-\frac{2(bx^2+cx^4)^{3/2}}{7bx^{10}}\right)}{3b}-\frac{2A(bx^2+cx^4)^{3/2}}{9bx^{12}}\right)$$

$$\Big\downarrow \text{1129}$$

$$\frac{1}{2}\left(\frac{(3bB-2Ac)\left(-\frac{4c\left(-\frac{2c\int \frac{\sqrt{cx^4+bx^2}}{x^6}dx^2}{5b}-\frac{2(bx^2+cx^4)^{3/2}}{5bx^8}\right)}{7b}-\frac{2(bx^2+cx^4)^{3/2}}{7bx^{10}}\right)}{3b}-\frac{2A(bx^2+cx^4)^{3/2}}{9bx^{12}}\right)$$

$$\Big\downarrow \text{1123}$$

$$\frac{1}{2}\left(\frac{\left(-\frac{4c\left(\frac{4c\left(bx^2+cx^4\right)^{3/2}}{15b^2x^6}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{5bx^8}\right)}{7b}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}\right)(3bB-2Ac)}{3b}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}}\right)$$

input
```
Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^11,x]
```

output
```
((-2*A*(b*x^2 + c*x^4)^(3/2))/(9*b*x^12) + ((3*b*B - 2*A*c)*((-2*(b*x^2 +
c*x^4)^(3/2))/(7*b*x^10) - (4*c*((-2*(b*x^2 + c*x^4)^(3/2))/(5*b*x^8) + (4
*c*(b*x^2 + c*x^4)^(3/2))/(15*b^2*x^6)))/(7*b)))/(3*b))/2
```

3.98.3.1 Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.98.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$-\frac{\left(\left(\frac{9x^{2}B}{7}+A\right)b^{3}-\frac{6x^{2}c\left(\frac{6x^{2}B}{5}+A\right)b^{2}}{7}+\frac{24c^{2}x^{4}\left(x^{2}B+A\right)b}{35}-\frac{16A\,c^{3}x^{6}}{35}\right)\sqrt{x^{2}(c\,x^{2}+b)}\,(c\,x^{2}+b)}{9b^{4}x^{10}}$
gosper	$-\frac{(c\,x^{2}+b)\left(-16A\,c^{3}x^{6}+24x^{6}Bb\,c^{2}+24Ab\,c^{2}x^{4}-36x^{4}B\,b^{2}c-30A\,b^{2}c\,x^{2}+45b^{3}B\,x^{2}+35b^{3}A\right)\sqrt{x^{4}c+b\,x^{2}}}{315b^{4}x^{10}}$
default	$-\frac{(c\,x^{2}+b)\left(-16A\,c^{3}x^{6}+24x^{6}Bb\,c^{2}+24Ab\,c^{2}x^{4}-36x^{4}B\,b^{2}c-30A\,b^{2}c\,x^{2}+45b^{3}B\,x^{2}+35b^{3}A\right)\sqrt{x^{4}c+b\,x^{2}}}{315b^{4}x^{10}}$
trager	$-\frac{\left(-16A\,x^{8}c^{4}+24B\,x^{8}b\,c^{3}+8A\,x^{6}b\,c^{3}-12B\,x^{6}b^{2}c^{2}-6A\,b^{2}c^{2}x^{4}+9B\,b^{3}c\,x^{4}+5A\,x^{2}b^{3}c+45B\,x^{2}b^{4}+35A\,b^{4}\right)\sqrt{x^{4}c+b\,x^{2}}}{315b^{4}x^{10}}$
risch	$-\frac{\sqrt{x^{2}(c\,x^{2}+b)}\left(-16A\,x^{8}c^{4}+24B\,x^{8}b\,c^{3}+8A\,x^{6}b\,c^{3}-12B\,x^{6}b^{2}c^{2}-6A\,b^{2}c^{2}x^{4}+9B\,b^{3}c\,x^{4}+5A\,x^{2}b^{3}c+45B\,x^{2}b^{4}+35A\,b^{4}\right)}{315x^{10}b^{4}}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/9*((9/7*x^2*B+A)*b^3-6/7*x^2*c*(6/5*x^2*B+A)*b^2+24/35*c^2*x^4*(B*x^2+A
)*b-16/35*A*c^3*x^6)*(x^2*(c*x^2+b))^(1/2)*(c*x^2+b)/b^4/x^10
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx =$$
$$-\frac{(8\,(3\,Bbc^3-2\,Ac^4)x^8-4\,(3\,Bb^2c^2-2\,Abc^3)x^6+35\,Ab^4+3\,(3\,Bb^3c-2\,Ab^2c^2)x^4+5\,(9\,Bb^4+Ab^3c)x^2}{315\,b^4x^{10}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="fricas")
```

output
```
-1/315*(8*(3*B*b*c^3 - 2*A*c^4)*x^8 - 4*(3*B*b^2*c^2 - 2*A*b*c^3)*x^6 + 35
*A*b^4 + 3*(3*B*b^3*c - 2*A*b^2*c^2)*x^4 + 5*(9*B*b^4 + A*b^3*c)*x^2)*sqrt
(c*x^4 + b*x^2)/(b^4*x^10)
```

3.98.6 Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x^{11}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**11,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**11, x)
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.57

$$\begin{aligned}&\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx\\&=-\frac{1}{105}\,B\left(\frac{8\sqrt{cx^4+bx^2}c^3}{b^3x^2}-\frac{4\sqrt{cx^4+bx^2}c^2}{b^2x^4}+\frac{3\sqrt{cx^4+bx^2}c}{bx^6}+\frac{15\sqrt{cx^4+bx^2}}{x^8}\right)\\&\quad+\frac{1}{315}\,A\left(\frac{16\sqrt{cx^4+bx^2}c^4}{b^4x^2}-\frac{8\sqrt{cx^4+bx^2}c^3}{b^3x^4}+\frac{6\sqrt{cx^4+bx^2}c^2}{b^2x^6}-\frac{5\sqrt{cx^4+bx^2}c}{bx^8}-\frac{35\sqrt{cx^4+bx^2}}{x^{10}}\right)\end{aligned}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="maxima")
```

output
```
-1/105*B*(8*sqrt(c*x^4 + b*x^2)*c^3/(b^3*x^2) - 4*sqrt(c*x^4 + b*x^2)*c^2/
(b^2*x^4) + 3*sqrt(c*x^4 + b*x^2)*c/(b*x^6) + 15*sqrt(c*x^4 + b*x^2)/x^8)
+ 1/315*A*(16*sqrt(c*x^4 + b*x^2)*c^4/(b^4*x^2) - 8*sqrt(c*x^4 + b*x^2)*c^
3/(b^3*x^4) + 6*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^6) - 5*sqrt(c*x^4 + b*x^2)*
c/(b*x^8) - 35*sqrt(c*x^4 + b*x^2)/x^10)
```

3.98.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(117) = 234$.

Time = 1.40 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.78

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx$$
$$= \frac{16\left(210\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Bc^{\frac{7}{2}}\mathrm{sgn}(x)-315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bbc^{\frac{7}{2}}\mathrm{sgn}(x)+630\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{1}\right.}{}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="giac")
```

output
```
16/315*(210*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*c^(7/2)*sgn(x) - 315*(sqrt(
c)*x - sqrt(c*x^2 + b))^10*B*b*c^(7/2)*sgn(x) + 630*(sqrt(c)*x - sqrt(c*x^
2 + b))^10*A*c^(9/2)*sgn(x) + 63*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^2*c^(
7/2)*sgn(x) + 378*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b*c^(9/2)*sgn(x) - 42*
(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^3*c^(7/2)*sgn(x) + 168*(sqrt(c)*x - sq
rt(c*x^2 + b))^6*A*b^2*c^(9/2)*sgn(x) + 108*(sqrt(c)*x - sqrt(c*x^2 + b))^
4*B*b^4*c^(7/2)*sgn(x) - 72*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^3*c^(9/2)*
sgn(x) - 27*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^5*c^(7/2)*sgn(x) + 18*(sqr
t(c)*x - sqrt(c*x^2 + b))^2*A*b^4*c^(9/2)*sgn(x) + 3*B*b^6*c^(7/2)*sgn(x)
- 2*A*b^5*c^(9/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^9
```

3.98.9 Mupad [B] (verification not implemented)

Time = 9.76 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.58

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx = \frac{2\,A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b^2\,x^6} - \frac{B\,\sqrt{c\,x^4+b\,x^2}}{7\,x^8} - \frac{A\,c\,\sqrt{c\,x^4+b\,x^2}}{63\,b\,x^8} - \frac{B\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,b\,x^6} - \frac{A\,\sqrt{c\,x^4+b\,x^2}}{9\,x^{10}} - \frac{8\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{315\,b^3\,x^4} + \frac{16\,A\,c^4\,\sqrt{c\,x^4+b\,x^2}}{315\,b^4\,x^2} + \frac{4\,B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b^2\,x^4} - \frac{8\,B\,c^3\,\sqrt{c\,x^4+b\,x^2}}{105\,b^3\,x^2}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^11,x)
```

output
```
(2*A*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^6) - (B*(b*x^2 + c*x^4)^(1/2))/
(7*x^8) - (A*c*(b*x^2 + c*x^4)^(1/2))/(63*b*x^8) - (B*c*(b*x^2 + c*x^4)^(1
/2))/(35*b*x^6) - (A*(b*x^2 + c*x^4)^(1/2))/(9*x^10) - (8*A*c^3*(b*x^2 + c
*x^4)^(1/2))/(315*b^3*x^4) + (16*A*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^4*x^2
) + (4*B*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^4) - (8*B*c^3*(b*x^2 + c*x^
4)^(1/2))/(105*b^3*x^2)
```

3.98. $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx$

3.99 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx$

3.99.1 Optimal result

Integrand size = 26, antiderivative size = 170

$$\begin{aligned}\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx =& -\frac{A(bx^2+cx^4)^{3/2}}{11bx^{14}}-\frac{(11bB-8Ac)\,(bx^2+cx^4)^{3/2}}{99b^2x^{12}}\\ &+\frac{2c(11bB-8Ac)\,(bx^2+cx^4)^{3/2}}{231b^3x^{10}}\\ &-\frac{8c^2(11bB-8Ac)\,(bx^2+cx^4)^{3/2}}{1155b^4x^8}\\ &+\frac{16c^3(11bB-8Ac)\,(bx^2+cx^4)^{3/2}}{3465b^5x^6}\end{aligned}$$

output
```
-1/11*A*(c*x^4+b*x^2)^(3/2)/b/x^14-1/99*(-8*A*c+11*B*b)*(c*x^4+b*x^2)^(3/2
)/b^2/x^12+2/231*c*(-8*A*c+11*B*b)*(c*x^4+b*x^2)^(3/2)/b^3/x^10-8/1155*c^2
*(-8*A*c+11*B*b)*(c*x^4+b*x^2)^(3/2)/b^4/x^8+16/3465*c^3*(-8*A*c+11*B*b)*(
c*x^4+b*x^2)^(3/2)/b^5/x^6
```

3.99.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.65

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx$$
$$= \frac{(x^2(b+cx^2))^{3/2}\,(11bBx^2(-35b^3+30b^2cx^2-24bc^2x^4+16c^3x^6)+A(-315b^4+280b^3cx^2-240b^2c^2x^4+1}{3465b^5x^{14}}$$

input
```
Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^13,x]
```

output
```
((x^2*(b + c*x^2))^(3/2)*(11*b*B*x^2*(-35*b^3 + 30*b^2*c*x^2 - 24*b*c^2*x^
4 + 16*c^3*x^6) + A*(-315*b^4 + 280*b^3*c*x^2 - 240*b^2*c^2*x^4 + 192*b*c^
3*x^6 - 128*c^4*x^8)))/(3465*b^5*x^14)
```

3.99.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1940, 1220, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx$$
$$\Big\downarrow \text{1940}$$
$$\frac{1}{2}\int \frac{(Bx^2+A)\sqrt{cx^4+bx^2}}{x^{14}}dx^2$$
$$\Big\downarrow \text{1220}$$
$$\frac{1}{2}\left(\frac{(11bB-8Ac)\int \frac{\sqrt{cx^4+bx^2}}{x^{12}}dx^2}{11b}-\frac{2A(bx^2+cx^4)^{3/2}}{11bx^{14}}\right)$$
$$\Big\downarrow \text{1129}$$
$$\frac{1}{2}\left(\frac{(11bB-8Ac)\left(-\frac{2c\int \frac{\sqrt{cx^4+bx^2}}{x^{10}}dx^2}{3b}-\frac{2(bx^2+cx^4)^{3/2}}{9bx^{12}}\right)}{11b}-\frac{2A(bx^2+cx^4)^{3/2}}{11bx^{14}}\right)$$

$$\Big\downarrow \text{1129}$$

$$\frac{1}{2}\left(\frac{(11bB-8Ac)\left(-\frac{2c\left(-\frac{4c\int \frac{\sqrt{cx^4+bx^2}}{x^8}dx^2}{7b}-\frac{2(bx^2+cx^4)^{3/2}}{7bx^{10}}\right)}{3b}-\frac{2(bx^2+cx^4)^{3/2}}{9bx^{12}}\right)}{11b}-\frac{2A(bx^2+cx^4)^{3/2}}{11bx^{14}}\right)$$

$$\Big\downarrow \text{1129}$$

$$\frac{1}{2}\left(\frac{(11bB-8Ac)\left(-\frac{2c\left(-\frac{4c\left(-\frac{2c\int\frac{\sqrt{cx^4+bx^2}}{x^6}dx^2}{5b}-\frac{2(bx^2+cx^4)^{3/2}}{5bx^8}\right)}{7b}-\frac{2(bx^2+cx^4)^{3/2}}{7bx^{10}}\right)}{3b}-\frac{2(bx^2+cx^4)^{3/2}}{9bx^{12}}\right)}{11b}-\frac{2A(bx^2+cx^4)^{3/2}}{11bx^{14}}\right.$$

$$\Big\downarrow \text{1123}$$

3.99. $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx$

$$\frac{1}{2}\left(\frac{\left(-\frac{2c\left(-\frac{4c\left(\frac{4c\left(bx^2+cx^4\right)^{3/2}}{15b^2x^6}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{5bx^8}\right)}{7b}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}\right)}{3b}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}}\right)(11bB-8Ac)}{11b}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}\right)$$

input
```
Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^13,x]
```

output
```
((-2*A*(b*x^2 + c*x^4)^(3/2))/(11*b*x^14) + ((11*b*B - 8*A*c)*((-2*(b*x^2
+ c*x^4)^(3/2))/(9*b*x^12) - (2*c*((-2*(b*x^2 + c*x^4)^(3/2))/(7*b*x^10) -
 (4*c*((-2*(b*x^2 + c*x^4)^(3/2))/(5*b*x^8) + (4*c*(b*x^2 + c*x^4)^(3/2))/
(15*b^2*x^6)))/(7*b)))/(3*b)))/(11*b))/2
```

3.99.3.1 Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.99.4 Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$-\frac{\left(\left(\frac{11 x^2 B}{9}+A\right) b^4-\frac{8 x^2 c \left(\frac{33 x^2 B}{28}+A\right) b^3}{9}+\frac{16 x^4 \left(\frac{11 x^2 B}{10}+A\right) c^2 b^2}{21}-\frac{64 \left(\frac{11 x^2 B}{12}+A\right) x^6 c^3 b}{105}+\frac{128 A x^8 c^4}{315}\right) \sqrt{x^2 (c x^2+b)}\, (c x^2+b)}{11 b^5 x^{12}}$
gosper	$-\frac{(c x^2+b) (128 A x^8 c^4-176 B x^8 b c^3-192 A x^6 b c^3+264 B x^6 b^2 c^2+240 A b^2 c^2 x^4-330 B b^3 c x^4-280 A x^2 b^3 c+385 B x^2 b^4+315}{3465 b^5 x^{12}}$
default	$-\frac{(c x^2+b) (128 A x^8 c^4-176 B x^8 b c^3-192 A x^6 b c^3+264 B x^6 b^2 c^2+240 A b^2 c^2 x^4-330 B b^3 c x^4-280 A x^2 b^3 c+385 B x^2 b^4+315}{3465 b^5 x^{12}}$
trager	$-\frac{(128 A x^{10} c^5-176 B b c^4 x^{10}-64 A x^8 b c^4+88 B b^2 c^3 x^8+48 A b^2 c^3 x^6-66 B b^3 c^2 x^6-40 A b^3 c^2 x^4+55 B x^4 b^4 c+35 A b^4 c x^2+38}{3465 b^5 x^{12}}$
risch	$-\frac{\sqrt{x^2 (c x^2+b)}\, (128 A x^{10} c^5-176 B b c^4 x^{10}-64 A x^8 b c^4+88 B b^2 c^3 x^8+48 A b^2 c^3 x^6-66 B b^3 c^2 x^6-40 A b^3 c^2 x^4+55 B x^4 b^4 c+3}{3465 x^{12} b^5}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/11*((11/9*x^2*B+A)*b^4-8/9*x^2*c*(33/28*x^2*B+A)*b^3+16/21*x^4*(11/10*x
^2*B+A)*c^2*b^2-64/105*(11/12*x^2*B+A)*x^6*c^3*b+128/315*A*x^8*c^4)*(x^2*(
c*x^2+b))^(1/2)*(c*x^2+b)/b^5/x^12
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx$$
$$= \frac{(16\,(11\,Bbc^4-8\,Ac^5)x^{10}-8\,(11\,Bb^2c^3-8\,Abc^4)x^8+6\,(11\,Bb^3c^2-8\,Ab^2c^3)x^6-315\,Ab^5-5\,(11\,Bb^4c-}{3465\,b^5x^{12}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="fricas")
```

output
```
1/3465*(16*(11*B*b*c^4 - 8*A*c^5)*x^10 - 8*(11*B*b^2*c^3 - 8*A*b*c^4)*x^8
+ 6*(11*B*b^3*c^2 - 8*A*b^2*c^3)*x^6 - 315*A*b^5 - 5*(11*B*b^4*c - 8*A*b^3
*c^2)*x^4 - 35*(11*B*b^5 + A*b^4*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^5*x^12)
```

3.99.6 Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x^{13}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**13,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**13, x)
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.51

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx$$
$$= \frac{1}{315}\,B\left(\frac{16\,\sqrt{cx^4+bx^2}c^4}{b^4x^2}-\frac{8\,\sqrt{cx^4+bx^2}c^3}{b^3x^4}+\frac{6\,\sqrt{cx^4+bx^2}c^2}{b^2x^6}-\frac{5\,\sqrt{cx^4+bx^2}c}{bx^8}-\frac{35\,\sqrt{cx^4+bx^2}}{x^{10}}\right)$$
$$-\frac{1}{3465}\,A\left(\frac{128\,\sqrt{cx^4+bx^2}c^5}{b^5x^2}-\frac{64\,\sqrt{cx^4+bx^2}c^4}{b^4x^4}+\frac{48\,\sqrt{cx^4+bx^2}c^3}{b^3x^6}-\frac{40\,\sqrt{cx^4+bx^2}c^2}{b^2x^8}+\frac{35\,\sqrt{cx^4+bx^2}}{bx^{10}}\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="maxima")
```

output
```
1/315*B*(16*sqrt(c*x^4 + b*x^2)*c^4/(b^4*x^2) - 8*sqrt(c*x^4 + b*x^2)*c^3/
(b^3*x^4) + 6*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^6) - 5*sqrt(c*x^4 + b*x^2)*c/
(b*x^8) - 35*sqrt(c*x^4 + b*x^2)/x^10) - 1/3465*A*(128*sqrt(c*x^4 + b*x^2)
*c^5/(b^5*x^2) - 64*sqrt(c*x^4 + b*x^2)*c^4/(b^4*x^4) + 48*sqrt(c*x^4 + b*
x^2)*c^3/(b^3*x^6) - 40*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^8) + 35*sqrt(c*x^4
+ b*x^2)*c/(b*x^10) + 315*sqrt(c*x^4 + b*x^2)/x^12)
```

3.99.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(150) = 300$.

Time = 1.85 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.53

$$\int \frac{(A + Bx^2)\sqrt{bx^2 + cx^4}}{x^{13}}\,dx$$
$$= \frac{32\left(3465\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^{14} Bc^{\frac{9}{2}}\operatorname{sgn}(x) - 4851\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^{12} Bbc^{\frac{9}{2}}\operatorname{sgn}(x) + 11088\left(\sqrt{c}x - \sqrt{cx^2\right.\right.}{}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="giac")
```

output
```
32/3465*(3465*(sqrt(c)*x - sqrt(c*x^2 + b))^14*B*c^(9/2)*sgn(x) - 4851*(sq
rt(c)*x - sqrt(c*x^2 + b))^12*B*b*c^(9/2)*sgn(x) + 11088*(sqrt(c)*x - sqrt
(c*x^2 + b))^12*A*c^(11/2)*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B
*b^2*c^(9/2)*sgn(x) + 7392*(sqrt(c)*x - sqrt(c*x^2 + b))^10*A*b*c^(11/2)*s
gn(x) - 165*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^3*c^(9/2)*sgn(x) + 2640*(s
qrt(c)*x - sqrt(c*x^2 + b))^8*A*b^2*c^(11/2)*sgn(x) + 1815*(sqrt(c)*x - sq
rt(c*x^2 + b))^6*B*b^4*c^(9/2)*sgn(x) - 1320*(sqrt(c)*x - sqrt(c*x^2 + b))
^6*A*b^3*c^(11/2)*sgn(x) - 605*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^5*c^(9/
2)*sgn(x) + 440*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^4*c^(11/2)*sgn(x) + 12
1*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^6*c^(9/2)*sgn(x) - 88*(sqrt(c)*x - s
qrt(c*x^2 + b))^2*A*b^5*c^(11/2)*sgn(x) - 11*B*b^7*c^(9/2)*sgn(x) + 8*A*b^
6*c^(11/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^11
```

3.99.9 Mupad [B] (verification not implemented)

Time = 10.22 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.53

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx = \frac{8\,A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{693\,b^2\,x^8} - \frac{B\,\sqrt{c\,x^4+b\,x^2}}{9\,x^{10}} - \frac{A\,c\,\sqrt{c\,x^4+b\,x^2}}{99\,b\,x^{10}} - \frac{B\,c\,\sqrt{c\,x^4+b\,x^2}}{63\,b\,x^8} - \frac{A\,\sqrt{c\,x^4+b\,x^2}}{11\,x^{12}} - \frac{16\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^3\,x^6} + \frac{64\,A\,c^4\,\sqrt{c\,x^4+b\,x^2}}{3465\,b^4\,x^4} - \frac{128\,A\,c^5\,\sqrt{c\,x^4+b\,x^2}}{3465\,b^5\,x^2} + \frac{2\,B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b^2\,x^6} - \frac{8\,B\,c^3\,\sqrt{c\,x^4+b\,x^2}}{315\,b^3\,x^4} + \frac{16\,B\,c^4\,\sqrt{c\,x^4+b\,x^2}}{315\,b^4\,x^2}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^13,x)
```

output
```
(8*A*c^2*(b*x^2 + c*x^4)^(1/2))/(693*b^2*x^8) - (B*(b*x^2 + c*x^4)^(1/2))/
(9*x^10) - (A*c*(b*x^2 + c*x^4)^(1/2))/(99*b*x^10) - (B*c*(b*x^2 + c*x^4)^
(1/2))/(63*b*x^8) - (A*(b*x^2 + c*x^4)^(1/2))/(11*x^12) - (16*A*c^3*(b*x^2
 + c*x^4)^(1/2))/(1155*b^3*x^6) + (64*A*c^4*(b*x^2 + c*x^4)^(1/2))/(3465*b
^4*x^4) - (128*A*c^5*(b*x^2 + c*x^4)^(1/2))/(3465*b^5*x^2) + (2*B*c^2*(b*x
^2 + c*x^4)^(1/2))/(105*b^2*x^6) - (8*B*c^3*(b*x^2 + c*x^4)^(1/2))/(315*b^
3*x^4) + (16*B*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^4*x^2)
```

3.100 $\int x^4(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

3.100.1 Optimal result

Integrand size = 26, antiderivative size = 131

$$\int x^4(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = -\frac{8b^2(2bB-3Ac)\,(bx^2+cx^4)^{3/2}}{315c^4x^3} + \frac{4b(2bB-3Ac)\,(bx^2+cx^4)^{3/2}}{105c^3x} - \frac{(2bB-3Ac)x(bx^2+cx^4)^{3/2}}{21c^2} + \frac{Bx^3(bx^2+cx^4)^{3/2}}{9c}$$

output
```
-8/315*b^2*(-3*A*c+2*B*b)*(c*x^4+b*x^2)^(3/2)/c^4/x^3+4/105*b*(-3*A*c+2*B*
b)*(c*x^4+b*x^2)^(3/2)/c^3/x-1/21*(-3*A*c+2*B*b)*x*(c*x^4+b*x^2)^(3/2)/c^2
+1/9*B*x^3*(c*x^4+b*x^2)^(3/2)/c
```

3.100.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.63

$$\int x^4(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{(x^2(b+cx^2))^{3/2}\,(-16b^3B+24b^2c(A+Bx^2)-6bc^2x^2(6A+5Bx^2)+5c^3x^4(9A+7Bx^2))}{315c^4x^3}$$

input
```
Integrate[x^4*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
((x^2*(b + c*x^2))^(3/2)*(-16*b^3*B + 24*b^2*c*(A + B*x^2) - 6*b*c^2*x^2*(
6*A + 5*B*x^2) + 5*c^3*x^4*(9*A + 7*B*x^2)))/(315*c^4*x^3)
```

3.100.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1945, 1421, 1421, 1398}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (A + Bx^2) \sqrt{bx^2 + cx^4} \, dx$$

$$\Big\downarrow \text{ 1945}$$

$$\frac{Bx^3 (bx^2 + cx^4)^{3/2}}{9c} - \frac{(2bB - 3Ac) \int x^4 \sqrt{cx^4 + bx^2} dx}{3c}$$

$$\Big\downarrow \text{ 1421}$$

$$\frac{Bx^3 (bx^2 + cx^4)^{3/2}}{9c} - \frac{(2bB - 3Ac) \left(\frac{x(bx^2+cx^4)^{3/2}}{7c} - \frac{4b \int x^2 \sqrt{cx^4+bx^2} dx}{7c}\right)}{3c}$$

$$\Big\downarrow \text{ 1421}$$

$$\frac{Bx^3 (bx^2 + cx^4)^{3/2}}{9c} - \frac{(2bB - 3Ac) \left(\frac{x(bx^2+cx^4)^{3/2}}{7c} - \frac{4b \left(\frac{(bx^2+cx^4)^{3/2}}{5cx} - \frac{2b \int \sqrt{cx^4+bx^2} dx}{5c}\right)}{7c}\right)}{3c}$$

$$\Big\downarrow \text{ 1398}$$

$$\frac{Bx^3 (bx^2 + cx^4)^{3/2}}{9c} - \frac{\left(\frac{x(bx^2+cx^4)^{3/2}}{7c} - \frac{4b \left(\frac{(bx^2+cx^4)^{3/2}}{5cx} - \frac{2b(bx^2+cx^4)^{3/2}}{15c^2x^3}\right)}{7c}\right)(2bB - 3Ac)}{3c}$$

input
```
Int[x^4*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(B*x^3*(b*x^2 + c*x^4)^(3/2))/(9*c) - ((2*b*B - 3*A*c)*((x*(b*x^2 + c*x^4)
^(3/2))/(7*c) - (4*b*((-2*b*(b*x^2 + c*x^4)^(3/2))/(15*c^2*x^3) + (b*x^2 +
 c*x^4)^(3/2)/(5*c*x)))/(7*c)))/(3*c)
```

3.100.3.1 Defintions of rubi rules used

rule 1398
```
Int[Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> Simp[(b*x^2 + c*x^4)^(3
/2)/(3*c*x^3), x] /; FreeQ[{b, c}, x]
```

rule 1421
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && IGtQ[Simplify[(m
 + 2*p - 1)/2], 0] && NeQ[m + 4*p + 1, 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.100.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.69

method	result	size
gosper	$\frac{\left(c\,x^{2}+b\right)\left(35B\,c^{3}x^{6}+45A\,c^{3}x^{4}-30Bb\,c^{2}x^{4}-36Ab\,c^{2}x^{2}+24B\,b^{2}c\,x^{2}+24b^{2}Ac-16B\,b^{3}\right)\sqrt{x^{4}c+b\,x^{2}}}{315c^{4}x}$	91
default	$\frac{\left(c\,x^{2}+b\right)\left(35B\,c^{3}x^{6}+45A\,c^{3}x^{4}-30Bb\,c^{2}x^{4}-36Ab\,c^{2}x^{2}+24B\,b^{2}c\,x^{2}+24b^{2}Ac-16B\,b^{3}\right)\sqrt{x^{4}c+b\,x^{2}}}{315c^{4}x}$	91
trager	$\frac{\left(35B\,x^{8}c^{4}+45A\,x^{6}c^{4}+5B\,x^{6}b\,c^{3}+9A\,x^{4}b\,c^{3}-6B\,x^{4}b^{2}c^{2}-12A\,x^{2}b^{2}c^{2}+8B\,x^{2}b^{3}c+24A\,b^{3}c-16B\,b^{4}\right)\sqrt{x^{4}c+b\,x^{2}}}{315c^{4}x}$	108
risch	$\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(35B\,x^{8}c^{4}+45A\,x^{6}c^{4}+5B\,x^{6}b\,c^{3}+9A\,x^{4}b\,c^{3}-6B\,x^{4}b^{2}c^{2}-12A\,x^{2}b^{2}c^{2}+8B\,x^{2}b^{3}c+24A\,b^{3}c-16B\,b^{4}\right)}{315x\,c^{4}}$	108

input
```
int(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*(c*x^2+b)*(35*B*c^3*x^6+45*A*c^3*x^4-30*B*b*c^2*x^4-36*A*b*c^2*x^2+2
4*B*b^2*c*x^2+24*A*b^2*c-16*B*b^3)*(c*x^4+b*x^2)^(1/2)/c^4/x
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.81

$$\int x^4(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{(35\,Bc^4x^8+5\,(Bbc^3+9\,Ac^4)x^6-16\,Bb^4+24\,Ab^3c-3\,(2\,Bb^2c^2-3\,Abc^3)x^4+4\,(2\,Bb^3c-3\,Ab^2c^2)x^2)\sqrt{}}{315\,c^4x}$$

input
```
integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/315*(35*B*c^4*x^8 + 5*(B*b*c^3 + 9*A*c^4)*x^6 - 16*B*b^4 + 24*A*b^3*c -
3*(2*B*b^2*c^2 - 3*A*b*c^3)*x^4 + 4*(2*B*b^3*c - 3*A*b^2*c^2)*x^2)*sqrt(c*
x^4 + b*x^2)/(c^4*x)
```

3.100.6 Sympy [F]

$$\int x^4(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \int x^4\sqrt{x^2\,(b+cx^2)}\,(A+Bx^2)\,\,dx$$

input
```
integrate(x**4*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**4*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.81

$$\int x^4(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{(15\,c^3x^6+3\,bc^2x^4-4\,b^2cx^2+8\,b^3)\sqrt{cx^2+b}A}{105\,c^3} + \frac{(35\,c^4x^8+5\,bc^3x^6-6\,b^2c^2x^4+8\,b^3cx^2-16\,b^4)\sqrt{cx^2+b}B}{315\,c^4}$$

input
```
integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/105*(15*c^3*x^6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^2 + b)*A/c
^3 + 1/315*(35*c^4*x^8 + 5*b*c^3*x^6 - 6*b^2*c^2*x^4 + 8*b^3*c*x^2 - 16*b^
4)*sqrt(c*x^2 + b)*B/c^4
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07

$$\int x^4(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{8\left(2\,Bb^{\frac{9}{2}}-3\,Ab^{\frac{7}{2}}c\right)\mathrm{sgn}(x)}{315\,c^4} + \frac{35\,(cx^2+b)^{\frac{9}{2}}B\mathrm{sgn}(x)-135\,(cx^2+b)^{\frac{7}{2}}Bb\mathrm{sgn}(x)+189\,(cx^2+b)^{\frac{5}{2}}Bb^2\mathrm{sgn}(x)-105\,(cx^2+b)^{\frac{3}{2}}Bb^3\mathrm{sgn}(x}{315\,c^4}$$

input
```
integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
8/315*(2*B*b^(9/2) - 3*A*b^(7/2)*c)*sgn(x)/c^4 + 1/315*(35*(c*x^2 + b)^(9/
2)*B*sgn(x) - 135*(c*x^2 + b)^(7/2)*B*b*sgn(x) + 189*(c*x^2 + b)^(5/2)*B*b
^2*sgn(x) - 105*(c*x^2 + b)^(3/2)*B*b^3*sgn(x) + 45*(c*x^2 + b)^(7/2)*A*c*
sgn(x) - 126*(c*x^2 + b)^(5/2)*A*b*c*sgn(x) + 105*(c*x^2 + b)^(3/2)*A*b^2*
c*sgn(x))/c^4
```

3.100.9 Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.79

$$\int x^4(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}\left(\frac{B\,x^8}{9}-\frac{16\,B\,b^4-24\,A\,b^3\,c}{315\,c^4}+\frac{x^6\,\left(45\,A\,c^4+5\,B\,b\,c^3\right)}{315\,c^4}-\frac{4\,b^2\,x^2\,\left(3\,A\,c-2\,B\,b\right)}{315\,c^3}+\frac{b\,x^4\,\left(3\,A\,c-2\,B\,b\right)}{105\,c^2}\right)}{x}$$

input
```
int(x^4*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

output
```
((b*x^2 + c*x^4)^(1/2)*((B*x^8)/9 - (16*B*b^4 - 24*A*b^3*c)/(315*c^4) + (x
^6*(45*A*c^4 + 5*B*b*c^3))/(315*c^4) - (4*b^2*x^2*(3*A*c - 2*B*b))/(315*c^
3) + (b*x^4*(3*A*c - 2*B*b))/(105*c^2)))/x
```

3.101 $\int x^2(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

3.101.1 Optimal result

Integrand size = 26, antiderivative size = 94

$$\int x^2(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{2b(4bB-7Ac)(bx^2+cx^4)^{3/2}}{105c^3x^3} - \frac{(4bB-7Ac)(bx^2+cx^4)^{3/2}}{35c^2x} + \frac{Bx(bx^2+cx^4)^{3/2}}{7c}$$

output
```
2/105*b*(-7*A*c+4*B*b)*(c*x^4+b*x^2)^(3/2)/c^3/x^3-1/35*(-7*A*c+4*B*b)*(c*
x^4+b*x^2)^(3/2)/c^2/x+1/7*B*x*(c*x^4+b*x^2)^(3/2)/c
```

3.101.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.68

$$\int x^2(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{(x^2(b+cx^2))^{3/2}(8b^2B+3c^2x^2(7A+5Bx^2)-2bc(7A+6Bx^2))}{105c^3x^3}$$

input
```
Integrate[x^2*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
((x^2*(b + c*x^2))^(3/2)*(8*b^2*B + 3*c^2*x^2*(7*A + 5*B*x^2) - 2*b*c*(7*A
 + 6*B*x^2)))/(105*c^3*x^3)
```

3.101.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1945, 1421, 1398}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (A+Bx^2) \sqrt{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{Bx(bx^2+cx^4)^{3/2}}{7c}-\frac{(4bB-7Ac)\int x^2\sqrt{cx^4+bx^2}dx}{7c}$$

$$\Big\downarrow \text{1421}$$

$$\frac{Bx(bx^2+cx^4)^{3/2}}{7c}-\frac{(4bB-7Ac)\left(\frac{(bx^2+cx^4)^{3/2}}{5cx}-\frac{2b\int\sqrt{cx^4+bx^2}dx}{5c}\right)}{7c}$$

$$\Big\downarrow \text{1398}$$

$$\frac{Bx(bx^2+cx^4)^{3/2}}{7c}-\frac{\left(\frac{(bx^2+cx^4)^{3/2}}{5cx}-\frac{2b(bx^2+cx^4)^{3/2}}{15c^2x^3}\right)(4bB-7Ac)}{7c}$$

input
```
Int[x^2*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(B*x*(b*x^2 + c*x^4)^(3/2))/(7*c) - ((4*b*B - 7*A*c)*((-2*b*(b*x^2 + c*x^4
)^(3/2))/(15*c^2*x^3) + (b*x^2 + c*x^4)^(3/2)/(5*c*x)))/(7*c)
```

3.101.3.1 Defintions of rubi rules used

rule 1398
```
Int[Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> Simp[(b*x^2 + c*x^4)^(3
/2)/(3*c*x^3), x] /; FreeQ[{b, c}, x]
```

rule 1421
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && IGtQ[Simplify[(m
 + 2*p - 1)/2], 0] && NeQ[m + 4*p + 1, 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.101.4 Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-15 B\,c^{2} x^{4}-21 A\,c^{2} x^{2}+12 B b c\,x^{2}+14 A b c -8 B\,b^{2}\right)\sqrt{x^{4} c +b\,x^{2}}}{105 c^{3} x}$	67
default	$-\frac{\left(c\,x^{2}+b\right)\left(-15 B\,c^{2} x^{4}-21 A\,c^{2} x^{2}+12 B b c\,x^{2}+14 A b c -8 B\,b^{2}\right)\sqrt{x^{4} c +b\,x^{2}}}{105 c^{3} x}$	67
trager	$-\frac{\left(-15 B\,c^{3} x^{6}-21 A\,c^{3} x^{4}-3 B b\,c^{2} x^{4}-7 A b\,c^{2} x^{2}+4 B\,b^{2} c\,x^{2}+14 b^{2} A c -8 B\,b^{3}\right)\sqrt{x^{4} c +b\,x^{2}}}{105 c^{3} x}$	84
risch	$-\frac{\sqrt{x^{2} \left(c\,x^{2}+b\right)}\,\left(-15 B\,c^{3} x^{6}-21 A\,c^{3} x^{4}-3 B b\,c^{2} x^{4}-7 A b\,c^{2} x^{2}+4 B\,b^{2} c\,x^{2}+14 b^{2} A c -8 B\,b^{3}\right)}{105 x\,c^{3}}$	84

input
```
int(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105*(c*x^2+b)*(-15*B*c^2*x^4-21*A*c^2*x^2+12*B*b*c*x^2+14*A*b*c-8*B*b^2
)*(c*x^4+b*x^2)^(1/2)/c^3/x
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\int x^2(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{(15\,Bc^3x^6+3\,(Bbc^2+7\,Ac^3)x^4+8\,Bb^3-14\,Ab^2c-(4\,Bb^2c-7\,Abc^2)x^2)\sqrt{cx^4+bx^2}}{105\,c^3x}$$

input
```
integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(15*B*c^3*x^6 + 3*(B*b*c^2 + 7*A*c^3)*x^4 + 8*B*b^3 - 14*A*b^2*c - (
4*B*b^2*c - 7*A*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^3*x)
```

3.101.6 Sympy [F]

$$\int x^2(A+Bx^2)\sqrt{bx^2+cx^4}\,dx=\int x^2\sqrt{x^2\,(b+cx^2)}(A+Bx^2)\,dx$$

input
```
integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**2*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int x^2(A+Bx^2)\sqrt{bx^2+cx^4}\,dx=\frac{(3\,c^2x^4+bcx^2-2\,b^2)\sqrt{cx^2+b}A}{15\,c^2}+\frac{(15\,c^3x^6+3\,bc^2x^4-4\,b^2cx^2+8\,b^3)\sqrt{cx^2+b}B}{105\,c^3}$$

input
```
integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^2 + b)*A/c^2 + 1/105*(15*c^3*x
^6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^2 + b)*B/c^3
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12

$$\int x^2(A+Bx^2)\sqrt{bx^2+cx^4}\,dx=-\frac{2\left(4\,Bb^{\frac{7}{2}}-7\,Ab^{\frac{5}{2}}c\right)\text{sgn}(x)}{105\,c^3}+\frac{15\,(cx^2+b)^{\frac{7}{2}}B\text{sgn}(x)-42\,(cx^2+b)^{\frac{5}{2}}Bb\text{sgn}(x)+35\,(cx^2+b)^{\frac{3}{2}}Bb^2\text{sgn}(x)+21\,(cx^2+b)^{\frac{5}{2}}Ac\text{sgn}(x)-3}{105\,c^3}$$

input
```
integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2/105*(4*B*b^(7/2) - 7*A*b^(5/2)*c)*sgn(x)/c^3 + 1/105*(15*(c*x^2 + b)^(7
/2)*B*sgn(x) - 42*(c*x^2 + b)^(5/2)*B*b*sgn(x) + 35*(c*x^2 + b)^(3/2)*B*b^
2*sgn(x) + 21*(c*x^2 + b)^(5/2)*A*c*sgn(x) - 35*(c*x^2 + b)^(3/2)*A*b*c*sg
n(x))/c^3
```

3.101.9 Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int x^2(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$
$$=\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{B\,x^6}{7}+\frac{8\,B\,b^3-14\,A\,b^2\,c}{105\,c^3}+\frac{x^4\,\left(21\,A\,c^3+3\,B\,b\,c^2\right)}{105\,c^3}+\frac{b\,x^2\,\left(7\,A\,c-4\,B\,b\right)}{105\,c^2}\right)}{x}$$

input
```
int(x^2*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

output
```
((b*x^2 + c*x^4)^(1/2)*((B*x^6)/7 + (8*B*b^3 - 14*A*b^2*c)/(105*c^3) + (x^
4*(21*A*c^3 + 3*B*b*c^2))/(105*c^3) + (b*x^2*(7*A*c - 4*B*b))/(105*c^2)))/
x
```

3.102 $\int (A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

3.102.1 Optimal result

Integrand size = 23, antiderivative size = 61

$$\int (A+Bx^2)\sqrt{bx^2+cx^4}\,dx=-\frac{(2bB-5Ac)\left(bx^2+cx^4\right)^{3/2}}{15c^2x^3}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{5cx}$$

output
```
-1/15*(-5*A*c+2*B*b)*(c*x^4+b*x^2)^(3/2)/c^2/x^3+1/5*B*(c*x^4+b*x^2)^(3/2)
/c/x
```

3.102.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67

$$\int (A+Bx^2)\sqrt{bx^2+cx^4}\,dx=\frac{\left(x^2(b+cx^2)\right)^{3/2}(-2bB+5Ac+3Bcx^2)}{15c^2x^3}$$

input
```
Integrate[(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
((x^2*(b + c*x^2))^(3/2)*(-2*b*B + 5*A*c + 3*B*c*x^2))/(15*c^2*x^3)
```

3.102.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1465, 1398}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A + Bx^2)\sqrt{bx^2 + cx^4}\,dx$$

$$\Big\downarrow \text{1465}$$

$$\frac{B(bx^2+cx^4)^{3/2}}{5cx} - \frac{(2bB-5Ac)\int \sqrt{cx^4+bx^2}dx}{5c}$$

$$\Big\downarrow \text{1398}$$

$$\frac{B(bx^2+cx^4)^{3/2}}{5cx} - \frac{(bx^2+cx^4)^{3/2}(2bB-5Ac)}{15c^2x^3}$$

input
```
Int[(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
-1/15*((2*b*B - 5*A*c)*(b*x^2 + c*x^4)^(3/2))/(c^2*x^3) + (B*(b*x^2 + c*x^
4)^(3/2))/(5*c*x)
```

3.102.3.1 Defintions of rubi rules used

rule 1398
```
Int[Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> Simp[(b*x^2 + c*x^4)^(3
/2)/(3*c*x^3), x] /; FreeQ[{b, c}, x]
```

rule 1465
```
Int[((d_) + (e_.)*(x_)^2)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
Simp[e*((b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 3)*x)), x] - Simp[(b*e*(2*p + 1)
- c*d*(4*p + 3))/(c*(4*p + 3))   Int[(b*x^2 + c*x^4)^p, x], x] /; FreeQ[{b,
 c, d, e, p}, x] &&  !IntegerQ[p] && NeQ[4*p + 3, 0] && NeQ[b*e*(2*p + 1) -
 c*d*(4*p + 3), 0]
```

3.102.4 Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

method	result	size
gosper	$\frac{(c\,x^2+b)\,(3Bc\,x^2+5Ac-2Bb)\sqrt{x^4c+b\,x^2}}{15c^2x}$	45
default	$\frac{(c\,x^2+b)\,(3Bc\,x^2+5Ac-2Bb)\sqrt{x^4c+b\,x^2}}{15c^2x}$	45
trager	$\frac{(3B\,c^2x^4+5A\,c^2x^2+Bbc\,x^2+5Abc-2B\,b^2)\sqrt{x^4c+b\,x^2}}{15c^2x}$	59
risch	$\frac{\sqrt{x^2(c\,x^2+b)}\,(3B\,c^2x^4+5A\,c^2x^2+Bbc\,x^2+5Abc-2B\,b^2)}{15x\,c^2}$	59

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(c*x^2+b)*(3*B*c*x^2+5*A*c-2*B*b)*(c*x^4+b*x^2)^(1/2)/c^2/x
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int (A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{(3\,Bc^2x^4-2\,Bb^2+5\,Abc+(Bbc+5\,Ac^2)x^2)\sqrt{cx^4+bx^2}}{15\,c^2x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*B*c^2*x^4 - 2*B*b^2 + 5*A*b*c + (B*b*c + 5*A*c^2)*x^2)*sqrt(c*x^4
+ b*x^2)/(c^2*x)
```

3.102.6 Sympy [F]

$$\int (A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \int \sqrt{x^2\,(b+cx^2)}(A+Bx^2)\ dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int (A + Bx^2)\sqrt{bx^2 + cx^4}\,dx = \frac{(cx^2 + b)^{\frac{3}{2}}A}{3\,c} + \frac{(3\,c^2x^4 + bcx^2 - 2\,b^2)\sqrt{cx^2 + b}B}{15\,c^2}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(c*x^2 + b)^(3/2)*A/c + 1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^2
+ b)*B/c^2
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int (A + Bx^2)\sqrt{bx^2 + cx^4}\,dx$$
$$= \frac{\left(2\,Bb^{\frac{5}{2}} - 5\,Ab^{\frac{3}{2}}c\right)\mathrm{sgn}(x)}{15\,c^2}$$
$$+ \frac{3\,(cx^2 + b)^{\frac{5}{2}}B\mathrm{sgn}(x) - 5\,(cx^2 + b)^{\frac{3}{2}}Bb\mathrm{sgn}(x) + 5\,(cx^2 + b)^{\frac{3}{2}}Ac\mathrm{sgn}(x)}{15\,c^2}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/15*(2*B*b^(5/2) - 5*A*b^(3/2)*c)*sgn(x)/c^2 + 1/15*(3*(c*x^2 + b)^(5/2)*
B*sgn(x) - 5*(c*x^2 + b)^(3/2)*B*b*sgn(x) + 5*(c*x^2 + b)^(3/2)*A*c*sgn(x)
)/c^2
```

3.102.9 Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int (A + Bx^2)\sqrt{bx^2 + cx^4}\,dx = \frac{\sqrt{c\,x^4 + b\,x^2}\,\left(\frac{B\,x^4}{5} - \frac{2\,B\,b^2 - 5\,A\,b\,c}{15\,c^2} + \frac{x^2\,\left(5\,A\,c^2 + B\,b\,c\right)}{15\,c^2}\right)}{x}$$

input
```
int((A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

output
```
((b*x^2 + c*x^4)^(1/2)*((B*x^4)/5 - (2*B*b^2 - 5*A*b*c)/(15*c^2) + (x^2*(5
*A*c^2 + B*b*c))/(15*c^2)))/x
```

3.102. $\int (A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

3.103 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2} dx$

3.103.1 Optimal result

Integrand size = 26, antiderivative size = 78

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2} dx = \frac{A\sqrt{bx^2+cx^4}}{x} + \frac{B(bx^2+cx^4)^{3/2}}{3cx^3} - A\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)$$

output
```
1/3*B*(c*x^4+b*x^2)^(3/2)/c/x^3-A*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))*b
^(1/2)+A*(c*x^4+b*x^2)^(1/2)/x
```

3.103.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2} dx = \frac{x\left((b+cx^2)(bB+3Ac+Bcx^2) - 3A\sqrt{b}c\sqrt{b+cx^2}\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{3c\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^2,x]
```

output
```
(x*((b + c*x^2)*(b*B + 3*A*c + B*c*x^2) - 3*A*Sqrt[b]*c*Sqrt[b + c*x^2]*Ar
cTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(3*c*Sqrt[x^2*(b + c*x^2)])
```

3.103.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1945, 1426, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2}dx$$

$$\Big\downarrow \text{1945}$$

$$A\int \frac{\sqrt{cx^4+bx^2}}{x^2}dx+\frac{B(bx^2+cx^4)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{1426}$$

$$A\left(b\int \frac{1}{\sqrt{cx^4+bx^2}}dx+\frac{\sqrt{bx^2+cx^4}}{x}\right)+\frac{B(bx^2+cx^4)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{1400}$$

$$A\left(\frac{\sqrt{bx^2+cx^4}}{x}-b\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}\right)+\frac{B(bx^2+cx^4)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{219}$$

$$A\left(\frac{\sqrt{bx^2+cx^4}}{x}-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)\right)+\frac{B(bx^2+cx^4)^{3/2}}{3cx^3}$$

input
```
Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^2,x]
```

output
```
(B*(b*x^2 + c*x^4)^(3/2))/(3*c*x^3) + A*(Sqrt[b*x^2 + c*x^4]/x - Sqrt[b]*A
rcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])
```

3.103.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.103.4 Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\sqrt{x^{4} c+b \,x^{2}}\, \left(-3 A \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c \,x^{2}+b}}{x}\right) \sqrt{b}\, c +B \left(c \,x^{2}+b \right)^{\frac{3}{2}}+3 A \sqrt{c \,x^{2}+b}\, c \right)}{3 x \sqrt{c \,x^{2}+b}\, c}$	84

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(c*x^4+b*x^2)^(1/2)*(-3*A*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b^(1/2)*
c+B*(c*x^2+b)^(3/2)+3*A*(c*x^2+b)^(1/2)*c)/x/(c*x^2+b)^(1/2)/c
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.04

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2}\,dx$$
$$=\left[\frac{3\,A\sqrt{b}cx\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}(Bcx^2+Bb+3\,Ac)}{6\,cx},\ \frac{3\,A\sqrt{-b}cx\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{}}{cx^3+bx}\right.}{}\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/6*(3*A*sqrt(b)*c*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))
/x^3) + 2*sqrt(c*x^4 + b*x^2)*(B*c*x^2 + B*b + 3*A*c))/(c*x), 1/3*(3*A*sqr
t(-b)*c*x*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4
+ b*x^2)*(B*c*x^2 + B*b + 3*A*c))/(c*x)]
```

3.103.6 Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2}\,dx=\int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x^2}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**2, x)
```

3.103.7 Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2}\,dx=\int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^2}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
A*integrate(sqrt(c*x^2 + b)/x, x) + 1/3*(c*x^2 + b)^(3/2)*B/c
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2}\,dx = \frac{Ab\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}} - \frac{\left(3\,Abc\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+B\sqrt{-b}b^{\frac{3}{2}}+3\,A\sqrt{-b}\sqrt{b}c\right)\mathrm{sgn}(x)}{3\sqrt{-b}c} + \frac{(cx^2+b)^{\frac{3}{2}}Bc^2\mathrm{sgn}(x)+3\sqrt{cx^2+b}Ac^3\mathrm{sgn}(x)}{3\,c^3}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
A*b*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - 1/3*(3*A*b*c*arctan
(sqrt(b)/sqrt(-b)) + B*sqrt(-b)*b^(3/2) + 3*A*sqrt(-b)*sqrt(b)*c)*sgn(x)/(
sqrt(-b)*c) + 1/3*((c*x^2 + b)^(3/2)*B*c^2*sgn(x) + 3*sqrt(c*x^2 + b)*A*c^
3*sgn(x))/c^3
```

3.103.9 Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2}\,dx = \frac{A\sqrt{c\,x^4+b\,x^2}}{x} + \frac{B\,(c\,x^2+b)\,\sqrt{c\,x^4+b\,x^2}}{3\,c\,x} + \frac{A\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,1\mathrm{i}}{\sqrt{c}\,x}\right)\sqrt{c\,x^4+b\,x^2}\,1\mathrm{i}}{\sqrt{c}\,x^2\sqrt{\frac{b}{c\,x^2}+1}}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^2,x)
```

output
```
(A*(b*x^2 + c*x^4)^(1/2))/x + (B*(b + c*x^2)*(b*x^2 + c*x^4)^(1/2))/(3*c*x
) + (A*b^(1/2)*asin((b^(1/2)*1i)/(c^(1/2)*x))*(b*x^2 + c*x^4)^(1/2)*1i)/(c
^(1/2)*x^2*(b/(c*x^2) + 1)^(1/2))
```

3.104 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx$

3.104.1 Optimal result

Integrand size = 26, antiderivative size = 100

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx = \frac{(2bB+Ac)\sqrt{bx^2+cx^4}}{2bx} - \frac{A(bx^2+cx^4)^{3/2}}{2bx^5} - \frac{(2bB+Ac)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}$$

output
```
-1/2*A*(c*x^4+b*x^2)^(3/2)/b/x^5-1/2*(A*c+2*B*b)*arctanh(x*b^(1/2)/(c*x^4+
b*x^2)^(1/2))/b^(1/2)+1/2*(A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/b/x
```

3.104.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx = -\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b}(A-2Bx^2)\sqrt{b+cx^2}+(2bB+Ac)x^2\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{2\sqrt{b}x^3\sqrt{b+cx^2}}$$

input
```
Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^4,x]
```

output
```
-1/2*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*(A - 2*B*x^2)*Sqrt[b + c*x^2] + (2*b*
B + A*c)*x^2*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(Sqrt[b]*x^3*Sqrt[b + c*x^
2])
```

3.104.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1944, 1426, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(Ac+2bB)\int \frac{\sqrt{cx^4+bx^2}}{x^2}dx}{2b}-\frac{A(bx^2+cx^4)^{3/2}}{2bx^5}$$

$$\Big\downarrow \text{1426}$$

$$\frac{(Ac+2bB)\left(b\int \frac{1}{\sqrt{cx^4+bx^2}}dx+\frac{\sqrt{bx^2+cx^4}}{x}\right)}{2b}-\frac{A(bx^2+cx^4)^{3/2}}{2bx^5}$$

$$\Big\downarrow \text{1400}$$

$$\frac{(Ac+2bB)\left(\frac{\sqrt{bx^2+cx^4}}{x}-b\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}\right)}{2b}-\frac{A(bx^2+cx^4)^{3/2}}{2bx^5}$$

$$\Big\downarrow \text{219}$$

$$\frac{(Ac+2bB)\left(\frac{\sqrt{bx^2+cx^4}}{x}-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)\right)}{2b}-\frac{A(bx^2+cx^4)^{3/2}}{2bx^5}$$

input
```
Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^4,x]
```

output
```
-1/2*(A*(b*x^2 + c*x^4)^(3/2))/(b*x^5) + ((2*b*B + A*c)*(Sqrt[b*x^2 + c*x^
4]/x - Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]))/(2*b)
```

3.104.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.104.4 Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{A\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{2x^{3}}+\frac{\left(B\sqrt{c\,x^{2}+b}-\frac{\left(Ac+2Bb\right)\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)}{2\sqrt{b}}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{x\sqrt{c\,x^{2}+b}}$
default	$-\frac{\sqrt{x^{4}c+b\,x^{2}}\,\left(A\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)c\,x^{2}+2B\,b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)x^{2}-A\sqrt{c\,x^{2}+b}\,c\,x^{2}-2B\sqrt{c\,x^{2}+b}\,b\,x^{2}+A\left(c\,x^{2}+b\right)^{\frac{3}{2}}\right)}{2x^{3}\sqrt{c\,x^{2}+b}\,b}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/x^3*(x^2*(c*x^2+b))^(1/2)+(B*(c*x^2+b)^(1/2)-1/2*(A*c+2*B*b)/b^(1/2
)*ln((2*b+2*b^(1/2)*(c*x^2+b)^(1/2))/x))*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)
^(1/2)
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.69

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx$$
$$= \left[\frac{(2\,Bb+Ac)\sqrt{b}x^3\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}(2\,Bbx^2-Ab)}{4\,bx^3}, \frac{(2\,Bb+Ac)\sqrt{-b}x^3\arctan\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/4*((2*B*b + A*c)*sqrt(b)*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2
)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*B*b*x^2 - A*b))/(b*x^3), 1/2*((
2*B*b + A*c)*sqrt(-b)*x^3*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x
)) + sqrt(c*x^4 + b*x^2)*(2*B*b*x^2 - A*b))/(b*x^3)]
```

3.104.6 Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x^4}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**4, x)
```

3.104.7 Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^4}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^4, x)
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx = \frac{2\sqrt{cx^2+b}Bc\,\mathrm{sgn}(x) + \frac{(2\,Bbc\,\mathrm{sgn}(x)+Ac^2\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{\sqrt{cx^2+b}Ac\,\mathrm{sgn}(x)}{x^2}}{2\,c}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
1/2*(2*sqrt(c*x^2 + b)*B*c*sgn(x) + (2*B*b*c*sgn(x) + A*c^2*sgn(x))*arctan
(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) - sqrt(c*x^2 + b)*A*c*sgn(x)/x^2)/c
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^4}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^4,x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^4, x)
```

3.105 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx$

3.105.1 Optimal result

Integrand size = 26, antiderivative size = 103

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx = -\frac{(4bB-Ac)\sqrt{bx^2+cx^4}}{8bx^3}-\frac{A(bx^2+cx^4)^{3/2}}{4bx^7} - \frac{c(4bB-Ac)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}$$

output
```
-1/4*A*(c*x^4+b*x^2)^(3/2)/b/x^7-1/8*c*(-A*c+4*B*b)*arctanh(x*b^(1/2)/(c*x
^4+b*x^2)^(1/2))/b^(3/2)-1/8*(-A*c+4*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^3
```

3.105.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx = -\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b}\sqrt{b+cx^2}(2Ab+4bBx^2+Acx^2)+c(4bB-Ac)x^4\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{8b^{3/2}x^5\sqrt{b+cx^2}}$$

input
```
Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^6,x]
```

output
```
-1/8*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(2*A*b + 4*b*B*x^2 +
A*c*x^2) + c*(4*b*B - A*c)*x^4*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(b^(3/2)
*x^5*Sqrt[b + c*x^2])
```

3.105.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1944, 1425, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(4bB-Ac)\int \frac{\sqrt{cx^4+bx^2}}{x^4}dx}{4b}-\frac{A(bx^2+cx^4)^{3/2}}{4bx^7}$$

$$\Big\downarrow \text{1425}$$

$$\frac{(4bB-Ac)\left(\frac{1}{2}c\int \frac{1}{\sqrt{cx^4+bx^2}}dx-\frac{\sqrt{bx^2+cx^4}}{2x^3}\right)}{4b}-\frac{A(bx^2+cx^4)^{3/2}}{4bx^7}$$

$$\Big\downarrow \text{1400}$$

$$\frac{(4bB-Ac)\left(-\frac{1}{2}c\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}-\frac{\sqrt{bx^2+cx^4}}{2x^3}\right)}{4b}-\frac{A(bx^2+cx^4)^{3/2}}{4bx^7}$$

$$\Big\downarrow \text{219}$$

$$\frac{(4bB-Ac)\left(-\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}-\frac{\sqrt{bx^2+cx^4}}{2x^3}\right)}{4b}-\frac{A(bx^2+cx^4)^{3/2}}{4bx^7}$$

input
```
Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^6,x]
```

output
```
-1/4*(A*(b*x^2 + c*x^4)^(3/2))/(b*x^7) + ((4*b*B - A*c)*(-1/2*Sqrt[b*x^2 +
 c*x^4]/x^3 - (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*Sqrt[b])))/(
4*b)
```

3.105.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.105.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{\left(A c \,x^{2}+4 b B \,x^{2}+2 A b \right) \sqrt{x^{2} \left(c \,x^{2}+b \right)}}{8 x^{5} b}+\frac{\left(A c -4 B b \right) c \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c \,x^{2}+b}}{x}\right) \sqrt{x^{2} \left(c \,x^{2}+b \right)}}{8 b^{\frac{3}{2}} x \sqrt{c \,x^{2}+b}}$
default	$-\frac{\sqrt{x^{4} c +b \,x^{2}}\, \left(-A \sqrt{b}\, \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c \,x^{2}+b}}{x}\right) c^{2} x^{4}+4 B \,b^{\frac{3}{2}} \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c \,x^{2}+b}}{x}\right) c \,x^{4}+A \sqrt{c \,x^{2}+b}\, c^{2} x^{4}-4 B \sqrt{c \,x^{2}+b}\, b c \,x^{4}-A \left(c \,x^{2}+b \right) \cdots\right.}{8 x^{5} \sqrt{c \,x^{2}+b}\, b^{2}}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(A*c*x^2+4*B*b*x^2+2*A*b)/x^5/b*(x^2*(c*x^2+b))^(1/2)+1/8*(A*c-4*B*b)
*c/b^(3/2)*ln((2*b+2*b^(1/2)*(c*x^2+b)^(1/2))/x)*(x^2*(c*x^2+b))^(1/2)/x/(
c*x^2+b)^(1/2)
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.92

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx$$
$$= \left[-\frac{(4\,Bbc-Ac^2)\sqrt{b}x^5\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}(2\,Ab^2+(4\,Bb^2+Abc)x^2)}{16\,b^2x^5}, \frac{(4\,Bbc-A}{}\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
[-1/16*((4*B*b*c - A*c^2)*sqrt(b)*x^5*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 +
 b*x^2)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*A*b^2 + (4*B*b^2 + A*b*c)
*x^2))/(b^2*x^5), 1/8*((4*B*b*c - A*c^2)*sqrt(-b)*x^5*arctan(sqrt(c*x^4 +
b*x^2)*sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^4 + b*x^2)*(2*A*b^2 + (4*B*b^2 +
 A*b*c)*x^2))/(b^2*x^5)]
```

3.105.6 Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x^6}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**6,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**6, x)
```

3.105.7 Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^6}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^6, x)
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.28

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx = \frac{\frac{(4\,Bbc^2\mathrm{sgn}(x)-Ac^3\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b} - \frac{4\,(cx^2+b)^{\frac{3}{2}}Bbc^2\mathrm{sgn}(x)-4\,\sqrt{cx^2+b}Bb^2c^2\mathrm{sgn}(x)+(cx^2+b)^{\frac{3}{2}}Ac^3\mathrm{sgn}(x)+\sqrt{cx^2+b}Abc^3\mathrm{sgn}(x)}{bc^2x^4}}{8\,c}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="giac")
```

output
```
1/8*((4*B*b*c^2*sgn(x) - A*c^3*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(s
qrt(-b)*b) - (4*(c*x^2 + b)^(3/2)*B*b*c^2*sgn(x) - 4*sqrt(c*x^2 + b)*B*b^2
*c^2*sgn(x) + (c*x^2 + b)^(3/2)*A*c^3*sgn(x) + sqrt(c*x^2 + b)*A*b*c^3*sgn
(x))/(b*c^2*x^4))/c
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^6}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^6,x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^6, x)
```

3.106 $\int x^5(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

3.106.1 Optimal result

Integrand size = 26, antiderivative size = 223

$$\int x^5(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{b^4(9bB-14Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{2048c^5}-\frac{b^2(9bB-14Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{768c^4}+\frac{b(9bB-14Ac)\left(bx^2+cx^4\right)^{5/2}}{240c^3}-\frac{(9bB-14Ac)x^2\left(bx^2+cx^4\right)^{5/2}}{168c^2}+\frac{Bx^4\left(bx^2+cx^4\right)^{5/2}}{14c}-\frac{b^6(9bB-14Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2048c^{11/2}}$$

output
```
-1/768*b^2*(-14*A*c+9*B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(3/2)/c^4+1/240*b*(-1
4*A*c+9*B*b)*(c*x^4+b*x^2)^(5/2)/c^3-1/168*(-14*A*c+9*B*b)*x^2*(c*x^4+b*x^
2)^(5/2)/c^2+1/14*B*x^4*(c*x^4+b*x^2)^(5/2)/c-1/2048*b^6*(-14*A*c+9*B*b)*a
rctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(11/2)+1/2048*b^4*(-14*A*c+9*B*b
)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^5
```

3.106.2 Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.11

$$\int x^5 \left(A+Bx^2\right) \left(bx^2+cx^4\right)^{3/2} dx = \frac{\left(x^2(b+cx^2)\right)^{3/2} \left(945b^6B - 1470Ab^5c - 630b^5Bcx^2 + 980Ab^4c^2x^2 + 504b^4Bc^2x^4 - 784Ab^3c^3x \cdots\right)}{21\cdots} - \frac{b^6(9bB-14Ac)\left(x^2(b+cx^2)\right)^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{b}+\sqrt{b+cx^2}}\right)}{1024c^{11/2}x^3\left(b+cx^2\right)^{3/2}}$$

input
```
Integrate[x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
((x^2*(b + c*x^2))^(3/2)*(945*b^6*B - 1470*A*b^5*c - 630*b^5*B*c*x^2 + 980
*A*b^4*c^2*x^2 + 504*b^4*B*c^2*x^4 - 784*A*b^3*c^3*x^4 - 432*b^3*B*c^3*x^6
 + 672*A*b^2*c^4*x^6 + 384*b^2*B*c^4*x^8 + 23296*A*b*c^5*x^8 + 19200*b*B*c
^5*x^10 + 17920*A*c^6*x^10 + 15360*B*c^6*x^12))/(215040*c^5*x^2*(b + c*x^2
)) - (b^6*(9*b*B - 14*A*c)*(x^2*(b + c*x^2))^(3/2)*ArcTanh[(Sqrt[c]*x)/(-S
qrt[b] + Sqrt[b + c*x^2])])/(1024*c^(11/2)*x^3*(b + c*x^2)^(3/2))
```

3.106.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1940, 1221, 1134, 1160, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(A+Bx^2\right) \left(bx^2+cx^4\right)^{3/2} dx$$

$$\Big\downarrow \text{ 1940}$$

$$\frac{1}{2}\int x^4\left(Bx^2+A\right)\left(cx^4+bx^2\right)^{3/2}dx^2$$

$$\Big\downarrow \text{ 1221}$$

$$\frac{1}{2}\left(\frac{Bx^4\left(bx^2+cx^4\right)^{5/2}}{7c}-\frac{(9bB-14Ac)\int x^4\left(cx^4+bx^2\right)^{3/2}dx^2}{14c}\right)$$

$$\downarrow 1134$$

$$\frac{1}{2}\left(\frac{Bx^4\left(bx^2+cx^4\right)^{5/2}}{7c}-\frac{(9bB-14Ac)\left(\frac{x^2\left(bx^2+cx^4\right)^{5/2}}{6c}-\frac{7b\int x^2\left(cx^4+bx^2\right)^{3/2}dx^2}{12c}\right)}{14c}\right)$$

$$\downarrow 1160$$

$$\frac{1}{2}\left(\frac{Bx^4\left(bx^2+cx^4\right)^{5/2}}{7c}-\frac{(9bB-14Ac)\left(\frac{x^2\left(bx^2+cx^4\right)^{5/2}}{6c}-\frac{7b\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{5c}-\frac{b\int\left(cx^4+bx^2\right)^{3/2}dx^2}{2c}\right)}{12c}\right)}{14c}\right)$$

$$\downarrow 1087$$

$$\frac{1}{2}\left(\frac{Bx^4\left(bx^2+cx^4\right)^{5/2}}{7c}-\frac{(9bB-14Ac)\left(\frac{x^2\left(bx^2+cx^4\right)^{5/2}}{6c}-\frac{7b\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{5c}-\frac{b\left(\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{8c}-\frac{3b^2\int\sqrt{cx^4+bx^2}dx^2}{16c}\right)}{2c}\right)}{12c}\right)}{14c}\right)$$

$$\downarrow 1087$$

3.106. $\int x^5\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx$

$$\frac{1}{2}\left(\frac{Bx^4\left(bx^2+cx^4\right)^{5/2}}{7c}-\frac{(9bB-14Ac)\left(\frac{x^2\left(bx^2+cx^4\right)^{5/2}}{6c}-\frac{7b\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{5c}-\frac{b\left(\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{\left(b+2cx^2\right)\sqrt{bx^2+c}}{4c}\right.}{}\right.}{2c}\right.}{12c}\right.}{14c}\right.$$

$\downarrow$ 1091

3.106. $\int x^5(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

$$\frac{1}{2}\left(\frac{Bx^4\left(bx^2+cx^4\right)^{5/2}}{7c}-\frac{(9bB-14Ac)\left(\frac{x^2\left(bx^2+cx^4\right)^{5/2}}{6c}-\frac{7b\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{5c}-\frac{b\left(\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{\left(b+2cx^2\right)\sqrt{bx^2+c}}{4c}\right.}{}\right.}{2c}\right.}{12c}\right.}{14c}\right.$$

↓ 219

3.106. $\int x^5(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

$$\frac{1}{2}\left(\frac{Bx^4\left(bx^2+cx^4\right)^{5/2}}{7c}-\frac{(9bB-14Ac)\left(\frac{x^2\left(bx^2+cx^4\right)^{5/2}}{6c}-\frac{7b\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{5c}-\frac{b\left(\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{\left(b+2cx^2\right)\sqrt{bx^2+c}}{4c}\right.}{}\right.}{2c}\right.}{12c}\right.}{14c}\right.$$

3.106. $\int x^5(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

input
```
Int[x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
((B*x^4*(b*x^2 + c*x^4)^(5/2))/(7*c) - ((9*b*B - 14*A*c)*((x^2*(b*x^2 + c*
x^4)^(5/2))/(6*c) - (7*b*((b*x^2 + c*x^4)^(5/2)/(5*c) - (b*(((b + 2*c*x^2)
*(b*x^2 + c*x^4)^(3/2))/(8*c) - (3*b^2*(((b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4]
)/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(4*c^(3/2))))/(
16*c)))/(2*c)))/(12*c)))/(14*c))/2
```

3.106.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.106.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{\frac{7\left(A\,b^{6}c-\frac{9}{14}B\,b^{7}\right)\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)}{2048}+\frac{7\left(\frac{832x^{8}\left(\frac{75x^{2}B}{91}+A\right)b\,c^{\frac{11}{2}}}{35}+\frac{128x^{10}\left(\frac{6x^{2}B}{7}+A\right)c^{\frac{13}{2}}}{7}+\left(-\frac{3\left(\frac{3x^{2}B}{7}+A\right)b^{3}c^{\frac{3}{2}}}{2}\right.\right.}{}}{c^{\frac{11}{2}}}$
risch	$-\frac{\left(-15360B\,c^{6}x^{12}-17920A\,c^{6}x^{10}-19200Bb\,c^{5}x^{10}-23296Ab\,c^{5}x^{8}-384B\,b^{2}c^{4}x^{8}-672A\,b^{2}c^{4}x^{6}+432B\,b^{3}c^{3}x^{6}+784A\,b^{3}c^{3}x\right.}{215040c^{5}}$
default	$\frac{\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}\left(15360B\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{9}{2}}x^{9}+17920A\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{9}{2}}x^{7}-11520B\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{7}{2}}b\,x^{7}-12544A\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{7}{2}}b\,x^{5}+806\right.}{}$

input
```
int(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
7/1536*(3/4*(A*b^6*c-9/14*B*b^7)*ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2)*c^(1/
2)+b)/c^(1/2))+(832/35*x^8*(75/91*x^2*B+A)*b*c^(11/2)+128/7*x^10*(6/7*x^2*
B+A)*c^(13/2)+(-3/2*(3/7*x^2*B+A)*b^3*c^(3/2)+b^2*x^2*(18/35*x^2*B+A)*c^(5
/2)-4/5*x^4*(27/49*x^2*B+A)*b*c^(7/2)+24/35*x^6*(4/7*x^2*B+A)*c^(9/2)+27/2
8*B*c^(1/2)*b^4)*b^2)*(x^2*(c*x^2+b))^(1/2)-3/4*ln(2)*b^6*(A*c-9/14*B*b))/
c^(11/2)
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.87

$$\int x^5(A+Bx^2)(bx^2+cx^4)^{3/2}\,dx = \left[-\frac{105\,(9\,Bb^7-14\,Ab^6c)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,(15360\,Bc^7x^{12}+1280\,(1\right.$$

input
```
integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/430080*(105*(9*B*b^7 - 14*A*b^6*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c
*x^4 + b*x^2)*sqrt(c)) - 2*(15360*B*c^7*x^12 + 1280*(15*B*b*c^6 + 14*A*c^7
)*x^10 + 128*(3*B*b^2*c^5 + 182*A*b*c^6)*x^8 + 945*B*b^6*c - 1470*A*b^5*c^
2 - 48*(9*B*b^3*c^4 - 14*A*b^2*c^5)*x^6 + 56*(9*B*b^4*c^3 - 14*A*b^3*c^4)*
x^4 - 70*(9*B*b^5*c^2 - 14*A*b^4*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^6, 1/215
040*(105*(9*B*b^7 - 14*A*b^6*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-
c)/(c*x^2 + b)) + (15360*B*c^7*x^12 + 1280*(15*B*b*c^6 + 14*A*c^7)*x^10 +
128*(3*B*b^2*c^5 + 182*A*b*c^6)*x^8 + 945*B*b^6*c - 1470*A*b^5*c^2 - 48*(9
*B*b^3*c^4 - 14*A*b^2*c^5)*x^6 + 56*(9*B*b^4*c^3 - 14*A*b^3*c^4)*x^4 - 70*
(9*B*b^5*c^2 - 14*A*b^4*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^6]
```

3.106.6 Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 726, normalized size of antiderivative = 3.26

$$\int x^5 (A+Bx^2) (bx^2+cx^4)^{3/2}\, dx = \frac{Ab\left(\begin{cases} \frac{7b^5\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise} \end{cases}\right)}{256c^4} + \sqrt{bx^2+cx^4}\left(-\frac{7b^4}{128c^4}+\frac{7b^3x^2}{192c^3}-\frac{7b^2x^4}{240c^2}+\frac{bx^6}{40c} \right. \\ \frac{2(bx^2)^{\frac{9}{2}}}{9b^4} \\ 0 \end{cases}\right)}{2}$$

$$+\frac{Ac\left(\begin{cases} -\frac{21b^6\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise} \end{cases}\right)}{1024c^5} + \sqrt{bx^2+cx^4}\cdot\left(\frac{21b^5}{512c^5}-\frac{7b^4x^2}{256c^4}+\frac{7b^3x^4}{320c^3}-\frac{3b^2x^6}{160c^2}+\frac{bx^8}{60c}+ \right. \\ \frac{2(bx^2)^{\frac{11}{2}}}{11b^5} \\ 0 \end{cases}\right)}{2}$$

$$+\frac{Bb\left(\begin{cases} -\frac{21b^6\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise} \end{cases}\right)}{1024c^5} + \sqrt{bx^2+cx^4}\cdot\left(\frac{21b^5}{512c^5}-\frac{7b^4x^2}{256c^4}+\frac{7b^3x^4}{320c^3}-\frac{3b^2x^6}{160c^2}+\frac{bx^8}{60c}+ \right. \\ \frac{2(bx^2)^{\frac{11}{2}}}{11b^5} \\ 0 \end{cases}\right)}{2}$$

$$\frac{Bc\left(\begin{cases} \frac{33b^7\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise} \end{cases}\right)}{2048c^6} + \sqrt{bx^2+cx^4}\left(-\frac{33b^6}{1024c^6}+\frac{11b^5x^2}{512c^5}-\frac{11b^4x^4}{640c^4}+\frac{33b^3x^6}{2240c^3}-\frac{11b^2}{840c} \right. \\ \frac{2(bx^2)^{\frac{13}{2}}}{13b^6} \\ 0 \end{cases}\right)}{2}$$

input
```
integrate(x**5*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

output
```
A*b*Piecewise((7*b**5*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) +
 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/
sqrt(c*(b/(2*c) + x**2)**2), True))/(256*c**4) + sqrt(b*x**2 + c*x**4)*(-7
*b**4/(128*c**4) + 7*b**3*x**2/(192*c**3) - 7*b**2*x**4/(240*c**2) + b*x**
6/(40*c) + x**8/5), Ne(c, 0)), (2*(b*x**2)**(9/2)/(9*b**4), Ne(b, 0)), (0,
 True))/2 + A*c*Piecewise((-21*b**6*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x*
*2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/
(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(1024*c**5) + sqrt(b*x**
2 + c*x**4)*(21*b**5/(512*c**5) - 7*b**4*x**2/(256*c**4) + 7*b**3*x**4/(32
0*c**3) - 3*b**2*x**6/(160*c**2) + b*x**8/(60*c) + x**10/6), Ne(c, 0)), (2
*(b*x**2)**(11/2)/(11*b**5), Ne(b, 0)), (0, True))/2 + B*b*Piecewise((-21*
b**6*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*c*x**2)/sqrt(c
), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c) +
 x**2)**2), True))/(1024*c**5) + sqrt(b*x**2 + c*x**4)*(21*b**5/(512*c**5)
 - 7*b**4*x**2/(256*c**4) + 7*b**3*x**4/(320*c**3) - 3*b**2*x**6/(160*c**2
) + b*x**8/(60*c) + x**10/6), Ne(c, 0)), (2*(b*x**2)**(11/2)/(11*b**5), Ne
(b, 0)), (0, True))/2 + B*c*Piecewise((33*b**7*Piecewise((log(b + 2*sqrt(c
)*sqrt(b*x**2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x
**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(2048*c**6) +
 sqrt(b*x**2 + c*x**4)*(-33*b**6/(1024*c**6) + 11*b**5*x**2/(512*c**5) ...
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.63

$$\int x^5 (A + Bx^2) \left(bx^2 + cx^4\right)^{3/2} dx =$$

$$-\frac{1}{30720}\left(\frac{420\sqrt{cx^4 + bx^2}b^4x^2}{c^3} - \frac{1120\left(cx^4 + bx^2\right)^{\frac{3}{2}}b^2x^2}{c^2} - \frac{2560\left(cx^4 + bx^2\right)^{\frac{5}{2}}x^2}{c} - \frac{105\,b^6\log\left(2\,cx^2 + b + 2\right.}{c^{\frac{9}{2}}}\right.$$

$$+\frac{1}{143360}\left(\frac{10240\left(cx^4 + bx^2\right)^{\frac{5}{2}}x^4}{c} + \frac{1260\sqrt{cx^4 + bx^2}b^5x^2}{c^4} - \frac{3360\left(cx^4 + bx^2\right)^{\frac{3}{2}}b^3x^2}{c^3} - \frac{7680\left(cx^4 + bx^2\right)^{\frac{5}{2}}bx^2}{c^2}\right.$$

input
```
integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/30720*(420*sqrt(c*x^4 + b*x^2)*b^4*x^2/c^3 - 1120*(c*x^4 + b*x^2)^(3/2)
*b^2*x^2/c^2 - 2560*(c*x^4 + b*x^2)^(5/2)*x^2/c - 105*b^6*log(2*c*x^2 + b
+ 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(9/2) + 210*sqrt(c*x^4 + b*x^2)*b^5/c^4
 - 560*(c*x^4 + b*x^2)^(3/2)*b^3/c^3 + 1792*(c*x^4 + b*x^2)^(5/2)*b/c^2)*A
 + 1/143360*(10240*(c*x^4 + b*x^2)^(5/2)*x^4/c + 1260*sqrt(c*x^4 + b*x^2)*
b^5*x^2/c^4 - 3360*(c*x^4 + b*x^2)^(3/2)*b^3*x^2/c^3 - 7680*(c*x^4 + b*x^2
)^(5/2)*b*x^2/c^2 - 315*b^7*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c
))/c^(11/2) + 630*sqrt(c*x^4 + b*x^2)*b^6/c^5 - 1680*(c*x^4 + b*x^2)^(3/2)
*b^4/c^4 + 5376*(c*x^4 + b*x^2)^(5/2)*b^2/c^3)*B
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.26

$$\int x^5(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{1}{215040}\left(2\left(4\left(2\left(8\left(10\left(12\,Bcx^2\mathrm{sgn}(x)+\frac{15\,Bbc^{12}\mathrm{sgn}(x)+14\,Ac^{13}\mathrm{sgn}(x)}{c^{12}}\right)x^2+\frac{3\,Bb^2c^{11}\cdots}{\cdots}\right.\right.\right.\right.\right.$$
$$+\frac{(9\,Bb^7\mathrm{sgn}(x)-14\,Ab^6c\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{2048\,c^{\frac{11}{2}}}$$
$$-\frac{(9\,Bb^7\log(|b|)-14\,Ab^6c\log(|b|))\mathrm{sgn}(x)}{4096\,c^{\frac{11}{2}}}$$

input
```
integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/215040*(2*(4*(2*(8*(10*(12*B*c*x^2*sgn(x) + (15*B*b*c^12*sgn(x) + 14*A*c
^13*sgn(x))/c^12)*x^2 + (3*B*b^2*c^11*sgn(x) + 182*A*b*c^12*sgn(x))/c^12)*
x^2 - 3*(9*B*b^3*c^10*sgn(x) - 14*A*b^2*c^11*sgn(x))/c^12)*x^2 + 7*(9*B*b^
4*c^9*sgn(x) - 14*A*b^3*c^10*sgn(x))/c^12)*x^2 - 35*(9*B*b^5*c^8*sgn(x) -
14*A*b^4*c^9*sgn(x))/c^12)*x^2 + 105*(9*B*b^6*c^7*sgn(x) - 14*A*b^5*c^8*sg
n(x))/c^12)*sqrt(c*x^2 + b)*x + 1/2048*(9*B*b^7*sgn(x) - 14*A*b^6*c*sgn(x)
)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(11/2) - 1/4096*(9*B*b^7*log(ab
s(b)) - 14*A*b^6*c*log(abs(b)))*sgn(x)/c^(11/2)
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int x^5 (A + Bx^2) \left(bx^2 + cx^4\right)^{3/2} dx = \int x^5 \left(B\,x^2 + A\right) \left(c\,x^4 + b\,x^2\right)^{3/2} dx$$

input
```
int(x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

3.107 $\int x^3(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

3.107.1 Optimal result

Integrand size = 26, antiderivative size = 167

$$\int x^3(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=-\frac{b^3(7bB-12Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^4}+\frac{b(7bB-12Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{384c^3}-\frac{\left(7bB-12Ac-10Bcx^2\right)\left(bx^2+cx^4\right)^{5/2}}{120c^2}+\frac{b^5(7bB-12Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{9/2}}$$

output
```
1/384*b*(-12*A*c+7*B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(3/2)/c^3-1/120*(-10*B*c
*x^2-12*A*c+7*B*b)*(c*x^4+b*x^2)^(5/2)/c^2+1/1024*b^5*(-12*A*c+7*B*b)*arct
anh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(9/2)-1/1024*b^3*(-12*A*c+7*B*b)*(2
*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^4
```

3.107.2 Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.35

$$\int x^3(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{\left(x^2(b+cx^2)\right)^{3/2}\left(-105b^5Bx+180Ab^4cx+70b^4Bcx^3-120Ab^3c^2x^3-56b^3Bc^2x^5+96Ab^2c^3x\right.}{15360c^4x^3\left(b+cx^2\right)}+\frac{b^5(7bB-12Ac)\left(x^2(b+cx^2)\right)^{3/2}\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{b}+\sqrt{b+cx^2}}\right)}{512c^{9/2}x^3\left(b+cx^2\right)^{3/2}}$$

input
```
Integrate[x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
((x^2*(b + c*x^2))^(3/2)*(-105*b^5*B*x + 180*A*b^4*c*x + 70*b^4*B*c*x^3 -
120*A*b^3*c^2*x^3 - 56*b^3*B*c^2*x^5 + 96*A*b^2*c^3*x^5 + 48*b^2*B*c^3*x^7
 + 2112*A*b*c^4*x^7 + 1664*b*B*c^4*x^9 + 1536*A*c^5*x^9 + 1280*B*c^5*x^11)
)/(15360*c^4*x^3*(b + c*x^2)) + (b^5*(7*b*B - 12*A*c)*(x^2*(b + c*x^2))^(3
/2)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] + Sqrt[b + c*x^2])])/(512*c^(9/2)*x^3*(b
 + c*x^2)^(3/2))
```

3.107.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1940, 1225, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (A+Bx^2) \left(bx^2+cx^4\right)^{3/2} dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int x^2 (Bx^2+A) \left(cx^4+bx^2\right)^{3/2} dx^2$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{2}\left(\frac{b(7bB-12Ac)\int \left(cx^4+bx^2\right)^{3/2} dx^2}{24c^2}-\frac{\left(bx^2+cx^4\right)^{5/2}\left(-12Ac+7bB-10Bcx^2\right)}{60c^2}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{b(7bB-12Ac)\left(\frac{(b+2cx^2)\left(bx^2+cx^4\right)^{3/2}}{8c}-\frac{3b^2\int \sqrt{cx^4+bx^2}dx^2}{16c}\right)}{24c^2}-\frac{\left(bx^2+cx^4\right)^{5/2}\left(-12Ac+7bB-10Bcx^2\right)}{60c^2}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{b(7bB-12Ac)\left(\frac{(b+2cx^2)(bx^2+cx^4)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^4+bx^2}}dx^2}{8c}\right)}{16c}\right)}{24c^2}-\frac{(bx^2+cx^4)^{5/2}(-12Ac+7b}{60c^2}\right.$$

$$\Big\downarrow \textbf{1091}$$

$$\frac{1}{2}\left(\frac{b(7bB-12Ac)\left(\frac{(b+2cx^2)(bx^2+cx^4)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int\frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{4c}\right)}{16c}\right)}{24c^2}-\frac{(bx^2+cx^4)^{5/2}(-12Ac}{60c^2}\right.$$

$$\Big\downarrow \textbf{219}$$

$$\frac{1}{2}\left(\frac{b(7bB-12Ac)\left(\frac{(b+2cx^2)(bx^2+cx^4)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{4c^{3/2}}\right)}{16c}\right)}{24c^2}-\frac{(bx^2+cx^4)^{5/2}(-12A}{60c}\right.$$

input
```
Int[x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-1/60*((7*b*B - 12*A*c - 10*B*c*x^2)*(b*x^2 + c*x^4)^(5/2))/c^2 + (b*(7*b
*B - 12*A*c)*(((b + 2*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(8*c) - (3*b^2*(((b +
2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^
2 + c*x^4]])/(4*c^(3/2))))/(16*c)))/(24*c^2))/2
```

3.107. $\int x^3(A+Bx^2)(bx^2+cx^4)^{3/2}\,dx$

3.107.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.107. $\int x^3(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

3.107.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{\frac{3\left(-\frac{1}{2}A\,b^{5}c+\frac{7}{24}B\,b^{6}\right)\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)}{256}+\frac{3\left(\frac{128x^{8}\left(\frac{5x^{2}B}{6}+A\right)c^{\frac{11}{2}}}{15}+\left(b^{3}\left(\frac{7x^{2}B}{18}+A\right)c^{\frac{3}{2}}-\frac{2x^{2}\left(\frac{7x^{2}B}{15}+A\right)b^{2}c^{\frac{5}{2}}}{3}+\frac{8x}{}\right.\right.}{256}}{c^{\frac{9}{2}}}$
risch	$\frac{(1280B\,c^{5}x^{10}+1536A\,c^{5}x^{8}+1664Bb\,c^{4}x^{8}+2112Ab\,c^{4}x^{6}+48B\,b^{2}c^{3}x^{6}+96A\,b^{2}c^{3}x^{4}-56B\,b^{3}c^{2}x^{4}-120A\,b^{3}c^{2}x^{2}+70B\,b^{4}c\,x^{2}-}{15360c^{4}}$
default	$\frac{\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}\left(1280B\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{7}{2}}x^{7}+1536A\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{7}{2}}x^{5}-896B\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{5}{2}}b\,x^{5}-960A\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{5}{2}}b\,x^{3}+560B\left(c\,x^{2}\right.\right.}{}$

input
```
int(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/256/c^(9/2)*((-1/2*A*b^5*c+7/24*B*b^6)*ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/
2)*c^(1/2)+b)/c^(1/2))+(128/15*x^8*(5/6*x^2*B+A)*c^(11/2)+(b^3*(7/18*x^2*B
+A)*c^(3/2)-2/3*x^2*(7/15*x^2*B+A)*b^2*c^(5/2)+8/15*x^4*(1/2*x^2*B+A)*b*c^
(7/2)+176/15*x^6*(26/33*x^2*B+A)*c^(9/2)-7/12*B*c^(1/2)*b^4)*b)*(x^2*(c*x^
2+b))^(1/2)+1/2*ln(2)*(A*c-7/12*B*b)*b^5)
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.21

$$\int x^{3}\left(A+Bx^{2}\right)\left(bx^{2}+cx^{4}\right)^{3/2}\,dx=\left[-\frac{15\,(7\,Bb^{6}-12\,Ab^{5}c)\sqrt{c}\log\left(-2\,cx^{2}-b+2\,\sqrt{cx^{4}+bx^{2}}\sqrt{c}\right)-2\,(1280\,Bc^{6}x^{10}+128\,(13\,B}{}\right.$$

$$-\frac{15\,(7\,Bb^{6}-12\,Ab^{5}c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\sqrt{-c}}{cx^{2}+b}\right)-(1280\,Bc^{6}x^{10}+128\,(13\,Bbc^{5}+12\,Ac^{6})x^{8}-105\,Bb^{5}c+}{1536}$$

input
```
integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/30720*(15*(7*B*b^6 - 12*A*b^5*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x
^4 + b*x^2)*sqrt(c)) - 2*(1280*B*c^6*x^10 + 128*(13*B*b*c^5 + 12*A*c^6)*x^
8 - 105*B*b^5*c + 180*A*b^4*c^2 + 48*(B*b^2*c^4 + 44*A*b*c^5)*x^6 - 8*(7*B
*b^3*c^3 - 12*A*b^2*c^4)*x^4 + 10*(7*B*b^4*c^2 - 12*A*b^3*c^3)*x^2)*sqrt(c
*x^4 + b*x^2))/c^5, -1/15360*(15*(7*B*b^6 - 12*A*b^5*c)*sqrt(-c)*arctan(sq
rt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (1280*B*c^6*x^10 + 128*(13*B*b*c
^5 + 12*A*c^6)*x^8 - 105*B*b^5*c + 180*A*b^4*c^2 + 48*(B*b^2*c^4 + 44*A*b*
c^5)*x^6 - 8*(7*B*b^3*c^3 - 12*A*b^2*c^4)*x^4 + 10*(7*B*b^4*c^2 - 12*A*b^3
*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^5]
```

3.107. $\int x^3(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

3.107.6 Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 672, normalized size of antiderivative = 4.02

$$\int x^3(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{Ab\left(\begin{cases} -\frac{5b^4\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{128c^3}+\sqrt{bx^2+cx^4}\cdot\left(\frac{5b^3}{64c^3}-\frac{5b^2x^2}{96c^2}+\frac{bx^4}{24c}+\frac{x^6}{4}\right) \\ \frac{2(bx^2)^{\frac{7}{2}}}{7b^3} \\ 0\end{cases}\right)}{2}$$

$$+\frac{Ac\left(\begin{cases} \frac{7b^5\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{256c^4}+\sqrt{bx^2+cx^4}\left(-\frac{7b^4}{128c^4}+\frac{7b^3x^2}{192c^3}-\frac{7b^2x^4}{240c^2}+\frac{bx^6}{40c}+\frac{x^8}{5}\right) & \text{for } c \\ \frac{2(bx^2)^{\frac{9}{2}}}{9b^4} & \text{for } b \\ 0 & \text{othe}\end{cases}\right)}{2}$$

$$+\frac{Bb\left(\begin{cases} \frac{7b^5\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{256c^4}+\sqrt{bx^2+cx^4}\left(-\frac{7b^4}{128c^4}+\frac{7b^3x^2}{192c^3}-\frac{7b^2x^4}{240c^2}+\frac{bx^6}{40c}+\frac{x^8}{5}\right) & \text{for } c \\ \frac{2(bx^2)^{\frac{9}{2}}}{9b^4} & \text{for } b \\ 0 & \text{othe}\end{cases}\right)}{2}$$

$$+\frac{Bc\left(\begin{cases} -\frac{21b^6\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{1024c^5}+\sqrt{bx^2+cx^4}\cdot\left(\frac{21b^5}{512c^5}-\frac{7b^4x^2}{256c^4}+\frac{7b^3x^4}{320c^3}-\frac{3b^2x^6}{160c^2}+\frac{bx^8}{60c}+\right. \\ \frac{2(bx^2)^{\frac{11}{2}}}{11b^5} \\ 0\end{cases}\right)}{2}$$

input
```
integrate(x**3*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

output
```
A*b*Piecewise((-5*b**4*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4)
+ 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)
/sqrt(c*(b/(2*c) + x**2)**2), True))/(128*c**3) + sqrt(b*x**2 + c*x**4)*(5
*b**3/(64*c**3) - 5*b**2*x**2/(96*c**2) + b*x**4/(24*c) + x**6/4), Ne(c, 0
)), (2*(b*x**2)**(7/2)/(7*b**3), Ne(b, 0)), (0, True))/2 + A*c*Piecewise((
7*b**5*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*c*x**2)/sqrt
(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c)
 + x**2)**2), True))/(256*c**4) + sqrt(b*x**2 + c*x**4)*(-7*b**4/(128*c**4
) + 7*b**3*x**2/(192*c**3) - 7*b**2*x**4/(240*c**2) + b*x**6/(40*c) + x**8
/5), Ne(c, 0)), (2*(b*x**2)**(9/2)/(9*b**4), Ne(b, 0)), (0, True))/2 + B*b
*Piecewise((7*b**5*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*
c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqr
t(c*(b/(2*c) + x**2)**2), True))/(256*c**4) + sqrt(b*x**2 + c*x**4)*(-7*b*
*4/(128*c**4) + 7*b**3*x**2/(192*c**3) - 7*b**2*x**4/(240*c**2) + b*x**6/(
40*c) + x**8/5), Ne(c, 0)), (2*(b*x**2)**(9/2)/(9*b**4), Ne(b, 0)), (0, Tr
ue))/2 + B*c*Piecewise((-21*b**6*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2
+ c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*
c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(1024*c**5) + sqrt(b*x**2 +
 c*x**4)*(21*b**5/(512*c**5) - 7*b**4*x**2/(256*c**4) + 7*b**3*x**4/(320*c
**3) - 3*b**2*x**6/(160*c**2) + b*x**8/(60*c) + x**10/6), Ne(c, 0)), (2...
```

3.107.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(147) = 294$.

Time = 0.23 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.89

$$\int x^3(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{1}{2560}\left(\frac{60\sqrt{cx^4+bx^2}b^3x^2}{c^2} - \frac{160\left(cx^4+bx^2\right)^{\frac{3}{2}}bx^2}{c} - \frac{15\,b^5\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{7}{2}}} \cdots\right.$$
$$-\frac{1}{30720}\left(\frac{420\sqrt{cx^4+bx^2}b^4x^2}{c^3} - \frac{1120\left(cx^4+bx^2\right)^{\frac{3}{2}}b^2x^2}{c^2} - \frac{2560\left(cx^4+bx^2\right)^{\frac{5}{2}}x^2}{c} - \frac{105\,b^6\log\left(2\,cx^2+b+2\cdots\right.}{c^{\frac{9}{2}}}\cdots\right.$$

input
```
integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2560*(60*sqrt(c*x^4 + b*x^2)*b^3*x^2/c^2 - 160*(c*x^4 + b*x^2)^(3/2)*b*x
^2/c - 15*b^5*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 3
0*sqrt(c*x^4 + b*x^2)*b^4/c^3 - 80*(c*x^4 + b*x^2)^(3/2)*b^2/c^2 + 256*(c*
x^4 + b*x^2)^(5/2)/c)*A - 1/30720*(420*sqrt(c*x^4 + b*x^2)*b^4*x^2/c^3 - 1
120*(c*x^4 + b*x^2)^(3/2)*b^2*x^2/c^2 - 2560*(c*x^4 + b*x^2)^(5/2)*x^2/c -
 105*b^6*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(9/2) + 210*sq
rt(c*x^4 + b*x^2)*b^5/c^4 - 560*(c*x^4 + b*x^2)^(3/2)*b^3/c^3 + 1792*(c*x^
4 + b*x^2)^(5/2)*b/c^2)*B
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.47

$$\int x^3(A+Bx^2)\,(bx^2+cx^4)^{3/2}\,dx = \frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,Bcx^2\mathrm{sgn}(x)+\frac{13\,Bbc^{10}\mathrm{sgn}(x)+12\,Ac^{11}\mathrm{sgn}(x)}{c^{10}}\right)x^2+\frac{3\,(Bb^2c^9\mathrm{sgn}(x)\right.\right.\right.\right.$$
$$-\frac{(7\,Bb^6\mathrm{sgn}(x)-12\,Ab^5c\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{1024\,c^{\frac{9}{2}}}$$
$$+\frac{(7\,Bb^6\log(|b|)-12\,Ab^5c\log(|b|))\mathrm{sgn}(x)}{2048\,c^{\frac{9}{2}}}$$

input
```
integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/15360*(2*(4*(2*(8*(10*B*c*x^2*sgn(x) + (13*B*b*c^10*sgn(x) + 12*A*c^11*s
gn(x))/c^10)*x^2 + 3*(B*b^2*c^9*sgn(x) + 44*A*b*c^10*sgn(x))/c^10)*x^2 - (
7*B*b^3*c^8*sgn(x) - 12*A*b^2*c^9*sgn(x))/c^10)*x^2 + 5*(7*B*b^4*c^7*sgn(x
) - 12*A*b^3*c^8*sgn(x))/c^10)*x^2 - 15*(7*B*b^5*c^6*sgn(x) - 12*A*b^4*c^7
*sgn(x))/c^10)*sqrt(c*x^2 + b)*x - 1/1024*(7*B*b^6*sgn(x) - 12*A*b^5*c*sgn
(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(9/2) + 1/2048*(7*B*b^6*log(
abs(b)) - 12*A*b^5*c*log(abs(b)))*sgn(x)/c^(9/2)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int x^3(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int x^3\,\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}\,dx$$

input
```
int(x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

3.108 $\int x(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

3.108.1 Optimal result

Integrand size = 24, antiderivative size = 148

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{3b^2(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{(bB-2Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{10c}-\frac{3b^4(bB-2Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}}$$

output
```
-1/32*(-2*A*c+B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(3/2)/c^2+1/10*B*(c*x^4+b*x^2
)^(5/2)/c-3/256*b^4*(-2*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/
c^(7/2)+3/256*b^2*(-2*A*c+B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^3
```

3.108.2 Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.34

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{\left(x^2(b+cx^2)\right)^{3/2}\left(15b^4B-30Ab^3c-10b^3Bcx^2+20Ab^2c^2x^2+8b^2Bc^2x^4+240Abc^3x^4+176b\right.}{1280c^3x^2\left(b+cx^2\right)}-\frac{3b^4(bB-2Ac)\left(x^2(b+cx^2)\right)^{3/2}\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{b}+\sqrt{b+cx^2}}\right)}{128c^{7/2}x^3\left(b+cx^2\right)^{3/2}}$$

input
```
Integrate[x*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
((x^2*(b + c*x^2))^(3/2)*(15*b^4*B - 30*A*b^3*c - 10*b^3*B*c*x^2 + 20*A*b^
2*c^2*x^2 + 8*b^2*B*c^2*x^4 + 240*A*b*c^3*x^4 + 176*b*B*c^3*x^6 + 160*A*c^
4*x^6 + 128*B*c^4*x^8))/(1280*c^3*x^2*(b + c*x^2)) - (3*b^4*(b*B - 2*A*c)*
(x^2*(b + c*x^2))^(3/2)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] + Sqrt[b + c*x^2])])
/(128*c^(7/2)*x^3*(b + c*x^2)^(3/2))
```

3.108.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1940, 1160, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \left(Bx^2+A\right)\left(cx^4+bx^2\right)^{3/2}dx^2$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{B\left(bx^2+cx^4\right)^{5/2}}{5c}-\frac{(bB-2Ac)\int \left(cx^4+bx^2\right)^{3/2}dx^2}{2c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{B\left(bx^2+cx^4\right)^{5/2}}{5c}-\frac{(bB-2Ac)\left(\frac{(b+2cx^2)\left(bx^2+cx^4\right)^{3/2}}{8c}-\frac{3b^2\int \sqrt{cx^4+bx^2}dx^2}{16c}\right)}{2c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{B\left(bx^2+cx^4\right)^{5/2}}{5c}-\frac{(bB-2Ac)\left(\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^4+bx^2}}dx^2}{8c}\right)}{16c}\right)}{2c}\right)$$

$\downarrow$ 1091

$$\frac{1}{2}\left(\frac{B\left(bx^2+cx^4\right)^{5/2}}{5c}-\frac{(bB-2Ac)\left(\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int\frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{4c}\right)}{16c}\right)}{2c}\right)$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{B\left(bx^2+cx^4\right)^{5/2}}{5c}-\frac{(bB-2Ac)\left(\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{4c^{3/2}}\right)}{16c}\right)}{2c}\right)$$

input
```
Int[x*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
((B*(b*x^2 + c*x^4)^(5/2))/(5*c) - ((b*B - 2*A*c)*(((b + 2*c*x^2)*(b*x^2 +
 c*x^4)^(3/2))/(8*c) - (3*b^2*(((b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(4*c) -
 (b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(4*c^(3/2))))/(16*c)))/(
2*c))/2
```

3.108. $\int x(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

3.108.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.108.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$-\frac{3\left(\left(-\frac{1}{2}A\,b^{4}c+\frac{1}{4}b^{5}B\right)\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)+\left(b^{3}\left(\frac{x^{2}B}{3}+A\right)c^{\frac{3}{2}}-\frac{2x^{2}\left(\frac{2x^{2}B}{5}+A\right)b^{2}c^{\frac{5}{2}}}{3}-8x^{4}\left(\frac{11x^{2}B}{15}+A\right)b\,c^{\frac{7}{2}}-\right.}{128c^{\frac{7}{2}}}$
risch	$-\frac{\left(-128B\,x^{8}c^{4}-160A\,x^{6}c^{4}-176B\,x^{6}b\,c^{3}-240A\,x^{4}b\,c^{3}-8B\,x^{4}b^{2}c^{2}-20A\,x^{2}b^{2}c^{2}+10B\,x^{2}b^{3}c+30A\,b^{3}c-15B\,b^{4}\right)\sqrt{x^{2}(c\,x^{2}+}}{1280c^{3}}$
default	$\frac{\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}\left(128B\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{5}{2}}x^{5}+160A\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{5}{2}}x^{3}-80B\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{3}{2}}b\,x^{3}-80A\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{3}{2}}bx+20A\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}\right.}{}$

input
```
int(x*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-3/128/c^(7/2)*((-1/2*A*b^4*c+1/4*b^5*B)*ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/
2)*c^(1/2)+b)/c^(1/2))+(b^3*(1/3*x^2*B+A)*c^(3/2)-2/3*x^2*(2/5*x^2*B+A)*b^
2*c^(5/2)-8*x^4*(11/15*x^2*B+A)*b*c^(7/2)-16/3*x^6*(4/5*x^2*B+A)*c^(9/2)-1
/2*B*c^(1/2)*b^4)*(x^2*(c*x^2+b))^(1/2)+1/2*ln(2)*(A*c-1/2*B*b)*b^4)
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.14

$$\int x(A+Bx^{2})\left(bx^{2}+cx^{4}\right)^{3/2}\,dx=\left[-\frac{15\left(Bb^{5}-2\,Ab^{4}c\right)\sqrt{c}\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\sqrt{c}\right)-2\left(128\,Bc^{5}x^{8}+16\left(11\,Bbc^{4}+\right.\right.}{2}\right.$$

input
```
integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2560*(15*(B*b^5 - 2*A*b^4*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 +
 b*x^2)*sqrt(c)) - 2*(128*B*c^5*x^8 + 16*(11*B*b*c^4 + 10*A*c^5)*x^6 + 15*
B*b^4*c - 30*A*b^3*c^2 + 8*(B*b^2*c^3 + 30*A*b*c^4)*x^4 - 10*(B*b^3*c^2 -
2*A*b^2*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4, 1/1280*(15*(B*b^5 - 2*A*b^4*c)
*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (128*B*c^5*x^
8 + 16*(11*B*b*c^4 + 10*A*c^5)*x^6 + 15*B*b^4*c - 30*A*b^3*c^2 + 8*(B*b^2*
c^3 + 30*A*b*c^4)*x^4 - 10*(B*b^3*c^2 - 2*A*b^2*c^3)*x^2)*sqrt(c*x^4 + b*x
^2))/c^4]
```

3.108. $\int x(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

3.108.6 Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 614, normalized size of antiderivative = 4.15

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{Ab\left(\begin{cases} \frac{b^3\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{16c^2} + \sqrt{bx^2+cx^4}\left(-\frac{b^2}{8c^2}+\frac{bx^2}{12c}+\frac{x^4}{3}\right) & \text{for } c\neq 0 \\ \frac{2(bx^2)^{\frac{5}{2}}}{5b^2} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{2}$$

$$+\frac{Ac\left(\begin{cases} -\frac{5b^4\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{128c^3} + \sqrt{bx^2+cx^4}\cdot\left(\frac{5b^3}{64c^3}-\frac{5b^2x^2}{96c^2}+\frac{bx^4}{24c}+\frac{x^6}{4}\right) & \text{for } c\neq 0 \\ \frac{2(bx^2)^{\frac{7}{2}}}{7b^3} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{2}$$

$$+\frac{Bb\left(\begin{cases} -\frac{5b^4\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{128c^3} + \sqrt{bx^2+cx^4}\cdot\left(\frac{5b^3}{64c^3}-\frac{5b^2x^2}{96c^2}+\frac{bx^4}{24c}+\frac{x^6}{4}\right) & \text{for } c\neq 0 \\ \frac{2(bx^2)^{\frac{7}{2}}}{7b^3} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{2}$$

$$\frac{Bc\left(\begin{cases} \frac{7b^5\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{256c^4} + \sqrt{bx^2+cx^4}\left(-\frac{7b^4}{128c^4}+\frac{7b^3x^2}{192c^3}-\frac{7b^2x^4}{240c^2}+\frac{bx^6}{40c}+\frac{x^8}{5}\right) & \text{for } c\neq 0 \\ \frac{2(bx^2)^{\frac{9}{2}}}{9b^4} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{2}$$

input
```
integrate(x*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

output
```
A*b*Piecewise((b**3*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2
*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sq
rt(c*(b/(2*c) + x**2)**2), True))/(16*c**2) + sqrt(b*x**2 + c*x**4)*(-b**2
/(8*c**2) + b*x**2/(12*c) + x**4/3), Ne(c, 0)), (2*(b*x**2)**(5/2)/(5*b**2
), Ne(b, 0)), (0, True))/2 + A*c*Piecewise((-5*b**4*Piecewise((log(b + 2*s
qrt(c)*sqrt(b*x**2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c
) + x**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(128*c**
3) + sqrt(b*x**2 + c*x**4)*(5*b**3/(64*c**3) - 5*b**2*x**2/(96*c**2) + b*x
**4/(24*c) + x**6/4), Ne(c, 0)), (2*(b*x**2)**(7/2)/(7*b**3), Ne(b, 0)), (
0, True))/2 + B*b*Piecewise((-5*b**4*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x
**2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b
/(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(128*c**3) + sqrt(b*x**
2 + c*x**4)*(5*b**3/(64*c**3) - 5*b**2*x**2/(96*c**2) + b*x**4/(24*c) + x*
*6/4), Ne(c, 0)), (2*(b*x**2)**(7/2)/(7*b**3), Ne(b, 0)), (0, True))/2 + B
*c*Piecewise((7*b**5*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) +
2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/s
qrt(c*(b/(2*c) + x**2)**2), True))/(256*c**4) + sqrt(b*x**2 + c*x**4)*(-7*
b**4/(128*c**4) + 7*b**3*x**2/(192*c**3) - 7*b**2*x**4/(240*c**2) + b*x**6
/(40*c) + x**8/5), Ne(c, 0)), (2*(b*x**2)**(9/2)/(9*b**4), Ne(b, 0)), (0,
True))/2
```

3.108.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(128) = 256$.

Time = 0.22 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.80

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{1}{256}\left(32\left(cx^4+bx^2\right)^{\frac{3}{2}}x^2 - \frac{12\sqrt{cx^4+bx^2}b^2x^2}{c} + \frac{3\,b^4\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}} - \frac{6\,\sqrt{\ }\cdots}{\cdots}\right. + \frac{1}{2560}\left(\frac{60\sqrt{cx^4+bx^2}b^3x^2}{c^2} - \frac{160\left(cx^4+bx^2\right)^{\frac{3}{2}}bx^2}{c} - \frac{15\,b^5\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{7}{2}}} + \frac{30\sqrt{cx^4+\cdots}}{c^3}\cdots\right.$$

input
```
integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/256*(32*(c*x^4 + b*x^2)^(3/2)*x^2 - 12*sqrt(c*x^4 + b*x^2)*b^2*x^2/c + 3
*b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2) - 6*sqrt(c*x
^4 + b*x^2)*b^3/c^2 + 16*(c*x^4 + b*x^2)^(3/2)*b/c)*A + 1/2560*(60*sqrt(c*
x^4 + b*x^2)*b^3*x^2/c^2 - 160*(c*x^4 + b*x^2)^(3/2)*b*x^2/c - 15*b^5*log(
2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 30*sqrt(c*x^4 + b*x
^2)*b^4/c^3 - 80*(c*x^4 + b*x^2)^(3/2)*b^2/c^2 + 256*(c*x^4 + b*x^2)^(5/2)
/c)*B
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.40

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{1}{1280}\left(2\left(4\left(2\left(8\,Bcx^2\mathrm{sgn}(x)+\frac{11\,Bbc^8\mathrm{sgn}(x)+10\,Ac^9\mathrm{sgn}(x)}{c^8}\right)x^2+\frac{Bb^2c^7\mathrm{sgn}(x)+30\,Ab}{c^8}\right.\right.\right.$$
$$+\frac{3\left(Bb^5\mathrm{sgn}(x)-2\,Ab^4c\mathrm{sgn}(x)\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{256\,c^{\frac{7}{2}}}$$
$$-\frac{3\left(Bb^5\log\left(|b|\right)-2\,Ab^4c\log\left(|b|\right)\right)\mathrm{sgn}(x)}{512\,c^{\frac{7}{2}}}$$

input
```
integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/1280*(2*(4*(2*(8*B*c*x^2*sgn(x) + (11*B*b*c^8*sgn(x) + 10*A*c^9*sgn(x))/
c^8)*x^2 + (B*b^2*c^7*sgn(x) + 30*A*b*c^8*sgn(x))/c^8)*x^2 - 5*(B*b^3*c^6*
sgn(x) - 2*A*b^2*c^7*sgn(x))/c^8)*x^2 + 15*(B*b^4*c^5*sgn(x) - 2*A*b^3*c^6
*sgn(x))/c^8)*sqrt(c*x^2 + b)*x + 3/256*(B*b^5*sgn(x) - 2*A*b^4*c*sgn(x))*
log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(7/2) - 3/512*(B*b^5*log(abs(b))
- 2*A*b^4*c*log(abs(b)))*sgn(x)/c^(7/2)
```

3.108.9 Mupad [B] (verification not implemented)

Time = 9.87 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.59

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{B\left(c\,x^4+b\,x^2\right)^{5/2}}{10\,c}+\frac{A\left(c\,x^4+b\,x^2\right)^{3/2}\left(c\,x^2+\frac{b}{2}\right)}{8\,c}-\frac{3\,A\,b^2\left(\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\sqrt{c\,x^4+b\,x^2}-\frac{b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{8\,c^{3/2}}\right)}{32\,c}-\frac{B\,b\left(\frac{x^2\,\left(c\,x^4+b\,x^2\right)^{3/2}}{4}-\frac{3\,b^2\left(\frac{\left(2\,c\,x^2+b\right)\,\sqrt{c\,x^4+b\,x^2}}{4\,c}-\frac{b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{8\,c^{3/2}}\right)}{16\,c}+\frac{b\,\left(c\,x^4+b\,x^2\right)^{3/2}}{8\,c}\right)}{4\,c}$$

input
```
int(x*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

output
```
(B*(b*x^2 + c*x^4)^(5/2))/(10*c) + (A*(b*x^2 + c*x^4)^(3/2)*(b/2 + c*x^2))
/(8*c) - (3*A*b^2*((b/(4*c) + x^2/2)*(b*x^2 + c*x^4)^(1/2) - (b^2*log((b/2
 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(8*c^(3/2))))/(32*c) - (B*b*((
x^2*(b*x^2 + c*x^4)^(3/2))/4 - (3*b^2*(((b + 2*c*x^2)*(b*x^2 + c*x^4)^(1/2
))/(4*c) - (b^2*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(8*c^(
3/2))))/(16*c) + (b*(b*x^2 + c*x^4)^(3/2))/(8*c)))/(4*c)
```

3.109 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x}\,dx$

3.109.1 Optimal result

Integrand size = 26, antiderivative size = 144

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x}\,dx = -\frac{b(3bB-8Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^2} - \frac{(3bB-8Ac)(bx^2+cx^4)^{3/2}}{48c} + \frac{B(bx^2+cx^4)^{5/2}}{8cx^2} + \frac{b^3(3bB-8Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}$$

output
```
-1/48*(-8*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/c+1/8*B*(c*x^4+b*x^2)^(5/2)/c/x^2
+1/128*b^3*(-8*A*c+3*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(5/2)
-1/128*b*(-8*A*c+3*B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^2
```

3.109.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x}\,dx = \frac{(x^2(b+cx^2))^{3/2}\left(\frac{\sqrt{c}x(-9b^3B+6b^2c(4A+Bx^2)+16c^3x^4(4A+3Bx^2)+8bc^2x^2(14A+9Bx^2))}{b+cx^2}\right.}{384c^{5/2}x^3}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x,x]
```

output
```
((x^2*(b + c*x^2))^(3/2)*((Sqrt[c]*x*(-9*b^3*B + 6*b^2*c*(4*A + B*x^2) + 1
6*c^3*x^4*(4*A + 3*B*x^2) + 8*b*c^2*x^2*(14*A + 9*B*x^2)))/(b + c*x^2) + (
6*b^3*(3*b*B - 8*A*c)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] + Sqrt[b + c*x^2])])/(
b + c*x^2)^(3/2)))/(384*c^(5/2)*x^3)
```

3.109.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1940, 1221, 1131, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)\left(cx^4+bx^2\right)^{3/2}}{x^2}dx^2$$

$$\Big\downarrow \text{1221}$$

$$\frac{1}{2}\left(\frac{B\left(bx^2+cx^4\right)^{5/2}}{4cx^2}-\frac{(3bB-8Ac)\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^2}dx^2}{8c}\right)$$

$$\Big\downarrow \text{1131}$$

$$\frac{1}{2}\left(\frac{B\left(bx^2+cx^4\right)^{5/2}}{4cx^2}-\frac{(3bB-8Ac)\left(\frac{1}{2}b\int \sqrt{cx^4+bx^2}dx^2+\frac{1}{3}\left(bx^2+cx^4\right)^{3/2}\right)}{8c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{B\left(bx^2+cx^4\right)^{5/2}}{4cx^2}-\frac{(3bB-8Ac)\left(\frac{1}{2}b\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2}{8c}\right)+\frac{1}{3}\left(bx^2+cx^4\right)^{3/2}\right)}{8c}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{B\left(bx^2+cx^4\right)^{5/2}}{4cx^2}-\frac{(3bB-8Ac)\left(\frac{1}{2}b\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int\frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{4c}\right)+\frac{1}{3}\left(bx^2+cx^4\right)^{3/2}\right)}{8c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{B\left(bx^2+cx^4\right)^{5/2}}{4cx^2}-\frac{(3bB-8Ac)\left(\frac{1}{2}b\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{4c^{3/2}}\right)+\frac{1}{3}\left(bx^2+cx^4\right)^{3/2}\right)}{8c}\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x,x]
```

output
```
((B*(b*x^2 + c*x^4)^(5/2))/(4*c*x^2) - ((3*b*B - 8*A*c)*((b*x^2 + c*x^4)^(
3/2)/3 + (b*(((b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(4*c) - (b^2*ArcTanh[(Sqr
t[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(4*c^(3/2))))/2))/(8*c))/2
```

3.109.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.109.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.97

method	result
risch	$\frac{\left(48B\,c^{3}x^{6}+64A\,c^{3}x^{4}+72Bb\,c^{2}x^{4}+112Ab\,c^{2}x^{2}+6B\,b^{2}c\,x^{2}+24b^{2}Ac-9B\,b^{3}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{384c^{2}}-\frac{b^{3}\left(8Ac-3Bb\right)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}}\right.}{128c^{\frac{5}{2}}\,x\sqrt{c\,x^{2}}}$
pseudoelliptic	$\frac{\frac{\left(-A\,b^{3}c+\frac{3}{8}B\,b^{4}\right)\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)}{2}+\left(b^{2}\left(\frac{x^{2}B}{4}+A\right)c^{\frac{3}{2}}+\frac{14x^{2}\left(\frac{9x^{2}B}{14}+A\right)b\,c^{\frac{5}{2}}}{3}+2\left(B\,x^{6}+\frac{4}{3}A\,x^{4}\right)c^{\frac{7}{2}}-\frac{3B\sqrt{c}\,b^{3}}{8}\right)}{16c^{\frac{5}{2}}}$
default	$\frac{\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}\left(48B\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{3}{2}}x^{3}+64A\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{3}{2}}x-24B\left(c\,x^{2}+b\right)^{\frac{5}{2}}\sqrt{c}\,bx-16A\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}bx+6B\left(c\,x^{2}+b\right)^{\frac{3}{2}}\sqrt{c}\,b^{2}x\right.}{384x^{3}\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{5}{2}}}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/384/c^2*(48*B*c^3*x^6+64*A*c^3*x^4+72*B*b*c^2*x^4+112*A*b*c^2*x^2+6*B*b^
2*c*x^2+24*A*b^2*c-9*B*b^3)*(x^2*(c*x^2+b))^(1/2)-1/128*b^3*(8*A*c-3*B*b)/
c^(5/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1
/2)
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.91

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x}\,dx = \left[-\frac{3\,(3\,Bb^4-8\,Ab^3c)\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,(48\,Bc^4x\right.$$

$$-\frac{3\,(3\,Bb^4-8\,Ab^3c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-(48\,Bc^4x^6-9\,Bb^3c+24\,Ab^2c^2+8\,(9\,Bbc^3+8\,Ac^4)x^4+2}{384\,c^3}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
[-1/768*(3*(3*B*b^4 - 8*A*b^3*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 +
 b*x^2)*sqrt(c)) - 2*(48*B*c^4*x^6 - 9*B*b^3*c + 24*A*b^2*c^2 + 8*(9*B*b*c
^3 + 8*A*c^4)*x^4 + 2*(3*B*b^2*c^2 + 56*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))
/c^3, -1/384*(3*(3*B*b^4 - 8*A*b^3*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*
sqrt(-c)/(c*x^2 + b)) - (48*B*c^4*x^6 - 9*B*b^3*c + 24*A*b^2*c^2 + 8*(9*B*
b*c^3 + 8*A*c^4)*x^4 + 2*(3*B*b^2*c^2 + 56*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^
2))/c^3]
```

3.109.6 Sympy [A] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 551, normalized size of antiderivative = 3.83

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x}\,dx = \frac{Ab\left(\begin{cases} -\frac{b^2\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{8c} + \left(\frac{b}{4c}+\frac{x^2}{2}\right)\sqrt{bx^2+c} \\ \frac{2(bx^2)^{\frac{3}{2}}}{3b} \\ 0 \end{cases}\right.}{2}$$

$$+\frac{Ac\left(\begin{cases} \frac{b^3\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{16c^2} + \sqrt{bx^2+cx^4}\left(-\frac{b^2}{8c^2}+\frac{bx^2}{12c}+\frac{x^4}{3}\right) & \text{for } c\neq 0\\ \frac{2(bx^2)^{\frac{5}{2}}}{5b^2} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{2}$$

$$+\frac{Bb\left(\begin{cases} \frac{b^3\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{16c^2} + \sqrt{bx^2+cx^4}\left(-\frac{b^2}{8c^2}+\frac{bx^2}{12c}+\frac{x^4}{3}\right) & \text{for } c\neq 0\\ \frac{2(bx^2)^{\frac{5}{2}}}{5b^2} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{2}$$

$$+\frac{Bc\left(\begin{cases} -\frac{5b^4\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{128c^3} + \sqrt{bx^2+cx^4}\cdot\left(\frac{5b^3}{64c^3}-\frac{5b^2x^2}{96c^2}+\frac{bx^4}{24c}+\frac{x^6}{4}\right) & \text{for } c\neq 0\\ \frac{2(bx^2)^{\frac{7}{2}}}{7b^3} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}\right.}{2}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x,x)
```

output
```
A*b*Piecewise((-b**2*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) +
2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/s
qrt(c*(b/(2*c) + x**2)**2), True))/(8*c) + (b/(4*c) + x**2/2)*sqrt(b*x**2
+ c*x**4), Ne(c, 0)), (2*(b*x**2)**(3/2)/(3*b), Ne(b, 0)), (0, True))/2 +
A*c*Piecewise((b**3*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2
*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sq
rt(c*(b/(2*c) + x**2)**2), True))/(16*c**2) + sqrt(b*x**2 + c*x**4)*(-b**2
/(8*c**2) + b*x**2/(12*c) + x**4/3), Ne(c, 0)), (2*(b*x**2)**(5/2)/(5*b**2
), Ne(b, 0)), (0, True))/2 + B*b*Piecewise((b**3*Piecewise((log(b + 2*sqrt
(c)*sqrt(b*x**2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) +
 x**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(16*c**2) +
 sqrt(b*x**2 + c*x**4)*(-b**2/(8*c**2) + b*x**2/(12*c) + x**4/3), Ne(c, 0)
), (2*(b*x**2)**(5/2)/(5*b**2), Ne(b, 0)), (0, True))/2 + B*c*Piecewise((-
5*b**4*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*c*x**2)/sqrt
(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c)
 + x**2)**2), True))/(128*c**3) + sqrt(b*x**2 + c*x**4)*(5*b**3/(64*c**3)
- 5*b**2*x**2/(96*c**2) + b*x**4/(24*c) + x**6/4), Ne(c, 0)), (2*(b*x**2)*
*(7/2)/(7*b**3), Ne(b, 0)), (0, True))/2
```

3.109.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.50

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x}\,dx = \frac{1}{96}\left(12\sqrt{cx^4+bx^2}bx^2 - \frac{3\,b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{3}{2}}} + 16\left(cx^4\right.\right.$$
$$+\frac{1}{256}\left(32\left(cx^4+bx^2\right)^{\frac{3}{2}}x^2 - \frac{12\sqrt{cx^4+bx^2}b^2x^2}{c} + \frac{3\,b^4\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}} - \frac{6\sqrt{cx^4+bx^2}b^3}{c^2}\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
1/96*(12*sqrt(c*x^4 + b*x^2)*b*x^2 - 3*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4
+ b*x^2)*sqrt(c))/c^(3/2) + 16*(c*x^4 + b*x^2)^(3/2) + 6*sqrt(c*x^4 + b*x^
2)*b^2/c)*A + 1/256*(32*(c*x^4 + b*x^2)^(3/2)*x^2 - 12*sqrt(c*x^4 + b*x^2)
*b^2*x^2/c + 3*b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2
) - 6*sqrt(c*x^4 + b*x^2)*b^3/c^2 + 16*(c*x^4 + b*x^2)^(3/2)*b/c)*B
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.24

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x}\,dx = \frac{1}{384}\left(2\left(4\left(6\,Bcx^2\mathrm{sgn}(x)+\frac{9\,Bbc^6\mathrm{sgn}(x)+8\,Ac^7\mathrm{sgn}(x)}{c^6}\right)x^2+\frac{3\,Bb^2c^5\mathrm{sg}}{}\right.\right.$$
$$-\frac{(3\,Bb^4\mathrm{sgn}(x)-8\,Ab^3c\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{128\,c^{\frac{5}{2}}}$$
$$+\frac{(3\,Bb^4\log(|b|)-8\,Ab^3c\log(|b|))\mathrm{sgn}(x)}{256\,c^{\frac{5}{2}}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x,x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*B*c*x^2*sgn(x) + (9*B*b*c^6*sgn(x) + 8*A*c^7*sgn(x))/c^6)*x
^2 + (3*B*b^2*c^5*sgn(x) + 56*A*b*c^6*sgn(x))/c^6)*x^2 - 3*(3*B*b^3*c^4*sg
n(x) - 8*A*b^2*c^5*sgn(x))/c^6)*sqrt(c*x^2 + b)*x - 1/128*(3*B*b^4*sgn(x)
- 8*A*b^3*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(5/2) + 1/256
*(3*B*b^4*log(abs(b)) - 8*A*b^3*c*log(abs(b)))*sgn(x)/c^(5/2)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x}\,dx = \int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2)^{3/2}}{x}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x,x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x, x)
```

3.110 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^3} dx$

3.110.1 Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^3} dx = \frac{(bB-6Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{16c} + \frac{(bB-6Ac)(bx^2+cx^4)^{3/2}}{6b} + \frac{A(bx^2+cx^4)^{5/2}}{bx^4} - \frac{b^2(bB-6Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}$$

output
```
1/6*(-6*A*c+B*b)*(c*x^4+b*x^2)^(3/2)/b+A*(c*x^4+b*x^2)^(5/2)/b/x^4-1/16*b^
2*(-6*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(3/2)+1/16*(-6*A
*c+B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c
```

3.110.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^3} dx = \frac{x\left(\sqrt{c}x(b+cx^2)(3b^2B+4c^2x^2(3A+2Bx^2)+2bc(15A+7Bx^2))+3b^2(b\right.}{48c^{3/2}\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^3,x]
```

output
```
(x*(Sqrt[c]*x*(b + c*x^2)*(3*b^2*B + 4*c^2*x^2*(3*A + 2*B*x^2) + 2*b*c*(15
*A + 7*B*x^2)) + 3*b^2*(b*B - 6*A*c)*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sq
rt[b + c*x^2]]))/(48*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

3.110.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1940, 1220, 1131, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{ 1940}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)\left(cx^4+bx^2\right)^{3/2}}{x^4}dx^2$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{1}{2}\left(\frac{(bB-6Ac)\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^2}dx^2}{b}+\frac{2A\left(bx^2+cx^4\right)^{5/2}}{bx^4}\right)$$

$$\Big\downarrow \text{ 1131}$$

$$\frac{1}{2}\left(\frac{(bB-6Ac)\left(\frac{1}{2}b\int \sqrt{cx^4+bx^2}dx^2+\frac{1}{3}\left(bx^2+cx^4\right)^{3/2}\right)}{b}+\frac{2A\left(bx^2+cx^4\right)^{5/2}}{bx^4}\right)$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{1}{2}\left(\frac{(bB-6Ac)\left(\frac{1}{2}b\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2}{8c}\right)+\frac{1}{3}\left(bx^2+cx^4\right)^{3/2}\right)}{b}+\frac{2A\left(bx^2+cx^4\right)^{5/2}}{bx^4}\right)$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{1}{2}\left(\frac{(bB-6Ac)\left(\frac{1}{2}b\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{4c}\right)+\frac{1}{3}\left(bx^2+cx^4\right)^{3/2}\right)}{b}+\frac{2A\left(bx^2+cx^4\right)^{5/2}}{bx^4}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{(bB-6Ac)\left(\frac{1}{2}b\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{4c^{3/2}}\right)+\frac{1}{3}\left(bx^2+cx^4\right)^{3/2}\right)}{b}+\frac{2A\left(bx^2+cx^4\right)^{5/2}}{bx^4}\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^3,x]
```

output
```
((2*A*(b*x^2 + c*x^4)^(5/2))/(b*x^4) + ((b*B - 6*A*c)*((b*x^2 + c*x^4)^(3/
2)/3 + (b*(((b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(4*c) - (b^2*ArcTanh[(Sqrt[
c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(4*c^(3/2))))/2))/b)/2
```

3.110.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.110.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

method	result
risch	$\frac{\left(8B\,c^{2}x^{4}+12A\,c^{2}x^{2}+14Bbc\,x^{2}+30Abc+3B\,b^{2}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{48c}+\frac{b^{2}\left(6Ac-Bb\right)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{16c^{\frac{3}{2}}\,x\sqrt{c\,x^{2}+b}}$
default	$\frac{\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}\left(8B\sqrt{c}\,\left(c\,x^{2}+b\right)^{\frac{5}{2}}x+12A\,c^{\frac{3}{2}}\left(c\,x^{2}+b\right)^{\frac{3}{2}}x-2B\left(c\,x^{2}+b\right)^{\frac{3}{2}}\sqrt{c}\,bx+18A\sqrt{c\,x^{2}+b}\,c^{\frac{3}{2}}bx-3B\sqrt{c\,x^{2}+b}\,\sqrt{c}\,b^{2}x+18A\right.}{48x^{3}\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}}$
pseudoelliptic	$\frac{16B\,c^{\frac{5}{2}}x^{4}\sqrt{x^{2}\left(c\,x^{2}+b\right)}+24A\,c^{\frac{5}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,x^{2}+28B\,c^{\frac{3}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,b\,x^{2}+60A\,c^{\frac{3}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,b-18A\ln\left(2\right)b^{2}c+18A\ln}{96c^{\frac{3}{2}}}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/48/c*(8*B*c^2*x^4+12*A*c^2*x^2+14*B*b*c*x^2+30*A*b*c+3*B*b^2)*(x^2*(c*x^
2+b))^(1/2)+1/16*b^2*(6*A*c-B*b)/c^(3/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*(x^
2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.64

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^3}\,dx = \left[-\frac{3\,(Bb^3-6\,Ab^2c)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,(8\,Bc^3x^4}{96\,c^2}\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[-1/96*(3*(B*b^3 - 6*A*b^2*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*
x^2)*sqrt(c)) - 2*(8*B*c^3*x^4 + 3*B*b^2*c + 30*A*b*c^2 + 2*(7*B*b*c^2 + 6
*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^2, 1/48*(3*(B*b^3 - 6*A*b^2*c)*sqrt(-c
)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (8*B*c^3*x^4 + 3*B*b^
2*c + 30*A*b*c^2 + 2*(7*B*b*c^2 + 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^2]
```

3.110.6 Sympy [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^3}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}\,(A+Bx^2)}{x^3}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**3,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**3, x)
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^3}\,dx = \frac{1}{16}\left(\frac{3\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{\sqrt{c}}+6\,\sqrt{cx^4+bx^2}b+\frac{4\,(cx^4+bx^2)}{x^2}\right.$$
$$+\frac{1}{96}\left(12\,\sqrt{cx^4+bx^2}bx^2-\frac{3\,b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{3}{2}}}+16\,(cx^4+bx^2)^{\frac{3}{2}}+\frac{6\,\sqrt{cx^4+bx^2}b^2}{c}\right)B$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
1/16*(3*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) + 6*s
qrt(c*x^4 + b*x^2)*b + 4*(c*x^4 + b*x^2)^(3/2)/x^2)*A + 1/96*(12*sqrt(c*x^
4 + b*x^2)*b*x^2 - 3*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/
c^(3/2) + 16*(c*x^4 + b*x^2)^(3/2) + 6*sqrt(c*x^4 + b*x^2)*b^2/c)*B
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^3}\,dx = \frac{1}{48}\left(2\left(4\,Bcx^2\text{sgn}(x)+\frac{7\,Bbc^4\text{sgn}(x)+6\,Ac^5\text{sgn}(x)}{c^4}\right)x^2+\frac{3\,(Bb^2c^3\text{sgn}(x)$$
$$+\frac{(Bb^3\text{sgn}(x)-6\,Ab^2c\text{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{16\,c^{\frac{3}{2}}}$$
$$-\frac{(Bb^3\log(|b|)-6\,Ab^2c\log(|b|))\text{sgn}(x)}{32\,c^{\frac{3}{2}}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/48*(2*(4*B*c*x^2*sgn(x) + (7*B*b*c^4*sgn(x) + 6*A*c^5*sgn(x))/c^4)*x^2 +
 3*(B*b^2*c^3*sgn(x) + 10*A*b*c^4*sgn(x))/c^4)*sqrt(c*x^2 + b)*x + 1/16*(B
*b^3*sgn(x) - 6*A*b^2*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(
3/2) - 1/32*(B*b^3*log(abs(b)) - 6*A*b^2*c*log(abs(b)))*sgn(x)/c^(3/2)
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^3}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^3}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^3,x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^3, x)
```

3.111 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^5} dx$

3.111.1 Optimal result

Integrand size = 26, antiderivative size = 128

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^5} dx = \frac{3}{8}(bB+4Ac)\sqrt{bx^2+cx^4} + \frac{(bB+4Ac)(bx^2+cx^4)^{3/2}}{4bx^2} - \frac{A(bx^2+cx^4)^{5/2}}{bx^6} + \frac{3b(bB+4Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}$$

output
```
1/4*(4*A*c+B*b)*(c*x^4+b*x^2)^(3/2)/b/x^2-A*(c*x^4+b*x^2)^(5/2)/b/x^6+3/8*
b*(4*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(1/2)+3/8*(4*A*c+
B*b)*(c*x^4+b*x^2)^(1/2)
```

3.111.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^5} dx = \frac{\sqrt{c}(b+cx^2)(-8Ab+5bBx^2+4Acx^2+2Bcx^4)+6b(bB+4Ac)x\sqrt{b+c}}{8\sqrt{c}\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^5,x]
```

output
```
(Sqrt[c]*(b + c*x^2)*(-8*A*b + 5*b*B*x^2 + 4*A*c*x^2 + 2*B*c*x^4) + 6*b*(b
*B + 4*A*c)*x*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] + Sqrt[b + c*x
^2])])/(8*Sqrt[c]*Sqrt[x^2*(b + c*x^2)])
```

3.111.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1940, 1220, 1131, 1131, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)(cx^4+bx^2)^{3/2}}{x^6}dx^2$$

$$\Big\downarrow \text{1220}$$

$$\frac{1}{2}\left(\frac{(4Ac+bB)\int \frac{(cx^4+bx^2)^{3/2}}{x^4}dx^2}{b}-\frac{2A(bx^2+cx^4)^{5/2}}{bx^6}\right)$$

$$\Big\downarrow \text{1131}$$

$$\frac{1}{2}\left(\frac{(4Ac+bB)\left(\frac{3}{4}b\int \frac{\sqrt{cx^4+bx^2}}{x^2}dx^2+\frac{(bx^2+cx^4)^{3/2}}{2x^2}\right)}{b}-\frac{2A(bx^2+cx^4)^{5/2}}{bx^6}\right)$$

$$\Big\downarrow \text{1131}$$

$$\frac{1}{2}\left(\frac{(4Ac+bB)\left(\frac{3}{4}b\left(\frac{1}{2}b\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2+\sqrt{bx^2+cx^4}\right)+\frac{(bx^2+cx^4)^{3/2}}{2x^2}\right)}{b}-\frac{2A(bx^2+cx^4)^{5/2}}{bx^6}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{(4Ac+bB)\left(\frac{3}{4}b\left(b\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}+\sqrt{bx^2+cx^4}\right)+\frac{(bx^2+cx^4)^{3/2}}{2x^2}\right)}{b}-\frac{2A(bx^2+cx^4)^{5/2}}{bx^6}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{(4Ac+bB)\left(\frac{3}{4}b\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}+\sqrt{bx^2+cx^4}\right)+\frac{(bx^2+cx^4)^{3/2}}{2x^2}\right)}{b}-\frac{2A(bx^2+cx^4)^{5/2}}{bx^6}\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^5,x]
```

output
```
((-2*A*(b*x^2 + c*x^4)^(5/2))/(b*x^6) + ((b*B + 4*A*c)*((b*x^2 + c*x^4)^(3
/2)/(2*x^2) + (3*b*(Sqrt[b*x^2 + c*x^4] + (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*
x^2 + c*x^4]])/Sqrt[c]))/4))/b)/2
```

3.111.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.111.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{\left(-2Bc\,x^{4}-4Ac\,x^{2}-5bB\,x^{2}+8Ab\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{8x^{2}}+\frac{3b\left(4Ac+Bb\right)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{8\sqrt{c}\,x\sqrt{c\,x^{2}+b}}$
pseudoelliptic	$\frac{\frac{x^{2}\left(\frac{x^{2}B}{2}+A\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,c^{\frac{3}{2}}}{2}+\frac{3\left(-\frac{4\sqrt{x^{2}\left(c\,x^{2}+b\right)}\left(-\frac{5x^{2}B}{8}+A\right)\sqrt{c}}{3}+\left(-\ln(2)+\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)\right)x^{2}\left(Ac+\frac{Bb}{4}\right)\right)}{4}}{x^{2}\sqrt{c}}$
default	$-\frac{\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}\left(-8A\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}x^{2}-2B\left(c\,x^{2}+b\right)^{\frac{3}{2}}\sqrt{c}\,b\,x^{2}+8A\left(c\,x^{2}+b\right)^{\frac{5}{2}}\sqrt{c}-12A\sqrt{c\,x^{2}+b}\,c^{\frac{3}{2}}b\,x^{2}-3B\sqrt{c\,x^{2}+b}\,\sqrt{c}\,b^{2}x^{2}\right.}{8x^{4}\left(c\,x^{2}+b\right)^{\frac{3}{2}}b\sqrt{c}}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-2*B*c*x^4-4*A*c*x^2-5*B*b*x^2+8*A*b)/x^2*(x^2*(c*x^2+b))^(1/2)+3/8*
b*(4*A*c+B*b)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))/c^(1/2)*(x^2*(c*x^2+b))^(1/2)/
x/(c*x^2+b)^(1/2)
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.63

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^5}\,dx = \left[\frac{3\,(Bb^2+4\,Abc)\sqrt{c}x^2\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,(2\,Bc^2x^4-}{16\,cx^2}\right.$$
$$\left.-\frac{3\,(Bb^2+4\,Abc)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-(2\,Bc^2x^4-8\,Abc+(5\,Bbc+4\,Ac^2)x^2)\sqrt{cx^4+bx^2}}{8\,cx^2}\right]$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/16*(3*(B*b^2 + 4*A*b*c)*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b
*x^2)*sqrt(c)) + 2*(2*B*c^2*x^4 - 8*A*b*c + (5*B*b*c + 4*A*c^2)*x^2)*sqrt(
c*x^4 + b*x^2))/(c*x^2), -1/8*(3*(B*b^2 + 4*A*b*c)*sqrt(-c)*x^2*arctan(sqr
t(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (2*B*c^2*x^4 - 8*A*b*c + (5*B*b*c
 + 4*A*c^2)*x^2)*sqrt(c*x^4 + b*x^2))/(c*x^2)]
```

3.111.6 Sympy [F]

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^5}\,dx = \int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}(A+Bx^2)}{x^5}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**5,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**5, x)
```

3.111.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.16

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^5}\,dx = \frac{1}{4}\left(3\,b\sqrt{c}\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right) - \frac{6\,\sqrt{cx^4+bx^2}b}{x^2} + \frac{2\,(cx^4+}{x^4}\right.$$
$$+ \frac{1}{16}\left(\frac{3\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{\sqrt{c}} + 6\,\sqrt{cx^4+bx^2}b + \frac{4\,(cx^4+bx^2)^{\frac{3}{2}}}{x^2}\right)B$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
1/4*(3*b*sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 6*sqrt
(c*x^4 + b*x^2)*b/x^2 + 2*(c*x^4 + b*x^2)^(3/2)/x^4)*A + 1/16*(3*b^2*log(2
*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) + 6*sqrt(c*x^4 + b*x^2
)*b + 4*(c*x^4 + b*x^2)^(3/2)/x^2)*B
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^5}\,dx = \frac{2\,Ab^2\sqrt{c}\mathrm{sgn}(x)}{\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b} + \frac{1}{8}\left(2\,Bcx^2\mathrm{sgn}(x)+\frac{5\,Bbc^2\mathrm{sgn}(x)+4\,Ac^3\mathrm{sgn}(x)}{c^2}\right)\sqrt{cx^2+b}x - \frac{3\,(Bb^2\mathrm{sgn}(x)+4\,Abc\mathrm{sgn}(x))\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)}{16\,\sqrt{c}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="giac")
```

output
```
2*A*b^2*sqrt(c)*sgn(x)/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b) + 1/8*(2*B*c*
x^2*sgn(x) + (5*B*b*c^2*sgn(x) + 4*A*c^3*sgn(x))/c^2)*sqrt(c*x^2 + b)*x -
3/16*(B*b^2*sgn(x) + 4*A*b*c*sgn(x))*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)/
sqrt(c)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^5}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^5}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^5,x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^5, x)
```

3.112 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7}\,dx$

3.112.1 Optimal result

Integrand size = 26, antiderivative size = 136

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7}\,dx = \frac{c(3bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{(3bB+2Ac)(bx^2+cx^4)^{3/2}}{3bx^4} - \frac{A(bx^2+cx^4)^{5/2}}{3bx^8} + \frac{1}{2}\sqrt{c}(3bB+2Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

output
```
-1/3*(2*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^4-1/3*A*(c*x^4+b*x^2)^(5/2)/b/x
^8+1/2*(2*A*c+3*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))*c^(1/2)+1/2*
c*(2*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/b
```

3.112.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7}\,dx = \frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b+cx^2}(-6bBx^2+3Bcx^4-2A(b+4cx^2))+6\sqrt{c}(3bB\right.}{6x^4\sqrt{b+cx^2}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^7,x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b + c*x^2]*(-6*b*B*x^2 + 3*B*c*x^4 - 2*A*(b +
 4*c*x^2)) + 6*Sqrt[c]*(3*b*B + 2*A*c)*x^3*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] +
 Sqrt[b + c*x^2])]))/(6*x^4*Sqrt[b + c*x^2])
```

3.112.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1940, 1220, 1125, 25, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)(cx^4+bx^2)^{3/2}}{x^8}dx^2$$

$$\Big\downarrow \text{1220}$$

$$\frac{1}{2}\left(\frac{(2Ac+3bB)\int \frac{(cx^4+bx^2)^{3/2}}{x^6}dx^2}{3b}-\frac{2A(bx^2+cx^4)^{5/2}}{3bx^8}\right)$$

$$\Big\downarrow \text{1125}$$

$$\frac{1}{2}\left(\frac{(2Ac+3bB)\left(-\int -\frac{c(cx^2+2b)}{\sqrt{cx^4+bx^2}}dx^2-\frac{2b\sqrt{bx^2+cx^4}}{x^2}\right)}{3b}-\frac{2A(bx^2+cx^4)^{5/2}}{3bx^8}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{(2Ac+3bB)\left(\int \frac{c(cx^2+2b)}{\sqrt{cx^4+bx^2}}dx^2-\frac{2b\sqrt{bx^2+cx^4}}{x^2}\right)}{3b}-\frac{2A(bx^2+cx^4)^{5/2}}{3bx^8}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{(2Ac+3bB)\left(c\int \frac{cx^2+2b}{\sqrt{cx^4+bx^2}}dx^2-\frac{2b\sqrt{bx^2+cx^4}}{x^2}\right)}{3b}-\frac{2A(bx^2+cx^4)^{5/2}}{3bx^8}\right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{(2Ac+3bB)\left(c\left(\frac{3}{2}b\int\frac{1}{\sqrt{cx^4+bx^2}}dx^2+\sqrt{bx^2+cx^4}\right)-\frac{2b\sqrt{bx^2+cx^4}}{x^2}\right)}{3b}-\frac{2A(bx^2+cx^4)^{5/2}}{3bx^8}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{(2Ac+3bB)\left(c\left(3b\int\frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}+\sqrt{bx^2+cx^4}\right)-\frac{2b\sqrt{bx^2+cx^4}}{x^2}\right)}{3b}-\frac{2A(bx^2+cx^4)^{5/2}}{3bx^8}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{(2Ac+3bB)\left(c\left(\frac{3b\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}+\sqrt{bx^2+cx^4}\right)-\frac{2b\sqrt{bx^2+cx^4}}{x^2}\right)}{3b}-\frac{2A(bx^2+cx^4)^{5/2}}{3bx^8}\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^7,x]
```

output
```
((-2*A*(b*x^2 + c*x^4)^(5/2))/(3*b*x^8) + ((3*b*B + 2*A*c)*((-2*b*Sqrt[b*x
^2 + c*x^4])/x^2 + c*(Sqrt[b*x^2 + c*x^4] + (3*b*ArcTanh[(Sqrt[c]*x^2)/Sqr
t[b*x^2 + c*x^4]])/Sqrt[c])))/(3*b))/2
```

3.112.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.112. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7} dx$

3.112.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{\left(-3Bc\,x^4+8Ac\,x^2+6bB\,x^2+2Ab\right)\sqrt{x^2\left(c\,x^2+b\right)}}{6x^4}+\frac{\left(2Ac+3Bb\right)\sqrt{c}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)\sqrt{x^2\left(c\,x^2+b\right)}}{2x\sqrt{c\,x^2+b}}$
pseudoelliptic	$\frac{3x^4\left(3\sqrt{c}\,Bb+2A\,c^{\frac{3}{2}}\right)\ln\left(\frac{2c\,x^2+2\sqrt{x^2\left(c\,x^2+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)+2\left(3Bc\,x^4+2(-4Ac-3Bb)x^2-2Ab\right)\sqrt{x^2\left(c\,x^2+b\right)}-6x^4\ln(2)\left(\frac{3\sqrt{c}\,Bb}{2}\right.}{12x^4}$
default	$-\frac{\left(x^4c+b\,x^2\right)^{\frac{3}{2}}\left(-4A\left(c\,x^2+b\right)^{\frac{3}{2}}c^{\frac{5}{2}}x^4-6B\left(c\,x^2+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}b\,x^4+4A\left(c\,x^2+b\right)^{\frac{5}{2}}c^{\frac{3}{2}}x^2-6A\sqrt{c\,x^2+b}\,c^{\frac{5}{2}}b\,x^4+6B\left(c\,x^2+b\right)^{\frac{5}{2}}\sqrt{c}\,b\,x\right.}{6x^6\left(c\,x^2+b\right)^{\frac{3}{2}}b^2}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-3*B*c*x^4+8*A*c*x^2+6*B*b*x^2+2*A*b)/x^4*(x^2*(c*x^2+b))^(1/2)+1/2*
(2*A*c+3*B*b)*c^(1/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*(x^2*(c*x^2+b))^(1/2)/
x/(c*x^2+b)^(1/2)
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.39

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^7}\,dx = \left[\frac{3\,(3\,Bb+2\,Ac)\sqrt{c}x^4\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,(3\,Bcx^4-}{12\,x^4}\right.$$

$$\left.-\frac{3\,(3\,Bb+2\,Ac)\sqrt{-c}x^4\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-(3\,Bcx^4-2\,(3\,Bb+4\,Ac)x^2-2\,Ab)\sqrt{cx^4+bx^2}}{6\,x^4}\right]$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[1/12*(3*(3*B*b + 2*A*c)*sqrt(c)*x^4*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x
^2)*sqrt(c)) + 2*(3*B*c*x^4 - 2*(3*B*b + 4*A*c)*x^2 - 2*A*b)*sqrt(c*x^4 +
b*x^2))/x^4, -1/6*(3*(3*B*b + 2*A*c)*sqrt(-c)*x^4*arctan(sqrt(c*x^4 + b*x^
2)*sqrt(-c)/(c*x^2 + b)) - (3*B*c*x^4 - 2*(3*B*b + 4*A*c)*x^2 - 2*A*b)*sqr
t(c*x^4 + b*x^2))/x^4]
```

3.112.6 Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)}{x^7}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**7,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**7, x)
```

3.112.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7}\,dx = \frac{1}{6}\left(3\,c^{\frac{3}{2}}\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right) - \frac{7\,\sqrt{cx^4+bx^2}c}{x^2} - \frac{\sqrt{cx^4+bx}}{x^4}\right. \\ + \frac{1}{4}\left(3\,b\sqrt{c}\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right) - \frac{6\,\sqrt{cx^4+bx^2}b}{x^2} + \frac{2\,(cx^4+bx^2)^{\frac{3}{2}}}{x^4}\right)B$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
1/6*(3*c^(3/2)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 7*sqrt(c
*x^4 + b*x^2)*c/x^2 - sqrt(c*x^4 + b*x^2)*b/x^4 - (c*x^4 + b*x^2)^(3/2)/x^
6)*A + 1/4*(3*b*sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) -
 6*sqrt(c*x^4 + b*x^2)*b/x^2 + 2*(c*x^4 + b*x^2)^(3/2)/x^4)*B
```

3.112.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.65

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7}\,dx = \frac{1}{2}\sqrt{cx^2+b}Bcx\mathrm{sgn}(x) \\ - \frac{1}{4}\left(3\,Bb\sqrt{c}\mathrm{sgn}(x) + 2\,Ac^{\frac{3}{2}}\mathrm{sgn}(x)\right)\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right) \\ + \frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Bb^2\sqrt{c}\mathrm{sgn}(x) + 6\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Abc^{\frac{3}{2}}\mathrm{sgn}(x) - 6\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2Bb^3\sqrt{cs}\right.}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="giac")
```

output
```
1/2*sqrt(c*x^2 + b)*B*c*x*sgn(x) - 1/4*(3*B*b*sqrt(c)*sgn(x) + 2*A*c^(3/2)
*sgn(x))*log((sqrt(c)*x - sqrt(c*x^2 + b))^2) + 2/3*(3*(sqrt(c)*x - sqrt(c
*x^2 + b))^4*B*b^2*sqrt(c)*sgn(x) + 6*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b*
c^(3/2)*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^3*sqrt(c)*sgn(x) -
6*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^2*c^(3/2)*sgn(x) + 3*B*b^4*sqrt(c)*s
gn(x) + 4*A*b^3*c^(3/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^3
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7}\,dx = \int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2)^{3/2}}{x^7}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^7,x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^7, x)
```

3.113 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9}\,dx$

3.113.1 Optimal result

Integrand size = 26, antiderivative size = 104

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9}\,dx = -\frac{Bc\sqrt{bx^2+cx^4}}{x^2}-\frac{B(bx^2+cx^4)^{3/2}}{3x^6} - \frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}}+Bc^{3/2}\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

output
```
-1/3*B*(c*x^4+b*x^2)^(3/2)/x^6-1/5*A*(c*x^4+b*x^2)^(5/2)/b/x^10+B*c^(3/2)*
arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))-B*c*(c*x^4+b*x^2)^(1/2)/x^2
```

3.113.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9}\,dx = -\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b+cx^2}\left(3A(b+cx^2)^2+5bBx^2(b+4cx^2)\right)+15bBc^{3/2}x^5\log\left(-\sqrt{c}x+\sqrt{b+cx^2}\right)\right)}{15bx^6\sqrt{b+cx^2}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9,x]
```

output
```
-1/15*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b + c*x^2]*(3*A*(b + c*x^2)^2 + 5*b*B*x
^2*(b + 4*c*x^2)) + 15*b*B*c^(3/2)*x^5*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]
))/(b*x^6*Sqrt[b + c*x^2])
```

3.113.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1940, 1220, 1130, 1125, 25, 27, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^9}dx$$

$$\Big\downarrow \ 1940$$

$$\frac{1}{2}\int \frac{(Bx^2+A)\left(cx^4+bx^2\right)^{3/2}}{x^{10}}dx^2$$

$$\Big\downarrow \ 1220$$

$$\frac{1}{2}\left(B\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^8}dx^2-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{10}}\right)$$

$$\Big\downarrow \ 1130$$

$$\frac{1}{2}\left(B\left(c\int \frac{\sqrt{cx^4+bx^2}}{x^4}dx^2-\frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^6}\right)-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{10}}\right)$$

$$\Big\downarrow \ 1125$$

$$\frac{1}{2}\left(B\left(c\left(-\int -\frac{c}{\sqrt{cx^4+bx^2}}dx^2-\frac{2\sqrt{bx^2+cx^4}}{x^2}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^6}\right)-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{10}}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(B\left(c\left(\int \frac{c}{\sqrt{cx^4+bx^2}}dx^2-\frac{2\sqrt{bx^2+cx^4}}{x^2}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^6}\right)-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{10}}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(B\left(c\left(c\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2-\frac{2\sqrt{bx^2+cx^4}}{x^2}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^6}\right)-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{10}}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(B\left(c\left(2c\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}-\frac{2\sqrt{bx^2+cx^4}}{x^2}\right)-\frac{2(bx^2+cx^4)^{3/2}}{3x^6}\right)-\frac{2A(bx^2+cx^4)^{5/2}}{5bx^{10}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(B\left(c\left(2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{2\sqrt{bx^2+cx^4}}{x^2}\right)-\frac{2(bx^2+cx^4)^{3/2}}{3x^6}\right)-\frac{2A(bx^2+cx^4)^{5/2}}{5bx^{10}}\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9,x]
```

output
```
((-2*A*(b*x^2 + c*x^4)^(5/2))/(5*b*x^10) + B*((-2*(b*x^2 + c*x^4)^(3/2))/(
3*x^6) + c*((-2*Sqrt[b*x^2 + c*x^4])/x^2 + 2*Sqrt[c]*ArcTanh[(Sqrt[c]*x^2)
/Sqrt[b*x^2 + c*x^4]])))/2
```

3.113.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.113.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{5x^6\left(-\ln(2)+\ln\left(\frac{2c\,x^2+2\sqrt{x^2\left(c\,x^2+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)\right)Bb\,c^{\frac{3}{2}}-2\left(\left(\frac{5x^2B}{3}+A\right)b^2+2x^2\left(\frac{10x^2B}{3}+A\right)cb+A\,c^2x^4\right)\sqrt{x^2(c\,x^2+b)}}{10b\,x^6}$
risch	$-\frac{(3A\,c^2x^4+20x^4Bbc+6Abc\,x^2+5b^2B\,x^2+3b^2A)\sqrt{x^2(c\,x^2+b)}}{15x^6b}+\frac{B\,c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)\sqrt{x^2(c\,x^2+b)}}{x\sqrt{c\,x^2+b}}$
default	$-\frac{(x^4c+b\,x^2)^{\frac{3}{2}}\left(-10B(c\,x^2+b)^{\frac{3}{2}}c^{\frac{5}{2}}x^6+10B(c\,x^2+b)^{\frac{5}{2}}c^{\frac{3}{2}}x^4-15B\sqrt{c\,x^2+b}\,c^{\frac{5}{2}}b\,x^6-15B\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)b^2c^2x^5+5B(\right.}{15x^8(c\,x^2+b)^{\frac{3}{2}}b^2\sqrt{c}}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/10*(5*x^6*(-ln(2)+ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2)*c^(1/2)+b)/c^(1/2)
))*B*b*c^(3/2)-2*((5/3*x^2*B+A)*b^2+2*x^2*(10/3*x^2*B+A)*c*b+A*c^2*x^4)*(x
^2*(c*x^2+b))^(1/2))/b/x^6
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.99

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9}\,dx = \left[\frac{15\,Bbc^{\frac{3}{2}}x^6\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,((20\,Bbc+3\,Ac^2)x^4}{30\,bx^6}\right.$$

$$\left.-\frac{15\,Bb\sqrt{-c}cx^6\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)+((20\,Bbc+3\,Ac^2)x^4+3\,Ab^2+(5\,Bb^2+6\,Abc)x^2)\sqrt{cx^4+bx^2}}{15\,bx^6}\right]$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
[1/30*(15*B*b*c^(3/2)*x^6*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)
) - 2*((20*B*b*c + 3*A*c^2)*x^4 + 3*A*b^2 + (5*B*b^2 + 6*A*b*c)*x^2)*sqrt(
c*x^4 + b*x^2))/(b*x^6), -1/15*(15*B*b*sqrt(-c)*c*x^6*arctan(sqrt(c*x^4 +
b*x^2)*sqrt(-c)/(c*x^2 + b)) + ((20*B*b*c + 3*A*c^2)*x^4 + 3*A*b^2 + (5*B*
b^2 + 6*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b*x^6)]
```

3.113.6 Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)}{x^9}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**9,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**9, x)
```

3.113.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(88) = 176.

Time = 0.21 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.70

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9}\,dx = \frac{1}{6}\left(3\,c^{\frac{3}{2}}\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-\frac{7\,\sqrt{cx^4+bx^2}c}{x^2}-\frac{\sqrt{cx^4+bx^2}\ldots}{x^4}\right.$$
$$-\frac{1}{10}\,A\left(\frac{2\,\sqrt{cx^4+bx^2}c^2}{bx^2}-\frac{\sqrt{cx^4+bx^2}c}{x^4}-\frac{3\,\sqrt{cx^4+bx^2}b}{x^6}+\frac{5\,(cx^4+bx^2)^{\frac{3}{2}}}{x^8}\right)$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
1/6*(3*c^(3/2)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 7*sqrt(c
*x^4 + b*x^2)*c/x^2 - sqrt(c*x^4 + b*x^2)*b/x^4 - (c*x^4 + b*x^2)^(3/2)/x^
6)*B - 1/10*A*(2*sqrt(c*x^4 + b*x^2)*c^2/(b*x^2) - sqrt(c*x^4 + b*x^2)*c/x
^4 - 3*sqrt(c*x^4 + b*x^2)*b/x^6 + 5*(c*x^4 + b*x^2)^(3/2)/x^8)
```

3.113.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(88) = 176.

Time = 0.83 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.44

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9}\,dx = -\frac{1}{2}\,Bc^{\frac{3}{2}}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)\mathrm{sgn}(x)$$
$$+\frac{2\left(30\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8Bbc^{\frac{3}{2}}\mathrm{sgn}(x)+15\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8Ac^{\frac{5}{2}}\mathrm{sgn}(x)-90\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6Bb^2c^{\frac{3}{2}}\mathrm{s}\ldots\right.}{\ldots}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="giac")
```

output
```
-1/2*B*c^(3/2)*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sgn(x) + 2/15*(30*(sqr
t(c)*x - sqrt(c*x^2 + b))^8*B*b*c^(3/2)*sgn(x) + 15*(sqrt(c)*x - sqrt(c*x^
2 + b))^8*A*c^(5/2)*sgn(x) - 90*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^2*c^(3
/2)*sgn(x) + 110*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^3*c^(3/2)*sgn(x) + 30
*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^2*c^(5/2)*sgn(x) - 70*(sqrt(c)*x - sq
rt(c*x^2 + b))^2*B*b^4*c^(3/2)*sgn(x) + 20*B*b^5*c^(3/2)*sgn(x) + 3*A*b^4*
c^(5/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^5
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^9}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9,x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9, x)
```

3.114 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11}}\,dx$

3.114.1 Optimal result

Integrand size = 26, antiderivative size = 61

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11}}\,dx = -\frac{A(bx^2+cx^4)^{5/2}}{7bx^{12}} - \frac{(7bB-2Ac)(bx^2+cx^4)^{5/2}}{35b^2x^{10}}$$

output
```
-1/7*A*(c*x^4+b*x^2)^(5/2)/b/x^12-1/35*(-2*A*c+7*B*b)*(c*x^4+b*x^2)^(5/2)/
b^2/x^10
```

3.114.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11}}\,dx = \frac{(x^2(b+cx^2))^{5/2}(-5Ab-7bBx^2+2Acx^2)}{35b^2x^{12}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^11,x]
```

output
```
((x^2*(b + c*x^2))^(5/2)*(-5*A*b - 7*b*B*x^2 + 2*A*c*x^2))/(35*b^2*x^12)
```

3.114.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1940, 1220, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11}}\,dx$$

$$\Big\downarrow \text{ 1940}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)(cx^4+bx^2)^{3/2}}{x^{12}}dx^2$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{1}{2}\left(\frac{(7bB-2Ac)\int \frac{(cx^4+bx^2)^{3/2}}{x^{10}}dx^2}{7b}-\frac{2A(bx^2+cx^4)^{5/2}}{7bx^{12}}\right)$$

$$\Big\downarrow \text{ 1123}$$

$$\frac{1}{2}\left(-\frac{2(bx^2+cx^4)^{5/2}(7bB-2Ac)}{35b^2x^{10}}-\frac{2A(bx^2+cx^4)^{5/2}}{7bx^{12}}\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^11,x]
```

output
```
((-2*A*(b*x^2 + c*x^4)^(5/2))/(7*b*x^12) - (2*(7*b*B - 2*A*c)*(b*x^2 + c*x
^4)^(5/2))/(35*b^2*x^10))/2
```

3.114.3.1 Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.114.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79

method	result	size
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-2A c\,x^{2}+7bB\,x^{2}+5Ab\right)\left(x^{4} c+b\,x^{2}\right)^{\frac{3}{2}}}{35b^{2} x^{10}}$	48
default	$-\frac{\left(c\,x^{2}+b\right)\left(-2A c\,x^{2}+7bB\,x^{2}+5Ab\right)\left(x^{4} c+b\,x^{2}\right)^{\frac{3}{2}}}{35b^{2} x^{10}}$	48
pseudoelliptic	$-\frac{\left(\left(\frac{7x^{2} B}{5}+A\right)b-\frac{2Ac\,x^{2}}{5}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(c\,x^{2}+b\right)^{2}}{7x^{8} b^{2}}$	49
trager	$-\frac{\left(-2A\,c^{3} x^{6}+7x^{6} Bb\,c^{2}+Ab\,c^{2} x^{4}+14x^{4} B\,b^{2} c+8A\,b^{2} c\,x^{2}+7b^{3} B\,x^{2}+5b^{3} A\right)\sqrt{x^{4} c+b\,x^{2}}}{35b^{2} x^{8}}$	86
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(-2A\,c^{3} x^{6}+7x^{6} Bb\,c^{2}+Ab\,c^{2} x^{4}+14x^{4} B\,b^{2} c+8A\,b^{2} c\,x^{2}+7b^{3} B\,x^{2}+5b^{3} A\right)}{35x^{8} b^{2}}$	86

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/35*(c*x^2+b)*(-2*A*c*x^2+7*B*b*x^2+5*A*b)*(c*x^4+b*x^2)^(3/2)/b^2/x^10
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{11}}\,dx = -\frac{((7\,Bbc^2-2\,Ac^3)x^6+(14\,Bb^2c+Abc^2)x^4+5\,Ab^3+(7\,Bb^3+8\,Ab^2c)x^2)\sqrt{cx^4+bx^2}}{35\,b^2x^8}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="fricas")
```

output
```
-1/35*((7*B*b*c^2 - 2*A*c^3)*x^6 + (14*B*b^2*c + A*b*c^2)*x^4 + 5*A*b^3 +
(7*B*b^3 + 8*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^2*x^8)
```

3.114.6 Sympy [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{11}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}\,(A+Bx^2)}{x^{11}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**11,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**11, x)
```

3.114.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(53) = 106.

Time = 0.23 (sec) , antiderivative size = 193, normalized size of antiderivative = 3.16

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{11}}\,dx = -\frac{1}{10}\,B\left(\frac{2\,\sqrt{cx^4+bx^2}c^2}{bx^2}-\frac{\sqrt{cx^4+bx^2}c}{x^4}-\frac{3\,\sqrt{cx^4+bx^2}b}{x^6}+\frac{5\,(cx^4+bx^2)^{\frac{3}{2}}}{x^8}\right) + \frac{1}{140}\,A\left(\frac{8\,\sqrt{cx^4+bx^2}c^3}{b^2x^2}-\frac{4\,\sqrt{cx^4+bx^2}c^2}{bx^4}+\frac{3\,\sqrt{cx^4+bx^2}c}{x^6}+\frac{15\,\sqrt{cx^4+bx^2}b}{x^8}-\frac{35\,(cx^4+bx^2)^{\frac{3}{2}}}{x^{10}}\right)$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="maxima")
```

output
```
-1/10*B*(2*sqrt(c*x^4 + b*x^2)*c^2/(b*x^2) - sqrt(c*x^4 + b*x^2)*c/x^4 - 3
*sqrt(c*x^4 + b*x^2)*b/x^6 + 5*(c*x^4 + b*x^2)^(3/2)/x^8) + 1/140*A*(8*sqr
t(c*x^4 + b*x^2)*c^3/(b^2*x^2) - 4*sqrt(c*x^4 + b*x^2)*c^2/(b*x^4) + 3*sqr
t(c*x^4 + b*x^2)*c/x^6 + 15*sqrt(c*x^4 + b*x^2)*b/x^8 - 35*(c*x^4 + b*x^2)
^(3/2)/x^10)
```

3.114.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(53) = 106$.

Time = 1.19 (sec) , antiderivative size = 370, normalized size of antiderivative = 6.07

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11}}\,dx = \frac{2\left(35\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Bc^{\frac{5}{2}}\mathrm{sgn}(x)-70\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bbc^{\frac{5}{2}}\mathrm{sgn}(\right.}{}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="giac")
```

output
```
2/35*(35*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*c^(5/2)*sgn(x) - 70*(sqrt(c)*x
 - sqrt(c*x^2 + b))^10*B*b*c^(5/2)*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + b
))^10*A*c^(7/2)*sgn(x) + 105*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^2*c^(5/2)
*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b*c^(7/2)*sgn(x) - 140*(sqr
t(c)*x - sqrt(c*x^2 + b))^6*B*b^3*c^(5/2)*sgn(x) + 140*(sqrt(c)*x - sqrt(c
*x^2 + b))^6*A*b^2*c^(7/2)*sgn(x) + 77*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b
^4*c^(5/2)*sgn(x) + 28*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^3*c^(7/2)*sgn(x
) - 14*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^5*c^(5/2)*sgn(x) + 14*(sqrt(c)*
x - sqrt(c*x^2 + b))^2*A*b^4*c^(7/2)*sgn(x) + 7*B*b^6*c^(5/2)*sgn(x) - 2*A
*b^5*c^(7/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^7
```

3.114.9 Mupad [B] (verification not implemented)

Time = 9.82 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.56

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{11}}\,dx=\frac{2\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{35\,b^2\,x^2}-\frac{8\,A\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,x^6}-\frac{B\,b\,\sqrt{c\,x^4+b\,x^2}}{5\,x^6}-\frac{2\,B\,c\,\sqrt{c\,x^4+b\,x^2}}{5\,x^4}-\frac{A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{35\,b\,x^4}-\frac{A\,b\,\sqrt{c\,x^4+b\,x^2}}{7\,x^8}-\frac{B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{5\,b\,x^2}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^11,x)
```

output
```
(2*A*c^3*(b*x^2 + c*x^4)^(1/2))/(35*b^2*x^2) - (8*A*c*(b*x^2 + c*x^4)^(1/2
))/(35*x^6) - (B*b*(b*x^2 + c*x^4)^(1/2))/(5*x^6) - (2*B*c*(b*x^2 + c*x^4)
^(1/2))/(5*x^4) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(35*b*x^4) - (A*b*(b*x^2 +
 c*x^4)^(1/2))/(7*x^8) - (B*c^2*(b*x^2 + c*x^4)^(1/2))/(5*b*x^2)
```

3.115 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}}\,dx$

3.115.1 Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = -\frac{A(bx^2+cx^4)^{5/2}}{9bx^{14}} - \frac{(9bB-4Ac)(bx^2+cx^4)^{5/2}}{63b^2x^{12}} + \frac{2c(9bB-4Ac)(bx^2+cx^4)^{5/2}}{315b^3x^{10}}$$

output
```
-1/9*A*(c*x^4+b*x^2)^(5/2)/b/x^14-1/63*(-4*A*c+9*B*b)*(c*x^4+b*x^2)^(5/2)/
b^2/x^12+2/315*c*(-4*A*c+9*B*b)*(c*x^4+b*x^2)^(5/2)/b^3/x^10
```

3.115.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \frac{(x^2(b+cx^2))^{5/2}(9bBx^2(-5b+2cx^2)+A(-35b^2+20bcx^2-8c^2x^4))}{315b^3x^{14}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^13,x]
```

output
```
((x^2*(b + c*x^2))^(5/2)*(9*b*B*x^2*(-5*b + 2*c*x^2) + A*(-35*b^2 + 20*b*c
*x^2 - 8*c^2*x^4)))/(315*b^3*x^14)
```

3.115.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1940, 1220, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{13}}\,dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)\left(cx^4+bx^2\right)^{3/2}}{x^{14}}dx^2$$

$$\Big\downarrow \text{1220}$$

$$\frac{1}{2}\left(\frac{(9bB-4Ac)\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^{12}}dx^2}{9b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}\right)$$

$$\Big\downarrow \text{1129}$$

$$\frac{1}{2}\left(\frac{(9bB-4Ac)\left(-\frac{2c\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^{10}}dx^2}{7b}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{7bx^{12}}\right)}{9b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}\right)$$

$$\Big\downarrow \text{1123}$$

$$\frac{1}{2}\left(\frac{\left(\frac{4c\left(bx^2+cx^4\right)^{5/2}}{35b^2x^{10}}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{7bx^{12}}\right)(9bB-4Ac)}{9b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^13,x]
```

output
```
((-2*A*(b*x^2 + c*x^4)^(5/2))/(9*b*x^14) + ((9*b*B - 4*A*c)*((-2*(b*x^2 +
c*x^4)^(5/2))/(7*b*x^12) + (4*c*(b*x^2 + c*x^4)^(5/2))/(35*b^2*x^10)))/(9*
b))/2
```

3.115.3.1 Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.115. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}}\,dx$

3.115.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.71

method	result	s
pseudoelliptic	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(c\,x^{2}+b\right)^{2}\left(\left(\frac{9x^{2}B}{7}+A\right)b^{2}-\frac{4x^{2}\left(\frac{9x^{2}B}{10}+A\right)cb}{7}+\frac{8A\,c^{2}x^{4}}{35}\right)}{9x^{10}b^{3}}$	6
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(8A\,c^{2}x^{4}-18x^{4}Bbc-20Abc\,x^{2}+45b^{2}B\,x^{2}+35b^{2}A\right)\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}}{315b^{3}x^{12}}$	7
default	$-\frac{\left(c\,x^{2}+b\right)\left(8A\,c^{2}x^{4}-18x^{4}Bbc-20Abc\,x^{2}+45b^{2}B\,x^{2}+35b^{2}A\right)\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}}{315b^{3}x^{12}}$	7
trager	$-\frac{\left(8A\,x^{8}c^{4}-18B\,x^{8}b\,c^{3}-4A\,x^{6}b\,c^{3}+9B\,x^{6}b^{2}c^{2}+3A\,b^{2}c^{2}x^{4}+72B\,b^{3}c\,x^{4}+50A\,x^{2}b^{3}c+45B\,x^{2}b^{4}+35A\,b^{4}\right)\sqrt{x^{4}c+b\,x^{2}}}{315b^{3}x^{10}}$	1
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(8A\,x^{8}c^{4}-18B\,x^{8}b\,c^{3}-4A\,x^{6}b\,c^{3}+9B\,x^{6}b^{2}c^{2}+3A\,b^{2}c^{2}x^{4}+72B\,b^{3}c\,x^{4}+50A\,x^{2}b^{3}c+45B\,x^{2}b^{4}+35A\,b^{4}\right)}{315x^{10}b^{3}}$	1

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/9*(x^2*(c*x^2+b))^(1/2)*(c*x^2+b)^2*((9/7*x^2*B+A)*b^2-4/7*x^2*(9/10*x^
2*B+A)*c*b+8/35*A*c^2*x^4)/x^10/b^3
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \frac{(2\,(9\,Bbc^3-4\,Ac^4)x^8-(9\,Bb^2c^2-4\,Abc^3)x^6-35\,Ab^4-3\,(24\,Bb^3c+Ab}{315\,b^3x^{10}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="fricas")
```

output
```
1/315*(2*(9*B*b*c^3 - 4*A*c^4)*x^8 - (9*B*b^2*c^2 - 4*A*b*c^3)*x^6 - 35*A*
b^4 - 3*(24*B*b^3*c + A*b^2*c^2)*x^4 - 5*(9*B*b^4 + 10*A*b^3*c)*x^2)*sqrt(
c*x^4 + b*x^2)/(b^3*x^10)
```

3.115.6 Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)}{x^{13}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**13,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**13, x)
```

3.115.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(84) = 168.

Time = 0.22 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.51

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \frac{1}{140}B\left(\frac{8\sqrt{cx^4+bx^2}c^3}{b^2x^2} - \frac{4\sqrt{cx^4+bx^2}c^2}{bx^4} + \frac{3\sqrt{cx^4+bx^2}c}{x^6} + \frac{15\sqrt{cx^4+\cdots}}{x^8}\right.$$
$$-\frac{1}{630}A\left(\frac{16\sqrt{cx^4+bx^2}c^4}{b^3x^2} - \frac{8\sqrt{cx^4+bx^2}c^3}{b^2x^4} + \frac{6\sqrt{cx^4+bx^2}c^2}{bx^6} - \frac{5\sqrt{cx^4+bx^2}c}{x^8} - \frac{35\sqrt{cx^4+bx^2}b}{x^{10}} + \frac{105\cdots}{\cdots}\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="maxima")
```

output
```
1/140*B*(8*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^2) - 4*sqrt(c*x^4 + b*x^2)*c^2/(
b*x^4) + 3*sqrt(c*x^4 + b*x^2)*c/x^6 + 15*sqrt(c*x^4 + b*x^2)*b/x^8 - 35*(
c*x^4 + b*x^2)^(3/2)/x^10) - 1/630*A*(16*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^2)
 - 8*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^4) + 6*sqrt(c*x^4 + b*x^2)*c^2/(b*x^6)
 - 5*sqrt(c*x^4 + b*x^2)*c/x^8 - 35*sqrt(c*x^4 + b*x^2)*b/x^10 + 105*(c*x^
4 + b*x^2)^(3/2)/x^12)
```

3.115.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(84) = 168$.

Time = 1.54 (sec) , antiderivative size = 430, normalized size of antiderivative = 4.48

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \frac{4\left(315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}Bc^{\frac{7}{2}}\mathrm{sgn}(x)-315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Bbc^{\frac{7}{2}}\mathrm{sg}\right.}{}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="giac")
```

output
```
4/315*(315*(sqrt(c)*x - sqrt(c*x^2 + b))^14*B*c^(7/2)*sgn(x) - 315*(sqrt(c
)*x - sqrt(c*x^2 + b))^12*B*b*c^(7/2)*sgn(x) + 840*(sqrt(c)*x - sqrt(c*x^2
 + b))^12*A*c^(9/2)*sgn(x) + 315*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*b^2*c^
(7/2)*sgn(x) + 1260*(sqrt(c)*x - sqrt(c*x^2 + b))^10*A*b*c^(9/2)*sgn(x) -
819*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^3*c^(7/2)*sgn(x) + 1764*(sqrt(c)*x
 - sqrt(c*x^2 + b))^8*A*b^2*c^(9/2)*sgn(x) + 441*(sqrt(c)*x - sqrt(c*x^2 +
 b))^6*B*b^4*c^(7/2)*sgn(x) + 504*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b^3*c^
(9/2)*sgn(x) - 9*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^5*c^(7/2)*sgn(x) + 14
4*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^4*c^(9/2)*sgn(x) + 81*(sqrt(c)*x - s
qrt(c*x^2 + b))^2*B*b^6*c^(7/2)*sgn(x) - 36*(sqrt(c)*x - sqrt(c*x^2 + b))^
2*A*b^5*c^(9/2)*sgn(x) - 9*B*b^7*c^(7/2)*sgn(x) + 4*A*b^6*c^(9/2)*sgn(x))/
((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^9
```

3.115.9 Mupad [B] (verification not implemented)

Time = 10.24 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.15

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \frac{4\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{315\,b^2\,x^4} - \frac{10\,A\,c\,\sqrt{c\,x^4+b\,x^2}}{63\,x^8} - \frac{B\,b\,\sqrt{c\,x^4+b\,x^2}}{7\,x^8} - \frac{8\,B\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,x^6} - \frac{A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b\,x^6} - \frac{A\,b\,\sqrt{c\,x^4+b\,x^2}}{9\,x^{10}} - \frac{8\,A\,c^4\,\sqrt{c\,x^4+b\,x^2}}{315\,b^3\,x^2} - \frac{B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{35\,b\,x^4} + \frac{2\,B\,c^3\,\sqrt{c\,x^4+b\,x^2}}{35\,b^2\,x^2}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^13,x)
```

output
```
(4*A*c^3*(b*x^2 + c*x^4)^(1/2))/(315*b^2*x^4) - (10*A*c*(b*x^2 + c*x^4)^(1
/2))/(63*x^8) - (B*b*(b*x^2 + c*x^4)^(1/2))/(7*x^8) - (8*B*c*(b*x^2 + c*x^
4)^(1/2))/(35*x^6) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b*x^6) - (A*b*(b*x
^2 + c*x^4)^(1/2))/(9*x^10) - (8*A*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^3*x^2
) - (B*c^2*(b*x^2 + c*x^4)^(1/2))/(35*b*x^4) + (2*B*c^3*(b*x^2 + c*x^4)^(1
/2))/(35*b^2*x^2)
```

3.115. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}}\,dx$

3.116 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15}}\,dx$

3.116.1 Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15}}\,dx = -\frac{A(bx^2+cx^4)^{5/2}}{11bx^{16}} - \frac{(11bB-6Ac)(bx^2+cx^4)^{5/2}}{99b^2x^{14}} + \frac{4c(11bB-6Ac)(bx^2+cx^4)^{5/2}}{693b^3x^{12}} - \frac{8c^2(11bB-6Ac)(bx^2+cx^4)^{5/2}}{3465b^4x^{10}}$$

output
```
-1/11*A*(c*x^4+b*x^2)^(5/2)/b/x^16-1/99*(-6*A*c+11*B*b)*(c*x^4+b*x^2)^(5/2
)/b^2/x^14+4/693*c*(-6*A*c+11*B*b)*(c*x^4+b*x^2)^(5/2)/b^3/x^12-8/3465*c^2
*(-6*A*c+11*B*b)*(c*x^4+b*x^2)^(5/2)/b^4/x^10
```

3.116.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.67

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15}}\,dx = -\frac{(x^2(b+cx^2))^{5/2}(11bBx^2(35b^2-20bcx^2+8c^2x^4)+3A(105b^3-70b^2cx^2+40bc^2x^4-16c^3x^6))}{3465b^4x^{16}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^15,x]
```

output
```
-1/3465*((x^2*(b + c*x^2))^(5/2)*(11*b*B*x^2*(35*b^2 - 20*b*c*x^2 + 8*c^2*
x^4) + 3*A*(105*b^3 - 70*b^2*c*x^2 + 40*b*c^2*x^4 - 16*c^3*x^6)))/(b^4*x^1
6)
```

3.116.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1940, 1220, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{15}}\,dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)\,(cx^4+bx^2)^{3/2}}{x^{16}}dx^2$$

$$\Big\downarrow \text{1220}$$

$$\frac{1}{2}\left(\frac{(11bB-6Ac)\int \frac{(cx^4+bx^2)^{3/2}}{x^{14}}dx^2}{11b}-\frac{2A(bx^2+cx^4)^{5/2}}{11bx^{16}}\right)$$

$$\Big\downarrow \text{1129}$$

$$\frac{1}{2}\left(\frac{(11bB-6Ac)\left(-\frac{4c\int \frac{(cx^4+bx^2)^{3/2}}{x^{12}}dx^2}{9b}-\frac{2(bx^2+cx^4)^{5/2}}{9bx^{14}}\right)}{11b}-\frac{2A(bx^2+cx^4)^{5/2}}{11bx^{16}}\right)$$

$$\Big\downarrow \text{1129}$$

$$\frac{1}{2}\left(\frac{(11bB-6Ac)\left(-\frac{4c\left(-\frac{2c\int\frac{\left(cx^4+bx^2\right)^{3/2}}{x^{10}}dx^2}{7b}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{7bx^{12}}\right)}{9b}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}\right)}{11b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}\right)$$

$$\Big\downarrow \text{1123}$$

$$\frac{1}{2}\left(\frac{\left(-\frac{4c\left(\frac{4c\left(bx^2+cx^4\right)^{5/2}}{35b^2x^{10}}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{7bx^{12}}\right)}{9b}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}\right)(11bB-6Ac)}{11b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^15,x]
```

output
```
((-2*A*(b*x^2 + c*x^4)^(5/2))/(11*b*x^16) + ((11*b*B - 6*A*c)*((-2*(b*x^2
+ c*x^4)^(5/2))/(9*b*x^14) - (4*c*((-2*(b*x^2 + c*x^4)^(5/2))/(7*b*x^12) +
 (4*c*(b*x^2 + c*x^4)^(5/2))/(35*b^2*x^10)))/(9*b)))/(11*b))/2
```

3.116.3.1 Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

3.116. $\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{15}}\,dx$

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.116.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$-\frac{\left(\left(\frac{11x^2B}{9}+A\right)b^3-\frac{2x^2\left(\frac{22x^2B}{21}+A\right)c\,b^2}{3}+\frac{8x^4\left(\frac{11x^2B}{15}+A\right)c^2b}{21}-\frac{16A\,c^3x^6}{105}\right)\sqrt{x^2(c\,x^2+b)}\,(c\,x^2+b)^2}{11x^{12}b^4}$
gosper	$-\frac{(c\,x^2+b)\left(-48A\,c^3x^6+88x^6Bb\,c^2+120Ab\,c^2x^4-220x^4B\,b^2c-210A\,b^2c\,x^2+385b^3B\,x^2+315b^3A\right)\left(x^4c+b\,x^2\right)^{\frac{3}{2}}}{3465b^4x^{14}}$
default	$-\frac{(c\,x^2+b)\left(-48A\,c^3x^6+88x^6Bb\,c^2+120Ab\,c^2x^4-220x^4B\,b^2c-210A\,b^2c\,x^2+385b^3B\,x^2+315b^3A\right)\left(x^4c+b\,x^2\right)^{\frac{3}{2}}}{3465b^4x^{14}}$
trager	$-\frac{(-48A\,x^{10}c^5+88Bb\,c^4x^{10}+24A\,x^8b\,c^4-44B\,b^2c^3x^8-18A\,b^2c^3x^6+33B\,b^3c^2x^6+15A\,b^3c^2x^4+550B\,b^4c\,x^4+420A\,b^4c\,x^2+}{3465b^4x^{12}}$
risch	$-\frac{\sqrt{x^2(c\,x^2+b)}\,(-48A\,x^{10}c^5+88Bb\,c^4x^{10}+24A\,x^8b\,c^4-44B\,b^2c^3x^8-18A\,b^2c^3x^6+33B\,b^3c^2x^6+15A\,b^3c^2x^4+550B\,b^4c\,x^4+}{3465x^{12}b^4}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^15,x,method=_RETURNVERBOSE)
```

output
```
-1/11*((11/9*x^2*B+A)*b^3-2/3*x^2*(22/21*x^2*B+A)*c*b^2+8/21*x^4*(11/15*x^
2*B+A)*c^2*b-16/105*A*c^3*x^6)*(x^2*(c*x^2+b))^(1/2)*(c*x^2+b)^2/x^12/b^4
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{15}}\,dx =$$
$$-\frac{(8\,(11\,Bbc^4-6\,Ac^5)x^{10}-4\,(11\,Bb^2c^3-6\,Abc^4)x^8+3\,(11\,Bb^3c^2-6\,Ab^2c^3)x^6+315\,Ab^5+5\,(110\,Bb^4c+3}{3465\,b^4x^{12}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="fricas")
```

output
```
-1/3465*(8*(11*B*b*c^4 - 6*A*c^5)*x^10 - 4*(11*B*b^2*c^3 - 6*A*b*c^4)*x^8
+ 3*(11*B*b^3*c^2 - 6*A*b^2*c^3)*x^6 + 315*A*b^5 + 5*(110*B*b^4*c + 3*A*b^
3*c^2)*x^4 + 35*(11*B*b^5 + 12*A*b^4*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^4*x^12
)
```

3.116.6 Sympy [F]

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{15}}\,dx = \int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}(A+Bx^2)}{x^{15}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**15,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**15, x)
```

3.116.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(117) = 234$.

Time = 0.21 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.17

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15}}\,dx =$$
$$-\frac{1}{630}B\left(\frac{16\sqrt{cx^4+bx^2}c^4}{b^3x^2} - \frac{8\sqrt{cx^4+bx^2}c^3}{b^2x^4} + \frac{6\sqrt{cx^4+bx^2}c^2}{bx^6} - \frac{5\sqrt{cx^4+bx^2}c}{x^8} - \frac{35\sqrt{cx^4+bx^2}b}{x^{10}} + \frac{105\,\cdots}{\cdots}\right.$$
$$+\frac{1}{9240}A\left(\frac{128\sqrt{cx^4+bx^2}c^5}{b^4x^2} - \frac{64\sqrt{cx^4+bx^2}c^4}{b^3x^4} + \frac{48\sqrt{cx^4+bx^2}c^3}{b^2x^6} - \frac{40\sqrt{cx^4+bx^2}c^2}{bx^8} + \frac{35\sqrt{cx^4+bx^2}c}{x^{10}} \cdots\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="maxima")
```

output
```
-1/630*B*(16*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^2) - 8*sqrt(c*x^4 + b*x^2)*c^3
/(b^2*x^4) + 6*sqrt(c*x^4 + b*x^2)*c^2/(b*x^6) - 5*sqrt(c*x^4 + b*x^2)*c/x
^8 - 35*sqrt(c*x^4 + b*x^2)*b/x^10 + 105*(c*x^4 + b*x^2)^(3/2)/x^12) + 1/9
240*A*(128*sqrt(c*x^4 + b*x^2)*c^5/(b^4*x^2) - 64*sqrt(c*x^4 + b*x^2)*c^4/
(b^3*x^4) + 48*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^6) - 40*sqrt(c*x^4 + b*x^2)*
c^2/(b*x^8) + 35*sqrt(c*x^4 + b*x^2)*c/x^10 + 315*sqrt(c*x^4 + b*x^2)*b/x^
12 - 1155*(c*x^4 + b*x^2)^(3/2)/x^14)
```

3.116.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(117) = 234$.

Time = 1.76 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.68

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15}}\,dx = \frac{16\left(2310\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{16}Bc^{\frac{9}{2}}\operatorname{sgn}(x) - 1155\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}Bbc\cdots\right.}{\cdots}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="giac")
```

output
```
16/3465*(2310*(sqrt(c)*x - sqrt(c*x^2 + b))^16*B*c^(9/2)*sgn(x) - 1155*(sq
rt(c)*x - sqrt(c*x^2 + b))^14*B*b*c^(9/2)*sgn(x) + 6930*(sqrt(c)*x - sqrt(
c*x^2 + b))^14*A*c^(11/2)*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*
b^2*c^(9/2)*sgn(x) + 12474*(sqrt(c)*x - sqrt(c*x^2 + b))^12*A*b*c^(11/2)*s
gn(x) - 4851*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*b^3*c^(9/2)*sgn(x) + 15246
*(sqrt(c)*x - sqrt(c*x^2 + b))^10*A*b^2*c^(11/2)*sgn(x) + 2475*(sqrt(c)*x
- sqrt(c*x^2 + b))^8*B*b^4*c^(9/2)*sgn(x) + 4950*(sqrt(c)*x - sqrt(c*x^2 +
 b))^8*A*b^3*c^(11/2)*sgn(x) + 495*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^5*c
^(9/2)*sgn(x) + 990*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b^4*c^(11/2)*sgn(x)
+ 605*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^6*c^(9/2)*sgn(x) - 330*(sqrt(c)*
x - sqrt(c*x^2 + b))^4*A*b^5*c^(11/2)*sgn(x) - 121*(sqrt(c)*x - sqrt(c*x^2
 + b))^2*B*b^7*c^(9/2)*sgn(x) + 66*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^6*c
^(11/2)*sgn(x) + 11*B*b^8*c^(9/2)*sgn(x) - 6*A*b^7*c^(11/2)*sgn(x))/((sqrt
(c)*x - sqrt(c*x^2 + b))^2 - b)^11
```

3.116.9 Mupad [B] (verification not implemented)

Time = 10.82 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.92

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{15}}\,dx = \frac{2\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{385\,b^2\,x^6}-\frac{4\,A\,c\,\sqrt{c\,x^4+b\,x^2}}{33\,x^{10}} - \frac{B\,b\,\sqrt{c\,x^4+b\,x^2}}{9\,x^{10}}-\frac{10\,B\,c\,\sqrt{c\,x^4+b\,x^2}}{63\,x^8}-\frac{A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{231\,b\,x^8} - \frac{A\,b\,\sqrt{c\,x^4+b\,x^2}}{11\,x^{12}}-\frac{8\,A\,c^4\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^3\,x^4}+\frac{16\,A\,c^5\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^4\,x^2} - \frac{B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b\,x^6}+\frac{4\,B\,c^3\,\sqrt{c\,x^4+b\,x^2}}{315\,b^2\,x^4}-\frac{8\,B\,c^4\,\sqrt{c\,x^4+b\,x^2}}{315\,b^3\,x^2}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^15,x)
```

output
```
(2*A*c^3*(b*x^2 + c*x^4)^(1/2))/(385*b^2*x^6) - (4*A*c*(b*x^2 + c*x^4)^(1/
2))/(33*x^10) - (B*b*(b*x^2 + c*x^4)^(1/2))/(9*x^10) - (10*B*c*(b*x^2 + c*
x^4)^(1/2))/(63*x^8) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(231*b*x^8) - (A*b*(b
*x^2 + c*x^4)^(1/2))/(11*x^12) - (8*A*c^4*(b*x^2 + c*x^4)^(1/2))/(1155*b^3
*x^4) + (16*A*c^5*(b*x^2 + c*x^4)^(1/2))/(1155*b^4*x^2) - (B*c^2*(b*x^2 +
c*x^4)^(1/2))/(105*b*x^6) + (4*B*c^3*(b*x^2 + c*x^4)^(1/2))/(315*b^2*x^4)
- (8*B*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^3*x^2)
```

3.117 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}}\,dx$

3.117.1 Optimal result

Integrand size = 26, antiderivative size = 170

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}}\,dx = -\frac{A(bx^2+cx^4)^{5/2}}{13bx^{18}} - \frac{(13bB-8Ac)(bx^2+cx^4)^{5/2}}{143b^2x^{16}} + \frac{2c(13bB-8Ac)(bx^2+cx^4)^{5/2}}{429b^3x^{14}} - \frac{8c^2(13bB-8Ac)(bx^2+cx^4)^{5/2}}{3003b^4x^{12}} + \frac{16c^3(13bB-8Ac)(bx^2+cx^4)^{5/2}}{15015b^5x^{10}}$$

output
```
-1/13*A*(c*x^4+b*x^2)^(5/2)/b/x^18-1/143*(-8*A*c+13*B*b)*(c*x^4+b*x^2)^(5/
2)/b^2/x^16+2/429*c*(-8*A*c+13*B*b)*(c*x^4+b*x^2)^(5/2)/b^3/x^14-8/3003*c^
2*(-8*A*c+13*B*b)*(c*x^4+b*x^2)^(5/2)/b^4/x^12+16/15015*c^3*(-8*A*c+13*B*b
)*(c*x^4+b*x^2)^(5/2)/b^5/x^10
```

3.117.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.65

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}}\,dx = \frac{(x^2(b+cx^2))^{5/2}(13bBx^2(-105b^3+70b^2cx^2-40bc^2x^4+16c^3x^6)+A(-1}{15015b^5x^{18}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^17,x]
```

output
```
((x^2*(b + c*x^2))^(5/2)*(13*b*B*x^2*(-105*b^3 + 70*b^2*c*x^2 - 40*b*c^2*x
^4 + 16*c^3*x^6) + A*(-1155*b^4 + 840*b^3*c*x^2 - 560*b^2*c^2*x^4 + 320*b*
c^3*x^6 - 128*c^4*x^8)))/(15015*b^5*x^18)
```

3.117.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1940, 1220, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{17}}dx$$

$$\Big\downarrow \text{ 1940}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)\left(cx^4+bx^2\right)^{3/2}}{x^{18}}dx^2$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{1}{2}\left(\frac{(13bB-8Ac)\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^{16}}dx^2}{13b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}}\right)$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{1}{2}\left(\frac{(13bB-8Ac)\left(-\frac{6c\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^{14}}dx^2}{11b}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}\right)}{13b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}}\right)$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{1}{2}\left(\frac{(13bB-8Ac)\left(-\frac{6c\left(-\frac{4c\int\frac{\left(cx^4+bx^2\right)^{3/2}}{x^{12}}dx^2}{9b}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}\right)}{11b}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}\right)}{13b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}}\right)$$

$\downarrow$ 1129

$$\frac{1}{2}\left(\frac{(13bB-8Ac)\left(-\frac{6c\left(-\frac{4c\left(-\frac{2c\int\frac{\left(cx^4+bx^2\right)^{3/2}}{x^{10}}dx^2}{7b}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{7bx^{12}}\right)}{9b}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}\right)}{11b}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}\right)}{13b}-\frac{2A\left(bx^2+cx^4\right)}{13bx^{18}}\right.$$

$\downarrow$ 1123

3.117. $\int\frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{17}}\,dx$

$$\frac{1}{2}\left(\frac{\left(-\frac{6c\left(-\frac{4c\left(\frac{4c\left(bx^2+cx^4\right)^{5/2}}{35b^2x^{10}}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{7bx^{12}}\right)}{9b}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}\right)}{11b}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}\right)(13bB-8Ac)}{13b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}}\right)$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^17,x]
```

output
```
((-2*A*(b*x^2 + c*x^4)^(5/2))/(13*b*x^18) + ((13*b*B - 8*A*c)*((-2*(b*x^2
+ c*x^4)^(5/2))/(11*b*x^16) - (6*c*((-2*(b*x^2 + c*x^4)^(5/2))/(9*b*x^14)
- (4*c*((-2*(b*x^2 + c*x^4)^(5/2))/(7*b*x^12) + (4*c*(b*x^2 + c*x^4)^(5/2)
)/(35*b^2*x^10)))/(9*b)))/(11*b)))/(13*b))/2
```

3.117.3.1 Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.117.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(\left(\frac{13x^{2}B}{11}+A\right)b^{4}-\frac{8x^{2}\left(\frac{13x^{2}B}{12}+A\right)c\,b^{3}}{11}+\frac{16x^{4}\left(\frac{13x^{2}B}{14}+A\right)c^{2}b^{2}}{33}-\frac{64x^{6}\left(\frac{13x^{2}B}{20}+A\right)c^{3}b}{231}+\frac{128A\,x^{8}c^{4}}{1155}\right)\left(c\,x^{2}+b\right.}{13x^{14}b^{5}}$
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(128A\,x^{8}c^{4}-208B\,x^{8}b\,c^{3}-320A\,x^{6}b\,c^{3}+520B\,x^{6}b^{2}c^{2}+560A\,b^{2}c^{2}x^{4}-910B\,b^{3}c\,x^{4}-840A\,x^{2}b^{3}c+1365B\,x^{2}b^{4}+11\right.}{15015b^{5}x^{16}}$
default	$-\frac{\left(c\,x^{2}+b\right)\left(128A\,x^{8}c^{4}-208B\,x^{8}b\,c^{3}-320A\,x^{6}b\,c^{3}+520B\,x^{6}b^{2}c^{2}+560A\,b^{2}c^{2}x^{4}-910B\,b^{3}c\,x^{4}-840A\,x^{2}b^{3}c+1365B\,x^{2}b^{4}+11\right.}{15015b^{5}x^{16}}$
trager	$-\frac{\left(128A\,c^{6}x^{12}-208Bb\,c^{5}x^{12}-64Ab\,c^{5}x^{10}+104B\,b^{2}c^{4}x^{10}+48A\,b^{2}c^{4}x^{8}-78B\,b^{3}c^{3}x^{8}-40A\,b^{3}c^{3}x^{6}+65B\,b^{4}c^{2}x^{6}+35A\,b^{4}c^{2}x^{4}\right.}{15015b^{5}x^{14}}$
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(128A\,c^{6}x^{12}-208Bb\,c^{5}x^{12}-64Ab\,c^{5}x^{10}+104B\,b^{2}c^{4}x^{10}+48A\,b^{2}c^{4}x^{8}-78B\,b^{3}c^{3}x^{8}-40A\,b^{3}c^{3}x^{6}+65B\,b^{4}c^{2}x\right.}{15015x^{14}b^{5}}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^17,x,method=_RETURNVERBOSE)
```

output
```
-1/13*(x^2*(c*x^2+b))^(1/2)*((13/11*x^2*B+A)*b^4-8/11*x^2*(13/12*x^2*B+A)*
c*b^3+16/33*x^4*(13/14*x^2*B+A)*c^2*b^2-64/231*x^6*(13/20*x^2*B+A)*c^3*b+1
28/1155*A*x^8*c^4)*(c*x^2+b)^2/x^14/b^5
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}}\,dx = \frac{(16\,(13\,Bbc^5-8\,Ac^6)x^{12}-8\,(13\,Bb^2c^4-8\,Abc^5)x^{10}+6\,(13\,Bb^3c^3-8\,Ab}{}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="fricas")
```

output
```
1/15015*(16*(13*B*b*c^5 - 8*A*c^6)*x^12 - 8*(13*B*b^2*c^4 - 8*A*b*c^5)*x^1
0 + 6*(13*B*b^3*c^3 - 8*A*b^2*c^4)*x^8 - 1155*A*b^6 - 5*(13*B*b^4*c^2 - 8*
A*b^3*c^3)*x^6 - 35*(52*B*b^5*c + A*b^4*c^2)*x^4 - 105*(13*B*b^6 + 14*A*b^
5*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^5*x^14)
```

3.117.6 Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)}{x^{17}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**17,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**17, x)
```

3.117.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(150) = 300.

Time = 0.22 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.98

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}}\,dx = \frac{1}{9240}B\left(\frac{128\sqrt{cx^4+bx^2}c^5}{b^4x^2}-\frac{64\sqrt{cx^4+bx^2}c^4}{b^3x^4}+\frac{48\sqrt{cx^4+bx^2}c^3}{b^2x^6}-\frac{40\sqrt{}}{}\right.$$

$$-\frac{1}{30030}A\left(\frac{256\sqrt{cx^4+bx^2}c^6}{b^5x^2}-\frac{128\sqrt{cx^4+bx^2}c^5}{b^4x^4}+\frac{96\sqrt{cx^4+bx^2}c^4}{b^3x^6}-\frac{80\sqrt{cx^4+bx^2}c^3}{b^2x^8}+\frac{70\sqrt{cx^4+bx^2}}{bx^{10}}\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="maxima")
```

output
```
1/9240*B*(128*sqrt(c*x^4 + b*x^2)*c^5/(b^4*x^2) - 64*sqrt(c*x^4 + b*x^2)*c
^4/(b^3*x^4) + 48*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^6) - 40*sqrt(c*x^4 + b*x^
2)*c^2/(b*x^8) + 35*sqrt(c*x^4 + b*x^2)*c/x^10 + 315*sqrt(c*x^4 + b*x^2)*b
/x^12 - 1155*(c*x^4 + b*x^2)^(3/2)/x^14) - 1/30030*A*(256*sqrt(c*x^4 + b*x
^2)*c^6/(b^5*x^2) - 128*sqrt(c*x^4 + b*x^2)*c^5/(b^4*x^4) + 96*sqrt(c*x^4
+ b*x^2)*c^4/(b^3*x^6) - 80*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^8) + 70*sqrt(c*
x^4 + b*x^2)*c^2/(b*x^10) - 63*sqrt(c*x^4 + b*x^2)*c/x^12 - 693*sqrt(c*x^4
 + b*x^2)*b/x^14 + 3003*(c*x^4 + b*x^2)^(3/2)/x^16)
```

3.117.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(150) = 300$.

Time = 2.16 (sec) , antiderivative size = 550, normalized size of antiderivative = 3.24

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}}\,dx = \frac{32\left(15015\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{18}Bc^{\frac{11}{2}}\operatorname{sgn}(x)-3003\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{16}B\right.}{}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="giac")
```

output
```
32/15015*(15015*(sqrt(c)*x - sqrt(c*x^2 + b))^18*B*c^(11/2)*sgn(x) - 3003*
(sqrt(c)*x - sqrt(c*x^2 + b))^16*B*b*c^(11/2)*sgn(x) + 48048*(sqrt(c)*x -
sqrt(c*x^2 + b))^16*A*c^(13/2)*sgn(x) - 6006*(sqrt(c)*x - sqrt(c*x^2 + b))
^14*B*b^2*c^(11/2)*sgn(x) + 96096*(sqrt(c)*x - sqrt(c*x^2 + b))^14*A*b*c^(
13/2)*sgn(x) - 28314*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*b^3*c^(11/2)*sgn(x
) + 109824*(sqrt(c)*x - sqrt(c*x^2 + b))^12*A*b^2*c^(13/2)*sgn(x) + 13728*
(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*b^4*c^(11/2)*sgn(x) + 37752*(sqrt(c)*x
- sqrt(c*x^2 + b))^10*A*b^3*c^(13/2)*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2
 + b))^8*B*b^5*c^(11/2)*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b^
4*c^(13/2)*sgn(x) + 3718*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^6*c^(11/2)*sg
n(x) - 2288*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b^5*c^(13/2)*sgn(x) - 1014*(
sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^7*c^(11/2)*sgn(x) + 624*(sqrt(c)*x - sq
rt(c*x^2 + b))^4*A*b^6*c^(13/2)*sgn(x) + 169*(sqrt(c)*x - sqrt(c*x^2 + b))
^2*B*b^8*c^(11/2)*sgn(x) - 104*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^7*c^(13
/2)*sgn(x) - 13*B*b^9*c^(11/2)*sgn(x) + 8*A*b^8*c^(13/2)*sgn(x))/((sqrt(c)
*x - sqrt(c*x^2 + b))^2 - b)^13
```

3.117.9 Mupad [B] (verification not implemented)

Time = 11.41 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.80

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}}\,dx = \frac{8\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{3003\,b^2\,x^8} - \frac{14\,A\,c\,\sqrt{c\,x^4+b\,x^2}}{143\,x^{12}} - \frac{B\,b\,\sqrt{c\,x^4+b\,x^2}}{11\,x^{12}} - \frac{4\,B\,c\,\sqrt{c\,x^4+b\,x^2}}{33\,x^{10}} - \frac{A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{429\,b\,x^{10}} - \frac{A\,b\,\sqrt{c\,x^4+b\,x^2}}{13\,x^{14}} - \frac{16\,A\,c^4\,\sqrt{c\,x^4+b\,x^2}}{5005\,b^3\,x^6} + \frac{64\,A\,c^5\,\sqrt{c\,x^4+b\,x^2}}{15015\,b^4\,x^4} - \frac{128\,A\,c^6\,\sqrt{c\,x^4+b\,x^2}}{15015\,b^5\,x^2} - \frac{B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{231\,b\,x^8} + \frac{2\,B\,c^3\,\sqrt{c\,x^4+b\,x^2}}{385\,b^2\,x^6} - \frac{8\,B\,c^4\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^3\,x^4} + \frac{16\,B\,c^5\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^4\,x^2}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^17,x)
```

output
```
(8*A*c^3*(b*x^2 + c*x^4)^(1/2))/(3003*b^2*x^8) - (14*A*c*(b*x^2 + c*x^4)^(
1/2))/(143*x^12) - (B*b*(b*x^2 + c*x^4)^(1/2))/(11*x^12) - (4*B*c*(b*x^2 +
 c*x^4)^(1/2))/(33*x^10) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(429*b*x^10) - (A
*b*(b*x^2 + c*x^4)^(1/2))/(13*x^14) - (16*A*c^4*(b*x^2 + c*x^4)^(1/2))/(50
05*b^3*x^6) + (64*A*c^5*(b*x^2 + c*x^4)^(1/2))/(15015*b^4*x^4) - (128*A*c^
6*(b*x^2 + c*x^4)^(1/2))/(15015*b^5*x^2) - (B*c^2*(b*x^2 + c*x^4)^(1/2))/(
231*b*x^8) + (2*B*c^3*(b*x^2 + c*x^4)^(1/2))/(385*b^2*x^6) - (8*B*c^4*(b*x
^2 + c*x^4)^(1/2))/(1155*b^3*x^4) + (16*B*c^5*(b*x^2 + c*x^4)^(1/2))/(1155
*b^4*x^2)
```

3.118 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}} \, dx$

3.118.1 Optimal result

Integrand size = 26, antiderivative size = 207

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}} \, dx = -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB-2Ac)(bx^2+cx^4)^{5/2}}{429b^3x^{16}} - \frac{16c^2(3bB-2Ac)(bx^2+cx^4)^{5/2}}{1287b^4x^{14}} + \frac{64c^3(3bB-2Ac)(bx^2+cx^4)^{5/2}}{9009b^5x^{12}} - \frac{128c^4(3bB-2Ac)(bx^2+cx^4)^{5/2}}{45045b^6x^{10}}$$

output
```
-1/15*A*(c*x^4+b*x^2)^(5/2)/b/x^20-1/39*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(5/2)
/b^2/x^18+8/429*c*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(5/2)/b^3/x^16-16/1287*c^2*
(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(5/2)/b^4/x^14+64/9009*c^3*(-2*A*c+3*B*b)*(c*
x^4+b*x^2)^(5/2)/b^5/x^12-128/45045*c^4*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(5/2)
/b^6/x^10
```

3.118.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.64

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}} \, dx = -\frac{(x^2(b+cx^2))^{5/2}(3bBx^2(1155b^4-840b^3cx^2+560b^2c^2x^4-320bc^3x^6+128c^4x^8)+A(3003b^5-2310b^4cx^2}{45045b^6x^{20}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^19,x]
```

output
```
-1/45045*((x^2*(b + c*x^2))^(5/2)*(3*b*B*x^2*(1155*b^4 - 840*b^3*c*x^2 + 5
60*b^2*c^2*x^4 - 320*b*c^3*x^6 + 128*c^4*x^8) + A*(3003*b^5 - 2310*b^4*c*x
^2 + 1680*b^3*c^2*x^4 - 1120*b^2*c^3*x^6 + 640*b*c^4*x^8 - 256*c^5*x^10)))
/(b^6*x^20)
```

3.118.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1940, 1220, 1129, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{19}}dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)\left(cx^4+bx^2\right)^{3/2}}{x^{20}}dx^2$$

$$\Big\downarrow \text{1220}$$

$$\frac{1}{2}\left(\frac{(3bB-2Ac)\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^{18}}dx^2}{3b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{15bx^{20}}\right)$$

$$\Big\downarrow \text{1129}$$

$$\frac{1}{2}\left(\frac{(3bB-2Ac)\left(-\frac{8c\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^{16}}dx^2}{13b}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}}\right)}{3b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{15bx^{20}}\right)$$

$$\Big\downarrow \text{1129}$$

$$\frac{1}{2}\left(\frac{(3bB-2Ac)\left(-\frac{8c\left(-\frac{6c\int\frac{(cx^4+bx^2)^{3/2}}{x^{14}}dx^2}{11b}-\frac{2(bx^2+cx^4)^{5/2}}{11bx^{16}}\right)}{13b}-\frac{2(bx^2+cx^4)^{5/2}}{13bx^{18}}\right)}{3b}-\frac{2A(bx^2+cx^4)^{5/2}}{15bx^{20}}\right)$$

$\downarrow$ 1129

$$\frac{1}{2}\left(\frac{(3bB-2Ac)\left(-\frac{8c\left(-\frac{6c\left(-\frac{4c\int\frac{(cx^4+bx^2)^{3/2}}{x^{12}}dx^2}{9b}-\frac{2(bx^2+cx^4)^{5/2}}{9bx^{14}}\right)}{11b}-\frac{2(bx^2+cx^4)^{5/2}}{11bx^{16}}\right)}{13b}-\frac{2(bx^2+cx^4)^{5/2}}{13bx^{18}}\right)}{3b}-\frac{2A(bx^2+cx^4)^{5}}{15bx^{20}}\right)$$

$\downarrow$ 1129

3.118. $\int\frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}}\,dx$

$$\frac{1}{2}\left(\frac{(3bB-2Ac)\left(-\frac{8c\left(-\frac{6c\left(-\frac{4c\left(-\frac{2c\int\frac{\left(cx^4+bx^2\right)^{3/2}}{x^{10}}dx^2}{7b}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{7bx^{12}}\right)}{9b}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}\right)}{11b}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}\right)}{13b}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}}\right)}{3b}\right.$$

$\downarrow$ 1123

3.118. $\int\frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{19}}\,dx$

$$\frac{1}{2}\left(\frac{\left(-\frac{8c\left(-\frac{6c\left(-\frac{4c\left(\frac{4c\left(bx^2+cx^4\right)^{5/2}}{35b^2x^{10}}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{7bx^{12}}\right)}{9b}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}\right)}{11b}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}\right)}{13b}-\frac{2\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}}\right)(3bB-2Ac)}{3b}-\frac{2A\cdots\right.$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^19,x]
```

output
```
((-2*A*(b*x^2 + c*x^4)^(5/2))/(15*b*x^20) + ((3*b*B - 2*A*c)*((-2*(b*x^2 +
 c*x^4)^(5/2))/(13*b*x^18) - (8*c*((-2*(b*x^2 + c*x^4)^(5/2))/(11*b*x^16)
- (6*c*((-2*(b*x^2 + c*x^4)^(5/2))/(9*b*x^14) - (4*c*((-2*(b*x^2 + c*x^4)^
(5/2))/(7*b*x^12) + (4*c*(b*x^2 + c*x^4)^(5/2))/(35*b^2*x^10)))/(9*b)))/(1
1*b)))/(13*b)))/(3*b))/2
```

3.118. $\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{19}}\,dx$

3.118.3.1 Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.118. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}}\,dx$

3.118.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.60

method	result
pseudoelliptic	$-\frac{\sqrt{x^2(c\,x^2+b)}\,(c\,x^2+b)^2\left(\left(\left(\frac{15x^2B}{13}+A\right)b^5-\frac{10x^2c\left(\frac{12x^2B}{11}+A\right)b^4}{13}+\frac{80c^2x^4\left(x^2B+A\right)b^3}{143}-\frac{160x^6\left(\frac{6x^2B}{7}+A\right)c^3b^2}{429}+\frac{640x^8c^4\left(\frac{3x}{}\right.}{300}\right.\right.}{15x^{16}b^6}$
gosper	$-\frac{(c\,x^2+b)\,(-256A\,c^5x^{10}+384Bb\,c^4x^{10}+640A\,x^8b\,c^4-960B\,b^2c^3x^8-1120A\,b^2c^3x^6+1680B\,b^3c^2x^6+1680A\,b^3c^2x^4-2520B}{45045x^{18}b^6}$
default	$-\frac{(c\,x^2+b)\,(-256A\,c^5x^{10}+384Bb\,c^4x^{10}+640A\,x^8b\,c^4-960B\,b^2c^3x^8-1120A\,b^2c^3x^6+1680B\,b^3c^2x^6+1680A\,b^3c^2x^4-2520B}{45045x^{18}b^6}$
trager	$-\frac{(-256A\,c^7x^{14}+384Bb\,c^6x^{14}+128Ab\,c^6x^{12}-192B\,b^2c^5x^{12}-96A\,b^2c^5x^{10}+144B\,b^3c^4x^{10}+80A\,b^3c^4x^8-120B\,b^4c^3x^8-70A}{45045b^6x^{16}}$
risch	$-\frac{\sqrt{x^2(c\,x^2+b)}\,(-256A\,c^7x^{14}+384Bb\,c^6x^{14}+128Ab\,c^6x^{12}-192B\,b^2c^5x^{12}-96A\,b^2c^5x^{10}+144B\,b^3c^4x^{10}+80A\,b^3c^4x^8-120}{45045x^{16}b^6}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^19,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(x^2*(c*x^2+b))^(1/2)*(c*x^2+b)^2*((15/13*x^2*B+A)*b^5-10/13*x^2*c*(
12/11*x^2*B+A)*b^4+80/143*c^2*x^4*(B*x^2+A)*b^3-160/429*x^6*(6/7*x^2*B+A)*
c^3*b^2+640/3003*x^8*c^4*(3/5*x^2*B+A)*b-256/3003*A*c^5*x^10)/x^16/b^6
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{19}}\,dx =$$

$$-\frac{(128\,(3\,Bbc^6-2\,Ac^7)x^{14}-64\,(3\,Bb^2c^5-2\,Abc^6)x^{12}+48\,(3\,Bb^3c^4-2\,Ab^2c^5)x^{10}-40\,(3\,Bb^4c^3-2\,Ab^3c^4)}{4504}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^19,x, algorithm="fricas")
```

output
```
-1/45045*(128*(3*B*b*c^6 - 2*A*c^7)*x^14 - 64*(3*B*b^2*c^5 - 2*A*b*c^6)*x^
12 + 48*(3*B*b^3*c^4 - 2*A*b^2*c^5)*x^10 - 40*(3*B*b^4*c^3 - 2*A*b^3*c^4)*
x^8 + 3003*A*b^7 + 35*(3*B*b^5*c^2 - 2*A*b^4*c^3)*x^6 + 63*(70*B*b^6*c + A
*b^5*c^2)*x^4 + 231*(15*B*b^7 + 16*A*b^6*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^6*
x^16)
```

3.118.6 Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)}{x^{19}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**19,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**19, x)
```

3.118.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(183) = 366.

Time = 0.22 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.86

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}}\,dx =$$
$$-\frac{1}{30030}B\left(\frac{256\sqrt{cx^4+bx^2}c^6}{b^5x^2} - \frac{128\sqrt{cx^4+bx^2}c^5}{b^4x^4} + \frac{96\sqrt{cx^4+bx^2}c^4}{b^3x^6} - \frac{80\sqrt{cx^4+bx^2}c^3}{b^2x^8} + \frac{70\sqrt{cx^4+bx^2}}{bx^{10}}\right.$$
$$+\frac{1}{180180}A\left(\frac{1024\sqrt{cx^4+bx^2}c^7}{b^6x^2} - \frac{512\sqrt{cx^4+bx^2}c^6}{b^5x^4} + \frac{384\sqrt{cx^4+bx^2}c^5}{b^4x^6} - \frac{320\sqrt{cx^4+bx^2}c^4}{b^3x^8} + \frac{280\sqrt{cx^4}}{b^2x}\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^19,x, algorithm="maxima")
```

output
```
-1/30030*B*(256*sqrt(c*x^4 + b*x^2)*c^6/(b^5*x^2) - 128*sqrt(c*x^4 + b*x^2
)*c^5/(b^4*x^4) + 96*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^6) - 80*sqrt(c*x^4 + b
*x^2)*c^3/(b^2*x^8) + 70*sqrt(c*x^4 + b*x^2)*c^2/(b*x^10) - 63*sqrt(c*x^4
+ b*x^2)*c/x^12 - 693*sqrt(c*x^4 + b*x^2)*b/x^14 + 3003*(c*x^4 + b*x^2)^(3
/2)/x^16) + 1/180180*A*(1024*sqrt(c*x^4 + b*x^2)*c^7/(b^6*x^2) - 512*sqrt(
c*x^4 + b*x^2)*c^6/(b^5*x^4) + 384*sqrt(c*x^4 + b*x^2)*c^5/(b^4*x^6) - 320
*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^8) + 280*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^10
) - 252*sqrt(c*x^4 + b*x^2)*c^2/(b*x^12) + 231*sqrt(c*x^4 + b*x^2)*c/x^14
+ 3003*sqrt(c*x^4 + b*x^2)*b/x^16 - 15015*(c*x^4 + b*x^2)^(3/2)/x^18)
```

3.118.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(183) = 366$.

Time = 2.31 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.81

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}}\,dx = \frac{256\left(18018\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{20}Bc^{\frac{13}{2}}\operatorname{sgn}(x)+60060\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{18}\right.}{}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^19,x, algorithm="giac")
```

output
```
256/45045*(18018*(sqrt(c)*x - sqrt(c*x^2 + b))^20*B*c^(13/2)*sgn(x) + 6006
0*(sqrt(c)*x - sqrt(c*x^2 + b))^18*A*c^(15/2)*sgn(x) - 12870*(sqrt(c)*x -
sqrt(c*x^2 + b))^16*B*b^2*c^(13/2)*sgn(x) + 128700*(sqrt(c)*x - sqrt(c*x^2
 + b))^16*A*b*c^(15/2)*sgn(x) - 32175*(sqrt(c)*x - sqrt(c*x^2 + b))^14*B*b
^3*c^(13/2)*sgn(x) + 141570*(sqrt(c)*x - sqrt(c*x^2 + b))^14*A*b^2*c^(15/2
)*sgn(x) + 15015*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*b^4*c^(13/2)*sgn(x) +
50050*(sqrt(c)*x - sqrt(c*x^2 + b))^12*A*b^3*c^(15/2)*sgn(x) + 9009*(sqrt(
c)*x - sqrt(c*x^2 + b))^10*B*b^5*c^(13/2)*sgn(x) + 6006*(sqrt(c)*x - sqrt(
c*x^2 + b))^10*A*b^4*c^(15/2)*sgn(x) + 4095*(sqrt(c)*x - sqrt(c*x^2 + b))^
8*B*b^6*c^(13/2)*sgn(x) - 2730*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b^5*c^(15
/2)*sgn(x) - 1365*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^7*c^(13/2)*sgn(x) +
910*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b^6*c^(15/2)*sgn(x) + 315*(sqrt(c)*x
 - sqrt(c*x^2 + b))^4*B*b^8*c^(13/2)*sgn(x) - 210*(sqrt(c)*x - sqrt(c*x^2
+ b))^4*A*b^7*c^(15/2)*sgn(x) - 45*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^9*c
^(13/2)*sgn(x) + 30*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^8*c^(15/2)*sgn(x)
+ 3*B*b^10*c^(13/2)*sgn(x) - 2*A*b^9*c^(15/2)*sgn(x))/((sqrt(c)*x - sqrt(c
*x^2 + b))^2 - b)^15
```

3.118.9 Mupad [B] (verification not implemented)

Time = 12.10 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.72

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}}\,dx = \frac{2\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{1287\,b^2\,x^{10}} - \frac{16\,A\,c\,\sqrt{c\,x^4+b\,x^2}}{195\,x^{14}} - \frac{B\,b\,\sqrt{c\,x^4+b\,x^2}}{13\,x^{14}} - \frac{14\,B\,c\,\sqrt{c\,x^4+b\,x^2}}{143\,x^{12}} - \frac{A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{715\,b\,x^{12}} - \frac{A\,b\,\sqrt{c\,x^4+b\,x^2}}{15\,x^{16}} - \frac{16\,A\,c^4\,\sqrt{c\,x^4+b\,x^2}}{9009\,b^3\,x^8} + \frac{32\,A\,c^5\,\sqrt{c\,x^4+b\,x^2}}{15015\,b^4\,x^6} - \frac{128\,A\,c^6\,\sqrt{c\,x^4+b\,x^2}}{45045\,b^5\,x^4} + \frac{256\,A\,c^7\,\sqrt{c\,x^4+b\,x^2}}{45045\,b^6\,x^2} - \frac{B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{429\,b\,x^{10}} + \frac{8\,B\,c^3\,\sqrt{c\,x^4+b\,x^2}}{3003\,b^2\,x^8} - \frac{16\,B\,c^4\,\sqrt{c\,x^4+b\,x^2}}{5005\,b^3\,x^6} + \frac{64\,B\,c^5\,\sqrt{c\,x^4+b\,x^2}}{15015\,b^4\,x^4} - \frac{128\,B\,c^6\,\sqrt{c\,x^4+b\,x^2}}{15015\,b^5\,x^2}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^19,x)
```

output
```
(2*A*c^3*(b*x^2 + c*x^4)^(1/2))/(1287*b^2*x^10) - (16*A*c*(b*x^2 + c*x^4)^
(1/2))/(195*x^14) - (B*b*(b*x^2 + c*x^4)^(1/2))/(13*x^14) - (14*B*c*(b*x^2
 + c*x^4)^(1/2))/(143*x^12) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(715*b*x^12) -
 (A*b*(b*x^2 + c*x^4)^(1/2))/(15*x^16) - (16*A*c^4*(b*x^2 + c*x^4)^(1/2))/
(9009*b^3*x^8) + (32*A*c^5*(b*x^2 + c*x^4)^(1/2))/(15015*b^4*x^6) - (128*A
*c^6*(b*x^2 + c*x^4)^(1/2))/(45045*b^5*x^4) + (256*A*c^7*(b*x^2 + c*x^4)^(
1/2))/(45045*b^6*x^2) - (B*c^2*(b*x^2 + c*x^4)^(1/2))/(429*b*x^10) + (8*B*
c^3*(b*x^2 + c*x^4)^(1/2))/(3003*b^2*x^8) - (16*B*c^4*(b*x^2 + c*x^4)^(1/2
))/(5005*b^3*x^6) + (64*B*c^5*(b*x^2 + c*x^4)^(1/2))/(15015*b^4*x^4) - (12
8*B*c^6*(b*x^2 + c*x^4)^(1/2))/(15015*b^5*x^2)
```

3.119 $\int x^4(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

3.119.1 Optimal result

Integrand size = 26, antiderivative size = 168

$$\int x^4(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{16b^3(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{15015c^5x^5}-\frac{8b^2(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3}+\frac{2b(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{429c^3x}-\frac{(8bB-13Ac)x\left(bx^2+cx^4\right)^{5/2}}{143c^2}+\frac{Bx^3\left(bx^2+cx^4\right)^{5/2}}{13c}$$

output
```
16/15015*b^3*(-13*A*c+8*B*b)*(c*x^4+b*x^2)^(5/2)/c^5/x^5-8/3003*b^2*(-13*A
*c+8*B*b)*(c*x^4+b*x^2)^(5/2)/c^4/x^3+2/429*b*(-13*A*c+8*B*b)*(c*x^4+b*x^2
)^(5/2)/c^3/x-1/143*(-13*A*c+8*B*b)*x*(c*x^4+b*x^2)^(5/2)/c^2+1/13*B*x^3*(
c*x^4+b*x^2)^(5/2)/c
```

3.119.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.67

$$\int x^4(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{x(b+cx^2)^3\left(128b^4B+105c^4x^6(13A+11Bx^2)-70bc^3x^4(13A+12Bx^2)+40b^2c^2x^2(13A+1\right.}{15015c^5\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[x^4*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(b + c*x^2)^3*(128*b^4*B + 105*c^4*x^6*(13*A + 11*B*x^2) - 70*b*c^3*x^4
*(13*A + 12*B*x^2) + 40*b^2*c^2*x^2*(13*A + 14*B*x^2) - 16*b^3*c*(13*A + 2
0*B*x^2)))/(15015*c^5*Sqrt[x^2*(b + c*x^2)])
```

3.119.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1945, 1421, 1421, 1399, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (A+Bx^2) \left(bx^2+cx^4\right)^{3/2} dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{Bx^3 \left(bx^2+cx^4\right)^{5/2}}{13c}-\frac{(8bB-13Ac)\int x^4 \left(cx^4+bx^2\right)^{3/2}dx}{13c}$$

$$\Big\downarrow \text{1421}$$

$$\frac{Bx^3 \left(bx^2+cx^4\right)^{5/2}}{13c}-\frac{(8bB-13Ac)\left(\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{6b\int x^2\left(cx^4+bx^2\right)^{3/2}dx}{11c}\right)}{13c}$$

$$\Big\downarrow \text{1421}$$

$$\frac{Bx^3 \left(bx^2+cx^4\right)^{5/2}}{13c}-\frac{(8bB-13Ac)\left(\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{6b\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{4b\int \left(cx^4+bx^2\right)^{3/2}dx}{9c}\right)}{11c}\right)}{13c}$$

$$\Big\downarrow \text{1399}$$

$$\frac{Bx^3\left(bx^2+cx^4\right)^{5/2}}{13c}-\frac{(8bB-13Ac)\left(\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{6b\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{4b\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b\int\frac{\left(cx^4+bx^2\right)^{3/2}}{x^2}dx}{7c}\right)}{9c}\right)}{11c}\right)}{13c}$$

$\downarrow$ 1420

$$\frac{Bx^3\left(bx^2+cx^4\right)^{5/2}}{13c}-\frac{\left(\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{6b\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{4b\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{35c^2x^5}\right)}{9c}\right)}{11c}\right)(8bB-13Ac)}{13c}$$

input
```
Int[x^4*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(B*x^3*(b*x^2 + c*x^4)^(5/2))/(13*c) - ((8*b*B - 13*A*c)*((x*(b*x^2 + c*x^
4)^(5/2))/(11*c) - (6*b*((b*x^2 + c*x^4)^(5/2)/(9*c*x) - (4*b*((-2*b*(b*x^
2 + c*x^4)^(5/2))/(35*c^2*x^5) + (b*x^2 + c*x^4)^(5/2)/(7*c*x^3)))/(9*c)))
/(11*c)))/(13*c)
```

3.119. $\int x^4(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

3.119.3.1 Defintions of rubi rules used

rule 1399
```
Int[((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(b*x^2 + c*x^4)^(
p + 1)/(c*(4*p + 1)*x^3), x] - Simp[b*((2*p - 1)/(c*(4*p + 1)))   Int[(b*x^
2 + c*x^4)^p/x^2, x], x] /; FreeQ[{b, c, p}, x] && IGtQ[p - 1/2, 0]
```

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

rule 1421
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && IGtQ[Simplify[(m
 + 2*p - 1)/2], 0] && NeQ[m + 4*p + 1, 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.119.4 Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.68

method	result
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-1155B\,x^{8}c^{4}-1365A\,x^{6}c^{4}+840B\,x^{6}b\,c^{3}+910A\,x^{4}b\,c^{3}-560B\,x^{4}b^{2}c^{2}-520A\,x^{2}b^{2}c^{2}+320B\,x^{2}b^{3}c+208A\,b^{3}c-128B\,b^{4}\right)}{15015c^{5}x^{3}}$
default	$-\frac{\left(c\,x^{2}+b\right)\left(-1155B\,x^{8}c^{4}-1365A\,x^{6}c^{4}+840B\,x^{6}b\,c^{3}+910A\,x^{4}b\,c^{3}-560B\,x^{4}b^{2}c^{2}-520A\,x^{2}b^{2}c^{2}+320B\,x^{2}b^{3}c+208A\,b^{3}c-128B\,b^{4}\right)}{15015c^{5}x^{3}}$
trager	$-\frac{\left(-1155B\,c^{6}x^{12}-1365A\,c^{6}x^{10}-1470Bb\,c^{5}x^{10}-1820Ab\,c^{5}x^{8}-35B\,b^{2}c^{4}x^{8}-65A\,b^{2}c^{4}x^{6}+40B\,b^{3}c^{3}x^{6}+78A\,b^{3}c^{3}x^{4}-48B\,b^{4}c^{2}x^{4}\right.}{15015c^{5}x}$
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(-1155B\,c^{6}x^{12}-1365A\,c^{6}x^{10}-1470Bb\,c^{5}x^{10}-1820Ab\,c^{5}x^{8}-35B\,b^{2}c^{4}x^{8}-65A\,b^{2}c^{4}x^{6}+40B\,b^{3}c^{3}x^{6}+78A\,b^{3}c^{3}x^{4}\right.}{15015x\,c^{5}}$

input
```
int(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15015*(c*x^2+b)*(-1155*B*c^4*x^8-1365*A*c^4*x^6+840*B*b*c^3*x^6+910*A*b
*c^3*x^4-560*B*b^2*c^2*x^4-520*A*b^2*c^2*x^2+320*B*b^3*c*x^2+208*A*b^3*c-1
28*B*b^4)*(c*x^4+b*x^2)^(3/2)/c^5/x^3
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.92

$$\int x^4(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{(1155\,Bc^6x^{12}+105\,(14\,Bbc^5+13\,Ac^6)x^{10}+35\,(Bb^2c^4+52\,Abc^5)x^8+128\,Bb^6-208\,Ab^5c-}{15015}$$

input
```
integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/15015*(1155*B*c^6*x^12 + 105*(14*B*b*c^5 + 13*A*c^6)*x^10 + 35*(B*b^2*c^
4 + 52*A*b*c^5)*x^8 + 128*B*b^6 - 208*A*b^5*c - 5*(8*B*b^3*c^3 - 13*A*b^2*
c^4)*x^6 + 6*(8*B*b^4*c^2 - 13*A*b^3*c^3)*x^4 - 8*(8*B*b^5*c - 13*A*b^4*c^
2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^5*x)
```

3.119.6 Sympy [F]

$$\int x^4(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int x^4\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)\,dx$$

input
```
integrate(x**4*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**4*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

3.119.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89

$$\int x^4(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{(105\,c^5x^{10}+140\,bc^4x^8+5\,b^2c^3x^6-6\,b^3c^2x^4+8\,b^4cx^2-16\,b^5)\sqrt{cx^2+b}A}{1155\,c^4} + \frac{(1155\,c^6x^{12}+1470\,bc^5x^{10}+35\,b^2c^4x^8-40\,b^3c^3x^6+48\,b^4c^2x^4-64\,b^5cx^2+128\,b^6)\sqrt{cx^2+b}B}{15015\,c^5}$$

input
```
integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 - 6*b^3*c^2*x^4 + 8*b
^4*c*x^2 - 16*b^5)*sqrt(c*x^2 + b)*A/c^4 + 1/15015*(1155*c^6*x^12 + 1470*b
*c^5*x^10 + 35*b^2*c^4*x^8 - 40*b^3*c^3*x^6 + 48*b^4*c^2*x^4 - 64*b^5*c*x^
2 + 128*b^6)*sqrt(c*x^2 + b)*B/c^5
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.04

$$\int x^4(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = -\frac{16\left(8\,Bb^{\frac{13}{2}}-13\,Ab^{\frac{11}{2}}c\right)\operatorname{sgn}(x)}{15015\,c^5} + \frac{1155\,(cx^2+b)^{\frac{13}{2}}B\operatorname{sgn}(x)-5460\,(cx^2+b)^{\frac{11}{2}}Bb\operatorname{sgn}(x)+10010\,(cx^2+b)^{\frac{9}{2}}Bb^2\operatorname{sgn}(x)-8580\,(cx^2+b)^{\frac{7}{2}}Bb^3}{}$$

input
```
integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
-16/15015*(8*B*b^(13/2) - 13*A*b^(11/2)*c)*sgn(x)/c^5 + 1/15015*(1155*(c*x
^2 + b)^(13/2)*B*sgn(x) - 5460*(c*x^2 + b)^(11/2)*B*b*sgn(x) + 10010*(c*x^
2 + b)^(9/2)*B*b^2*sgn(x) - 8580*(c*x^2 + b)^(7/2)*B*b^3*sgn(x) + 3003*(c*
x^2 + b)^(5/2)*B*b^4*sgn(x) + 1365*(c*x^2 + b)^(11/2)*A*c*sgn(x) - 5005*(c
*x^2 + b)^(9/2)*A*b*c*sgn(x) + 6435*(c*x^2 + b)^(7/2)*A*b^2*c*sgn(x) - 300
3*(c*x^2 + b)^(5/2)*A*b^3*c*sgn(x))/c^5
```

3.119.9 Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.85

$$\int x^4(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{128\,B\,b^6-208\,A\,b^5\,c}{15015\,c^5}+\frac{x^{10}\,\left(1365\,A\,c^6+1470\,B\,b\,c^5\right)}{15015\,c^5}+\frac{B\,c\,x^{12}}{13}+\frac{b^2\,x^6\,\left(13\,A\,c-8\,B\,b\right)}{3003\,c^2}-\frac{2\,b^3\,x^4\,\left(13\,A\,c\right.}{5005\,c^3}\right.}{x}$$

input
```
int(x^4*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

output
```
((b*x^2 + c*x^4)^(1/2)*((128*B*b^6 - 208*A*b^5*c)/(15015*c^5) + (x^10*(136
5*A*c^6 + 1470*B*b*c^5))/(15015*c^5) + (B*c*x^12)/13 + (b^2*x^6*(13*A*c -
8*B*b))/(3003*c^2) - (2*b^3*x^4*(13*A*c - 8*B*b))/(5005*c^3) + (8*b^4*x^2*
(13*A*c - 8*B*b))/(15015*c^4) + (b*x^8*(52*A*c + B*b))/(429*c)))/x
```

3.120 $\int x^2(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

3.120.1 Optimal result

Integrand size = 26, antiderivative size = 131

$$\int x^2\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=-\frac{8b^2(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{3465c^4x^5}+\frac{4b(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{693c^3x^3}-\frac{(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{99c^2x}+\frac{Bx(bx^2+cx^4)^{5/2}}{11c}$$

output
```
-8/3465*b^2*(-11*A*c+6*B*b)*(c*x^4+b*x^2)^(5/2)/c^4/x^5+4/693*b*(-11*A*c+6
*B*b)*(c*x^4+b*x^2)^(5/2)/c^3/x^3-1/99*(-11*A*c+6*B*b)*(c*x^4+b*x^2)^(5/2)
/c^2/x+1/11*B*x*(c*x^4+b*x^2)^(5/2)/c
```

3.120.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.72

$$\int x^2\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{(b+cx^2)\left(x^2(b+cx^2)\right)^{3/2}\left(-48b^3B+88Ab^2c+120b^2Bcx^2-220Abc^2x^2-210bBc^2x^4+385A\ldots\right.}{3465c^4x^3}$$

input
```
Integrate[x^2*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
((b + c*x^2)*(x^2*(b + c*x^2))^(3/2)*(-48*b^3*B + 88*A*b^2*c + 120*b^2*B*c
*x^2 - 220*A*b*c^2*x^2 - 210*b*B*c^2*x^4 + 385*A*c^3*x^4 + 315*B*c^3*x^6))
/(3465*c^4*x^3)
```

3.120.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1945, 1421, 1399, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (A+Bx^2) \left(bx^2+cx^4\right)^{3/2} dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{Bx\left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{(6bB-11Ac)\int x^2\left(cx^4+bx^2\right)^{3/2}dx}{11c}$$

$$\Big\downarrow \text{1421}$$

$$\frac{Bx\left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{(6bB-11Ac)\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{4b\int\left(cx^4+bx^2\right)^{3/2}dx}{9c}\right)}{11c}$$

$$\Big\downarrow \text{1399}$$

$$\frac{Bx\left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{(6bB-11Ac)\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{4b\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b\int\frac{\left(cx^4+bx^2\right)^{3/2}}{x^2}dx}{7c}\right)}{9c}\right)}{11c}$$

$$\Big\downarrow \text{1420}$$

$$\frac{Bx\left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{4b\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{35c^2x^5}\right)}{9c}\right)(6bB-11Ac)}{11c}$$

input
```
Int[x^2*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(B*x*(b*x^2 + c*x^4)^(5/2))/(11*c) - ((6*b*B - 11*A*c)*((b*x^2 + c*x^4)^(5
/2)/(9*c*x) - (4*b*((-2*b*(b*x^2 + c*x^4)^(5/2))/(35*c^2*x^5) + (b*x^2 + c
*x^4)^(5/2)/(7*c*x^3)))/(9*c)))/(11*c)
```

3.120.3.1 Defintions of rubi rules used

rule 1399
```
Int[((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(b*x^2 + c*x^4)^(
p + 1)/(c*(4*p + 1)*x^3), x] - Simp[b*((2*p - 1)/(c*(4*p + 1)))   Int[(b*x^
2 + c*x^4)^p/x^2, x], x] /; FreeQ[{b, c, p}, x] && IGtQ[p - 1/2, 0]
```

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

rule 1421
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && IGtQ[Simplify[(m
 + 2*p - 1)/2], 0] && NeQ[m + 4*p + 1, 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.120.4 Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.69

method	result
gosper	$\frac{\left(c\,x^{2}+b\right)\left(315B\,c^{3}x^{6}+385A\,c^{3}x^{4}-210Bb\,c^{2}x^{4}-220Ab\,c^{2}x^{2}+120B\,b^{2}c\,x^{2}+88b^{2}Ac-48B\,b^{3}\right)\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}}{3465c^{4}x^{3}}$
default	$\frac{\left(c\,x^{2}+b\right)\left(315B\,c^{3}x^{6}+385A\,c^{3}x^{4}-210Bb\,c^{2}x^{4}-220Ab\,c^{2}x^{2}+120B\,b^{2}c\,x^{2}+88b^{2}Ac-48B\,b^{3}\right)\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}}{3465c^{4}x^{3}}$
trager	$\frac{(315B\,c^{5}x^{10}+385A\,c^{5}x^{8}+420Bb\,c^{4}x^{8}+550Ab\,c^{4}x^{6}+15B\,b^{2}c^{3}x^{6}+33A\,b^{2}c^{3}x^{4}-18B\,b^{3}c^{2}x^{4}-44A\,b^{3}c^{2}x^{2}+24B\,b^{4}c\,x^{2}+88A\,b^{4}c-48b^{5}}{3465c^{4}x}$
risch	$\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,(315B\,c^{5}x^{10}+385A\,c^{5}x^{8}+420Bb\,c^{4}x^{8}+550Ab\,c^{4}x^{6}+15B\,b^{2}c^{3}x^{6}+33A\,b^{2}c^{3}x^{4}-18B\,b^{3}c^{2}x^{4}-44A\,b^{3}c^{2}x^{2}+24B\,b^{4}c\,x^{2}+}{3465x\,c^{4}}$

input
```
int(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3465*(c*x^2+b)*(315*B*c^3*x^6+385*A*c^3*x^4-210*B*b*c^2*x^4-220*A*b*c^2*
x^2+120*B*b^2*c*x^2+88*A*b^2*c-48*B*b^3)*(c*x^4+b*x^2)^(3/2)/c^4/x^3
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00

$$\int x^2(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{(315\,Bc^5x^{10}+35\,(12\,Bbc^4+11\,Ac^5)x^8+5\,(3\,Bb^2c^3+110\,Abc^4)x^6-48\,Bb^5+88\,Ab^4c-3\,(6}{3465\,c^4x}$$

input
```
integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3465*(315*B*c^5*x^10 + 35*(12*B*b*c^4 + 11*A*c^5)*x^8 + 5*(3*B*b^2*c^3 +
 110*A*b*c^4)*x^6 - 48*B*b^5 + 88*A*b^4*c - 3*(6*B*b^3*c^2 - 11*A*b^2*c^3)
*x^4 + 4*(6*B*b^4*c - 11*A*b^3*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^4*x)
```

3.120.6 Sympy [F]

$$\int x^2(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int x^2\left(x^2(b+cx^2)\right)^{\frac{3}{2}}(A+Bx^2)\,dx$$

input
```
integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**2*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

3.120.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int x^2(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{(35\,c^4x^8+50\,bc^3x^6+3\,b^2c^2x^4-4\,b^3cx^2+8\,b^4)\sqrt{cx^2+b}A}{315\,c^3}+\frac{(105\,c^5x^{10}+140\,bc^4x^8+5\,b^2c^3x^6-6\,b^3c^2x^4+8\,b^4cx^2-16\,b^5)\sqrt{cx^2+b}B}{1155\,c^4}$$

input
```
integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*sq
rt(c*x^2 + b)*A/c^3 + 1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6
 - 6*b^3*c^2*x^4 + 8*b^4*c*x^2 - 16*b^5)*sqrt(c*x^2 + b)*B/c^4
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07

$$\int x^2(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{8\left(6\,Bb^{\frac{11}{2}}-11\,Ab^{\frac{9}{2}}c\right)\mathrm{sgn}(x)}{3465\,c^4}$$
$$+\frac{315\,(cx^2+b)^{\frac{11}{2}}B\mathrm{sgn}(x)-1155\,(cx^2+b)^{\frac{9}{2}}Bb\mathrm{sgn}(x)+1485\,(cx^2+b)^{\frac{7}{2}}Bb^2\mathrm{sgn}(x)-693\,(cx^2+b)^{\frac{5}{2}}Bb^3\mathrm{sgn}(}{3465\,c^4}$$

input
```
integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
8/3465*(6*B*b^(11/2) - 11*A*b^(9/2)*c)*sgn(x)/c^4 + 1/3465*(315*(c*x^2 + b
)^(11/2)*B*sgn(x) - 1155*(c*x^2 + b)^(9/2)*B*b*sgn(x) + 1485*(c*x^2 + b)^(
7/2)*B*b^2*sgn(x) - 693*(c*x^2 + b)^(5/2)*B*b^3*sgn(x) + 385*(c*x^2 + b)^(
9/2)*A*c*sgn(x) - 990*(c*x^2 + b)^(7/2)*A*b*c*sgn(x) + 693*(c*x^2 + b)^(5/
2)*A*b^2*c*sgn(x))/c^4
```

3.120.9 Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95

$$\int x^2(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{\sqrt{c\,x^4+b\,x^2}\left(\frac{x^8\,(385\,A\,c^5+420\,B\,b\,c^4)}{3465\,c^4}-\frac{48\,B\,b^5-88\,A\,b^4\,c}{3465\,c^4}+\frac{B\,c\,x^{10}}{11}+\frac{b^2\,x^4\,(11\,A\,c-6\,B\,b)}{1155\,c^2}-\frac{4\,b^3\,x^2\,(11\,A\,c-6\,B}{3465\,c^3}\right.}{x}$$

input
```
int(x^2*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

output
```
((b*x^2 + c*x^4)^(1/2)*((x^8*(385*A*c^5 + 420*B*b*c^4))/(3465*c^4) - (48*B
*b^5 - 88*A*b^4*c)/(3465*c^4) + (B*c*x^10)/11 + (b^2*x^4*(11*A*c - 6*B*b))
/(1155*c^2) - (4*b^3*x^2*(11*A*c - 6*B*b))/(3465*c^3) + (b*x^6*(110*A*c +
3*B*b))/(693*c)))/x
```

3.120. $\int x^2(A + Bx^2)\,(bx^2 + cx^4)^{3/2}\,dx$

3.121 $\int (A+Bx^2)(bx^2+cx^4)^{3/2}\,dx$

3.121.1 Optimal result

Integrand size = 23, antiderivative size = 96

$$\int (A+Bx^2)(bx^2+cx^4)^{3/2}\,dx = \frac{2b(4bB-9Ac)(bx^2+cx^4)^{5/2}}{315c^3x^5} - \frac{(4bB-9Ac)(bx^2+cx^4)^{5/2}}{63c^2x^3} + \frac{B(bx^2+cx^4)^{5/2}}{9cx}$$

output
```
2/315*b*(-9*A*c+4*B*b)*(c*x^4+b*x^2)^(5/2)/c^3/x^5-1/63*(-9*A*c+4*B*b)*(c*
x^4+b*x^2)^(5/2)/c^2/x^3+1/9*B*(c*x^4+b*x^2)^(5/2)/c/x
```

3.121.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.73

$$\int (A+Bx^2)(bx^2+cx^4)^{3/2}\,dx = \frac{(b+cx^2)(x^2(b+cx^2))^{3/2}(8b^2B-18Abc-20bBcx^2+45Ac^2x^2+35Bc^2x^4)}{315c^3x^3}$$

input
```
Integrate[(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
((b + c*x^2)*(x^2*(b + c*x^2))^(3/2)*(8*b^2*B - 18*A*b*c - 20*b*B*c*x^2 +
45*A*c^2*x^2 + 35*B*c^2*x^4))/(315*c^3*x^3)
```

3.121.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1465, 1399, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx^2)\left(bx^2+cx^4\right)^{3/2}dx$$

$$\Big\downarrow \; 1465$$

$$\frac{B\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{(4bB-9Ac)\int \left(cx^4+bx^2\right)^{3/2}dx}{9c}$$

$$\Big\downarrow \; 1399$$

$$\frac{B\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{(4bB-9Ac)\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^2}dx}{7c}\right)}{9c}$$

$$\Big\downarrow \; 1420$$

$$\frac{B\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{35c^2x^5}\right)(4bB-9Ac)}{9c}$$

input
```
Int[(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(B*(b*x^2 + c*x^4)^(5/2))/(9*c*x) - ((4*b*B - 9*A*c)*((-2*b*(b*x^2 + c*x^4
)^(5/2))/(35*c^2*x^5) + (b*x^2 + c*x^4)^(5/2)/(7*c*x^3)))/(9*c)
```

3.121.3.1 Defintions of rubi rules used

rule 1399
```
Int[((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(b*x^2 + c*x^4)^(
p + 1)/(c*(4*p + 1)*x^3), x] - Simp[b*((2*p - 1)/(c*(4*p + 1)))   Int[(b*x^
2 + c*x^4)^p/x^2, x], x] /; FreeQ[{b, c, p}, x] && IGtQ[p - 1/2, 0]
```

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

rule 1465
```
Int[((d_) + (e_.)*(x_)^2)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
Simp[e*((b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 3)*x)), x] - Simp[(b*e*(2*p + 1)
- c*d*(4*p + 3))/(c*(4*p + 3))   Int[(b*x^2 + c*x^4)^p, x], x] /; FreeQ[{b,
 c, d, e, p}, x] &&  !IntegerQ[p] && NeQ[4*p + 3, 0] && NeQ[b*e*(2*p + 1) -
 c*d*(4*p + 3), 0]
```

3.121.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.70

method	result	size
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-35B\,c^{2}x^{4}-45A\,c^{2}x^{2}+20Bbc\,x^{2}+18Abc-8B\,b^{2}\right)\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}}{315c^{3}x^{3}}$	67
default	$-\frac{\left(c\,x^{2}+b\right)\left(-35B\,c^{2}x^{4}-45A\,c^{2}x^{2}+20Bbc\,x^{2}+18Abc-8B\,b^{2}\right)\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}}{315c^{3}x^{3}}$	67
trager	$-\frac{\left(-35B\,x^{8}c^{4}-45A\,x^{6}c^{4}-50B\,x^{6}b\,c^{3}-72A\,x^{4}b\,c^{3}-3B\,x^{4}b^{2}c^{2}-9A\,x^{2}b^{2}c^{2}+4B\,x^{2}b^{3}c+18A\,b^{3}c-8B\,b^{4}\right)\sqrt{x^{4}c+b\,x^{2}}}{315c^{3}x}$	108
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(-35B\,x^{8}c^{4}-45A\,x^{6}c^{4}-50B\,x^{6}b\,c^{3}-72A\,x^{4}b\,c^{3}-3B\,x^{4}b^{2}c^{2}-9A\,x^{2}b^{2}c^{2}+4B\,x^{2}b^{3}c+18A\,b^{3}c-8B\,b^{4}\right)}{315x\,c^{3}}$	108

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/315*(c*x^2+b)*(-35*B*c^2*x^4-45*A*c^2*x^2+20*B*b*c*x^2+18*A*b*c-8*B*b^2
)*(c*x^4+b*x^2)^(3/2)/c^3/x^3
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.10

$$\int (A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{(35\,Bc^4x^8+5\,(10\,Bbc^3+9\,Ac^4)x^6+8\,Bb^4-18\,Ab^3c+3\,(Bb^2c^2+24\,Abc^3)x^4-(4\,Bb^3c-9}{315\,c^3x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/315*(35*B*c^4*x^8 + 5*(10*B*b*c^3 + 9*A*c^4)*x^6 + 8*B*b^4 - 18*A*b^3*c
+ 3*(B*b^2*c^2 + 24*A*b*c^3)*x^4 - (4*B*b^3*c - 9*A*b^2*c^2)*x^2)*sqrt(c*x
^4 + b*x^2)/(c^3*x)
```

3.121.6 Sympy [F]

$$\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\int \left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.09

$$\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{\left(5\,c^3x^6+8\,bc^2x^4+b^2cx^2-2\,b^3\right)\sqrt{cx^2+b}A}{35\,c^2}+\frac{\left(35\,c^4x^8+50\,bc^3x^6+3\,b^2c^2x^4-4\,b^3cx^2+8\,b^4\right)\sqrt{cx^2+b}B}{315\,c^3}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/35*(5*c^3*x^6 + 8*b*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^2 + b)*A/c^2 +
 1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*s
qrt(c*x^2 + b)*B/c^3
```

3.121.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.09

$$\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=-\frac{2\left(4\,Bb^{\frac{9}{2}}-9\,Ab^{\frac{7}{2}}c\right)\mathrm{sgn}(x)}{315\,c^3}+\frac{35\left(cx^2+b\right)^{\frac{9}{2}}B\mathrm{sgn}(x)-90\left(cx^2+b\right)^{\frac{7}{2}}Bb\mathrm{sgn}(x)+63\left(cx^2+b\right)^{\frac{5}{2}}Bb^2\mathrm{sgn}(x)+45\left(cx^2+b\right)^{\frac{7}{2}}Ac\mathrm{sgn}(x)-63\,(}{315\,c^3}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
-2/315*(4*B*b^(9/2) - 9*A*b^(7/2)*c)*sgn(x)/c^3 + 1/315*(35*(c*x^2 + b)^(9
/2)*B*sgn(x) - 90*(c*x^2 + b)^(7/2)*B*b*sgn(x) + 63*(c*x^2 + b)^(5/2)*B*b^
2*sgn(x) + 45*(c*x^2 + b)^(7/2)*A*c*sgn(x) - 63*(c*x^2 + b)^(5/2)*A*b*c*sg
n(x))/c^3
```

3.121.9 Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.07

$$\int (A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{8\,B\,b^4-18\,A\,b^3\,c}{315\,c^3}+\frac{x^6\,\left(45\,A\,c^4+50\,B\,b\,c^3\right)}{315\,c^3}+\frac{B\,c\,x^8}{9}+\frac{b^2\,x^2\,\left(9\,A\,c-4\,B\,b\right)}{315\,c^2}+\frac{b\,x^4\,\left(24\,A\,c+B\,b\right)}{105\,c}\right)}{x}$$

input
```
int((A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

output
```
((b*x^2 + c*x^4)^(1/2)*((8*B*b^4 - 18*A*b^3*c)/(315*c^3) + (x^6*(45*A*c^4
+ 50*B*b*c^3))/(315*c^3) + (B*c*x^8)/9 + (b^2*x^2*(9*A*c - 4*B*b))/(315*c^
2) + (b*x^4*(24*A*c + B*b))/(105*c)))/x
```

3.122 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2}\,dx$

3.122.1 Optimal result

Integrand size = 26, antiderivative size = 61

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2}\,dx = -\frac{(2bB-7Ac)(bx^2+cx^4)^{5/2}}{35c^2x^5}+\frac{B(bx^2+cx^4)^{5/2}}{7cx^3}$$

output
```
-1/35*(-7*A*c+2*B*b)*(c*x^4+b*x^2)^(5/2)/c^2/x^5+1/7*B*(c*x^4+b*x^2)^(5/2)
/c/x^3
```

3.122.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2}\,dx = \frac{(x^2(b+cx^2))^{5/2}(-2bB+7Ac+5Bcx^2)}{35c^2x^5}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^2,x]
```

output
```
((x^2*(b + c*x^2))^(5/2)*(-2*b*B + 7*A*c + 5*B*c*x^2))/(35*c^2*x^5)
```

3.122.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1945, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{B\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{(2bB-7Ac)\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^2}dx}{7c}$$

$$\Big\downarrow \text{1420}$$

$$\frac{B\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{\left(bx^2+cx^4\right)^{5/2}(2bB-7Ac)}{35c^2x^5}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^2,x]
```

output
```
-1/35*((2*b*B - 7*A*c)*(b*x^2 + c*x^4)^(5/2))/(c^2*x^5) + (B*(b*x^2 + c*x^
4)^(5/2))/(7*c*x^3)
```

3.122.3.1 Defintions of rubi rules used

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.122.4 Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

method	result	size
gosper	$\frac{(c\,x^2+b)\,(5Bc\,x^2+7Ac-2Bb)\,(x^4c+b\,x^2)^{\frac{3}{2}}}{35c^2x^3}$	45
default	$\frac{(c\,x^2+b)\,(5Bc\,x^2+7Ac-2Bb)\,(x^4c+b\,x^2)^{\frac{3}{2}}}{35c^2x^3}$	45
trager	$\frac{(5B\,c^3x^6+7A\,c^3x^4+8Bb\,c^2x^4+14Ab\,c^2x^2+B\,b^2c\,x^2+7b^2Ac-2B\,b^3)\sqrt{x^4c+b\,x^2}}{35c^2x}$	83
risch	$\frac{\sqrt{x^2(c\,x^2+b)}\,(5B\,c^3x^6+7A\,c^3x^4+8Bb\,c^2x^4+14Ab\,c^2x^2+B\,b^2c\,x^2+7b^2Ac-2B\,b^3)}{35x\,c^2}$	83

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/35*(c*x^2+b)*(5*B*c*x^2+7*A*c-2*B*b)*(c*x^4+b*x^2)^(3/2)/c^2/x^3
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^2}\,dx = \frac{(5\,Bc^3x^6+(8\,Bbc^2+7\,Ac^3)x^4-2\,Bb^3+7\,Ab^2c+(Bb^2c+14\,Abc^2)x^2)\sqrt{}}{35\,c^2x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
1/35*(5*B*c^3*x^6 + (8*B*b*c^2 + 7*A*c^3)*x^4 - 2*B*b^3 + 7*A*b^2*c + (B*b
^2*c + 14*A*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^2*x)
```

3.122.6 Sympy [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^2}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}\,(A+Bx^2)}{x^2}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**2,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**2, x)
```

3.122.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2}\,dx = \frac{(c^2x^4+2\,bcx^2+b^2)\sqrt{cx^2+b}A}{5\,c} + \frac{(5\,c^3x^6+8\,bc^2x^4+b^2cx^2-2\,b^3)\sqrt{cx^2+b}B}{35\,c^2}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^2 + b)*A/c + 1/35*(5*c^3*x^6 + 8*
b*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^2 + b)*B/c^2
```

3.122.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2}\,dx = \frac{\left(2\,Bb^{\frac{7}{2}}-7\,Ab^{\frac{5}{2}}c\right)\mathrm{sgn}(x)}{35\,c^2} + \frac{5\,(cx^2+b)^{\frac{7}{2}}B\mathrm{sgn}(x)-7\,(cx^2+b)^{\frac{5}{2}}Bb\mathrm{sgn}(x)+7\,(cx^2+b)^{\frac{5}{2}}Ac\mathrm{sgn}(x)}{35\,c^2}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/35*(2*B*b^(7/2) - 7*A*b^(5/2)*c)*sgn(x)/c^2 + 1/35*(5*(c*x^2 + b)^(7/2)*
B*sgn(x) - 7*(c*x^2 + b)^(5/2)*B*b*sgn(x) + 7*(c*x^2 + b)^(5/2)*A*c*sgn(x)
)/c^2
```

3.122.9 Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.36

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{x^4\,(7\,A\,c^3+8\,B\,b\,c^2)}{35\,c^2}-\frac{2\,B\,b^3-7\,A\,b^2\,c}{35\,c^2}+\frac{B\,c\,x^6}{7}+\frac{b\,x^2\,(14\,A\,c+B\,b)}{35\,c}\right)}{x}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^2,x)
```

output
```
((b*x^2 + c*x^4)^(1/2)*((x^4*(7*A*c^3 + 8*B*b*c^2))/(35*c^2) - (2*B*b^3 -
7*A*b^2*c)/(35*c^2) + (B*c*x^6)/7 + (b*x^2*(14*A*c + B*b))/(35*c)))/x
```

3.122. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2}\,dx$

3.123 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4} \, dx$

3.123.1 Optimal result

Integrand size = 26, antiderivative size = 102

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4} \, dx = \frac{Ab\sqrt{bx^2+cx^4}}{x} + \frac{A(bx^2+cx^4)^{3/2}}{3x^3} + \frac{B(bx^2+cx^4)^{5/2}}{5cx^5} - Ab^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)$$

output
```
1/3*A*(c*x^4+b*x^2)^(3/2)/x^3+1/5*B*(c*x^4+b*x^2)^(5/2)/c/x^5-A*b^(3/2)*ar
ctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))+A*b*(c*x^4+b*x^2)^(1/2)/x
```

3.123.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4} \, dx = \frac{x\left((b+cx^2)(3b^2B+c^2x^2(5A+3Bx^2)+b(20Ac+6Bcx^2)) - 15Ab^{3/2}c\sqrt{\ }\right.}{15c\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^4,x]
```

output
```
(x*((b + c*x^2)*(3*b^2*B + c^2*x^2*(5*A + 3*B*x^2) + b*(20*A*c + 6*B*c*x^2
)) - 15*A*b^(3/2)*c*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(15
*c*Sqrt[x^2*(b + c*x^2)])
```

3.123.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1945, 1426, 1426, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{1945}$$

$$A\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^4}dx+\frac{B\left(bx^2+cx^4\right)^{5/2}}{5cx^5}$$

$$\Big\downarrow \text{1426}$$

$$A\left(b\int \frac{\sqrt{cx^4+bx^2}}{x^2}dx+\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^3}\right)+\frac{B\left(bx^2+cx^4\right)^{5/2}}{5cx^5}$$

$$\Big\downarrow \text{1426}$$

$$A\left(b\left(b\int \frac{1}{\sqrt{cx^4+bx^2}}dx+\frac{\sqrt{bx^2+cx^4}}{x}\right)+\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^3}\right)+\frac{B\left(bx^2+cx^4\right)^{5/2}}{5cx^5}$$

$$\Big\downarrow \text{1400}$$

$$A\left(b\left(\frac{\sqrt{bx^2+cx^4}}{x}-b\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}\right)+\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^3}\right)+\frac{B\left(bx^2+cx^4\right)^{5/2}}{5cx^5}$$

$$\Big\downarrow \text{219}$$

$$A\left(b\left(\frac{\sqrt{bx^2+cx^4}}{x}-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)\right)+\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^3}\right)+\frac{B\left(bx^2+cx^4\right)^{5/2}}{5cx^5}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^4,x]
```

output
```
(B*(b*x^2 + c*x^4)^(5/2))/(5*c*x^5) + A*((b*x^2 + c*x^4)^(3/2)/(3*x^3) + b
*(Sqrt[b*x^2 + c*x^4]/x - Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
))
```

3.123.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.123.4 Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\left(x^4 c+b\,x^2\right)^{\frac{3}{2}}\left(3B\left(c\,x^2+b\right)^{\frac{5}{2}}+5A\left(c\,x^2+b\right)^{\frac{3}{2}}c-15A\,b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)c+15A\sqrt{c\,x^2+b}\,bc\right)}{15x^3\left(c\,x^2+b\right)^{\frac{3}{2}}c}$	99

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/15*(c*x^4+b*x^2)^(3/2)*(3*B*(c*x^2+b)^(5/2)+5*A*(c*x^2+b)^(3/2)*c-15*A*b
^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c+15*A*(c*x^2+b)^(1/2)*b*c)/x^3
/(c*x^2+b)^(3/2)/c
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.02

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4}\,dx = \left[\frac{15\,Ab^{\frac{3}{2}}cx\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,(3\,Bc^2x^4+3\,Bb^2+20\,Abc+\cdots}{30\,cx}\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/30*(15*A*b^(3/2)*c*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) + 2*(3*B*c^2*x^4 + 3*B*b^2 + 20*A*b*c + (6*B*b*c + 5*A*c^2)*x^2)*s
qrt(c*x^4 + b*x^2))/(c*x), 1/15*(15*A*sqrt(-b)*b*c*x*arctan(sqrt(c*x^4 + b
*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (3*B*c^2*x^4 + 3*B*b^2 + 20*A*b*c + (6*B*b
*c + 5*A*c^2)*x^2)*sqrt(c*x^4 + b*x^2))/(c*x)]
```

3.123.6 Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}\,(A+Bx^2)}{x^4}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**4,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**4, x)
```

3.123.7 Maxima [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^4}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^4, x)
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.37

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4}\,dx = \frac{Ab^2\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}} - \frac{\left(15\,Ab^2c\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+3\,B\sqrt{-b}b^{\frac{5}{2}}+20\,A\sqrt{-b}b^{\frac{3}{2}}c\right)\mathrm{sgn}(x)}{15\sqrt{-b}c} + \frac{3\,(cx^2+b)^{\frac{5}{2}}Bc^4\mathrm{sgn}(x)+5\,(cx^2+b)^{\frac{3}{2}}Ac^5\mathrm{sgn}(x)+15\sqrt{cx^2+b}Abc^5\mathrm{sgn}(x)}{15\,c^5}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
A*b^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - 1/15*(15*A*b^2*c*
arctan(sqrt(b)/sqrt(-b)) + 3*B*sqrt(-b)*b^(5/2) + 20*A*sqrt(-b)*b^(3/2)*c)
*sgn(x)/(sqrt(-b)*c) + 1/15*(3*(c*x^2 + b)^(5/2)*B*c^4*sgn(x) + 5*(c*x^2 +
 b)^(3/2)*A*c^5*sgn(x) + 15*sqrt(c*x^2 + b)*A*b*c^5*sgn(x))/c^5
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4}\,dx = \int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2)^{3/2}}{x^4}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^4,x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^4, x)
```

3.124 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^6} \, dx$

3.124.1 Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^6} \, dx = \frac{(2bB+3Ac)\sqrt{bx^2+cx^4}}{2x} + \frac{(2bB+3Ac)(bx^2+cx^4)^{3/2}}{6bx^3} - \frac{A(bx^2+cx^4)^{5/2}}{2bx^7} - \frac{1}{2}\sqrt{b}(2bB+3Ac)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)$$

output
```
1/6*(3*A*c+2*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^3-1/2*A*(c*x^4+b*x^2)^(5/2)/b/x^
7-1/2*(3*A*c+2*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))*b^(1/2)+1/2*(3*
A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/x
```

3.124.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^6} \, dx = \frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b+cx^2}(-3Ab+8bBx^2+6Acx^2+2Bcx^4)-3\sqrt{b}(2bB\right.}{6x^3\sqrt{b+cx^2}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^6,x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b + c*x^2]*(-3*A*b + 8*b*B*x^2 + 6*A*c*x^2 +
2*B*c*x^4) - 3*Sqrt[b]*(2*b*B + 3*A*c)*x^2*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]
]))/(6*x^3*Sqrt[b + c*x^2])
```

3.124.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1944, 1426, 1426, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^6}dx$$

$$\Big\downarrow \; 1944$$

$$\frac{(3Ac+2bB)\int \frac{(cx^4+bx^2)^{3/2}}{x^4}dx}{2b}-\frac{A(bx^2+cx^4)^{5/2}}{2bx^7}$$

$$\Big\downarrow \; 1426$$

$$\frac{(3Ac+2bB)\left(b\int \frac{\sqrt{cx^4+bx^2}}{x^2}dx+\frac{(bx^2+cx^4)^{3/2}}{3x^3}\right)}{2b}-\frac{A(bx^2+cx^4)^{5/2}}{2bx^7}$$

$$\Big\downarrow \; 1426$$

$$\frac{(3Ac+2bB)\left(b\left(b\int \frac{1}{\sqrt{cx^4+bx^2}}dx+\frac{\sqrt{bx^2+cx^4}}{x}\right)+\frac{(bx^2+cx^4)^{3/2}}{3x^3}\right)}{2b}-\frac{A(bx^2+cx^4)^{5/2}}{2bx^7}$$

$$\Big\downarrow \; 1400$$

$$\frac{(3Ac+2bB)\left(b\left(\frac{\sqrt{bx^2+cx^4}}{x}-b\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}\right)+\frac{(bx^2+cx^4)^{3/2}}{3x^3}\right)}{2b}-\frac{A(bx^2+cx^4)^{5/2}}{2bx^7}$$

$$\Big\downarrow \; 219$$

$$\frac{(3Ac+2bB)\left(b\left(\frac{\sqrt{bx^2+cx^4}}{x}-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)\right)+\frac{(bx^2+cx^4)^{3/2}}{3x^3}\right)}{2b}-\frac{A(bx^2+cx^4)^{5/2}}{2bx^7}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^6,x]
```

output
```
-1/2*(A*(b*x^2 + c*x^4)^(5/2))/(b*x^7) + ((2*b*B + 3*A*c)*((b*x^2 + c*x^4)
^(3/2)/(3*x^3) + b*(Sqrt[b*x^2 + c*x^4]/x - Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sq
rt[b*x^2 + c*x^4]])))/(2*b)
```

3.124.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.124. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^6}\,dx$

3.124.4 Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{bA\sqrt{x^2(c x^2+b)}}{2x^3}+\frac{\left(B c^2\left(\frac{x^2\sqrt{c x^2+b}}{3c}-\frac{2b\sqrt{c x^2+b}}{3c^2}\right)+A\sqrt{c x^2+b}\,c+2Bb\sqrt{c x^2+b}-\frac{\sqrt{b}\,(3Ac+2Bb)\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c x^2+b}}{x}\right)}{2}\right)\sqrt{x^2(c x}{x\sqrt{c x^2+b}}$
default	$-\frac{(x^4c+b x^2)^{\frac{3}{2}}\left(-3A(c x^2+b)^{\frac{3}{2}} c x^2+9A b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c x^2+b}}{x}\right) c x^2-2B(c x^2+b)^{\frac{3}{2}} b x^2+6B b^{\frac{5}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c x^2+b}}{x}\right) x^2+3A(c x^2}{6x^5(c x^2+b)^{\frac{3}{2}} b}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*A/x^3*(x^2*(c*x^2+b))^(1/2)+(B*c^2*(1/3*x^2/c*(c*x^2+b)^(1/2)-2/3*b
/c^2*(c*x^2+b)^(1/2))+A*(c*x^2+b)^(1/2)*c+2*B*b*(c*x^2+b)^(1/2)-1/2*b^(1/2
)*(3*A*c+2*B*b)*ln((2*b+2*b^(1/2)*(c*x^2+b)^(1/2))/x))*(x^2*(c*x^2+b))^(1/
2)/x/(c*x^2+b)^(1/2)
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.47

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^6}\,dx = \left[\frac{3\,(2\,Bb+3\,Ac)\sqrt{b}x^3\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,(2\,Bcx^4+2\,(4\,Bb}{12\,x^3}\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
[1/12*(3*(2*B*b + 3*A*c)*sqrt(b)*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 +
b*x^2)*sqrt(b))/x^3) + 2*(2*B*c*x^4 + 2*(4*B*b + 3*A*c)*x^2 - 3*A*b)*sqrt(
c*x^4 + b*x^2))/x^3, 1/6*(3*(2*B*b + 3*A*c)*sqrt(-b)*x^3*arctan(sqrt(c*x^4
 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (2*B*c*x^4 + 2*(4*B*b + 3*A*c)*x^2 - 3
*A*b)*sqrt(c*x^4 + b*x^2))/x^3]
```

3.124.6 Sympy [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^6}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}\,(A+Bx^2)}{x^6}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**6,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**6, x)
```

3.124.7 Maxima [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^6}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^6}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^6, x)
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^6}\,dx = \frac{2\,(cx^2+b)^{\frac{3}{2}}\,Bc\mathrm{sgn}(x)+6\,\sqrt{cx^2+b}Bbc\mathrm{sgn}(x)+6\,\sqrt{cx^2+b}Ac^2\mathrm{sgn}(x)-}{6\,c}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="giac")
```

output
```
1/6*(2*(c*x^2 + b)^(3/2)*B*c*sgn(x) + 6*sqrt(c*x^2 + b)*B*b*c*sgn(x) + 6*s
qrt(c*x^2 + b)*A*c^2*sgn(x) - 3*sqrt(c*x^2 + b)*A*b*c*sgn(x)/x^2 + 3*(2*B*
b^2*c*sgn(x) + 3*A*b*c^2*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b)
)/c
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^6}\,dx=\int \frac{\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^6}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^6,x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^6, x)
```

3.125 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8}\,dx$

3.125.1 Optimal result

Integrand size = 26, antiderivative size = 135

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8}\,dx = \frac{3c(4bB+Ac)\sqrt{bx^2+cx^4}}{8bx} - \frac{(4bB+Ac)(bx^2+cx^4)^{3/2}}{8bx^5} - \frac{A(bx^2+cx^4)^{5/2}}{4bx^9} - \frac{3c(4bB+Ac)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}$$

output
```
-1/8*(A*c+4*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^5-1/4*A*(c*x^4+b*x^2)^(5/2)/b/x^9
-3/8*c*(A*c+4*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(1/2)+3/8*c*(A
*c+4*B*b)*(c*x^4+b*x^2)^(1/2)/b/x
```

3.125.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8}\,dx = -\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b}\sqrt{b+cx^2}(2Ab+4bBx^2+5Acx^2-8Bcx^4)+3c(4bB+Ac)x^4\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{8\sqrt{b}x^5\sqrt{b+cx^2}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^8,x]
```

output
```
-1/8*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(2*A*b + 4*b*B*x^2 +
5*A*c*x^2 - 8*B*c*x^4) + 3*c*(4*b*B + A*c)*x^4*ArcTanh[Sqrt[b + c*x^2]/Sqr
t[b]]))/(Sqrt[b]*x^5*Sqrt[b + c*x^2])
```

3.125.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1944, 1425, 1426, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(Ac+4bB)\int \frac{(cx^4+bx^2)^{3/2}}{x^6}dx}{4b}-\frac{A(bx^2+cx^4)^{5/2}}{4bx^9}$$

$$\Big\downarrow \text{1425}$$

$$\frac{(Ac+4bB)\left(\frac{3}{2}c\int \frac{\sqrt{cx^4+bx^2}}{x^2}dx-\frac{(bx^2+cx^4)^{3/2}}{2x^5}\right)}{4b}-\frac{A(bx^2+cx^4)^{5/2}}{4bx^9}$$

$$\Big\downarrow \text{1426}$$

$$\frac{(Ac+4bB)\left(\frac{3}{2}c\left(b\int \frac{1}{\sqrt{cx^4+bx^2}}dx+\frac{\sqrt{bx^2+cx^4}}{x}\right)-\frac{(bx^2+cx^4)^{3/2}}{2x^5}\right)}{4b}-\frac{A(bx^2+cx^4)^{5/2}}{4bx^9}$$

$$\Big\downarrow \text{1400}$$

$$\frac{(Ac+4bB)\left(\frac{3}{2}c\left(\frac{\sqrt{bx^2+cx^4}}{x}-b\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}\right)-\frac{(bx^2+cx^4)^{3/2}}{2x^5}\right)}{4b}-\frac{A(bx^2+cx^4)^{5/2}}{4bx^9}$$

$$\Big\downarrow \text{219}$$

$$\frac{(Ac+4bB)\left(\frac{3}{2}c\left(\frac{\sqrt{bx^2+cx^4}}{x}-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)\right)-\frac{(bx^2+cx^4)^{3/2}}{2x^5}\right)}{4b}-\frac{A(bx^2+cx^4)^{5/2}}{4bx^9}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^8,x]
```

output
```
-1/4*(A*(b*x^2 + c*x^4)^(5/2))/(b*x^9) + ((4*b*B + A*c)*(-1/2*(b*x^2 + c*x
^4)^(3/2)/x^5 + (3*c*(Sqrt[b*x^2 + c*x^4]/x - Sqrt[b]*ArcTanh[(Sqrt[b]*x)/
Sqrt[b*x^2 + c*x^4]]))/2))/(4*b)
```

3.125.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.125.4 Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{(5Ac\,x^2+4bB\,x^2+2Ab)\sqrt{x^2(c\,x^2+b)}}{8x^5}+\frac{c\left(8B\sqrt{c\,x^2+b}-\frac{(3Ac+12Bb)\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)}{\sqrt{b}}\right)\sqrt{x^2(c\,x^2+b)}}{8x\sqrt{c\,x^2+b}}$
default	$-\frac{(x^4c+b\,x^2)^{\frac{3}{2}}\left(-A(c\,x^2+b)^{\frac{3}{2}}c^2x^4+3A\,b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)c^2x^4-4B(c\,x^2+b)^{\frac{3}{2}}bc\,x^4+12B\,b^{\frac{5}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)c\,x^4+A(c\right.}{8x^7(c\,x^2+b)^{\frac{3}{2}}b^2}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(5*A*c*x^2+4*B*b*x^2+2*A*b)/x^5*(x^2*(c*x^2+b))^(1/2)+1/8*c*(8*B*(c*x
^2+b)^(1/2)-(3*A*c+12*B*b)/b^(1/2)*ln((2*b+2*b^(1/2)*(c*x^2+b)^(1/2))/x))*
(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.61

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^8}\,dx = \left[\frac{3\,(4\,Bbc+Ac^2)\sqrt{b}x^5\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,(8\,Bbcx^4-2\,Ab^2}{16\,bx^5}\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
[1/16*(3*(4*B*b*c + A*c^2)*sqrt(b)*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4
+ b*x^2)*sqrt(b))/x^3) + 2*(8*B*b*c*x^4 - 2*A*b^2 - (4*B*b^2 + 5*A*b*c)*x^
2)*sqrt(c*x^4 + b*x^2))/(b*x^5), 1/8*(3*(4*B*b*c + A*c^2)*sqrt(-b)*x^5*arc
tan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (8*B*b*c*x^4 - 2*A*b^2 -
 (4*B*b^2 + 5*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b*x^5)]
```

3.125.6 Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)}{x^8}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**8,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**8, x)
```

3.125.7 Maxima [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^8}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^8, x)
```

3.125.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8}\,dx = \frac{8\sqrt{cx^2+b}Bc^2\mathrm{sgn}(x) + \frac{3(4Bbc^2\mathrm{sgn}(x)+Ac^3\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{4(cx^2+b)^{\frac{3}{2}}Bb}{}}{8c}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="giac")
```

output
```
1/8*(8*sqrt(c*x^2 + b)*B*c^2*sgn(x) + 3*(4*B*b*c^2*sgn(x) + A*c^3*sgn(x))*
arctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) - (4*(c*x^2 + b)^(3/2)*B*b*c^2*s
gn(x) - 4*sqrt(c*x^2 + b)*B*b^2*c^2*sgn(x) + 5*(c*x^2 + b)^(3/2)*A*c^3*sgn
(x) - 3*sqrt(c*x^2 + b)*A*b*c^3*sgn(x))/(c^2*x^4))/c
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8}\,dx=\int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2)^{3/2}}{x^8}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^8,x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^8, x)
```

3.126 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{10}}\,dx$

3.126.1 Optimal result

Integrand size = 26, antiderivative size = 140

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{10}}\,dx = -\frac{c(6bB-Ac)\sqrt{bx^2+cx^4}}{16bx^3} - \frac{(6bB-Ac)(bx^2+cx^4)^{3/2}}{24bx^7} - \frac{A(bx^2+cx^4)^{5/2}}{6bx^{11}} - \frac{c^2(6bB-Ac)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}$$

output
```
-1/24*(-A*c+6*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^7-1/6*A*(c*x^4+b*x^2)^(5/2)/b/x
^11-1/16*c^2*(-A*c+6*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(3/2)-1
/16*c*(-A*c+6*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^3
```

3.126.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{10}}\,dx = \frac{\sqrt{x^2(b+cx^2)}\left(-\sqrt{b}\sqrt{b+cx^2}(6bBx^2(2b+5cx^2)+A(8b^2+14bcx^2+3c^2x\right.}{48b^{3/2}x^7\sqrt{b+cx^2}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^10,x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(-(Sqrt[b]*Sqrt[b + c*x^2]*(6*b*B*x^2*(2*b + 5*c*x^
2) + A*(8*b^2 + 14*b*c*x^2 + 3*c^2*x^4))) + 3*c^2*(-6*b*B + A*c)*x^6*ArcTa
nh[Sqrt[b + c*x^2]/Sqrt[b]]))/(48*b^(3/2)*x^7*Sqrt[b + c*x^2])
```

3.126.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1944, 1425, 1425, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{10}}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(6bB-Ac)\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^8}dx}{6b}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{6bx^{11}}$$

$$\Big\downarrow \text{1425}$$

$$\frac{(6bB-Ac)\left(\frac{3}{4}c\int \frac{\sqrt{cx^4+bx^2}}{x^4}dx-\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}\right)}{6b}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{6bx^{11}}$$

$$\Big\downarrow \text{1425}$$

$$\frac{(6bB-Ac)\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{cx^4+bx^2}}dx-\frac{\sqrt{bx^2+cx^4}}{2x^3}\right)-\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}\right)}{6b}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{6bx^{11}}$$

$$\Big\downarrow \text{1400}$$

$$\frac{(6bB-Ac)\left(\frac{3}{4}c\left(-\frac{1}{2}c\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}-\frac{\sqrt{bx^2+cx^4}}{2x^3}\right)-\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}\right)}{6b}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{6bx^{11}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(6bB-Ac)\left(\frac{3}{4}c\left(-\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}-\frac{\sqrt{bx^2+cx^4}}{2x^3}\right)-\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}\right)}{6b}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{6bx^{11}}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^10,x]
```

output
```
-1/6*(A*(b*x^2 + c*x^4)^(5/2))/(b*x^11) + ((6*b*B - A*c)*(-1/4*(b*x^2 + c*
x^4)^(3/2)/x^7 + (3*c*(-1/2*Sqrt[b*x^2 + c*x^4]/x^3 - (c*ArcTanh[(Sqrt[b]*
x)/Sqrt[b*x^2 + c*x^4]])/(2*Sqrt[b])))/4))/(6*b)
```

3.126.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.126.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{\left(3A\,c^{2}x^{4}+30x^{4}Bbc+14Abc\,x^{2}+12b^{2}B\,x^{2}+8b^{2}A\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{48x^{7}b}+\frac{\left(Ac-6Bb\right)c^{2}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{16b^{\frac{3}{2}}x\sqrt{c\,x^{2}+b}}$
default	$-\frac{\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}\left(A\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{3}x^{6}-3A\,b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)c^{3}x^{6}-6B\left(c\,x^{2}+b\right)^{\frac{3}{2}}b\,c^{2}x^{6}+18B\,b^{\frac{5}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)c^{2}x^{6}-A(c}{48x^{9}}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(3*A*c^2*x^4+30*B*b*c*x^4+14*A*b*c*x^2+12*B*b^2*x^2+8*A*b^2)/x^7/b*(
x^2*(c*x^2+b))^(1/2)+1/16*(A*c-6*B*b)*c^2/b^(3/2)*ln((2*b+2*b^(1/2)*(c*x^2
+b)^(1/2))/x)*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.79

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{10}}\,dx = \left[-\frac{3\,(6\,Bbc^2-Ac^3)\sqrt{b}x^7\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,(3\,(10\,Bb^2c+A}{96\,b^2x^7}\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="fricas")
```

output
```
[-1/96*(3*(6*B*b*c^2 - A*c^3)*sqrt(b)*x^7*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x
^4 + b*x^2)*sqrt(b))/x^3) + 2*(3*(10*B*b^2*c + A*b*c^2)*x^4 + 8*A*b^3 + 2*
(6*B*b^3 + 7*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b^2*x^7), 1/48*(3*(6*B*b*
c^2 - A*c^3)*sqrt(-b)*x^7*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x
)) - (3*(10*B*b^2*c + A*b*c^2)*x^4 + 8*A*b^3 + 2*(6*B*b^3 + 7*A*b^2*c)*x^2
)*sqrt(c*x^4 + b*x^2))/(b^2*x^7)]
```

3.126.6 Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{10}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)}{x^{10}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**10,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**10, x)
```

3.126.7 Maxima [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{10}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{10}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^10, x)
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.25

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{10}}\,dx = \frac{\frac{3\,(6\,Bbc^3\mathrm{sgn}(x)-Ac^4\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b} - \frac{30\,(cx^2+b)^{\frac{5}{2}}Bbc^3\mathrm{sgn}(x)-48\,(cx^2+b)^{\frac{3}{2}}Bb^2c^3\mathrm{sg}}{}}{4}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="giac")
```

output
```
1/48*(3*(6*B*b*c^3*sgn(x) - A*c^4*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-b))
/(sqrt(-b)*b) - (30*(c*x^2 + b)^(5/2)*B*b*c^3*sgn(x) - 48*(c*x^2 + b)^(3/2
)*B*b^2*c^3*sgn(x) + 18*sqrt(c*x^2 + b)*B*b^3*c^3*sgn(x) + 3*(c*x^2 + b)^(
5/2)*A*c^4*sgn(x) + 8*(c*x^2 + b)^(3/2)*A*b*c^4*sgn(x) - 3*sqrt(c*x^2 + b)
*A*b^2*c^4*sgn(x))/(b*c^3*x^6))/c
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{10}}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{10}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^10,x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^10, x)
```

3.127 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}}\,dx$

3.127.1 Optimal result

Integrand size = 26, antiderivative size = 177

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}}\,dx = -\frac{c(8bB-3Ac)\sqrt{bx^2+cx^4}}{64bx^5} - \frac{c^2(8bB-3Ac)\sqrt{bx^2+cx^4}}{128b^2x^3} - \frac{(8bB-3Ac)(bx^2+cx^4)^{3/2}}{48bx^9} - \frac{A(bx^2+cx^4)^{5/2}}{8bx^{13}} + \frac{c^3(8bB-3Ac)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}$$

output
```
-1/48*(-3*A*c+8*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^9-1/8*A*(c*x^4+b*x^2)^(5/2)/b
/x^13+1/128*c^3*(-3*A*c+8*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(5
/2)-1/64*c*(-3*A*c+8*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^5-1/128*c^2*(-3*A*c+8*B*
b)*(c*x^4+b*x^2)^(1/2)/b^2/x^3
```

3.127.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}}\,dx = -\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b}\sqrt{b+cx^2}(8bBx^2(8b^2+14bcx^2+3c^2x^4)+A(48b^3+72b^2cx^2+6bc^2x^4-9c^3x^6))+3c^3(\right.}{384b^{5/2}x^9\sqrt{b+cx^2}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^12,x]
```

output
```
-1/384*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(8*b*B*x^2*(8*b^2 +
 14*b*c*x^2 + 3*c^2*x^4) + A*(48*b^3 + 72*b^2*c*x^2 + 6*b*c^2*x^4 - 9*c^3*
x^6)) + 3*c^3*(-8*b*B + 3*A*c)*x^8*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(b^(
5/2)*x^9*Sqrt[b + c*x^2])
```

3.127.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1944, 1425, 1425, 1430, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{12}}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(8bB-3Ac)\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^{10}}dx}{8b}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{8bx^{13}}$$

$$\Big\downarrow \text{1425}$$

$$\frac{(8bB-3Ac)\left(\frac{1}{2}c\int \frac{\sqrt{cx^4+bx^2}}{x^6}dx-\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}\right)}{8b}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{8bx^{13}}$$

$$\Big\downarrow \text{1425}$$

$$\frac{(8bB-3Ac)\left(\frac{1}{2}c\left(\frac{1}{4}c\int \frac{1}{x^2\sqrt{cx^4+bx^2}}dx-\frac{\sqrt{bx^2+cx^4}}{4x^5}\right)-\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}\right)}{8b}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{8bx^{13}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{(8bB-3Ac)\left(\frac{1}{2}c\left(\frac{1}{4}c\left(-\frac{c\int \frac{1}{\sqrt{cx^4+bx^2}}dx}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)-\frac{\sqrt{bx^2+cx^4}}{4x^5}\right)-\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}\right)}{8b}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{8bx^{13}}$$

$$\Big\downarrow \text{1400}$$

$$\frac{(8bB-3Ac)\left(\frac{1}{2}c\left(\frac{1}{4}c\left(\frac{c\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)-\frac{\sqrt{bx^2+cx^4}}{4x^5}\right)-\frac{(bx^2+cx^4)^{3/2}}{6x^9}\right)}{8b}-\frac{A(bx^2+cx^4)^{5/2}}{8bx^{13}}$$

$$\Big\downarrow \; 219$$

$$\frac{(8bB-3Ac)\left(\frac{1}{2}c\left(\frac{1}{4}c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)-\frac{\sqrt{bx^2+cx^4}}{4x^5}\right)-\frac{(bx^2+cx^4)^{3/2}}{6x^9}\right)}{8b}-\frac{A(bx^2+cx^4)^{5/2}}{8bx^{13}}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^12,x]
```

output
```
-1/8*(A*(b*x^2 + c*x^4)^(5/2))/(b*x^13) + ((8*b*B - 3*A*c)*(-1/6*(b*x^2 +
c*x^4)^(3/2)/x^9 + (c*(-1/4*Sqrt[b*x^2 + c*x^4]/x^5 + (c*(-1/2*Sqrt[b*x^2
+ c*x^4]/(b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(3/2)
)))/4))/2))/(8*b)
```

3.127.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.127.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\left(-9 A\,c^{3} x^{6}+24 x^{6} B b\,c^{2}+6 A b\,c^{2} x^{4}+112 x^{4} B\,b^{2} c+72 A\,b^{2} c\,x^{2}+64 b^{3} B\,x^{2}+48 b^{3} A\right) \sqrt{x^{2} \left(c\,x^{2}+b\right)}}{384 x^{9} b^{2}}-\frac{\left(3 A c-8 B b\right) c^{3} \ln \left(\frac{2 b+2 \sqrt{b}\,\sqrt{c\,x^{2}}}{x}\right.}{128 b^{\frac{5}{2}} x \sqrt{c\,x^{2}+}}$
default	$-\frac{\left(x^{4} c+b\,x^{2}\right)^{\frac{3}{2}} \left(-3 A \left(c\,x^{2}+b\right)^{\frac{3}{2}} c^{4} x^{8}+9 A\,b^{\frac{3}{2}} \ln \left(\frac{2 b+2 \sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right) c^{4} x^{8}+8 B \left(c\,x^{2}+b\right)^{\frac{3}{2}} b\,c^{3} x^{8}-24 B\,b^{\frac{5}{2}} \ln \left(\frac{2 b+2 \sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right) c^{3} x^{8}+}{}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/384*(-9*A*c^3*x^6+24*B*b*c^2*x^6+6*A*b*c^2*x^4+112*B*b^2*c*x^4+72*A*b^2
*c*x^2+64*B*b^3*x^2+48*A*b^3)/x^9/b^2*(x^2*(c*x^2+b))^(1/2)-1/128*(3*A*c-8
*B*b)*c^3/b^(5/2)*ln((2*b+2*b^(1/2)*(c*x^2+b)^(1/2))/x)*(x^2*(c*x^2+b))^(1
/2)/x/(c*x^2+b)^(1/2)
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.69

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}}\,dx = \left[-\frac{3\,(8\,Bbc^3-3\,Ac^4)\sqrt{b}x^9\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,(3\,(8\,Bb^2c^2-\right.$$

$$-\frac{3\,(8\,Bbc^3-3\,Ac^4)\sqrt{-b}x^9\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+(3\,(8\,Bb^2c^2-3\,Abc^3)x^6+48\,Ab^4+2\,(56\,Bb^3c+3\,Ab^2c}{384\,b^3x^9}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="fricas")
```

output
```
[-1/768*(3*(8*B*b*c^3 - 3*A*c^4)*sqrt(b)*x^9*log(-(c*x^3 + 2*b*x - 2*sqrt(
c*x^4 + b*x^2)*sqrt(b))/x^3) + 2*(3*(8*B*b^2*c^2 - 3*A*b*c^3)*x^6 + 48*A*b
^4 + 2*(56*B*b^3*c + 3*A*b^2*c^2)*x^4 + 8*(8*B*b^4 + 9*A*b^3*c)*x^2)*sqrt(
c*x^4 + b*x^2))/(b^3*x^9), -1/384*(3*(8*B*b*c^3 - 3*A*c^4)*sqrt(-b)*x^9*ar
ctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (3*(8*B*b^2*c^2 - 3*A*b
*c^3)*x^6 + 48*A*b^4 + 2*(56*B*b^3*c + 3*A*b^2*c^2)*x^4 + 8*(8*B*b^4 + 9*A
*b^3*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b^3*x^9)]
```

3.127.6 Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}\,(A+Bx^2)}{x^{12}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**12,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**12, x)
```

3.127.7 Maxima [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{12}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^12, x)
```

3.127.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}}\,dx =$$
$$-\frac{\frac{3\,(8\,Bbc^4\mathrm{sgn}(x)-3\,Ac^5\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2}+\frac{24\,(cx^2+b)^{\frac{7}{2}}Bbc^4\mathrm{sgn}(x)+40\,(cx^2+b)^{\frac{5}{2}}Bb^2c^4\mathrm{sgn}(x)-88\,(cx^2+b)^{\frac{3}{2}}Bb^3c^4\mathrm{sgn}(x)+24\,\sqrt{cx^2\cdots}}{\cdots}}{384\,c}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="giac")
```

output
```
-1/384*(3*(8*B*b*c^4*sgn(x) - 3*A*c^5*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(
-b))/(sqrt(-b)*b^2) + (24*(c*x^2 + b)^(7/2)*B*b*c^4*sgn(x) + 40*(c*x^2 + b
)^(5/2)*B*b^2*c^4*sgn(x) - 88*(c*x^2 + b)^(3/2)*B*b^3*c^4*sgn(x) + 24*sqrt
(c*x^2 + b)*B*b^4*c^4*sgn(x) - 9*(c*x^2 + b)^(7/2)*A*c^5*sgn(x) + 33*(c*x^
2 + b)^(5/2)*A*b*c^5*sgn(x) + 33*(c*x^2 + b)^(3/2)*A*b^2*c^5*sgn(x) - 9*sq
rt(c*x^2 + b)*A*b^3*c^5*sgn(x))/(b^2*c^4*x^8))/c
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}}\,dx = \int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2)^{3/2}}{x^{12}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^12,x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^12, x)
```

3.128 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}} \, dx$

3.128.1 Optimal result

Integrand size = 26, antiderivative size = 214

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}} \, dx = -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{c^2(2bB-Ac)\sqrt{bx^2+cx^4}}{128b^2x^5} + \frac{3c^3(2bB-Ac)\sqrt{bx^2+cx^4}}{256b^3x^3} - \frac{(2bB-Ac)(bx^2+cx^4)^{3/2}}{16bx^{11}} - \frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}} - \frac{3c^4(2bB-Ac)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}}$$

output
```
-1/16*(-A*c+2*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^11-1/10*A*(c*x^4+b*x^2)^(5/2)/b
/x^15-3/256*c^4*(-A*c+2*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(7/2
)-1/32*c*(-A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^7-1/128*c^2*(-A*c+2*B*b)*(c*
x^4+b*x^2)^(1/2)/b^2/x^5+3/256*c^3*(-A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^
3
```

3.128.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}} \, dx = \frac{-\sqrt{b}(b+cx^2)(10bBx^2(16b^3+24b^2cx^2+2bc^2x^4-3c^3x^6)+A(128b^4+17}{1280}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^14,x]
```

output
```
(-(Sqrt[b]*(b + c*x^2)*(10*b*B*x^2*(16*b^3 + 24*b^2*c*x^2 + 2*b*c^2*x^4 -
3*c^3*x^6) + A*(128*b^4 + 176*b^3*c*x^2 + 8*b^2*c^2*x^4 - 10*b*c^3*x^6 + 1
5*c^4*x^8))) + 15*c^4*(-2*b*B + A*c)*x^10*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b +
 c*x^2]/Sqrt[b]])/(1280*b^(7/2)*x^9*Sqrt[x^2*(b + c*x^2)])
```

3.128.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1944, 1425, 1425, 1430, 1430, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{14}}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(2bB-Ac)\int \frac{(cx^4+bx^2)^{3/2}}{x^{12}}dx}{2b}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{10bx^{15}}$$

$$\Big\downarrow \text{1425}$$

$$\frac{(2bB-Ac)\left(\frac{3}{8}c\int \frac{\sqrt{cx^4+bx^2}}{x^8}dx-\frac{(bx^2+cx^4)^{3/2}}{8x^{11}}\right)}{2b}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{10bx^{15}}$$

$$\Big\downarrow \text{1425}$$

$$\frac{(2bB-Ac)\left(\frac{3}{8}c\left(\frac{1}{6}c\int \frac{1}{x^4\sqrt{cx^4+bx^2}}dx-\frac{\sqrt{bx^2+cx^4}}{6x^7}\right)-\frac{(bx^2+cx^4)^{3/2}}{8x^{11}}\right)}{2b}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{10bx^{15}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{(2bB-Ac)\left(\frac{3}{8}c\left(\frac{1}{6}c\left(-\frac{3c\int \frac{1}{x^2\sqrt{cx^4+bx^2}}dx}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)-\frac{\sqrt{bx^2+cx^4}}{6x^7}\right)-\frac{(bx^2+cx^4)^{3/2}}{8x^{11}}\right)}{2b}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{10bx^{15}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{(2bB-Ac)\left(\frac{3}{8}c\left(\frac{1}{6}c\left(-\frac{3c\left(-\frac{c\int\frac{1}{\sqrt{cx^4+bx^2}}dx}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)-\frac{\sqrt{bx^2+cx^4}}{6x^7}\right)-\frac{(bx^2+cx^4)^{3/2}}{8x^{11}}\right)}{2b}-\frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}}$$

$\downarrow$ 1400

$$\frac{(2bB-Ac)\left(\frac{3}{8}c\left(\frac{1}{6}c\left(-\frac{3c\left(\frac{c\int\frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)-\frac{\sqrt{bx^2+cx^4}}{6x^7}\right)-\frac{(bx^2+cx^4)^{3/2}}{8x^{11}}\right)}{2b}-\frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}}$$

$\downarrow$ 219

$$\frac{(2bB-Ac)\left(\frac{3}{8}c\left(\frac{1}{6}c\left(-\frac{3c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)-\frac{\sqrt{bx^2+cx^4}}{6x^7}\right)-\frac{(bx^2+cx^4)^{3/2}}{8x^{11}}\right)}{2b}-\frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^14,x]
```

output
```
-1/10*(A*(b*x^2 + c*x^4)^(5/2))/(b*x^15) + ((2*b*B - A*c)*(-1/8*(b*x^2 + c
*x^4)^(3/2)/x^11 + (3*c*(-1/6*Sqrt[b*x^2 + c*x^4]/x^7 + (c*(-1/4*Sqrt[b*x^
2 + c*x^4]/(b*x^5) - (3*c*(-1/2*Sqrt[b*x^2 + c*x^4]/(b*x^3) + (c*ArcTanh[(
Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(3/2))))/(4*b)))/6))/8))/(2*b)
```

3.128. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}}\,dx$

3.128.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.128. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}} dx$

3.128.4 Maple [A] (verified)

Time = 3.20 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{\left(15A\,x^{8}c^{4}-30B\,x^{8}b\,c^{3}-10A\,x^{6}b\,c^{3}+20B\,x^{6}b^{2}c^{2}+8A\,b^{2}c^{2}x^{4}+240B\,b^{3}c\,x^{4}+176A\,x^{2}b^{3}c+160B\,x^{2}b^{4}+128A\,b^{4}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{1280x^{11}b^{3}}+$
default	$\frac{\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}\left(15A\,b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)c^{5}x^{10}-5A\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{5}x^{10}-30B\,b^{\frac{5}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)c^{4}x^{10}+10B\left(c\,x^{2}+b\right)^{\frac{3}{2}}b\,c^{4}x^{10}\right.}{}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/1280*(15*A*c^4*x^8-30*B*b*c^3*x^8-10*A*b*c^3*x^6+20*B*b^2*c^2*x^6+8*A*b
^2*c^2*x^4+240*B*b^3*c*x^4+176*A*b^3*c*x^2+160*B*b^4*x^2+128*A*b^4)/x^11/b
^3*(x^2*(c*x^2+b))^(1/2)+3/256*(A*c-2*B*b)*c^4/b^(7/2)*ln((2*b+2*b^(1/2)*(
c*x^2+b)^(1/2))/x)*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

3.128.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.61

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{14}}\,dx=\left[-\frac{15\left(2\,Bbc^4-Ac^5\right)\sqrt{b}x^{11}\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)-2\left(15\left(2\,Bb^2c^3\right.\right.}{}\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="fricas")
```

output
```
[-1/2560*(15*(2*B*b*c^4 - A*c^5)*sqrt(b)*x^11*log(-(c*x^3 + 2*b*x + 2*sqrt
(c*x^4 + b*x^2)*sqrt(b))/x^3) - 2*(15*(2*B*b^2*c^3 - A*b*c^4)*x^8 - 10*(2*
B*b^3*c^2 - A*b^2*c^3)*x^6 - 128*A*b^5 - 8*(30*B*b^4*c + A*b^3*c^2)*x^4 -
16*(10*B*b^5 + 11*A*b^4*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b^4*x^11), 1/1280*(1
5*(2*B*b*c^4 - A*c^5)*sqrt(-b)*x^11*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c
*x^3 + b*x)) + (15*(2*B*b^2*c^3 - A*b*c^4)*x^8 - 10*(2*B*b^3*c^2 - A*b^2*c
^3)*x^6 - 128*A*b^5 - 8*(30*B*b^4*c + A*b^3*c^2)*x^4 - 16*(10*B*b^5 + 11*A
*b^4*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b^4*x^11)]
```

3.128.6 Sympy [F]

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{14}}\,dx=\int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}(A+Bx^2)}{x^{14}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**14,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**14, x)
```

3.128.7 Maxima [F]

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{14}}\,dx=\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}(Bx^2+A)}{x^{14}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^14, x)
```

3.128.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{14}}\,dx=\frac{\frac{15\left(2\,Bbc^5\mathrm{sgn}(x)-Ac^6\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3}+\frac{30\left(cx^2+b\right)^{\frac{9}{2}}Bbc^5\mathrm{sgn}(x)-140\left(cx^2+b\right)^{\frac{7}{2}}Bb^2c^5}{}}{}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="giac")
```

output
```
1/1280*(15*(2*B*b*c^5*sgn(x) - A*c^6*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-
b))/(sqrt(-b)*b^3) + (30*(c*x^2 + b)^(9/2)*B*b*c^5*sgn(x) - 140*(c*x^2 + b
)^(7/2)*B*b^2*c^5*sgn(x) + 140*(c*x^2 + b)^(3/2)*B*b^4*c^5*sgn(x) - 30*sqr
t(c*x^2 + b)*B*b^5*c^5*sgn(x) - 15*(c*x^2 + b)^(9/2)*A*c^6*sgn(x) + 70*(c*
x^2 + b)^(7/2)*A*b*c^6*sgn(x) - 128*(c*x^2 + b)^(5/2)*A*b^2*c^6*sgn(x) - 7
0*(c*x^2 + b)^(3/2)*A*b^3*c^6*sgn(x) + 15*sqrt(c*x^2 + b)*A*b^4*c^6*sgn(x)
)/(b^3*c^5*x^10))/c
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{14}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^14,x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^14, x)
```

3.129 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{16}}\,dx$

3.129.1 Optimal result

Integrand size = 26, antiderivative size = 251

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{16}}\,dx = -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} + \frac{c^3(12bB-7Ac)\sqrt{bx^2+cx^4}}{1536b^3x^5} - \frac{c^4(12bB-7Ac)\sqrt{bx^2+cx^4}}{1024b^4x^3} - \frac{(12bB-7Ac)(bx^2+cx^4)^{3/2}}{120bx^{13}} - \frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}} + \frac{c^5(12bB-7Ac)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{1024b^{9/2}}$$

output
```
-1/120*(-7*A*c+12*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^13-1/12*A*(c*x^4+b*x^2)^(5/
2)/b/x^17+1/1024*c^5*(-7*A*c+12*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2)
)/b^(9/2)-1/320*c*(-7*A*c+12*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^9-1/1920*c^2*(-7
*A*c+12*B*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^7+1/1536*c^3*(-7*A*c+12*B*b)*(c*x^4
+b*x^2)^(1/2)/b^3/x^5-1/1024*c^4*(-7*A*c+12*B*b)*(c*x^4+b*x^2)^(1/2)/b^4/x
^3
```

3.129.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.79

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{16}}\,dx =$$
$$-\frac{\sqrt{x^2(b+cx^2)}\Big(\sqrt{b}\sqrt{b+cx^2}(12bBx^2(128b^4+176b^3cx^2+8b^2c^2x^4-10bc^3x^6+15c^4x^8)+A(1280b^5+1664\cdots}{15360b^{9/2}x^{13}\sqrt{\cdots}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^16,x]
```

output
```
-1/15360*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(12*b*B*x^2*(128*
b^4 + 176*b^3*c*x^2 + 8*b^2*c^2*x^4 - 10*b*c^3*x^6 + 15*c^4*x^8) + A*(1280
*b^5 + 1664*b^4*c*x^2 + 48*b^3*c^2*x^4 - 56*b^2*c^3*x^6 + 70*b*c^4*x^8 - 1
05*c^5*x^10)) + 15*c^5*(-12*b*B + 7*A*c)*x^12*ArcTanh[Sqrt[b + c*x^2]/Sqrt
[b]]))/(b^(9/2)*x^13*Sqrt[b + c*x^2])
```

3.129.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1944, 1425, 1425, 1430, 1430, 1430, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{16}}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(12bB-7Ac)\int \frac{(cx^4+bx^2)^{3/2}}{x^{14}}dx}{12b}-\frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}}$$

$$\Big\downarrow \text{1425}$$

$$\frac{(12bB-7Ac)\left(\frac{3}{10}c\int \frac{\sqrt{cx^4+bx^2}}{x^{10}}dx-\frac{(bx^2+cx^4)^{3/2}}{10x^{13}}\right)}{12b}-\frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}}$$

$$\Big\downarrow \text{1425}$$

$$\frac{(12bB-7Ac)\left(\frac{3}{10}c\left(\frac{1}{8}c\int\frac{1}{x^6\sqrt{cx^4+bx^2}}dx-\frac{\sqrt{bx^2+cx^4}}{8x^9}\right)-\frac{(bx^2+cx^4)^{3/2}}{10x^{13}}\right)}{12b}-\frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{(12bB-7Ac)\left(\frac{3}{10}c\left(\frac{1}{8}c\left(-\frac{5c\int\frac{1}{x^4\sqrt{cx^4+bx^2}}dx}{6b}-\frac{\sqrt{bx^2+cx^4}}{6bx^7}\right)-\frac{\sqrt{bx^2+cx^4}}{8x^9}\right)-\frac{(bx^2+cx^4)^{3/2}}{10x^{13}}\right)}{12b}-\frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{(12bB-7Ac)\left(\frac{3}{10}c\left(\frac{1}{8}c\left(-\frac{5c\left(-\frac{3c\int\frac{1}{x^2\sqrt{cx^4+bx^2}}dx}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)}{6b}-\frac{\sqrt{bx^2+cx^4}}{6bx^7}\right)-\frac{\sqrt{bx^2+cx^4}}{8x^9}\right)-\frac{(bx^2+cx^4)^{3/2}}{10x^{13}}\right)}{12b}-\frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{(12bB-7Ac)\left(\frac{3}{10}c\left(\frac{1}{8}c\left(-\frac{5c\left(-\frac{3c\left(-\frac{c\int\frac{1}{\sqrt{cx^4+bx^2}}dx}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)}{6b}-\frac{\sqrt{bx^2+cx^4}}{6bx^7}\right)-\frac{\sqrt{bx^2+cx^4}}{8x^9}\right)-\frac{(bx^2+cx^4)^{3/}}{10x^{13}}\right)}{12b}-\frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}}$$

$$\Big\downarrow \text{1400}$$

3.129. $\int\frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{16}}dx$

$$\frac{(12bB-7Ac)\left(\frac{3}{10}c\left(\frac{1}{8}c\left(-\frac{5c\left(-\frac{3c\left(\frac{c\int\frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)}{6b}-\frac{\sqrt{bx^2+cx^4}}{6bx^7}\right)-\frac{\sqrt{bx^2+cx^4}}{8x^9}\right)-\frac{(bx^2 \ldots}{\ldots}\right)}{12b}$$

$$\frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}}$$

$\Big\downarrow$ 219

$$\frac{(12bB-7Ac)\left(\frac{3}{10}c\left(\frac{1}{8}c\left(-\frac{5c\left(-\frac{3c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)}{6b}-\frac{\sqrt{bx^2+cx^4}}{6bx^7}\right)-\frac{\sqrt{bx^2+cx^4}}{8x^9}\right)-\frac{(bx^2+\ldots}{10\ldots}\right)}{12b}$$

$$\frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^16,x]
```

output
```
-1/12*(A*(b*x^2 + c*x^4)^(5/2))/(b*x^17) + ((12*b*B - 7*A*c)*(-1/10*(b*x^2
 + c*x^4)^(3/2)/x^13 + (3*c*(-1/8*Sqrt[b*x^2 + c*x^4]/x^9 + (c*(-1/6*Sqrt[
b*x^2 + c*x^4]/(b*x^7) - (5*c*(-1/4*Sqrt[b*x^2 + c*x^4]/(b*x^5) - (3*c*(-1
/2*Sqrt[b*x^2 + c*x^4]/(b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4
]])/(2*b^(3/2))))/(4*b)))/(6*b)))/8))/10))/(12*b)
```

3.129. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{16}}\,dx$

3.129.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.129. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{16}}\,dx$

3.129.4 Maple [A] (verified)

Time = 3.64 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{(-105A\,c^5x^{10}+180Bb\,c^4x^{10}+70A\,x^8b\,c^4-120B\,b^2c^3x^8-56A\,b^2c^3x^6+96B\,b^3c^2x^6+48A\,b^3c^2x^4+2112B\,b^4c\,x^4+1664A\,b^4c\,x^2+153}{15360x^{13}b^4}$
default	$-\frac{(x^4c+b\,x^2)^{\frac{3}{2}}\left(-35A(c\,x^2+b)^{\frac{3}{2}}c^6x^{12}+105A\,b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)c^6x^{12}+60B(c\,x^2+b)^{\frac{3}{2}}b\,c^5x^{12}-180B\,b^{\frac{5}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right.\right.}{}$

input

```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^16,x,method=_RETURNVERBOSE)
```

output

```
-1/15360*(-105*A*c^5*x^10+180*B*b*c^4*x^10+70*A*b*c^4*x^8-120*B*b^2*c^3*x^
8-56*A*b^2*c^3*x^6+96*B*b^3*c^2*x^6+48*A*b^3*c^2*x^4+2112*B*b^4*c*x^4+1664
*A*b^4*c*x^2+1536*B*b^5*x^2+1280*A*b^5)/x^13/b^4*(x^2*(c*x^2+b))^(1/2)-1/1
024*(7*A*c-12*B*b)*c^5/b^(9/2)*ln((2*b+2*b^(1/2)*(c*x^2+b)^(1/2))/x)*(x^2*
(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.57

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{16}}\,dx = \left[-\frac{15\,(12\,Bbc^5-7\,Ac^6)\sqrt{b}x^{13}\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,(15\,(12\,Bb}{}\right.$$

$$-\frac{15\,(12\,Bbc^5-7\,Ac^6)\sqrt{-b}x^{13}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+(15\,(12\,Bb^2c^4-7\,Abc^5)x^{10}-10\,(12\,Bb^3c^3-7\,Ab^2c^4}{1536}$$

input

```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^16,x, algorithm="fricas")
```

output
```
[-1/30720*(15*(12*B*b*c^5 - 7*A*c^6)*sqrt(b)*x^13*log(-(c*x^3 + 2*b*x - 2*
sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3) + 2*(15*(12*B*b^2*c^4 - 7*A*b*c^5)*x^10
- 10*(12*B*b^3*c^3 - 7*A*b^2*c^4)*x^8 + 1280*A*b^6 + 8*(12*B*b^4*c^2 - 7*A
*b^3*c^3)*x^6 + 48*(44*B*b^5*c + A*b^4*c^2)*x^4 + 128*(12*B*b^6 + 13*A*b^5
*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b^5*x^13), -1/15360*(15*(12*B*b*c^5 - 7*A*c
^6)*sqrt(-b)*x^13*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (15
*(12*B*b^2*c^4 - 7*A*b*c^5)*x^10 - 10*(12*B*b^3*c^3 - 7*A*b^2*c^4)*x^8 + 1
280*A*b^6 + 8*(12*B*b^4*c^2 - 7*A*b^3*c^3)*x^6 + 48*(44*B*b^5*c + A*b^4*c^
2)*x^4 + 128*(12*B*b^6 + 13*A*b^5*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b^5*x^13)]
```

3.129.6 Sympy [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{16}}\,dx = \int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}(A+Bx^2)}{x^{16}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**16,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**16, x)
```

3.129.7 Maxima [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{16}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{16}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^16,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^16, x)
```

3.129.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{16}}\,dx =$$

$$-\frac{\frac{15\,(12\,Bbc^6\mathrm{sgn}(x)-7\,Ac^7\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4}+\frac{180\,(cx^2+b)^{\frac{11}{2}}Bbc^6\mathrm{sgn}(x)-1020\,(cx^2+b)^{\frac{9}{2}}Bb^2c^6\mathrm{sgn}(x)+2376\,(cx^2+b)^{\frac{7}{2}}Bb^3c^6\mathrm{sgn}(x)-}{}}{}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^16,x, algorithm="giac")
```

output
```
-1/15360*(15*(12*B*b*c^6*sgn(x) - 7*A*c^7*sgn(x))*arctan(sqrt(c*x^2 + b)/s
qrt(-b))/(sqrt(-b)*b^4) + (180*(c*x^2 + b)^(11/2)*B*b*c^6*sgn(x) - 1020*(c
*x^2 + b)^(9/2)*B*b^2*c^6*sgn(x) + 2376*(c*x^2 + b)^(7/2)*B*b^3*c^6*sgn(x)
 - 696*(c*x^2 + b)^(5/2)*B*b^4*c^6*sgn(x) - 1020*(c*x^2 + b)^(3/2)*B*b^5*c
^6*sgn(x) + 180*sqrt(c*x^2 + b)*B*b^6*c^6*sgn(x) - 105*(c*x^2 + b)^(11/2)*
A*c^7*sgn(x) + 595*(c*x^2 + b)^(9/2)*A*b*c^7*sgn(x) - 1386*(c*x^2 + b)^(7/
2)*A*b^2*c^7*sgn(x) + 1686*(c*x^2 + b)^(5/2)*A*b^3*c^7*sgn(x) + 595*(c*x^2
 + b)^(3/2)*A*b^4*c^7*sgn(x) - 105*sqrt(c*x^2 + b)*A*b^5*c^7*sgn(x))/(b^4*
c^6*x^12))/c
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{16}}\,dx = \int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2)^{3/2}}{x^{16}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^16,x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^16, x)
```

3.130 $\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

3.130.1 Optimal result

Integrand size = 26, antiderivative size = 176

$$\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{5b^2(7bB-8Ac)\sqrt{bx^2+cx^4}}{128c^4}+\frac{5b(7bB-8Ac)x^2\sqrt{bx^2+cx^4}}{192c^3} - \frac{(7bB-8Ac)x^4\sqrt{bx^2+cx^4}}{48c^2}+\frac{Bx^6\sqrt{bx^2+cx^4}}{8c} + \frac{5b^3(7bB-8Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{9/2}}$$

output
```
5/128*b^3*(-8*A*c+7*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(9/2)-
5/128*b^2*(-8*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)/c^4+5/192*b*(-8*A*c+7*B*b)*x^
2*(c*x^4+b*x^2)^(1/2)/c^3-1/48*(-8*A*c+7*B*b)*x^4*(c*x^4+b*x^2)^(1/2)/c^2+
1/8*B*x^6*(c*x^4+b*x^2)^(1/2)/c
```

3.130.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.88

$$\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{x\left(\sqrt{c}(b+cx^2)\left(-105b^3Bx+16c^3x^5(4A+3Bx^2)-8bc^2x^3(10A+7Bx^2)+10b^2cx(12A+7Bx^2)\right)+30b^3\right.}{384c^{9/2}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[(x^7*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(x*(Sqrt[c]*(b + c*x^2)*(-105*b^3*B*x + 16*c^3*x^5*(4*A + 3*B*x^2) - 8*b*c
^2*x^3*(10*A + 7*B*x^2) + 10*b^2*c*x*(12*A + 7*B*x^2)) + 30*b^3*(7*b*B - 8
*A*c)*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] + Sqrt[b + c*x^2])]))/
(384*c^(9/2)*Sqrt[x^2*(b + c*x^2)])
```

3.130.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1940, 1221, 1134, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 (A+Bx^2)}{\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{x^6 (Bx^2+A)}{\sqrt{cx^4+bx^2}}dx^2$$

$$\Big\downarrow \text{1221}$$

$$\frac{1}{2}\left(\frac{Bx^6\sqrt{bx^2+cx^4}}{4c}-\frac{(7bB-8Ac)\int \frac{x^6}{\sqrt{cx^4+bx^2}}dx^2}{8c}\right)$$

$$\Big\downarrow \text{1134}$$

$$\frac{1}{2}\left(\frac{Bx^6\sqrt{bx^2+cx^4}}{4c}-\frac{(7bB-8Ac)\left(\frac{x^4\sqrt{bx^2+cx^4}}{3c}-\frac{5b\int \frac{x^4}{\sqrt{cx^4+bx^2}}dx^2}{6c}\right)}{8c}\right)$$

$$\Big\downarrow \text{1134}$$

$$\frac{1}{2}\left(\frac{Bx^6\sqrt{bx^2+cx^4}}{4c}-\frac{(7bB-8Ac)\left(\frac{x^4\sqrt{bx^2+cx^4}}{3c}-\frac{5b\left(\frac{x^2\sqrt{bx^2+cx^4}}{2c}-\frac{3b\int\frac{x^2}{\sqrt{cx^4+bx^2}}dx^2}{4c}\right)}{6c}\right)}{8c}\right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{Bx^6\sqrt{bx^2+cx^4}}{4c}-\frac{(7bB-8Ac)\left(\frac{x^4\sqrt{bx^2+cx^4}}{3c}-\frac{5b\left(\frac{x^2\sqrt{bx^2+cx^4}}{2c}-\frac{3b\left(\frac{\sqrt{bx^2+cx^4}}{c}-\frac{b\int\frac{1}{\sqrt{cx^4+bx^2}}dx^2}{2c}\right)}{4c}\right)}{6c}\right)}{8c}\right)$$

$$\Big\downarrow \text{1091}$$

3.130. $\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

$$\frac{1}{2}\left(\frac{Bx^6\sqrt{bx^2+cx^4}}{4c}-\frac{(7bB-8Ac)\left(\frac{x^4\sqrt{bx^2+cx^4}}{3c}-\frac{5b\left(\frac{x^2\sqrt{bx^2+cx^4}}{2c}-\frac{3b\left(\frac{\sqrt{bx^2+cx^4}}{c}-\frac{b\int\frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{c}\right)}{4c}\right)}{6c}\right)}{8c}\right)$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(\frac{Bx^6\sqrt{bx^2+cx^4}}{4c}-\frac{(7bB-8Ac)\left(\frac{x^4\sqrt{bx^2+cx^4}}{3c}-\frac{5b\left(\frac{x^2\sqrt{bx^2+cx^4}}{2c}-\frac{3b\left(\frac{\sqrt{bx^2+cx^4}}{c}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}\right)}{4c}\right)}{6c}\right)}{8c}\right)$$

input

```
Int[(x^7*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

3.130. $\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

output
```
((B*x^6*Sqrt[b*x^2 + c*x^4])/(4*c) - ((7*b*B - 8*A*c)*((x^4*Sqrt[b*x^2 + c
*x^4])/(3*c) - (5*b*((x^2*Sqrt[b*x^2 + c*x^4])/(2*c) - (3*b*(Sqrt[b*x^2 +
c*x^4]/c - (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/c^(3/2)))/(4*c))
)/(6*c)))/(8*c))/2
```

3.130.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.130.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{\frac{5\left(-\frac{1}{2} A\, b^{3} c+\frac{7}{16} B\, b^{4}\right) \ln\left(\frac{2 c\, x^{2}+2 \sqrt{x^{2} \left(c\, x^{2}+b\right)}\, \sqrt{c}+b}{\sqrt{c}}\right)}{16}+\frac{5\left(b^{2}\left(\frac{7 x^{2} B}{12}+A\right) c^{\frac{3}{2}}-\frac{2 x^{2}\left(\frac{7 x^{2} B}{10}+A\right) b\, c^{\frac{5}{2}}}{3}+\frac{8 x^{4}\left(\frac{3 x^{2} B}{4}+A\right) c^{\frac{7}{2}}}{15}-\frac{7 B \sqrt{c}\, b^{3}}{8}\right.}{16}}{c^{\frac{9}{2}}}$
risch	$\frac{x^{2}\left(48 B\, c^{3} x^{6}+64 A\, c^{3} x^{4}-56 B b\, c^{2} x^{4}-80 A b\, c^{2} x^{2}+70 B\, b^{2} c\, x^{2}+120 b^{2} A c-105 B\, b^{3}\right)\left(c\, x^{2}+b\right)}{384 c^{4} \sqrt{x^{2}\left(c\, x^{2}+b\right)}}-\frac{5 b^{3}\left(8 A c-7 B b\right) \ln\left(\sqrt{c}\, x+\sqrt{c}\right.}{128 c^{\frac{9}{2}} \sqrt{x^{2}\left(c\, x\right.}}$
default	$\frac{x \sqrt{c\, x^{2}+b}\, \left(48 B\, c^{\frac{9}{2}} \sqrt{c\, x^{2}+b}\, x^{7}+64 A\, c^{\frac{9}{2}} \sqrt{c\, x^{2}+b}\, x^{5}-56 B\, c^{\frac{7}{2}} \sqrt{c\, x^{2}+b}\, b\, x^{5}-80 A\, c^{\frac{7}{2}} \sqrt{c\, x^{2}+b}\, b\, x^{3}+70 B\, c^{\frac{5}{2}} \sqrt{c\, x^{2}+b}\, b^{2} x^{3}+12\right.}{384 \sqrt{x^{4} c+b\, x^{2}}\, c^{\frac{11}{2}}}$

input
```
int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
5/16/c^(9/2)*((-1/2*A*b^3*c+7/16*B*b^4)*ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2
)*c^(1/2)+b)/c^(1/2))+(b^2*(7/12*x^2*B+A)*c^(3/2)-2/3*x^2*(7/10*x^2*B+A)*b
*c^(5/2)+8/15*x^4*(3/4*x^2*B+A)*c^(7/2)-7/8*B*c^(1/2)*b^3)*(x^2*(c*x^2+b))
^(1/2)+1/2*ln(2)*b^3*(A*c-7/8*B*b))
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.56

$$\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \left[-\frac{15\,(7\,Bb^4-8\,Ab^3c)\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,(48\,Bc^4x^6-105\,Bb^3c+120\,Ab^2c^2-8}{768\,c^5}\right.$$
$$-\frac{15\,(7\,Bb^4-8\,Ab^3c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-(48\,Bc^4x^6-105\,Bb^3c+120\,Ab^2c^2-8\,(7\,Bbc^3-8\,Ac}{384\,c^5}$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/768*(15*(7*B*b^4 - 8*A*b^3*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4
+ b*x^2)*sqrt(c)) - 2*(48*B*c^4*x^6 - 105*B*b^3*c + 120*A*b^2*c^2 - 8*(7*B
*b*c^3 - 8*A*c^4)*x^4 + 10*(7*B*b^2*c^2 - 8*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x
^2))/c^5, -1/384*(15*(7*B*b^4 - 8*A*b^3*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*
x^2)*sqrt(-c)/(c*x^2 + b)) - (48*B*c^4*x^6 - 105*B*b^3*c + 120*A*b^2*c^2 -
 8*(7*B*b*c^3 - 8*A*c^4)*x^4 + 10*(7*B*b^2*c^2 - 8*A*b*c^3)*x^2)*sqrt(c*x^
4 + b*x^2))/c^5]
```

3.130.6 Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.27

$$\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\begin{cases} -\frac{5b^3\left(A-\frac{7Bb}{8c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{16c^3}+\sqrt{bx^2+cx^4}\left(\frac{Bx^6}{4c}+\frac{5b^2\left(A-\frac{7Bb}{8c}\right)}{8c^3}-\frac{5bx^2\left(A-\frac{7Bb}{8c}\right)}{12c^2}+\cdots\right. \\ \frac{\frac{2A\left(bx^2\right)^{\frac{7}{2}}}{7b^3}+\frac{2B\left(bx^2\right)^{\frac{9}{2}}}{9b^4}}{b} \\ \tilde{\infty}\left(\frac{Ax^8}{4}+\frac{Bx^{10}}{5}\right)\end{cases}}{2}$$

input
```
integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Piecewise((-5*b**3*(A - 7*B*b/(8*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x
**2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b
/(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(16*c**3) + sqrt(b*x**2
 + c*x**4)*(B*x**6/(4*c) + 5*b**2*(A - 7*B*b/(8*c))/(8*c**3) - 5*b*x**2*(A
 - 7*B*b/(8*c))/(12*c**2) + x**4*(A - 7*B*b/(8*c))/(3*c)), Ne(c, 0)), ((2*
A*(b*x**2)**(7/2)/(7*b**3) + 2*B*(b*x**2)**(9/2)/(9*b**4))/b, Ne(b, 0)), (
zoo*(A*x**8/4 + B*x**10/5), True))/2
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.31

$$\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{1}{96}\left(\frac{16\sqrt{cx^4+bx^2}x^4}{c} - \frac{20\sqrt{cx^4+bx^2}bx^2}{c^2} - \frac{15\,b^3\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{7}{2}}} + \frac{30\sqrt{cx^4+bx^2}b^2}{c^3}\right.$$
$$+\frac{1}{768}\left(\frac{96\sqrt{cx^4+bx^2}x^6}{c} - \frac{112\sqrt{cx^4+bx^2}bx^4}{c^2} + \frac{140\sqrt{cx^4+bx^2}b^2x^2}{c^3} + \frac{105\,b^4\log\left(2\,cx^2+b+2\sqrt{cx^4+\cdots}\right.}{c^{\frac{9}{2}}}\right.$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/96*(16*sqrt(c*x^4 + b*x^2)*x^4/c - 20*sqrt(c*x^4 + b*x^2)*b*x^2/c^2 - 15
*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 30*sqrt(c*
x^4 + b*x^2)*b^2/c^3)*A + 1/768*(96*sqrt(c*x^4 + b*x^2)*x^6/c - 112*sqrt(c
*x^4 + b*x^2)*b*x^4/c^2 + 140*sqrt(c*x^4 + b*x^2)*b^2*x^2/c^3 + 105*b^4*lo
g(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(9/2) - 210*sqrt(c*x^4 +
b*x^2)*b^3/c^4)*B
```

3.130.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.05

$$\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{1}{384}\left(2\left(4\,x^2\left(\frac{6\,Bx^2}{c\mathrm{sgn}\,(x)} - \frac{7\,Bbc^5\mathrm{sgn}(x) - 8\,Ac^6\mathrm{sgn}(x)}{c^7}\right) + \frac{5\,(7\,Bb^2c^4\mathrm{sgn}(x) - 8\,Abc^5\mathrm{sgn}(x))}{c^7}\right)x^2 - \frac{15\,(7\cdots}{}\right.$$
$$+\frac{5\,(7\,Bb^4\log\left(|b|\right) - 8\,Ab^3c\log\left(|b|\right))\mathrm{sgn}(x)}{256\,c^{\frac{9}{2}}} - \frac{5\,(7\,Bb^4 - 8\,Ab^3c)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{128\,c^{\frac{9}{2}}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/384*(2*(4*x^2*(6*B*x^2/(c*sgn(x)) - (7*B*b*c^5*sgn(x) - 8*A*c^6*sgn(x))/
c^7) + 5*(7*B*b^2*c^4*sgn(x) - 8*A*b*c^5*sgn(x))/c^7)*x^2 - 15*(7*B*b^3*c^
3*sgn(x) - 8*A*b^2*c^4*sgn(x))/c^7)*sqrt(c*x^2 + b)*x + 5/256*(7*B*b^4*log
(abs(b)) - 8*A*b^3*c*log(abs(b)))*sgn(x)/c^(9/2) - 5/128*(7*B*b^4 - 8*A*b^
3*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(9/2)*sgn(x))
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{x^7\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.131 $\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

3.131.1 Optimal result

Integrand size = 26, antiderivative size = 139

$$\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{b(5bB-6Ac)\sqrt{bx^2+cx^4}}{16c^3} - \frac{(5bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} - \frac{b^2(5bB-6Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}$$

output
```
-1/16*b^2*(-6*A*c+5*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(7/2)+
1/16*b*(-6*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/c^3-1/24*(-6*A*c+5*B*b)*x^2*(c*x
^4+b*x^2)^(1/2)/c^2+1/6*B*x^4*(c*x^4+b*x^2)^(1/2)/c
```

3.131.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{x\left(\sqrt{c}x(b+cx^2)\left(15b^2B+4c^2x^2(3A+2Bx^2)-2bc(9A+5Bx^2)\right)+6b^2(5bB-6Ac)\sqrt{b+cx^2}\text{arctanh}\left(\frac{}{\sqrt{b}\cdots}\right.\right.}{48c^{7/2}\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(x^5*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(x*(Sqrt[c]*x*(b + c*x^2)*(15*b^2*B + 4*c^2*x^2*(3*A + 2*B*x^2) - 2*b*c*(9
*A + 5*B*x^2)) + 6*b^2*(5*b*B - 6*A*c)*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)
/(Sqrt[b] - Sqrt[b + c*x^2])]))/(48*c^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

3.131.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1940, 1221, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx^2)}{\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 1940}$$

$$\frac{1}{2}\int \frac{x^4 (Bx^2+A)}{\sqrt{cx^4+bx^2}}dx^2$$

$$\Big\downarrow \text{ 1221}$$

$$\frac{1}{2}\left(\frac{Bx^4\sqrt{bx^2+cx^4}}{3c}-\frac{(5bB-6Ac)\int \frac{x^4}{\sqrt{cx^4+bx^2}}dx^2}{6c}\right)$$

$$\Big\downarrow \text{ 1134}$$

$$\frac{1}{2}\left(\frac{Bx^4\sqrt{bx^2+cx^4}}{3c}-\frac{(5bB-6Ac)\left(\frac{x^2\sqrt{bx^2+cx^4}}{2c}-\frac{3b\int \frac{x^2}{\sqrt{cx^4+bx^2}}dx^2}{4c}\right)}{6c}\right)$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{1}{2}\left(\frac{Bx^4\sqrt{bx^2+cx^4}}{3c}-\frac{(5bB-6Ac)\left(\frac{x^2\sqrt{bx^2+cx^4}}{2c}-\frac{3b\left(\frac{\sqrt{bx^2+cx^4}}{c}-\frac{b\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2}{2c}\right)}{4c}\right)}{6c}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{Bx^4\sqrt{bx^2+cx^4}}{3c}-\frac{(5bB-6Ac)\left(\frac{x^2\sqrt{bx^2+cx^4}}{2c}-\frac{3b\left(\frac{\sqrt{bx^2+cx^4}}{c}-\frac{b\int\frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{c}\right)}{4c}\right)}{6c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{Bx^4\sqrt{bx^2+cx^4}}{3c}-\frac{(5bB-6Ac)\left(\frac{x^2\sqrt{bx^2+cx^4}}{2c}-\frac{3b\left(\frac{\sqrt{bx^2+cx^4}}{c}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}\right)}{4c}\right)}{6c}\right)$$

input
```
Int[(x^5*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
((B*x^4*Sqrt[b*x^2 + c*x^4])/(3*c) - ((5*b*B - 6*A*c)*((x^2*Sqrt[b*x^2 + c
*x^4])/(2*c) - (3*b*(Sqrt[b*x^2 + c*x^4]/c - (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt
[b*x^2 + c*x^4]])/c^(3/2)))/(4*c)))/(6*c))/2
```

3.131.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

3.131. $\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.131.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{3\left(\left(-\frac{1}{2}b^2Ac+\frac{5}{12}B\,b^3\right)\ln\left(\frac{2c\,x^2+2\sqrt{x^2\left(c\,x^2+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)+\left(b\left(\frac{5x^2B}{9}+A\right)c^{\frac{3}{2}}-\frac{2x^2\left(\frac{2x^2B}{3}+A\right)c^{\frac{5}{2}}}{3}-\frac{5B\sqrt{c}\,b^2}{6}\right)\sqrt{x^2(c\,x^2+b)}$ … $}{8c^{\frac{7}{2}}}$
risch	$-\frac{x^2\left(-8B\,c^2x^4-12A\,c^2x^2+10Bbc\,x^2+18Abc-15B\,b^2\right)\left(c\,x^2+b\right)}{48c^3\sqrt{x^2(c\,x^2+b)}}+\frac{b^2(6Ac-5Bb)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)x\sqrt{c\,x^2+b}}{16c^{\frac{7}{2}}\sqrt{x^2(c\,x^2+b)}}$
default	$\frac{x\sqrt{c\,x^2+b}\left(8B\sqrt{c\,x^2+b}\,c^{\frac{7}{2}}x^5+12A\sqrt{c\,x^2+b}\,c^{\frac{7}{2}}x^3-10B\sqrt{c\,x^2+b}\,c^{\frac{5}{2}}b\,x^3-18A\sqrt{c\,x^2+b}\,c^{\frac{5}{2}}bx+15B\sqrt{c\,x^2+b}\,c^{\frac{3}{2}}b^2x+18A\ln\left(\right.\right.}{48\sqrt{x^4c+b\,x^2}\,c^{\frac{9}{2}}}$

input
```
int(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-3/8/c^(7/2)*((-1/2*b^2*A*c+5/12*B*b^3)*ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2
)*c^(1/2)+b)/c^(1/2))+(b*(5/9*x^2*B+A)*c^(3/2)-2/3*x^2*(2/3*x^2*B+A)*c^(5/
2)-5/6*B*c^(1/2)*b^2)*(x^2*(c*x^2+b))^(1/2)+1/2*ln(2)*b^2*(A*c-5/6*B*b))
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.63

$$\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$
$$=\left[-\frac{3\left(5\,Bb^3-6\,Ab^2c\right)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(8\,Bc^3x^4+15\,Bb^2c-18\,Abc^2-2\left(5\,Bbc\right.\right.}{96\,c^4}\right.$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(5*B*b^3 - 6*A*b^2*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 +
b*x^2)*sqrt(c)) - 2*(8*B*c^3*x^4 + 15*B*b^2*c - 18*A*b*c^2 - 2*(5*B*b*c^2
- 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4, 1/48*(3*(5*B*b^3 - 6*A*b^2*c)*sq
rt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (8*B*c^3*x^4 + 1
5*B*b^2*c - 18*A*b*c^2 - 2*(5*B*b*c^2 - 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))
/c^4]
```

3.131.6 Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.45

$$\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{\begin{cases} \frac{3b^2\left(A-\frac{5Bb}{6c}\right)\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise} \end{cases}\right)}{8c^2} + \sqrt{bx^2+cx^4}\left(\frac{Bx^4}{3c}-\frac{3b\left(A-\frac{5Bb}{6c}\right)}{4c^2}+\frac{x^2\left(A-\frac{5Bb}{6c}\right)}{2c}\right) & \text{for} \\ \frac{\frac{2A(bx^2)^{\frac{5}{2}}}{5b^2}+\frac{2B(bx^2)^{\frac{7}{2}}}{7b^3}}{b} & \text{for} \\ \tilde{\infty}\left(\frac{Ax^6}{3}+\frac{Bx^8}{4}\right) & \text{oth} \end{cases}}{2}$$

input
```
integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Piecewise((3*b**2*(A - 5*B*b/(6*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x*
*2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/
(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(8*c**2) + sqrt(b*x**2 +
 c*x**4)*(B*x**4/(3*c) - 3*b*(A - 5*B*b/(6*c))/(4*c**2) + x**2*(A - 5*B*b/
(6*c))/(2*c)), Ne(c, 0)), ((2*A*(b*x**2)**(5/2)/(5*b**2) + 2*B*(b*x**2)**(
7/2)/(7*b**3))/b, Ne(b, 0)), (zoo*(A*x**6/3 + B*x**8/4), True))/2
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.32

$$\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{1}{16}\left(\frac{4\sqrt{cx^4+bx^2}x^2}{c}+\frac{3\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}}-\frac{6\,\sqrt{cx^4+bx^2}b}{c^2}\right)A$$
$$+\frac{1}{96}\left(\frac{16\,\sqrt{cx^4+bx^2}x^4}{c}-\frac{20\,\sqrt{cx^4+bx^2}bx^2}{c^2}-\frac{15\,b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{7}{2}}}+\frac{30\,\sqrt{cx^4+bx^2}}{c^3}\right.$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/16*(4*sqrt(c*x^4 + b*x^2)*x^2/c + 3*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 +
 b*x^2)*sqrt(c))/c^(5/2) - 6*sqrt(c*x^4 + b*x^2)*b/c^2)*A + 1/96*(16*sqrt(
c*x^4 + b*x^2)*x^4/c - 20*sqrt(c*x^4 + b*x^2)*b*x^2/c^2 - 15*b^3*log(2*c*x
^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 30*sqrt(c*x^4 + b*x^2)*b
^2/c^3)*B
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.08

$$\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{1}{48}\sqrt{cx^2+b}\left(2\,x^2\left(\frac{4\,Bx^2}{c\mathrm{sgn}\,(x)} - \frac{5\,Bbc^3\mathrm{sgn}(x) - 6\,Ac^4\mathrm{sgn}(x)}{c^5}\right) + \frac{3\,(5\,Bb^2c^2\mathrm{sgn}(x) - 6\,Abc^3\mathrm{sgn}(x))}{c^5}\right)x$$
$$- \frac{(5\,Bb^3\log(|b|) - 6\,Ab^2c\log(|b|))\mathrm{sgn}(x)}{32\,c^{\frac{7}{2}}} + \frac{(5\,Bb^3 - 6\,Ab^2c)\log\left(\left|-\sqrt{c}x + \sqrt{cx^2+b}\right|\right)}{16\,c^{\frac{7}{2}}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/48*sqrt(c*x^2 + b)*(2*x^2*(4*B*x^2/(c*sgn(x)) - (5*B*b*c^3*sgn(x) - 6*A*
c^4*sgn(x))/c^5) + 3*(5*B*b^2*c^2*sgn(x) - 6*A*b*c^3*sgn(x))/c^5)*x - 1/32
*(5*B*b^3*log(abs(b)) - 6*A*b^2*c*log(abs(b)))*sgn(x)/c^(7/2) + 1/16*(5*B*
b^3 - 6*A*b^2*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(7/2)*sgn(x))
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^5\,(B\,x^2+A)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.132 $\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

3.132.1 Optimal result

Integrand size = 26, antiderivative size = 83

$$\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{(3bB-4Ac-2Bcx^2)\sqrt{bx^2+cx^4}}{8c^2}+\frac{b(3bB-4Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}$$

output
```
1/8*b*(-4*A*c+3*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(5/2)-1/8*
(-2*B*c*x^2-4*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/c^2
```

3.132.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.30

$$\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{x\left(\sqrt{c}x(b+cx^2)\left(-3bB+4Ac+2Bcx^2\right)+2b(3bB-4Ac)\sqrt{b+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{b}+\sqrt{b+cx^2}}\right)\right)}{8c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[(x^3*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(x*(Sqrt[c]*x*(b + c*x^2)*(-3*b*B + 4*A*c + 2*B*c*x^2) + 2*b*(3*b*B - 4*A*
c)*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] + Sqrt[b + c*x^2])]))/(8*
c^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

3.132.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1940, 1225, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (A+Bx^2)}{\sqrt{bx^2+cx^4}} \, dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{x^2 (Bx^2+A)}{\sqrt{cx^4+bx^2}}dx^2$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{2}\left(\frac{b(3bB-4Ac)\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2}{8c^2}-\frac{\sqrt{bx^2+cx^4}\,(-4Ac+3bB-2Bcx^2)}{4c^2}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{b(3bB-4Ac)\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{4c^2}-\frac{\sqrt{bx^2+cx^4}\,(-4Ac+3bB-2Bcx^2)}{4c^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{b(3bB-4Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{4c^{5/2}}-\frac{\sqrt{bx^2+cx^4}\,(-4Ac+3bB-2Bcx^2)}{4c^2}\right)$$

input
```
Int[(x^3*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(-1/4*((3*b*B - 4*A*c - 2*B*c*x^2)*Sqrt[b*x^2 + c*x^4])/c^2 + (b*(3*b*B -
4*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(4*c^(5/2)))/2
```

3.132.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.132.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.20

method	result
risch	$\frac{x^{2} \left(2 B c \,x^{2}+4 A c -3 B b \right) \left(c \,x^{2}+b \right)}{8 c^{2} \sqrt{x^{2} \left(c \,x^{2}+b \right)}}-\frac{b \left(4 A c -3 B b \right) \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+b}\right) x \sqrt{c \,x^{2}+b}}{8 c^{\frac{5}{2}} \sqrt{x^{2} \left(c \,x^{2}+b \right)}}$
default	$\frac{x \sqrt{c \,x^{2}+b}\, \left(2 B \sqrt{c \,x^{2}+b}\, c^{\frac{5}{2}} x^{3}+4 A \sqrt{c \,x^{2}+b}\, c^{\frac{5}{2}} x -3 B \sqrt{c \,x^{2}+b}\, c^{\frac{3}{2}} b x -4 A \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+b}\right) b \,c^{2}+3 B \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+b}\right) b^{2} c}{8 \sqrt{x^{4} c +b \,x^{2}}\, c^{\frac{7}{2}}}$
pseudoelliptic	$\frac{4 B \,c^{\frac{3}{2}} x^{2} \sqrt{x^{2} \left(c \,x^{2}+b \right)}+8 A \,c^{\frac{3}{2}} \sqrt{x^{2} \left(c \,x^{2}+b \right)}-4 A \ln \left(\frac{2 c \,x^{2}+2 \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, \sqrt{c}+b}{\sqrt{c}}\right) b c +4 A \ln \left(2\right) b c -6 B b \sqrt{c}\, \sqrt{x^{2} \left(c \,x^{2}+b \right)}+3 B}{16 c^{\frac{5}{2}}}$

input
```
int(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x^2*(2*B*c*x^2+4*A*c-3*B*b)*(c*x^2+b)/c^2/(x^2*(c*x^2+b))^(1/2)-1/8*b*
(4*A*c-3*B*b)/c^(5/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*x/(x^2*(c*x^2+b))^(1/2
)*(c*x^2+b)^(1/2)
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.13

$$\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \left[-\frac{(3\,Bb^2-4\,Abc)\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,(2\,Bc^2x^2-3\,Bbc+4\,Ac^2)\sqrt{cx^4+bx^2}}{16\,c^3}, -\frac{(3\,Bb^2-4\,Abc)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-(2\,Bc^2x^2-3\,Bbc+4\,Ac^2)\sqrt{cx^4+bx^2}}{8\,c^3}\right]$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((3*B*b^2 - 4*A*b*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^
2)*sqrt(c)) - 2*(2*B*c^2*x^2 - 3*B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2))/c^3
, -1/8*((3*B*b^2 - 4*A*b*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(
c*x^2 + b)) - (2*B*c^2*x^2 - 3*B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2))/c^3]
```

3.132.6 Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.05

$$\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\begin{cases} -\frac{b\left(A-\frac{3Bb}{4c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{2c}+\sqrt{bx^2+cx^4}\left(\frac{Bx^2}{2c}+\frac{A-\frac{3Bb}{4c}}{c}\right) & \text{for } c\neq 0\\ \frac{\frac{2A(bx^2)^{\frac{3}{2}}}{3b}+\frac{2B(bx^2)^{\frac{5}{2}}}{5b^2}}{b} & \text{for } b\neq 0\\ \tilde{\infty}\left(\frac{Ax^4}{2}+\frac{Bx^6}{3}\right) & \text{otherwise}\end{cases}}{2}$$

input
```
integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Piecewise((-b*(A - 3*B*b/(4*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 +
 c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c
) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(2*c) + sqrt(b*x**2 + c*x**4
)*(B*x**2/(2*c) + (A - 3*B*b/(4*c))/c), Ne(c, 0)), ((2*A*(b*x**2)**(3/2)/(
3*b) + 2*B*(b*x**2)**(5/2)/(5*b**2))/b, Ne(b, 0)), (zoo*(A*x**4/2 + B*x**6
/3), True))/2
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.61

$$\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{1}{16}\left(\frac{4\sqrt{cx^4+bx^2}x^2}{c}+\frac{3b^2\log\left(2cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}}-\frac{6\sqrt{cx^4+bx^2}b}{c^2}\right)B - \frac{1}{4}A\left(\frac{b\log\left(2cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{3}{2}}}-\frac{2\sqrt{cx^4+bx^2}}{c}\right)$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/16*(4*sqrt(c*x^4 + b*x^2)*x^2/c + 3*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 +
 b*x^2)*sqrt(c))/c^(5/2) - 6*sqrt(c*x^4 + b*x^2)*b/c^2)*B - 1/4*A*(b*log(2
*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(3/2) - 2*sqrt(c*x^4 + b*x^2
)/c)
```

3.132.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{1}{8}\sqrt{cx^2+b}x\left(\frac{2\,Bx^2}{c\mathrm{sgn}\left(x\right)}-\frac{3\,Bbc\mathrm{sgn}(x)-4\,Ac^2\mathrm{sgn}(x)}{c^3}\right) + \frac{(3\,Bb^2\log\left(|b|\right)-4\,Abc\log\left(|b|\right))\mathrm{sgn}(x)}{16\,c^{\frac{5}{2}}} - \frac{(3\,Bb^2-4\,Abc)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{8\,c^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(c*x^2 + b)*x*(2*B*x^2/(c*sgn(x)) - (3*B*b*c*sgn(x) - 4*A*c^2*sgn(
x))/c^3) + 1/16*(3*B*b^2*log(abs(b)) - 4*A*b*c*log(abs(b)))*sgn(x)/c^(5/2)
 - 1/8*(3*B*b^2 - 4*A*b*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(5/2)
*sgn(x))
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^3\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.133 $\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

3.133.1 Optimal result

Integrand size = 24, antiderivative size = 66

$$\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\frac{B\sqrt{bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

output
```
-1/2*(-2*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(3/2)+1/2*B*(
c*x^4+b*x^2)^(1/2)/c
```

3.133.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.38

$$\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\frac{x\left(B\sqrt{c}x(b+cx^2)+2(bB-2Ac)\sqrt{b+cx^2}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{b}-\sqrt{b+cx^2}}\right)\right)}{2c^{3/2}\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(x*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(x*(B*Sqrt[c]*x*(b + c*x^2) + 2*(b*B - 2*A*c)*Sqrt[b + c*x^2]*ArcTanh[(Sqr
t[c]*x)/(Sqrt[b] - Sqrt[b + c*x^2])]))/(2*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

3.133.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1940, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}}dx^2$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{B\sqrt{bx^2+cx^4}}{c}-\frac{(bB-2Ac)\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2}{2c}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{B\sqrt{bx^2+cx^4}}{c}-\frac{(bB-2Ac)\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{B\sqrt{bx^2+cx^4}}{c}-\frac{(bB-2Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}\right)$$

input
```
Int[(x*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
((B*Sqrt[b*x^2 + c*x^4])/c - ((b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x
^2 + c*x^4]])/c^(3/2))/2
```

3.133.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.133.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.27

method	result	siz
risch	$\frac{B\,x^{2}\left(c\,x^{2}+b\right)}{2c\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{\left(2Ac-Bb\right)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)x\sqrt{c\,x^{2}+b}}{2c^{\frac{3}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	84
default	$\frac{x\sqrt{c\,x^{2}+b}\,\left(B\,c^{\frac{3}{2}}\sqrt{c\,x^{2}+b}\,x+2A\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)c^{2}-B\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)bc\right)}{2\sqrt{x^{4}c+b\,x^{2}}\,c^{\frac{5}{2}}}$	88
pseudoelliptic	$\frac{2A\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)c-2A\ln\left(2\right)c-B\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)b+B\ln\left(2\right)b+2B\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}}{4c^{\frac{3}{2}}}$	10

input
```
int(x*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*B*x^2/c*(c*x^2+b)/(x^2*(c*x^2+b))^(1/2)+1/2*(2*A*c-B*b)/c^(3/2)*ln(c^(
1/2)*x+(c*x^2+b)^(1/2))*x/(x^2*(c*x^2+b))^(1/2)*(c*x^2+b)^(1/2)
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.98

$$\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \left[\frac{2\sqrt{cx^4+bx^2}Bc-(Bb-2Ac)\sqrt{c}\log\left(-2cx^2-b-2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{4c^2}, \frac{\sqrt{cx^4+bx^2}Bc+(Bb-2Ac)\sqrt{-}}{2c^2}\right.$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(c*x^4 + b*x^2)*B*c - (B*b - 2*A*c)*sqrt(c)*log(-2*c*x^2 - b -
 2*sqrt(c*x^4 + b*x^2)*sqrt(c)))/c^2, 1/2*(sqrt(c*x^4 + b*x^2)*B*c + (B*b
- 2*A*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)))/c^2]
```

3.133.6 Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.11

$$\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\begin{cases} \frac{B\sqrt{bx^2+cx^4}}{c}+\left(A-\frac{Bb}{2c}\right)\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise} \end{cases}\right) & \text{for } c\neq 0 \\ \frac{2A\sqrt{bx^2}+\frac{2B\left(bx^2\right)^{\frac{3}{2}}}{3b}}{b} & \text{for } b\neq 0 \\ \tilde{\infty}\left(Ax^2+\frac{Bx^4}{2}\right) & \text{otherwise} \end{cases}}{2}$$

input
```
integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Piecewise((B*sqrt(b*x**2 + c*x**4)/c + (A - B*b/(2*c))*Piecewise((log(b +
2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(
2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), True)), Ne(c
, 0)), ((2*A*sqrt(b*x**2) + 2*B*(b*x**2)**(3/2)/(3*b))/b, Ne(b, 0)), (zoo*
(A*x**2 + B*x**4/2), True))/2
```

3.133.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.33

$$\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{1}{4}B\left(\frac{b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{3}{2}}}-\frac{2\,\sqrt{cx^4+bx^2}}{c}\right) + \frac{A\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{2\,\sqrt{c}}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*B*(b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(3/2) - 2*sqr
t(c*x^4 + b*x^2)/c) + 1/2*A*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c
))/sqrt(c)
```

3.133.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

$$\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{cx^2+b}Bx}{2\,c\mathrm{sgn}\left(x\right)} - \frac{\left(Bb\log\left(|b|\right)-2\,Ac\log\left(|b|\right)\right)\mathrm{sgn}(x)}{4\,c^{\frac{3}{2}}} + \frac{\left(Bb-2\,Ac\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{2\,c^{\frac{3}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(c*x^2 + b)*B*x/(c*sgn(x)) - 1/4*(B*b*log(abs(b)) - 2*A*c*log(abs(
b)))*sgn(x)/c^(3/2) + 1/2*(B*b - 2*A*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 +
b)))/(c^(3/2)*sgn(x))
```

3.133.9 Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.35

$$\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{B\sqrt{c\,x^4+b\,x^2}}{2\,c}+\frac{A\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\,\sqrt{c}} - \frac{B\,b\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{4\,c^{3/2}}$$

input
```
int((x*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
(B*(b*x^2 + c*x^4)^(1/2))/(2*c) + (A*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 +
c*x^4)^(1/2)))/(2*c^(1/2)) - (B*b*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x
^4)^(1/2)))/(4*c^(3/2))
```

3.134 $\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}\,dx$

3.134.1 Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}\,dx=-\frac{A\sqrt{bx^2+cx^4}}{bx^2}+\frac{B\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

output
```
B*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(1/2)-A*(c*x^4+b*x^2)^(1/2)/b
/x^2
```

3.134.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}\,dx=\frac{-A\sqrt{c}(b+cx^2)-bBx\sqrt{b+cx^2}\log\left(-\sqrt{c}x+\sqrt{b+cx^2}\right)}{b\sqrt{c}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[(A + B*x^2)/(x*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-(A*Sqrt[c]*(b + c*x^2)) - b*B*x*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[
b + c*x^2]])/(b*Sqrt[c]*Sqrt[x^2*(b + c*x^2)])
```

3.134.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1940, 1220, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 1940}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^2\sqrt{cx^4+bx^2}}dx^2$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{1}{2}\left(B\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2-\frac{2A\sqrt{bx^2+cx^4}}{bx^2}\right)$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{1}{2}\left(2B\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}-\frac{2A\sqrt{bx^2+cx^4}}{bx^2}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{2B\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}-\frac{2A\sqrt{bx^2+cx^4}}{bx^2}\right)$$

input
```
Int[(A + B*x^2)/(x*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
((-2*A*Sqrt[b*x^2 + c*x^4])/(b*x^2) + (2*B*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^
2 + c*x^4]])/Sqrt[c])/2
```

3.134.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.134.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

method	result	size
default	$-\frac{\sqrt{c\,x^{2}+b}\,\left(-B\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)bx+A\sqrt{c\,x^{2}+b}\,\sqrt{c}\right)}{\sqrt{x^{4}c+b\,x^{2}}\,\sqrt{c}\,b}$	67
risch	$-\frac{A\left(c\,x^{2}+b\right)}{b\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{B\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)x\sqrt{c\,x^{2}+b}}{\sqrt{c}\,\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	72
pseudoelliptic	$-\frac{-\frac{B\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)b\,x^{2}}{2}+\frac{B\ln\left(2\right)b\,x^{2}}{2}+A\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}}{\sqrt{c}\,b\,x^{2}}$	78

input
```
int((B*x^2+A)/x/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2+b)^(1/2)*(-B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b*x+A*(c*x^2+b)^(1/2)*c
^(1/2))/(c*x^4+b*x^2)^(1/2)/c^(1/2)/b
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.39

$$\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}\,dx = \left[\frac{Bb\sqrt{c}x^2\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}Ac}{2\,bcx^2},\right.$$
$$\left.-\frac{Bb\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}Ac}{bcx^2}\right]$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(B*b*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) -
2*sqrt(c*x^4 + b*x^2)*A*c)/(b*c*x^2), -(B*b*sqrt(-c)*x^2*arctan(sqrt(c*x^4
 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*A*c)/(b*c*x^2)]
```

3.134.6 Sympy [F]

$$\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate((B*x**2+A)/x/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x*sqrt(x**2*(b + c*x**2))), x)
```

3.134.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}\,dx = \frac{B\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{2\,\sqrt{c}} - \frac{\sqrt{cx^4+bx^2}A}{bx^2}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*B*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) - sqrt(c*x^
4 + b*x^2)*A/(b*x^2)
```

3.134.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}\,dx = -\frac{B\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)}{2\,\sqrt{c}\mathrm{sgn}\,(x)} + \frac{2\,A\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)\mathrm{sgn}\,(x)}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*B*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)/(sqrt(c)*sgn(x)) + 2*A*sqrt(c)
/(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)*sgn(x))
```

3.134.9 Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}\,dx = \frac{B\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\,\sqrt{c}} - \frac{A\,\sqrt{c\,x^4+b\,x^2}}{b\,x^2}$$

input
```
int((A + B*x^2)/(x*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
(B*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(2*c^(1/2)) - (A*(b
*x^2 + c*x^4)^(1/2))/(b*x^2)
```

3.135 $\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx$

3.135.1 Optimal result

Integrand size = 26, antiderivative size = 61

$$\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx=-\frac{A\sqrt{bx^2+cx^4}}{3bx^4}-\frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{3b^2x^2}$$

output
```
-1/3*A*(c*x^4+b*x^2)^(1/2)/b/x^4-1/3*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/b^
2/x^2
```

3.135.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx=-\frac{\sqrt{x^2\left(b+cx^2\right)}(3bBx^2+A(b-2cx^2))}{3b^2x^4}$$

input
```
Integrate[(A + B*x^2)/(x^3*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
-1/3*(Sqrt[x^2*(b + c*x^2)]*(3*b*B*x^2 + A*(b - 2*c*x^2)))/(b^2*x^4)
```

3.135.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1940, 1220, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^4\sqrt{cx^4+bx^2}}dx^2$$

$$\Big\downarrow \text{1220}$$

$$\frac{1}{2}\left(\frac{(3bB-2Ac)\int \frac{1}{x^2\sqrt{cx^4+bx^2}}dx^2}{3b}-\frac{2A\sqrt{bx^2+cx^4}}{3bx^4}\right)$$

$$\Big\downarrow \text{1123}$$

$$\frac{1}{2}\left(-\frac{2\sqrt{bx^2+cx^4}(3bB-2Ac)}{3b^2x^2}-\frac{2A\sqrt{bx^2+cx^4}}{3bx^4}\right)$$

input
```
Int[(A + B*x^2)/(x^3*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
((-2*A*Sqrt[b*x^2 + c*x^4])/(3*b*x^4) - (2*(3*b*B - 2*A*c)*Sqrt[b*x^2 + c*
x^4])/(3*b^2*x^2))/2
```

3.135.3.1 Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.135.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

method	result	size
trager	$-\frac{\left(-2 A c \,x^{2}+3 b B \,x^{2}+A b \right) \sqrt{x^{4} c +b \,x^{2}}}{3 b^{2} x^{4}}$	40
gosper	$-\frac{\left(c \,x^{2}+b \right) \left(-2 A c \,x^{2}+3 b B \,x^{2}+A b \right)}{3 x^{2} b^{2} \sqrt{x^{4} c +b \,x^{2}}}$	47
default	$-\frac{\left(c \,x^{2}+b \right) \left(-2 A c \,x^{2}+3 b B \,x^{2}+A b \right)}{3 x^{2} b^{2} \sqrt{x^{4} c +b \,x^{2}}}$	47
risch	$-\frac{\left(c \,x^{2}+b \right) \left(-2 A c \,x^{2}+3 b B \,x^{2}+A b \right)}{3 x^{2} \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, b^{2}}$	47
pseudoelliptic	$-\frac{\left(\left(3 x^{2} B +A \right) b -2 A c \,x^{2}\right) \left(c \,x^{2}+b \right)}{3 \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, x^{2} b^{2}}$	47

input
```
int((B*x^2+A)/x^3/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-2*A*c*x^2+3*B*b*x^2+A*b)/b^2/x^4*(c*x^4+b*x^2)^(1/2)
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.62

$$\int \frac{A + Bx^2}{x^3\sqrt{bx^2 + cx^4}}\,dx = -\frac{\sqrt{cx^4 + bx^2}((3\,Bb - 2\,Ac)x^2 + Ab)}{3\,b^2x^4}$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(c*x^4 + b*x^2)*((3*B*b - 2*A*c)*x^2 + A*b)/(b^2*x^4)
```

3.135.6 Sympy [F]

$$\int \frac{A + Bx^2}{x^3\sqrt{bx^2 + cx^4}}\,dx = \int \frac{A + Bx^2}{x^3\sqrt{x^2\,(b + cx^2)}}\,dx$$

input
```
integrate((B*x**2+A)/x**3/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x**3*sqrt(x**2*(b + c*x**2))), x)
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int \frac{A + Bx^2}{x^3\sqrt{bx^2 + cx^4}}\,dx = \frac{1}{3}\,A\left(\frac{2\,\sqrt{cx^4 + bx^2}c}{b^2x^2} - \frac{\sqrt{cx^4 + bx^2}}{bx^4}\right) - \frac{\sqrt{cx^4 + bx^2}B}{bx^2}$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*A*(2*sqrt(c*x^4 + b*x^2)*c/(b^2*x^2) - sqrt(c*x^4 + b*x^2)/(b*x^4)) -
sqrt(c*x^4 + b*x^2)*B/(b*x^2)
```

3.135.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(53) = 106$.

Time = 0.39 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.03

$$\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx = \frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4B\sqrt{c}-6\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2Bb\sqrt{c}+6\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2Ac^{\frac{3}{2}}+3\,Bb^2\sqrt{c}-2\,A}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*sqrt(c) - 6*(sqrt(c)*x - sqrt(c*x
^2 + b))^2*B*b*sqrt(c) + 6*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*c^(3/2) + 3*B
*b^2*sqrt(c) - 2*A*b*c^(3/2))/(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^3*sgn
(x))
```

3.135.9 Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{c\,x^4+b\,x^2}\,\left(A\,b-2\,A\,c\,x^2+3\,B\,b\,x^2\right)}{3\,b^2\,x^4}$$

input
```
int((A + B*x^2)/(x^3*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
-((b*x^2 + c*x^4)^(1/2)*(A*b - 2*A*c*x^2 + 3*B*b*x^2))/(3*b^2*x^4)
```

3.136 $\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx$

3.136.1 Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx = -\frac{A\sqrt{bx^2+cx^4}}{5bx^6}-\frac{(5bB-4Ac)\sqrt{bx^2+cx^4}}{15b^2x^4}+\frac{2c(5bB-4Ac)\sqrt{bx^2+cx^4}}{15b^3x^2}$$

output
```
-1/5*A*(c*x^4+b*x^2)^(1/2)/b/x^6-1/15*(-4*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/b
^2/x^4+2/15*c*(-4*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^2
```

3.136.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

$$\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{x^2(b+cx^2)}(-5bBx^2(b-2cx^2)+A(-3b^2+4bcx^2-8c^2x^4))}{15b^3x^6}$$

input
```
Integrate[(A + B*x^2)/(x^5*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(-5*b*B*x^2*(b - 2*c*x^2) + A*(-3*b^2 + 4*b*c*x^2 -
 8*c^2*x^4)))/(15*b^3*x^6)
```

3.136.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1940, 1220, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}dx$$
$$\Big\downarrow \text{1940}$$
$$\frac{1}{2}\int \frac{Bx^2+A}{x^6\sqrt{cx^4+bx^2}}dx^2$$
$$\Big\downarrow \text{1220}$$
$$\frac{1}{2}\left(\frac{(5bB-4Ac)\int \frac{1}{x^4\sqrt{cx^4+bx^2}}dx^2}{5b}-\frac{2A\sqrt{bx^2+cx^4}}{5bx^6}\right)$$
$$\Big\downarrow \text{1129}$$
$$\frac{1}{2}\left(\frac{(5bB-4Ac)\left(-\frac{2c\int \frac{1}{x^2\sqrt{cx^4+bx^2}}dx^2}{3b}-\frac{2\sqrt{bx^2+cx^4}}{3bx^4}\right)}{5b}-\frac{2A\sqrt{bx^2+cx^4}}{5bx^6}\right)$$
$$\Big\downarrow \text{1123}$$
$$\frac{1}{2}\left(\frac{\left(\frac{4c\sqrt{bx^2+cx^4}}{3b^2x^2}-\frac{2\sqrt{bx^2+cx^4}}{3bx^4}\right)(5bB-4Ac)}{5b}-\frac{2A\sqrt{bx^2+cx^4}}{5bx^6}\right)$$

input
```
Int[(A + B*x^2)/(x^5*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
((-2*A*Sqrt[b*x^2 + c*x^4])/(5*b*x^6) + ((5*b*B - 4*A*c)*((-2*Sqrt[b*x^2 +
 c*x^4])/(3*b*x^4) + (4*c*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^2)))/(5*b))/2
```

3.136.3.1 Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.136.4 Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

method	result	size
trager	$-\frac{\left(8A\,c^{2}x^{4}-10x^{4}Bbc-4Abc\,x^{2}+5b^{2}B\,x^{2}+3b^{2}A\right)\sqrt{x^{4}c+b\,x^{2}}}{15b^{3}x^{6}}$	63
pseudoelliptic	$-\frac{\left(\left(\frac{5x^{2}B}{3}+A\right)b^{2}-\frac{4x^{2}\left(\frac{5x^{2}B}{2}+A\right)cb}{3}+\frac{8A\,c^{2}x^{4}}{3}\right)\left(c\,x^{2}+b\right)}{5\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,x^{4}b^{3}}$	66
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(8A\,c^{2}x^{4}-10x^{4}Bbc-4Abc\,x^{2}+5b^{2}B\,x^{2}+3b^{2}A\right)}{15x^{4}b^{3}\sqrt{x^{4}c+b\,x^{2}}}$	70
default	$-\frac{\left(c\,x^{2}+b\right)\left(8A\,c^{2}x^{4}-10x^{4}Bbc-4Abc\,x^{2}+5b^{2}B\,x^{2}+3b^{2}A\right)}{15x^{4}b^{3}\sqrt{x^{4}c+b\,x^{2}}}$	70
risch	$-\frac{\left(c\,x^{2}+b\right)\left(8A\,c^{2}x^{4}-10x^{4}Bbc-4Abc\,x^{2}+5b^{2}B\,x^{2}+3b^{2}A\right)}{15x^{4}\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,b^{3}}$	70

input
```
int((B*x^2+A)/x^5/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(8*A*c^2*x^4-10*B*b*c*x^4-4*A*b*c*x^2+5*B*b^2*x^2+3*A*b^2)/b^3/x^6*(
c*x^4+b*x^2)^(1/2)
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx = \frac{\left(2\left(5\,Bbc-4\,Ac^2\right)x^4-3\,Ab^2-\left(5\,Bb^2-4\,Abc\right)x^2\right)\sqrt{cx^4+bx^2}}{15\,b^3x^6}$$

input
```
integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(2*(5*B*b*c - 4*A*c^2)*x^4 - 3*A*b^2 - (5*B*b^2 - 4*A*b*c)*x^2)*sqrt(
c*x^4 + b*x^2)/(b^3*x^6)
```

3.136.6 Sympy [F]

$$\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx=\int \frac{A+Bx^2}{x^5\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate((B*x**2+A)/x**5/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x**5*sqrt(x**2*(b + c*x**2))), x)
```

3.136.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx=\frac{1}{3}\,B\left(\frac{2\,\sqrt{cx^4+bx^2}c}{b^2x^2}-\frac{\sqrt{cx^4+bx^2}}{bx^4}\right)-\frac{1}{15}\,A\left(\frac{8\,\sqrt{cx^4+bx^2}c^2}{b^3x^2}-\frac{4\,\sqrt{cx^4+bx^2}c}{b^2x^4}+\frac{3\,\sqrt{cx^4+bx^2}}{bx^6}\right)$$

input
```
integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*B*(2*sqrt(c*x^4 + b*x^2)*c/(b^2*x^2) - sqrt(c*x^4 + b*x^2)/(b*x^4)) -
1/15*A*(8*sqrt(c*x^4 + b*x^2)*c^2/(b^3*x^2) - 4*sqrt(c*x^4 + b*x^2)*c/(b^2
*x^4) + 3*sqrt(c*x^4 + b*x^2)/(b*x^6))
```

3.136.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(84) = 168.

Time = 0.58 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.88

$$\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx=\frac{4\left(15\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6Bc^{\frac{3}{2}}-35\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Bbc^{\frac{3}{2}}+40\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Ac^{\frac{5}{2}}+25\left(\sqrt{c}x-\right.\right.}{15\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^5\operatorname{sgn}\left(x\right.}$$

input
```
integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
4/15*(15*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*c^(3/2) - 35*(sqrt(c)*x - sqrt(
c*x^2 + b))^4*B*b*c^(3/2) + 40*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*c^(5/2) +
 25*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^2*c^(3/2) - 20*(sqrt(c)*x - sqrt(c
*x^2 + b))^2*A*b*c^(5/2) - 5*B*b^3*c^(3/2) + 4*A*b^2*c^(5/2))/(((sqrt(c)*x
 - sqrt(c*x^2 + b))^2 - b)^5*sgn(x))
```

3.136.9 Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx=-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(5\,B\,b^2\,x^2+3\,A\,b^2-10\,B\,b\,c\,x^4-4\,A\,b\,c\,x^2+8\,A\,c^2\,x^4\right)}{15\,b^3\,x^6}$$

input
```
int((A + B*x^2)/(x^5*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
-((b*x^2 + c*x^4)^(1/2)*(3*A*b^2 + 5*B*b^2*x^2 + 8*A*c^2*x^4 - 4*A*b*c*x^2
 - 10*B*b*c*x^4))/(15*b^3*x^6)
```

3.137 $\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}}\,dx$

3.137.1 Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}}\,dx = -\frac{A\sqrt{bx^2+cx^4}}{7bx^8}-\frac{(7bB-6Ac)\sqrt{bx^2+cx^4}}{35b^2x^6}+\frac{4c(7bB-6Ac)\sqrt{bx^2+cx^4}}{105b^3x^4}-\frac{8c^2(7bB-6Ac)\sqrt{bx^2+cx^4}}{105b^4x^2}$$

output
```
-1/7*A*(c*x^4+b*x^2)^(1/2)/b/x^8-1/35*(-6*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)/b
^2/x^6+4/105*c*(-6*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^4-8/105*c^2*(-6*A*
c+7*B*b)*(c*x^4+b*x^2)^(1/2)/b^4/x^2
```

3.137.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.67

$$\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{x^2(b+cx^2)}\left(7bBx^2(3b^2-4bcx^2+8c^2x^4)+3A(5b^3-6b^2cx^2+8bc^2x^4-16c^3x^6)\right)}{105b^4x^8}$$

input
```
Integrate[(A + B*x^2)/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
-1/105*(Sqrt[x^2*(b + c*x^2)]*(7*b*B*x^2*(3*b^2 - 4*b*c*x^2 + 8*c^2*x^4) +
 3*A*(5*b^3 - 6*b^2*c*x^2 + 8*b*c^2*x^4 - 16*c^3*x^6)))/(b^4*x^8)
```

3.137.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1940, 1220, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^8\sqrt{cx^4+bx^2}}dx^2$$

$$\Big\downarrow \text{1220}$$

$$\frac{1}{2}\left(\frac{(7bB-6Ac)\int \frac{1}{x^6\sqrt{cx^4+bx^2}}dx^2}{7b}-\frac{2A\sqrt{bx^2+cx^4}}{7bx^8}\right)$$

$$\Big\downarrow \text{1129}$$

$$\frac{1}{2}\left(\frac{(7bB-6Ac)\left(-\frac{4c\int \frac{1}{x^4\sqrt{cx^4+bx^2}}dx^2}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^6}\right)}{7b}-\frac{2A\sqrt{bx^2+cx^4}}{7bx^8}\right)$$

$$\Big\downarrow \text{1129}$$

$$\frac{1}{2}\left(\frac{(7bB-6Ac)\left(-\frac{4c\left(-\frac{2c\int \frac{1}{x^2\sqrt{cx^4+bx^2}}dx^2}{3b}-\frac{2\sqrt{bx^2+cx^4}}{3bx^4}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^6}\right)}{7b}-\frac{2A\sqrt{bx^2+cx^4}}{7bx^8}\right)$$

$$\Big\downarrow \text{1123}$$

$$\frac{1}{2}\left(\frac{\left(-\frac{4c\left(\frac{4c\sqrt{bx^2+cx^4}}{3b^2x^2}-\frac{2\sqrt{bx^2+cx^4}}{3bx^4}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^6}\right)(7bB-6Ac)}{7b}-\frac{2A\sqrt{bx^2+cx^4}}{7bx^8}\right)$$

input
```
Int[(A + B*x^2)/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
((-2*A*Sqrt[b*x^2 + c*x^4])/(7*b*x^8) + ((7*b*B - 6*A*c)*((-2*Sqrt[b*x^2 +
 c*x^4])/(5*b*x^6) - (4*c*((-2*Sqrt[b*x^2 + c*x^4])/(3*b*x^4) + (4*c*Sqrt[
b*x^2 + c*x^4])/(3*b^2*x^2)))/(5*b)))/(7*b))/2
```

3.137.3.1 Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.137.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$-\frac{\left(\left(\frac{7x^2B}{5}+A\right)b^3-\frac{6x^2c\left(\frac{14x^2B}{9}+A\right)b^2}{5}+\frac{8x^4\left(\frac{7x^2B}{3}+A\right)c^2b}{5}-\frac{16Ac^3x^6}{5}\right)(c\,x^2+b)}{7\sqrt{x^2(c\,x^2+b)}\,x^6b^4}$	85
trager	$-\frac{(-48A\,c^3x^6+56x^6Bb\,c^2+24Ab\,c^2x^4-28x^4B\,b^2c-18A\,b^2c\,x^2+21b^3B\,x^2+15b^3A)\sqrt{x^4c+b\,x^2}}{105x^8b^4}$	87
gosper	$-\frac{(c\,x^2+b)(-48A\,c^3x^6+56x^6Bb\,c^2+24Ab\,c^2x^4-28x^4B\,b^2c-18A\,b^2c\,x^2+21b^3B\,x^2+15b^3A)}{105x^6b^4\sqrt{x^4c+b\,x^2}}$	94
default	$-\frac{(c\,x^2+b)(-48A\,c^3x^6+56x^6Bb\,c^2+24Ab\,c^2x^4-28x^4B\,b^2c-18A\,b^2c\,x^2+21b^3B\,x^2+15b^3A)}{105x^6b^4\sqrt{x^4c+b\,x^2}}$	94
risch	$-\frac{(c\,x^2+b)(-48A\,c^3x^6+56x^6Bb\,c^2+24Ab\,c^2x^4-28x^4B\,b^2c-18A\,b^2c\,x^2+21b^3B\,x^2+15b^3A)}{105x^6\sqrt{x^2(c\,x^2+b)}\,b^4}$	94

input
```
int((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7*((7/5*x^2*B+A)*b^3-6/5*x^2*c*(14/9*x^2*B+A)*b^2+8/5*x^4*(7/3*x^2*B+A)
*c^2*b-16/5*A*c^3*x^6)/(x^2*(c*x^2+b))^(1/2)*(c*x^2+b)/x^6/b^4
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}}\,dx = -\frac{(8\,(7\,Bbc^2-6\,Ac^3)x^6-4\,(7\,Bb^2c-6\,Abc^2)x^4+15\,Ab^3+3\,(7\,Bb^3-6\,Ab^2c)x^2)\sqrt{cx^4+bx^2}}{105\,b^4x^8}$$

input
```
integrate((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/105*(8*(7*B*b*c^2 - 6*A*c^3)*x^6 - 4*(7*B*b^2*c - 6*A*b*c^2)*x^4 + 15*A
*b^3 + 3*(7*B*b^3 - 6*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^4*x^8)
```

3.137.6 Sympy [F]

$$\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^7\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate((B*x**2+A)/x**7/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x**7*sqrt(x**2*(b + c*x**2))), x)
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}}\,dx$$
$$= -\frac{1}{15}\,B\left(\frac{8\sqrt{cx^4+bx^2}c^2}{b^3x^2} - \frac{4\sqrt{cx^4+bx^2}c}{b^2x^4} + \frac{3\sqrt{cx^4+bx^2}}{bx^6}\right) + \frac{1}{35}\,A\left(\frac{16\sqrt{cx^4+bx^2}c^3}{b^4x^2} - \frac{8\sqrt{cx^4+bx^2}c^2}{b^3x^4} + \frac{6\sqrt{cx^4+bx^2}c}{b^2x^6} - \frac{5\sqrt{cx^4+bx^2}}{bx^8}\right)$$

input
```
integrate((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/15*B*(8*sqrt(c*x^4 + b*x^2)*c^2/(b^3*x^2) - 4*sqrt(c*x^4 + b*x^2)*c/(b^
2*x^4) + 3*sqrt(c*x^4 + b*x^2)/(b*x^6)) + 1/35*A*(16*sqrt(c*x^4 + b*x^2)*c
^3/(b^4*x^2) - 8*sqrt(c*x^4 + b*x^2)*c^2/(b^3*x^4) + 6*sqrt(c*x^4 + b*x^2)
*c/(b^2*x^6) - 5*sqrt(c*x^4 + b*x^2)/(b*x^8))
```

3.137.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(117) = 234$.

Time = 0.73 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.77

$$\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{16\left(70\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8Bc^{\frac{5}{2}} - 175\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6Bbc^{\frac{5}{2}} + 210\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6Ac^{\frac{7}{2}} + 147\left(\sqrt{c}\right.\right.}{10}$$

input
```
integrate((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
16/105*(70*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*c^(5/2) - 175*(sqrt(c)*x - sq
rt(c*x^2 + b))^6*B*b*c^(5/2) + 210*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*c^(7/
2) + 147*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^2*c^(5/2) - 126*(sqrt(c)*x -
sqrt(c*x^2 + b))^4*A*b*c^(7/2) - 49*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^3*
c^(5/2) + 42*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^2*c^(7/2) + 7*B*b^4*c^(5/
2) - 6*A*b^3*c^(7/2))/(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^7*sgn(x))
```

3.137.9 Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}}\,dx = \frac{(6\,A\,c-7\,B\,b)\,\sqrt{c\,x^4+b\,x^2}}{35\,b^2\,x^6} - \frac{A\,\sqrt{c\,x^4+b\,x^2}}{7\,b\,x^8} - \frac{(24\,A\,c^2-28\,B\,b\,c)\,\sqrt{c\,x^4+b\,x^2}}{105\,b^3\,x^4} + \frac{(48\,A\,c^3-56\,B\,b\,c^2)\,\sqrt{c\,x^4+b\,x^2}}{105\,b^4\,x^2}$$

input
```
int((A + B*x^2)/(x^7*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
((6*A*c - 7*B*b)*(b*x^2 + c*x^4)^(1/2))/(35*b^2*x^6) - (A*(b*x^2 + c*x^4)^
(1/2))/(7*b*x^8) - ((24*A*c^2 - 28*B*b*c)*(b*x^2 + c*x^4)^(1/2))/(105*b^3*
x^4) + ((48*A*c^3 - 56*B*b*c^2)*(b*x^2 + c*x^4)^(1/2))/(105*b^4*x^2)
```

3.138 $\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx$

3.138.1 Optimal result

Integrand size = 26, antiderivative size = 170

$$\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx = -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} - \frac{(9bB-8Ac)\sqrt{bx^2+cx^4}}{63b^2x^8} + \frac{2c(9bB-8Ac)\sqrt{bx^2+cx^4}}{105b^3x^6} - \frac{8c^2(9bB-8Ac)\sqrt{bx^2+cx^4}}{315b^4x^4} + \frac{16c^3(9bB-8Ac)\sqrt{bx^2+cx^4}}{315b^5x^2}$$

output
```
-1/9*A*(c*x^4+b*x^2)^(1/2)/b/x^10-1/63*(-8*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/
b^2/x^8+2/105*c*(-8*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^6-8/315*c^2*(-8*A
*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b^4/x^4+16/315*c^3*(-8*A*c+9*B*b)*(c*x^4+b*x
^2)^(1/2)/b^5/x^2
```

3.138.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{x^2(b+cx^2)}(9bBx^2(-5b^3+6b^2cx^2-8bc^2x^4+16c^3x^6)+A(-35b^4+40b^3cx^2-48b^2c^2x^4+64bc^3x^6-1}{315b^5x^{10}}$$

input
```
Integrate[(A + B*x^2)/(x^9*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(9*b*B*x^2*(-5*b^3 + 6*b^2*c*x^2 - 8*b*c^2*x^4 + 16
*c^3*x^6) + A*(-35*b^4 + 40*b^3*c*x^2 - 48*b^2*c^2*x^4 + 64*b*c^3*x^6 - 12
8*c^4*x^8)))/(315*b^5*x^10)
```

3.138.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1940, 1220, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^{10}\sqrt{cx^4+bx^2}}dx^2$$

$$\Big\downarrow \text{1220}$$

$$\frac{1}{2}\left(\frac{(9bB-8Ac)\int \frac{1}{x^8\sqrt{cx^4+bx^2}}dx^2}{9b}-\frac{2A\sqrt{bx^2+cx^4}}{9bx^{10}}\right)$$

$$\Big\downarrow \text{1129}$$

$$\frac{1}{2}\left(\frac{(9bB-8Ac)\left(-\frac{6c\int \frac{1}{x^6\sqrt{cx^4+bx^2}}dx^2}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^8}\right)}{9b}-\frac{2A\sqrt{bx^2+cx^4}}{9bx^{10}}\right)$$

$$\Big\downarrow \text{1129}$$

$$\frac{1}{2}\left(\frac{(9bB-8Ac)\left(-\frac{6c\left(-\frac{4c\int \frac{1}{x^4\sqrt{cx^4+bx^2}}dx^2}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^6}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^8}\right)}{9b}-\frac{2A\sqrt{bx^2+cx^4}}{9bx^{10}}\right)$$

$$\Big\downarrow \text{1129}$$

$$\frac{1}{2}\left(\frac{(9bB-8Ac)\left(-\frac{6c\left(-\frac{4c\left(-\frac{2c\int\frac{1}{x^2\sqrt{cx^4+bx^2}}dx^2}{3b}-\frac{2\sqrt{bx^2+cx^4}}{3bx^4}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^6}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^8}\right)}{9b}-\frac{2A\sqrt{bx^2+cx^4}}{9bx^{10}}\right)$$

$$\Big\downarrow \text{1123}$$

$$\frac{1}{2}\left(\frac{\left(-\frac{6c\left(-\frac{4c\left(\frac{4c\sqrt{bx^2+cx^4}}{3b^2x^2}-\frac{2\sqrt{bx^2+cx^4}}{3bx^4}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^6}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^8}\right)(9bB-8Ac)}{9b}-\frac{2A\sqrt{bx^2+cx^4}}{9bx^{10}}\right)$$

input
```
Int[(A + B*x^2)/(x^9*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
((-2*A*Sqrt[b*x^2 + c*x^4])/(9*b*x^10) + ((9*b*B - 8*A*c)*((-2*Sqrt[b*x^2
+ c*x^4])/(7*b*x^8) - (6*c*((-2*Sqrt[b*x^2 + c*x^4])/(5*b*x^6) - (4*c*((-2
*Sqrt[b*x^2 + c*x^4])/(3*b*x^4) + (4*c*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^2)))/
(5*b))/(7*b)))/(9*b))/2
```

3.138. $\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx$

3.138.3.1 Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.138. $\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx$

3.138.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$-\frac{\left(\left(\frac{9x^2B}{7}+A\right)b^4-\frac{8x^2c\left(\frac{27x^2B}{20}+A\right)b^3}{7}+\frac{48x^4\left(\frac{3x^2B}{2}+A\right)c^2b^2}{35}-\frac{64x^6\left(\frac{9x^2B}{4}+A\right)c^3b}{35}+\frac{128A\,x^8c^4}{35}\right)(c\,x^2+b)}{9\sqrt{x^2(c\,x^2+b)}\,x^8b^5}$
trager	$-\frac{(128A\,x^8c^4-144B\,x^8b\,c^3-64A\,x^6b\,c^3+72B\,x^6b^2c^2+48A\,b^2c^2x^4-54B\,b^3c\,x^4-40A\,x^2b^3c+45B\,x^2b^4+35A\,b^4)\sqrt{x^4c+b\,x^2}}{315b^5x^{10}}$
gosper	$-\frac{(c\,x^2+b)\,(128A\,x^8c^4-144B\,x^8b\,c^3-64A\,x^6b\,c^3+72B\,x^6b^2c^2+48A\,b^2c^2x^4-54B\,b^3c\,x^4-40A\,x^2b^3c+45B\,x^2b^4+35A\,b^4)}{315x^8b^5\sqrt{x^4c+b\,x^2}}$
default	$-\frac{(c\,x^2+b)\,(128A\,x^8c^4-144B\,x^8b\,c^3-64A\,x^6b\,c^3+72B\,x^6b^2c^2+48A\,b^2c^2x^4-54B\,b^3c\,x^4-40A\,x^2b^3c+45B\,x^2b^4+35A\,b^4)}{315x^8b^5\sqrt{x^4c+b\,x^2}}$
risch	$-\frac{(c\,x^2+b)\,(128A\,x^8c^4-144B\,x^8b\,c^3-64A\,x^6b\,c^3+72B\,x^6b^2c^2+48A\,b^2c^2x^4-54B\,b^3c\,x^4-40A\,x^2b^3c+45B\,x^2b^4+35A\,b^4)}{315x^8\sqrt{x^2(c\,x^2+b)}\,b^5}$

input
```
int((B*x^2+A)/x^9/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*((9/7*x^2*B+A)*b^4-8/7*x^2*c*(27/20*x^2*B+A)*b^3+48/35*x^4*(3/2*x^2*B
+A)*c^2*b^2-64/35*x^6*(9/4*x^2*B+A)*c^3*b+128/35*A*x^8*c^4)/(x^2*(c*x^2+b)
)^(1/2)*(c*x^2+b)/x^8/b^5
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx$$
$$=\frac{(16\,(9\,Bbc^3-8\,Ac^4)x^8-8\,(9\,Bb^2c^2-8\,Abc^3)x^6-35\,Ab^4+6\,(9\,Bb^3c-8\,Ab^2c^2)x^4-5\,(9\,Bb^4-8\,Ab^3c)x}{315\,b^5x^{10}}$$

input
```
integrate((B*x^2+A)/x^9/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/315*(16*(9*B*b*c^3 - 8*A*c^4)*x^8 - 8*(9*B*b^2*c^2 - 8*A*b*c^3)*x^6 - 35
*A*b^4 + 6*(9*B*b^3*c - 8*A*b^2*c^2)*x^4 - 5*(9*B*b^4 - 8*A*b^3*c)*x^2)*sq
rt(c*x^4 + b*x^2)/(b^5*x^10)
```

3.138.6 Sympy [F]

$$\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx=\int \frac{A+Bx^2}{x^9\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate((B*x**2+A)/x**9/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x**9*sqrt(x**2*(b + c*x**2))), x)
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.26

$$\begin{aligned}&\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx\\&=\frac{1}{35}\,B\left(\frac{16\,\sqrt{cx^4+bx^2}c^3}{b^4x^2}-\frac{8\,\sqrt{cx^4+bx^2}c^2}{b^3x^4}+\frac{6\,\sqrt{cx^4+bx^2}c}{b^2x^6}-\frac{5\,\sqrt{cx^4+bx^2}}{bx^8}\right)\\&\quad-\frac{1}{315}\,A\left(\frac{128\,\sqrt{cx^4+bx^2}c^4}{b^5x^2}-\frac{64\,\sqrt{cx^4+bx^2}c^3}{b^4x^4}+\frac{48\,\sqrt{cx^4+bx^2}c^2}{b^3x^6}-\frac{40\,\sqrt{cx^4+bx^2}c}{b^2x^8}+\frac{35\,\sqrt{cx^4+bx^2}}{bx^{10}}\right)\end{aligned}$$

input
```
integrate((B*x^2+A)/x^9/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/35*B*(16*sqrt(c*x^4 + b*x^2)*c^3/(b^4*x^2) - 8*sqrt(c*x^4 + b*x^2)*c^2/(
b^3*x^4) + 6*sqrt(c*x^4 + b*x^2)*c/(b^2*x^6) - 5*sqrt(c*x^4 + b*x^2)/(b*x^
8)) - 1/315*A*(128*sqrt(c*x^4 + b*x^2)*c^4/(b^5*x^2) - 64*sqrt(c*x^4 + b*x
^2)*c^3/(b^4*x^4) + 48*sqrt(c*x^4 + b*x^2)*c^2/(b^3*x^6) - 40*sqrt(c*x^4 +
 b*x^2)*c/(b^2*x^8) + 35*sqrt(c*x^4 + b*x^2)/(b*x^10))
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.72

$$\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{32\left(315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bc^{\frac{7}{2}}-819\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Bbc^{\frac{7}{2}}+1008\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Ac^{\frac{9}{2}}+756\left(\right.\right.}{}$$

input
```
integrate((B*x^2+A)/x^9/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
32/315*(315*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*c^(7/2) - 819*(sqrt(c)*x -
sqrt(c*x^2 + b))^8*B*b*c^(7/2) + 1008*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*c^
(9/2) + 756*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^2*c^(7/2) - 672*(sqrt(c)*x
 - sqrt(c*x^2 + b))^6*A*b*c^(9/2) - 324*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*
b^3*c^(7/2) + 288*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^2*c^(9/2) + 81*(sqrt
(c)*x - sqrt(c*x^2 + b))^2*B*b^4*c^(7/2) - 72*(sqrt(c)*x - sqrt(c*x^2 + b)
)^2*A*b^3*c^(9/2) - 9*B*b^5*c^(7/2) + 8*A*b^4*c^(9/2))/(((sqrt(c)*x - sqrt
(c*x^2 + b))^2 - b)^9*sgn(x))
```

3.138.9 Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx = \frac{(8\,A\,c-9\,B\,b)\,\sqrt{c\,x^4+b\,x^2}}{63\,b^2\,x^8}-\frac{A\,\sqrt{c\,x^4+b\,x^2}}{9\,b\,x^{10}}-\frac{(16\,A\,c^2-18\,B\,b\,c)\,\sqrt{c\,x^4+b\,x^2}}{105\,b^3\,x^6}+\frac{(64\,A\,c^3-72\,B\,b\,c^2)\,\sqrt{c\,x^4+b\,x^2}}{315\,b^4\,x^4}-\frac{(128\,A\,c^4-144\,B\,b\,c^3)\,\sqrt{c\,x^4+b\,x^2}}{315\,b^5\,x^2}$$

input
```
int((A + B*x^2)/(x^9*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
((8*A*c - 9*B*b)*(b*x^2 + c*x^4)^(1/2))/(63*b^2*x^8) - (A*(b*x^2 + c*x^4)^
(1/2))/(9*b*x^10) - ((16*A*c^2 - 18*B*b*c)*(b*x^2 + c*x^4)^(1/2))/(105*b^3
*x^6) + ((64*A*c^3 - 72*B*b*c^2)*(b*x^2 + c*x^4)^(1/2))/(315*b^4*x^4) - ((
128*A*c^4 - 144*B*b*c^3)*(b*x^2 + c*x^4)^(1/2))/(315*b^5*x^2)
```

3.139 $\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

3.139.1 Optimal result

Integrand size = 26, antiderivative size = 131

$$\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{8b^2(6bB-7Ac)\sqrt{bx^2+cx^4}}{105c^4x}+\frac{4b(6bB-7Ac)x\sqrt{bx^2+cx^4}}{105c^3} -\frac{(6bB-7Ac)x^3\sqrt{bx^2+cx^4}}{35c^2}+\frac{Bx^5\sqrt{bx^2+cx^4}}{7c}$$

output
```
-8/105*b^2*(-7*A*c+6*B*b)*(c*x^4+b*x^2)^(1/2)/c^4/x+4/105*b*(-7*A*c+6*B*b)
*x*(c*x^4+b*x^2)^(1/2)/c^3-1/35*(-7*A*c+6*B*b)*x^3*(c*x^4+b*x^2)^(1/2)/c^2
+1/7*B*x^5*(c*x^4+b*x^2)^(1/2)/c
```

3.139.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.65

$$\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{x^2(b+cx^2)}(-48b^3B+8b^2c(7A+3Bx^2)+3c^3x^4(7A+5Bx^2)-2bc^2x^2(14A+9Bx^2))}{105c^4x}$$

input
```
Integrate[(x^6*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(-48*b^3*B + 8*b^2*c*(7*A + 3*B*x^2) + 3*c^3*x^4*(7
*A + 5*B*x^2) - 2*b*c^2*x^2*(14*A + 9*B*x^2)))/(105*c^4*x)
```

3.139.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1945, 1421, 1421, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (A+Bx^2)}{\sqrt{bx^2+cx^4}} \, dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{Bx^5\sqrt{bx^2+cx^4}}{7c}-\frac{(6bB-7Ac)\int \frac{x^6}{\sqrt{cx^4+bx^2}}dx}{7c}$$

$$\Big\downarrow \text{1421}$$

$$\frac{Bx^5\sqrt{bx^2+cx^4}}{7c}-\frac{(6bB-7Ac)\left(\frac{x^3\sqrt{bx^2+cx^4}}{5c}-\frac{4b\int \frac{x^4}{\sqrt{cx^4+bx^2}}dx}{5c}\right)}{7c}$$

$$\Big\downarrow \text{1421}$$

$$\frac{Bx^5\sqrt{bx^2+cx^4}}{7c}-\frac{(6bB-7Ac)\left(\frac{x^3\sqrt{bx^2+cx^4}}{5c}-\frac{4b\left(\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\int \frac{x^2}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{5c}\right)}{7c}$$

$$\Big\downarrow \text{1420}$$

$$\frac{Bx^5\sqrt{bx^2+cx^4}}{7c}-\frac{\left(\frac{x^3\sqrt{bx^2+cx^4}}{5c}-\frac{4b\left(\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}\right)}{5c}\right)(6bB-7Ac)}{7c}$$

input
```
Int[(x^6*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(B*x^5*Sqrt[b*x^2 + c*x^4])/(7*c) - ((6*b*B - 7*A*c)*((x^3*Sqrt[b*x^2 + c*
x^4])/(5*c) - (4*b*((-2*b*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (x*Sqrt[b*x^2 +
 c*x^4])/(3*c)))/(5*c)))/(7*c)
```

3.139.3.1 Defintions of rubi rules used

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

rule 1421
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && IGtQ[Simplify[(m
 + 2*p - 1)/2], 0] && NeQ[m + 4*p + 1, 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.139.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.64

method	result	size
trager	$\frac{\left(15B\,c^{3}x^{6}+21A\,c^{3}x^{4}-18Bb\,c^{2}x^{4}-28Ab\,c^{2}x^{2}+24B\,b^{2}c\,x^{2}+56b^{2}Ac-48B\,b^{3}\right)\sqrt{x^{4}c+b\,x^{2}}}{105c^{4}x}$	84
gosper	$\frac{\left(c\,x^{2}+b\right)\left(15B\,c^{3}x^{6}+21A\,c^{3}x^{4}-18Bb\,c^{2}x^{4}-28Ab\,c^{2}x^{2}+24B\,b^{2}c\,x^{2}+56b^{2}Ac-48B\,b^{3}\right)x}{105c^{4}\sqrt{x^{4}c+b\,x^{2}}}$	89
default	$\frac{\left(c\,x^{2}+b\right)\left(15B\,c^{3}x^{6}+21A\,c^{3}x^{4}-18Bb\,c^{2}x^{4}-28Ab\,c^{2}x^{2}+24B\,b^{2}c\,x^{2}+56b^{2}Ac-48B\,b^{3}\right)x}{105c^{4}\sqrt{x^{4}c+b\,x^{2}}}$	89
risch	$\frac{x\left(c\,x^{2}+b\right)\left(15B\,c^{3}x^{6}+21A\,c^{3}x^{4}-18Bb\,c^{2}x^{4}-28Ab\,c^{2}x^{2}+24B\,b^{2}c\,x^{2}+56b^{2}Ac-48B\,b^{3}\right)}{105\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,c^{4}}$	89

input
```
int(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(15*B*c^3*x^6+21*A*c^3*x^4-18*B*b*c^2*x^4-28*A*b*c^2*x^2+24*B*b^2*c*
x^2+56*A*b^2*c-48*B*b^3)/c^4/x*(c*x^4+b*x^2)^(1/2)
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.63

$$\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{(15\,Bc^3x^6-3\,(6\,Bbc^2-7\,Ac^3)x^4-48\,Bb^3+56\,Ab^2c+4\,(6\,Bb^2c-7\,Abc^2)x^2)\sqrt{cx^4+bx^2}}{105\,c^4x}$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(15*B*c^3*x^6 - 3*(6*B*b*c^2 - 7*A*c^3)*x^4 - 48*B*b^3 + 56*A*b^2*c
+ 4*(6*B*b^2*c - 7*A*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^4*x)
```

3.139.6 Sympy [F]

$$\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^6(A+Bx^2)}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**6*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

3.139.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.81

$$\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{(3\,c^3x^6-bc^2x^4+4\,b^2cx^2+8\,b^3)A}{15\,\sqrt{cx^2+b}c^3} + \frac{(5\,c^4x^8-bc^3x^6+2\,b^2c^2x^4-8\,b^3cx^2-16\,b^4)B}{35\,\sqrt{cx^2+b}c^4}$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/15*(3*c^3*x^6 - b*c^2*x^4 + 4*b^2*c*x^2 + 8*b^3)*A/(sqrt(c*x^2 + b)*c^3)
 + 1/35*(5*c^4*x^8 - b*c^3*x^6 + 2*b^2*c^2*x^4 - 8*b^3*c*x^2 - 16*b^4)*B/(
sqrt(c*x^2 + b)*c^4)
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.99

$$\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{8\left(6\,Bb^{\frac{7}{2}}-7\,Ab^{\frac{5}{2}}c\right)\mathrm{sgn}(x)}{105\,c^4} - \frac{(Bb^3-Ab^2c)\sqrt{cx^2+b}}{c^4\mathrm{sgn}\,(x)} + \frac{15\,(cx^2+b)^{\frac{7}{2}}B-63\,(cx^2+b)^{\frac{5}{2}}Bb+105\,(cx^2+b)^{\frac{3}{2}}Bb^2+21\,(cx^2+b)^{\frac{5}{2}}Ac-70\,(cx^2+b)^{\frac{3}{2}}Abc}{105\,c^4\mathrm{sgn}\,(x)}$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
8/105*(6*B*b^(7/2) - 7*A*b^(5/2)*c)*sgn(x)/c^4 - (B*b^3 - A*b^2*c)*sqrt(c*
x^2 + b)/(c^4*sgn(x)) + 1/105*(15*(c*x^2 + b)^(7/2)*B - 63*(c*x^2 + b)^(5/
2)*B*b + 105*(c*x^2 + b)^(3/2)*B*b^2 + 21*(c*x^2 + b)^(5/2)*A*c - 70*(c*x^
2 + b)^(3/2)*A*b*c)/(c^4*sgn(x))
```

3.139.9 Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.66

$$\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{48\,B\,b^3-56\,A\,b^2\,c}{105\,c^4}-\frac{B\,x^6}{7\,c}-\frac{x^4\,\left(21\,A\,c^3-18\,B\,b\,c^2\right)}{105\,c^4}+\frac{4\,b\,x^2\,\left(7\,A\,c-6\,B\,b\right)}{105\,c^3}\right)}{x}$$

input
```
int((x^6*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
-((b*x^2 + c*x^4)^(1/2)*((48*B*b^3 - 56*A*b^2*c)/(105*c^4) - (B*x^6)/(7*c)
 - (x^4*(21*A*c^3 - 18*B*b*c^2))/(105*c^4) + (4*b*x^2*(7*A*c - 6*B*b))/(10
5*c^3)))/x
```

3.140 $\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

3.140.1 Optimal result

Integrand size = 26, antiderivative size = 94

$$\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2b(4bB-5Ac)\sqrt{bx^2+cx^4}}{15c^3x} - \frac{(4bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{bx^2+cx^4}}{5c}$$

output
```
2/15*b*(-5*A*c+4*B*b)*(c*x^4+b*x^2)^(1/2)/c^3/x-1/15*(-5*A*c+4*B*b)*x*(c*x
^4+b*x^2)^(1/2)/c^2+1/5*B*x^3*(c*x^4+b*x^2)^(1/2)/c
```

3.140.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.67

$$\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{x^2(b+cx^2)}(8b^2B-2bc(5A+2Bx^2)+c^2x^2(5A+3Bx^2))}{15c^3x}$$

input
```
Integrate[(x^4*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(8*b^2*B - 2*b*c*(5*A + 2*B*x^2) + c^2*x^2*(5*A + 3
*B*x^2)))/(15*c^3*x)
```

3.140.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1945, 1421, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$
$$\Big\downarrow \text{ 1945}$$
$$\frac{Bx^3\sqrt{bx^2+cx^4}}{5c}-\frac{(4bB-5Ac)\int \frac{x^4}{\sqrt{cx^4+bx^2}}dx}{5c}$$
$$\Big\downarrow \text{ 1421}$$
$$\frac{Bx^3\sqrt{bx^2+cx^4}}{5c}-\frac{(4bB-5Ac)\left(\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\int \frac{x^2}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{5c}$$
$$\Big\downarrow \text{ 1420}$$
$$\frac{Bx^3\sqrt{bx^2+cx^4}}{5c}-\frac{\left(\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}\right)(4bB-5Ac)}{5c}$$

input
```
Int[(x^4*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(B*x^3*Sqrt[b*x^2 + c*x^4])/(5*c) - ((4*b*B - 5*A*c)*((-2*b*Sqrt[b*x^2 + c
*x^4])/(3*c^2*x) + (x*Sqrt[b*x^2 + c*x^4])/(3*c)))/(5*c)
```

3.140.3.1 Defintions of rubi rules used

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

rule 1421
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && IGtQ[Simplify[(m
 + 2*p - 1)/2], 0] && NeQ[m + 4*p + 1, 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.140.4 Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.64

method	result	size
trager	$-\frac{\left(-3B\,c^{2}x^{4}-5A\,c^{2}x^{2}+4Bbc\,x^{2}+10Abc-8B\,b^{2}\right)\sqrt{x^{4}c+b\,x^{2}}}{15c^{3}x}$	60
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-3B\,c^{2}x^{4}-5A\,c^{2}x^{2}+4Bbc\,x^{2}+10Abc-8B\,b^{2}\right)x}{15c^{3}\sqrt{x^{4}c+b\,x^{2}}}$	65
default	$-\frac{\left(c\,x^{2}+b\right)\left(-3B\,c^{2}x^{4}-5A\,c^{2}x^{2}+4Bbc\,x^{2}+10Abc-8B\,b^{2}\right)x}{15c^{3}\sqrt{x^{4}c+b\,x^{2}}}$	65
risch	$-\frac{x\left(c\,x^{2}+b\right)\left(-3B\,c^{2}x^{4}-5A\,c^{2}x^{2}+4Bbc\,x^{2}+10Abc-8B\,b^{2}\right)}{15\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,c^{3}}$	65

input
```
int(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-3*B*c^2*x^4-5*A*c^2*x^2+4*B*b*c*x^2+10*A*b*c-8*B*b^2)/c^3/x*(c*x^4
+b*x^2)^(1/2)
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.63

$$\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{(3\,Bc^2x^4+8\,Bb^2-10\,Abc-(4\,Bbc-5\,Ac^2)x^2)\sqrt{cx^4+bx^2}}{15\,c^3x}$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*B*c^2*x^4 + 8*B*b^2 - 10*A*b*c - (4*B*b*c - 5*A*c^2)*x^2)*sqrt(c*x
^4 + b*x^2)/(c^3*x)
```

3.140.6 Sympy [F]

$$\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^4(A+Bx^2)}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**4*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{(c^2x^4-bcx^2-2\,b^2)A}{3\,\sqrt{cx^2+b}c^2} + \frac{(3\,c^3x^6-bc^2x^4+4\,b^2cx^2+8\,b^3)B}{15\,\sqrt{cx^2+b}c^3}$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)*A/(sqrt(c*x^2 + b)*c^2) + 1/15*(3*c^3*x^6
- b*c^2*x^4 + 4*b^2*c*x^2 + 8*b^3)*B/(sqrt(c*x^2 + b)*c^3)
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left(4\,Bb^{\frac{5}{2}}-5\,Ab^{\frac{3}{2}}c\right)\mathrm{sgn}(x)}{15\,c^3}+\frac{(Bb^2-Abc)\sqrt{cx^2+b}}{c^3\mathrm{sgn}\,(x)}+\frac{3\,(cx^2+b)^{\frac{5}{2}}B-10\,(cx^2+b)^{\frac{3}{2}}Bb+5\,(cx^2+b)^{\frac{3}{2}}Ac}{15\,c^3\mathrm{sgn}\,(x)}$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2/15*(4*B*b^(5/2) - 5*A*b^(3/2)*c)*sgn(x)/c^3 + (B*b^2 - A*b*c)*sqrt(c*x^
2 + b)/(c^3*sgn(x)) + 1/15*(3*(c*x^2 + b)^(5/2)*B - 10*(c*x^2 + b)^(3/2)*B
*b + 5*(c*x^2 + b)^(3/2)*A*c)/(c^3*sgn(x))
```

3.140.9 Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.68

$$\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{8\,B\,b^2-10\,A\,b\,c}{15\,c^3}+\frac{x^2\,\left(5\,A\,c^2-4\,B\,b\,c\right)}{15\,c^3}+\frac{B\,x^4}{5\,c}\right)}{x}$$

input
```
int((x^4*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
((b*x^2 + c*x^4)^(1/2)*((8*B*b^2 - 10*A*b*c)/(15*c^3) + (x^2*(5*A*c^2 - 4*
B*b*c))/(15*c^3) + (B*x^4)/(5*c)))/x
```

3.141 $\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

3.141.1 Optimal result

Integrand size = 26, antiderivative size = 59

$$\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{3c^2x}+\frac{Bx\sqrt{bx^2+cx^4}}{3c}$$

output
```
-1/3*(-3*A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/c^2/x+1/3*B*x*(c*x^4+b*x^2)^(1/2)/
c
```

3.141.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{x^2(b+cx^2)}(-2bB+3Ac+Bcx^2)}{3c^2x}$$

input
```
Integrate[(x^2*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(-2*b*B + 3*A*c + B*c*x^2))/(3*c^2*x)
```

3.141.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1945, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^2)}{\sqrt{bx^2+cx^4}} \, dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{Bx\sqrt{bx^2+cx^4}}{3c} - \frac{(2bB-3Ac)\int \frac{x^2}{\sqrt{cx^4+bx^2}}dx}{3c}$$

$$\Big\downarrow \text{1420}$$

$$\frac{Bx\sqrt{bx^2+cx^4}}{3c} - \frac{\sqrt{bx^2+cx^4}(2bB-3Ac)}{3c^2x}$$

input
```
Int[(x^2*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
-1/3*((2*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(c^2*x) + (B*x*Sqrt[b*x^2 + c*x
^4])/(3*c)
```

3.141.3.1 Defintions of rubi rules used

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.141.4 Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.63

method	result	size
trager	$\frac{(B c\,x^{2}+3 A c -2 B b)\sqrt{x^{4} c +b\,x^{2}}}{3 c^{2} x}$	37
gosper	$\frac{(c\,x^{2}+b)\,(B c\,x^{2}+3 A c -2 B b) x}{3 c^{2} \sqrt{x^{4} c +b\,x^{2}}}$	42
default	$\frac{(c\,x^{2}+b)\,(B c\,x^{2}+3 A c -2 B b) x}{3 c^{2} \sqrt{x^{4} c +b\,x^{2}}}$	42
risch	$\frac{x (c\,x^{2}+b)\,(B c\,x^{2}+3 A c -2 B b)}{3 \sqrt{x^{2} (c\,x^{2}+b)}\, c^{2}}$	42

input
```
int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(B*c*x^2+3*A*c-2*B*b)/c^2/x*(c*x^4+b*x^2)^(1/2)
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{cx^4+bx^2}(Bcx^2-2\,Bb+3\,Ac)}{3\,c^2x}$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(c*x^4 + b*x^2)*(B*c*x^2 - 2*B*b + 3*A*c)/(c^2*x)
```

3.141.6 Sympy [F]

$$\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^2(A+Bx^2)}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**2*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

3.141.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{cx^2+b}A}{c} + \frac{(c^2x^4-bcx^2-2\,b^2)B}{3\,\sqrt{cx^2+b}c^2}$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(c*x^2 + b)*A/c + 1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)*B/(sqrt(c*x^2 + b)*c
^2)
```

3.141.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\left(2\,Bb^{\frac{3}{2}}-3\,A\sqrt{b}c\right)\operatorname{sgn}(x)}{3\,c^2} + \frac{(cx^2+b)^{\frac{3}{2}}B}{3\,c^2\operatorname{sgn}(x)} - \frac{\sqrt{cx^2+b}(Bb-Ac)}{c^2\operatorname{sgn}(x)}$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*(2*B*b^(3/2) - 3*A*sqrt(b)*c)*sgn(x)/c^2 + 1/3*(c*x^2 + b)^(3/2)*B/(c^
2*sgn(x)) - sqrt(c*x^2 + b)*(B*b - A*c)/(c^2*sgn(x))
```

3.141.9 Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\left(\frac{3\,A\,c-2\,B\,b}{3\,c^2}+\frac{B\,x^2}{3\,c}\right)\,\sqrt{c\,x^4+b\,x^2}}{x}$$

input
```
int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
(((3*A*c - 2*B*b)/(3*c^2) + (B*x^2)/(3*c))*(b*x^2 + c*x^4)^(1/2))/x
```

3.142 $\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}}\,dx$

3.142.1 Optimal result

Integrand size = 23, antiderivative size = 55

$$\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}}\,dx = \frac{B\sqrt{bx^2+cx^4}}{cx} - \frac{A\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

output
```
-A*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(1/2)+B*(c*x^4+b*x^2)^(1/2)/c/
x
```

3.142.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}}\,dx = \frac{x\left(\sqrt{b}B(b+cx^2) - Ac\sqrt{b+cx^2}\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{\sqrt{b}c\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(A + B*x^2)/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(x*(Sqrt[b]*B*(b + c*x^2) - A*c*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sq
rt[b]]))/(Sqrt[b]*c*Sqrt[x^2*(b + c*x^2)])
```

3.142.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1465, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1465}$$

$$A\int \frac{1}{\sqrt{cx^4+bx^2}}dx+\frac{B\sqrt{bx^2+cx^4}}{cx}$$

$$\Big\downarrow \text{1400}$$

$$\frac{B\sqrt{bx^2+cx^4}}{cx}-A\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{B\sqrt{bx^2+cx^4}}{cx}-\frac{A\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

input
```
Int[(A + B*x^2)/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(B*Sqrt[b*x^2 + c*x^4])/(c*x) - (A*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]
])/Sqrt[b]
```

3.142.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1465
```
Int[((d_) + (e_.)*(x_)^2)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
Simp[e*((b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 3)*x)), x] - Simp[(b*e*(2*p + 1)
- c*d*(4*p + 3))/(c*(4*p + 3))   Int[(b*x^2 + c*x^4)^p, x], x] /; FreeQ[{b,
 c, d, e, p}, x] &&  !IntegerQ[p] && NeQ[4*p + 3, 0] && NeQ[b*e*(2*p + 1) -
 c*d*(4*p + 3), 0]
```

3.142.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.31

method	result	size
default	$-\frac{x\sqrt{c\,x^{2}+b}\,\left(A\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)c-B\sqrt{c\,x^{2}+b}\,\sqrt{b}\right)}{\sqrt{x^{4}c+b\,x^{2}}\,c\sqrt{b}}$	72

input
```
int((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-x*(c*x^2+b)^(1/2)*(A*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c-B*(c*x^2+b)^(1
/2)*b^(1/2))/(c*x^4+b*x^2)^(1/2)/c/b^(1/2)
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.51

$$\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}}\,dx = \left[\frac{A\sqrt{b}cx\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}Bb}{2\,bcx}, \frac{A\sqrt{-b}cx\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}B\right.$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(A*sqrt(b)*c*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x
^3) + 2*sqrt(c*x^4 + b*x^2)*B*b)/(b*c*x), (A*sqrt(-b)*c*x*arctan(sqrt(c*x^
4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*B*b)/(b*c*x)]
```

3.142.6 Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{A+Bx^2}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate((B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

3.142.7 Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2), x)
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

$$\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}}\,dx=-\frac{\left(Ac\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+B\sqrt{-b}\sqrt{b}\right)\mathrm{sgn}(x)}{\sqrt{-b}c}+\frac{\frac{A\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+\frac{\sqrt{cx^2+b}B}{c}}{\mathrm{sgn}\left(x\right)}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
-(A*c*arctan(sqrt(b)/sqrt(-b)) + B*sqrt(-b)*sqrt(b))*sgn(x)/(sqrt(-b)*c) +
 (A*arctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) + sqrt(c*x^2 + b)*B/c)/sgn(x
)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{B\,x^2+A}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int((A + B*x^2)/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((A + B*x^2)/(b*x^2 + c*x^4)^(1/2), x)
```

3.143 $\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx$

3.143.1 Optimal result

Integrand size = 26, antiderivative size = 68

$$\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx=-\frac{A\sqrt{bx^2+cx^4}}{2bx^3}-\frac{(2bB-Ac)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}$$

output
```
-1/2*(-A*c+2*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(3/2)-1/2*A*(c*
x^4+b*x^2)^(1/2)/b/x^3
```

3.143.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx=\frac{-A\sqrt{b}(b+cx^2)-(2bB-Ac)x^2\sqrt{b+cx^2}\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{2b^{3/2}x\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(A + B*x^2)/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-(A*Sqrt[b]*(b + c*x^2)) - (2*b*B - A*c)*x^2*Sqrt[b + c*x^2]*ArcTanh[Sqrt
[b + c*x^2]/Sqrt[b]])/(2*b^(3/2)*x*Sqrt[x^2*(b + c*x^2)])
```

3.143.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1944, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(2bB-Ac)\int \frac{1}{\sqrt{cx^4+bx^2}}dx}{2b}-\frac{A\sqrt{bx^2+cx^4}}{2bx^3}$$

$$\Big\downarrow \text{1400}$$

$$-\frac{(2bB-Ac)\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}}{2b}-\frac{A\sqrt{bx^2+cx^4}}{2bx^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{(2bB-Ac)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{A\sqrt{bx^2+cx^4}}{2bx^3}$$

input
```
Int[(A + B*x^2)/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
-1/2*(A*Sqrt[b*x^2 + c*x^4])/(b*x^3) - ((2*b*B - A*c)*ArcTanh[(Sqrt[b]*x)/
Sqrt[b*x^2 + c*x^4]])/(2*b^(3/2))
```

3.143.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.143.4 Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.32

method	result	size
risch	$-\frac{A\left(c\,x^{2}+b\right)}{2 b x \sqrt{x^{2} \left(c\,x^{2}+b\right)}}+\frac{\left(A c -2 B b \right) \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c\,x^{2}+b}}{x}\right) x \sqrt{c\,x^{2}+b}}{2 b^{\frac{3}{2}} \sqrt{x^{2} \left(c\,x^{2}+b\right)}}$	90
default	$-\frac{\sqrt{c\,x^{2}+b}\, \left(2 B \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c\,x^{2}+b}}{x}\right) b^{2} x^{2}-A \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c\,x^{2}+b}}{x}\right) b c\,x^{2}+A \sqrt{c\,x^{2}+b}\, b^{\frac{3}{2}}\right)}{2 x \sqrt{x^{4} c +b\,x^{2}}\, b^{\frac{5}{2}}}$	105

input
```
int((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*A*(c*x^2+b)/x/(x^2*(c*x^2+b))^(1/2)+1/2*(A*c-2*B*b)/b^(3/2)*ln((2*b
+2*b^(1/2)*(c*x^2+b)^(1/2))/x)*x/(x^2*(c*x^2+b))^(1/2)*(c*x^2+b)^(1/2)
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.24

$$\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx = \left[-\frac{(2\,Bb-Ac)\sqrt{b}x^3\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}Ab}{4\,b^2x^3}, \frac{(2\,Bb-Ac)\sqrt{-b}x^3\arctan\left(\frac{\sqrt{cx^4+bx^2}}{cx^3+b}\right.}{2\,b^2x^3}\right.$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*((2*B*b - A*c)*sqrt(b)*x^3*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^
2)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*A*b)/(b^2*x^3), 1/2*((2*B*b - A*c
)*sqrt(-b)*x^3*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) - sqrt(c
*x^4 + b*x^2)*A*b)/(b^2*x^3)]
```

3.143.6 Sympy [F]

$$\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^2\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate((B*x**2+A)/x**2/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x**2*sqrt(x**2*(b + c*x**2))), x)
```

3.143.7 Maxima [F]

$$\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^2}\,dx$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^2), x)
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx = \frac{\frac{(2\,Bbc-Ac^2)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b} - \frac{\sqrt{cx^2+b}Ac}{bx^2}}{2\,\mathrm{csgn}\,(x)}$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*((2*B*b*c - A*c^2)*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b) - sqr
t(c*x^2 + b)*A*c/(b*x^2))/(c*sgn(x))
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx=\int \frac{B\,x^2+A}{x^2\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(1/2)), x)
```

3.144 $\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx$

3.144.1 Optimal result

Integrand size = 26, antiderivative size = 103

$$\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx=-\frac{A\sqrt{bx^2+cx^4}}{4bx^5}-\frac{(4bB-3Ac)\sqrt{bx^2+cx^4}}{8b^2x^3}+\frac{c(4bB-3Ac)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}$$

output
```
1/8*c*(-3*A*c+4*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(5/2)-1/4*A*
(c*x^4+b*x^2)^(1/2)/b/x^5-1/8*(-3*A*c+4*B*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^3
```

3.144.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx=\frac{-\sqrt{b}(b+cx^2)\left(2Ab+4bBx^2-3Acx^2\right)+c(4bB-3Ac)x^4\sqrt{b+cx^2}\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{8b^{5/2}x^3\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[(A + B*x^2)/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-(Sqrt[b]*(b + c*x^2)*(2*A*b + 4*b*B*x^2 - 3*A*c*x^2)) + c*(4*b*B - 3*A*c
)*x^4*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])/(8*b^(5/2)*x^3*Sqr
t[x^2*(b + c*x^2)])
```

3.144.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1944, 1430, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(4bB-3Ac)\int \frac{1}{x^2\sqrt{cx^4+bx^2}}dx}{4b}-\frac{A\sqrt{bx^2+cx^4}}{4bx^5}$$

$$\Big\downarrow \text{1430}$$

$$\frac{(4bB-3Ac)\left(-\frac{c\int \frac{1}{\sqrt{cx^4+bx^2}}dx}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{A\sqrt{bx^2+cx^4}}{4bx^5}$$

$$\Big\downarrow \text{1400}$$

$$\frac{(4bB-3Ac)\left(\frac{c\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{A\sqrt{bx^2+cx^4}}{4bx^5}$$

$$\Big\downarrow \text{219}$$

$$\frac{(4bB-3Ac)\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{A\sqrt{bx^2+cx^4}}{4bx^5}$$

input
```
Int[(A + B*x^2)/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
-1/4*(A*Sqrt[b*x^2 + c*x^4])/(b*x^5) + ((4*b*B - 3*A*c)*(-1/2*Sqrt[b*x^2 +
 c*x^4]/(b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(3/2))
))/(4*b)
```

3.144.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.144.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{(c\,x^2+b)\,(-3Ac\,x^2+4bB\,x^2+2Ab)}{8b^2x^3\sqrt{x^2(c\,x^2+b)}}-\frac{(3Ac-4Bb)c\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)x\sqrt{c\,x^2+b}}{8b^{\frac{5}{2}}\sqrt{x^2(c\,x^2+b)}}$
default	$-\frac{\sqrt{c\,x^2+b}\,\left(3A\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)b\,c^2x^4-4B\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)b^2c\,x^4-3A\sqrt{c\,x^2+b}\,b^{\frac{3}{2}}c\,x^2+4B\sqrt{c\,x^2+b}\,b^{\frac{5}{2}}x^2+2A\sqrt{c\,x^2+b}\,b}{8x^3\sqrt{x^4c+b\,x^2}\,b^{\frac{7}{2}}}$

input
```
int((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(c*x^2+b)*(-3*A*c*x^2+4*B*b*x^2+2*A*b)/b^2/x^3/(x^2*(c*x^2+b))^(1/2)-
1/8*(3*A*c-4*B*b)*c/b^(5/2)*ln((2*b+2*b^(1/2)*(c*x^2+b)^(1/2))/x)*x/(x^2*(
c*x^2+b))^(1/2)*(c*x^2+b)^(1/2)
```

3.144.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.93

$$\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx$$
$$=\left[-\frac{(4\,Bbc-3\,Ac^2)\sqrt{b}x^5\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}(2\,Ab^2+(4\,Bb^2-3\,Abc)x^2)}{16\,b^3x^5},\right.$$
$$\left.-\frac{(4\,Bbc-3\,Ac^2)\sqrt{-b}x^5\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}(2\,Ab^2+(4\,Bb^2-3\,Abc)x^2)}{8\,b^3x^5}\right]$$

input
```
integrate((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((4*B*b*c - 3*A*c^2)*sqrt(b)*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4
 + b*x^2)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*A*b^2 + (4*B*b^2 - 3*A*
b*c)*x^2))/(b^3*x^5), -1/8*((4*B*b*c - 3*A*c^2)*sqrt(-b)*x^5*arctan(sqrt(c
*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*(2*A*b^2 + (4*
B*b^2 - 3*A*b*c)*x^2))/(b^3*x^5)]
```

3.144.6 Sympy [F]

$$\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^4\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate((B*x**2+A)/x**4/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x**4*sqrt(x**2*(b + c*x**2))), x)
```

3.144.7 Maxima [F]

$$\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}\,x^4}\,dx$$

input
```
integrate((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^4), x)
```

3.144.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx = -\frac{\frac{\left(4\,Bbc^2-3\,Ac^3\right)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2}+\frac{4\left(cx^2+b\right)^{\frac{3}{2}}Bbc^2-4\sqrt{cx^2+b}Bb^2c^2-3\left(cx^2+b\right)^{\frac{3}{2}}Ac^3+5\sqrt{cx^2+b}Abc^3}{b^2c^2x^4}}{8\,\mathrm{csgn}\left(x\right)}$$

input
```
integrate((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/8*((4*B*b*c^2 - 3*A*c^3)*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^2
) + (4*(c*x^2 + b)^(3/2)*B*b*c^2 - 4*sqrt(c*x^2 + b)*B*b^2*c^2 - 3*(c*x^2
+ b)^(3/2)*A*c^3 + 5*sqrt(c*x^2 + b)*A*b*c^3)/(b^2*c^2*x^4))/(c*sgn(x))
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx=\int \frac{B\,x^2+A}{x^4\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int((A + B*x^2)/(x^4*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2)/(x^4*(b*x^2 + c*x^4)^(1/2)), x)
```

3.145 $\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.145.1 Optimal result

Integrand size = 26, antiderivative size = 184

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}}+\frac{5b(7bB-6Ac)\sqrt{bx^2+cx^4}}{16c^4} - \frac{5(7bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^3}+\frac{(7bB-6Ac)x^4\sqrt{bx^2+cx^4}}{6bc^2} - \frac{5b^2(7bB-6Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{9/2}}$$

output
```
-5/16*b^2*(-6*A*c+7*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(9/2)-
(-A*c+B*b)*x^8/b/c/(c*x^4+b*x^2)^(1/2)+5/16*b*(-6*A*c+7*B*b)*(c*x^4+b*x^2)
^(1/2)/c^4-5/24*(-6*A*c+7*B*b)*x^2*(c*x^4+b*x^2)^(1/2)/c^3+1/6*(-6*A*c+7*B
*b)*x^4*(c*x^4+b*x^2)^(1/2)/b/c^2
```

3.145.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.85

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^3\left(\sqrt{c}x(b+cx^2)\left(105b^3B+4c^3x^4(3A+2Bx^2)-2bc^2x^2(15A+7Bx^2)+b^2(-90Ac\right.\right.}{48c^{9/2}\left(x^2\left(b+cx^2\right)\right)}$$

input
```
Integrate[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x^3*(Sqrt[c]*x*(b + c*x^2)*(105*b^3*B + 4*c^3*x^4*(3*A + 2*B*x^2) - 2*b*c
^2*x^2*(15*A + 7*B*x^2) + b^2*(-90*A*c + 35*B*c*x^2)) - 30*b^2*(7*b*B - 6*
A*c)*(b + c*x^2)^(3/2)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] + Sqrt[b + c*x^2])]))
/(48*c^(9/2)*(x^2*(b + c*x^2))^(3/2))
```

3.145.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1940, 1211, 25, 2192, 27, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9 \left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1940}$$

$$\frac{1}{2}\int \frac{x^8 \left(Bx^2+A\right)}{\left(cx^4+bx^2\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 1211}$$

$$\frac{1}{2}\left(\frac{\int -\frac{-Bc^3x^6+c^2(bB-Ac)x^4-bc(bB-Ac)x^2+b^2(bB-Ac)}{\sqrt{cx^4+bx^2}}dx^2}{c^4}+\frac{2b^2x^2(bB-Ac)}{c^4\sqrt{bx^2+cx^4}}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{2b^2x^2(bB-Ac)}{c^4\sqrt{bx^2+cx^4}}-\frac{\int \frac{-Bc^3x^6+c^2(bB-Ac)x^4-bc(bB-Ac)x^2+b^2(bB-Ac)}{\sqrt{cx^4+bx^2}}dx^2}{c^4}\right)$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{1}{2}\left(\frac{2b^2x^2(bB-Ac)}{c^4\sqrt{bx^2+cx^4}}-\frac{\frac{\int \frac{c^3(11bB-6Ac)x^4-6bc^2(bB-Ac)x^2+6b^2c(bB-Ac)}{2\sqrt{cx^4+bx^2}}dx^2}{3c}-\frac{1}{3}Bc^2x^4\sqrt{bx^2+cx^4}}{c^4}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{2b^2x^2(bB-Ac)}{c^4\sqrt{bx^2+cx^4}}-\frac{\frac{\int\frac{c^3(11bB-6Ac)x^4-6bc^2(bB-Ac)x^2+6b^2c(bB-Ac)}{\sqrt{cx^4+bx^2}}dx^2}{6c}-\frac{1}{3}Bc^2x^4\sqrt{bx^2+cx^4}}{c^4}\right)$$

$\big\downarrow$ 2192

$$\frac{1}{2}\left(\frac{2b^2x^2(bB-Ac)}{c^4\sqrt{bx^2+cx^4}}-\frac{\frac{\frac{\int\frac{3bc^2\left(8b(bB-Ac)-c(19bB-14Ac)x^2\right)}{2\sqrt{cx^4+bx^2}}dx^2}{2c}+\frac{1}{2}c^2x^2\sqrt{bx^2+cx^4}(11bB-6Ac)}{6c}-\frac{1}{3}Bc^2x^4\sqrt{bx^2+cx^4}}{c^4}\right)$$

$\big\downarrow$ 27

$$\frac{1}{2}\left(\frac{2b^2x^2(bB-Ac)}{c^4\sqrt{bx^2+cx^4}}-\frac{\frac{\frac{3}{4}bc\int\frac{8b(bB-Ac)-c(19bB-14Ac)x^2}{\sqrt{cx^4+bx^2}}dx^2+\frac{1}{2}c^2x^2\sqrt{bx^2+cx^4}(11bB-6Ac)}{6c}-\frac{1}{3}Bc^2x^4\sqrt{bx^2+cx^4}}{c^4}\right)$$

$\big\downarrow$ 1160

$$\frac{1}{2}\left(\frac{2b^2x^2(bB-Ac)}{c^4\sqrt{bx^2+cx^4}}-\frac{\frac{\frac{3}{4}bc\left(\frac{5}{2}b(7bB-6Ac)\int\frac{1}{\sqrt{cx^4+bx^2}}dx^2-\sqrt{bx^2+cx^4}(19bB-14Ac)\right)+\frac{1}{2}c^2x^2\sqrt{bx^2+cx^4}(11bB-6Ac)}{6c}-\frac{1}{3}Bc^2x^4\sqrt{bx}}{c^4}\right.$$

$\big\downarrow$ 1091

$$\frac{1}{2}\left(\frac{2b^2x^2(bB-Ac)}{c^4\sqrt{bx^2+cx^4}}-\frac{\frac{\frac{3}{4}bc\left(5b(7bB-6Ac)\int\frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}-\sqrt{bx^2+cx^4}(19bB-14Ac)\right)+\frac{1}{2}c^2x^2\sqrt{bx^2+cx^4}(11bB-6Ac)}{6c}-\frac{1}{3}Bc^2x^4\sqrt{}}{c^4}\right.$$

$\big\downarrow$ 219

$$\frac{1}{2}\left(\frac{2b^2x^2(bB-Ac)}{c^4\sqrt{bx^2+cx^4}}-\frac{\frac{\frac{3}{4}bc\left(\frac{5b(7bB-6Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}-\sqrt{bx^2+cx^4}(19bB-14Ac)\right)+\frac{1}{2}c^2x^2\sqrt{bx^2+cx^4}(11bB-6Ac)}{6c}-\frac{1}{3}Bc^2x}{c^4}\right.$$

input
```
Int[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

3.145. $\int\frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}dx$

output
```
((2*b^2*(b*B - A*c)*x^2)/(c^4*Sqrt[b*x^2 + c*x^4]) - (-1/3*(B*c^2*x^4*Sqrt
[b*x^2 + c*x^4]) + ((c^2*(11*b*B - 6*A*c)*x^2*Sqrt[b*x^2 + c*x^4])/2 + (3*
b*c*(-((19*b*B - 14*A*c)*Sqrt[b*x^2 + c*x^4]) + (5*b*(7*b*B - 6*A*c)*ArcTa
nh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/Sqrt[c]))/4)/(6*c))/c^4)/2
```

3.145.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

3.145. $\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.145.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{-\frac{15x^{2}\left(-\frac{7x^{2}B}{18}+A\right)b^{2}c^{\frac{3}{2}}}{8}-\frac{5x^{4}\left(\frac{7x^{2}B}{15}+A\right)b\,c^{\frac{5}{2}}}{8}+\frac{x^{6}\left(\frac{2x^{2}B}{3}+A\right)c^{\frac{7}{2}}}{4}+\frac{15\left(\frac{7Bb\,x^{2}\sqrt{c}}{3}+\left(Ac-\frac{7Bb}{6}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}\left(-\ln(2)+\ln\left(\frac{2c\,x^{2}}{}\right.\right.\right.}{16}}{c^{\frac{9}{2}}\sqrt{x^{2}(c\,x^{2}+b)}}$
default	$\frac{x^{3}\left(c\,x^{2}+b\right)\left(8x^{7}B\,c^{\frac{9}{2}}+12A\,c^{\frac{9}{2}}x^{5}-14x^{5}Bb\,c^{\frac{7}{2}}-30A\,c^{\frac{7}{2}}b\,x^{3}+35b^{2}x^{3}B\,c^{\frac{5}{2}}-90A\,c^{\frac{5}{2}}b^{2}x+105Bx\,b^{3}c^{\frac{3}{2}}+90A\sqrt{c\,x^{2}+b}\,\ln\left(\sqrt{c}\,x\right.\right.}{48(x^{4}c+b\,x^{2})^{\frac{3}{2}}c^{\frac{11}{2}}}$
risch	$-\frac{x^{2}\left(-8B\,c^{2}x^{4}-12A\,c^{2}x^{2}+22Bbc\,x^{2}+42Abc-57B\,b^{2}\right)\left(c\,x^{2}+b\right)}{48c^{4}\sqrt{x^{2}(c\,x^{2}+b)}}+\frac{b^{2}\left(-\frac{19Bbx}{\sqrt{c\,x^{2}+b}}+\frac{14Acx}{\sqrt{c\,x^{2}+b}}+(30A\,c^{2}-35Bbc)\left(-\frac{x}{c\sqrt{c\,x^{2}}}\right.\right.}{16c^{4}\sqrt{x^{2}(c\,x^{2}+b)}}$

input
```
int(x^9*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
15/16*(-2*x^2*(-7/18*x^2*B+A)*b^2*c^(3/2)-2/3*x^4*(7/15*x^2*B+A)*b*c^(5/2)
+4/15*x^6*(2/3*x^2*B+A)*c^(7/2)+(7/3*B*b*x^2*c^(1/2)+(A*c-7/6*B*b)*(x^2*(c
*x^2+b))^(1/2)*(-ln(2)+ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2)*c^(1/2)+b)/c^(1
/2))))*b^2)/c^(9/2)/(x^2*(c*x^2+b))^(1/2)
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.85

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \left[-\frac{15\,(7\,Bb^4-6\,Ab^3c+(7\,Bb^3c-6\,Ab^2c^2)x^2)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right.}{}\right.$$

input
```
integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/96*(15*(7*B*b^4 - 6*A*b^3*c + (7*B*b^3*c - 6*A*b^2*c^2)*x^2)*sqrt(c)*l
og(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*(8*B*c^4*x^6 + 105*B*
b^3*c - 90*A*b^2*c^2 - 2*(7*B*b*c^3 - 6*A*c^4)*x^4 + 5*(7*B*b^2*c^2 - 6*A*
b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/(c^6*x^2 + b*c^5), 1/48*(15*(7*B*b^4 - 6*
A*b^3*c + (7*B*b^3*c - 6*A*b^2*c^2)*x^2)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^
2)*sqrt(-c)/(c*x^2 + b)) + (8*B*c^4*x^6 + 105*B*b^3*c - 90*A*b^2*c^2 - 2*(
7*B*b*c^3 - 6*A*c^4)*x^4 + 5*(7*B*b^2*c^2 - 6*A*b*c^3)*x^2)*sqrt(c*x^4 + b
*x^2))/(c^6*x^2 + b*c^5)]
```

3.145.6 Sympy [F]

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^9(A+Bx^2)}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**9*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.145.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.29

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{1}{16}\left(\frac{4\,x^6}{\sqrt{cx^4+bx^2}c}-\frac{10\,bx^4}{\sqrt{cx^4+bx^2}c^2}-\frac{30\,b^2x^2}{\sqrt{cx^4+bx^2}c^3}+\frac{15\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4}\right.}{c^{\frac{7}{2}}}\right.$$
$$+\frac{1}{96}\left(\frac{16\,x^8}{\sqrt{cx^4+bx^2}c}-\frac{28\,bx^6}{\sqrt{cx^4+bx^2}c^2}+\frac{70\,b^2x^4}{\sqrt{cx^4+bx^2}c^3}+\frac{210\,b^3x^2}{\sqrt{cx^4+bx^2}c^4}-\frac{105\,b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx}\right.}{c^{\frac{9}{2}}}\right.$$

input
```
integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/16*(4*x^6/(sqrt(c*x^4 + b*x^2)*c) - 10*b*x^4/(sqrt(c*x^4 + b*x^2)*c^2) -
 30*b^2*x^2/(sqrt(c*x^4 + b*x^2)*c^3) + 15*b^2*log(2*c*x^2 + b + 2*sqrt(c*
x^4 + b*x^2)*sqrt(c))/c^(7/2))*A + 1/96*(16*x^8/(sqrt(c*x^4 + b*x^2)*c) -
28*b*x^6/(sqrt(c*x^4 + b*x^2)*c^2) + 70*b^2*x^4/(sqrt(c*x^4 + b*x^2)*c^3)
+ 210*b^3*x^2/(sqrt(c*x^4 + b*x^2)*c^4) - 105*b^3*log(2*c*x^2 + b + 2*sqrt
(c*x^4 + b*x^2)*sqrt(c))/c^(9/2))*B
```

3.145.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.99

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\left(\left(2\,x^2\left(\frac{4\,Bx^2}{c\mathrm{sgn}(x)} - \frac{7\,Bbc^5\mathrm{sgn}(x)-6\,Ac^6\mathrm{sgn}(x)}{c^7}\right) + \frac{5\,(7\,Bb^2c^4\mathrm{sgn}(x)-6\,Abc^5\mathrm{sgn}(x))}{c^7}\right)x^2 + \frac{15\,(7\,Bb^3c^3\mathrm{sgn}(}{}}{48\,\sqrt{cx^2+b}}$$

$$- \frac{5\,(7\,Bb^3\log(|b|) - 6\,Ab^2c\log(|b|))\mathrm{sgn}(x)}{32\,c^{\frac{9}{2}}} + \frac{5\,(7\,Bb^3 - 6\,Ab^2c)\log\left(\left|-\sqrt{c}x + \sqrt{cx^2+b}\right|\right)}{16\,c^{\frac{9}{2}}\mathrm{sgn}(x)}$$

input
```
integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/48*((2*x^2*(4*B*x^2/(c*sgn(x)) - (7*B*b*c^5*sgn(x) - 6*A*c^6*sgn(x))/c^7
) + 5*(7*B*b^2*c^4*sgn(x) - 6*A*b*c^5*sgn(x))/c^7)*x^2 + 15*(7*B*b^3*c^3*s
gn(x) - 6*A*b^2*c^4*sgn(x))/c^7)*x/sqrt(c*x^2 + b) - 5/32*(7*B*b^3*log(abs
(b)) - 6*A*b^2*c*log(abs(b)))*sgn(x)/c^(9/2) + 5/16*(7*B*b^3 - 6*A*b^2*c)*
log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(9/2)*sgn(x))
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^9\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int((x^9*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((x^9*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.146 $\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.146.1 Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} - \frac{3(5bB-4Ac)\sqrt{bx^2+cx^4}}{8c^3} + \frac{(5bB-4Ac)x^2\sqrt{bx^2+cx^4}}{4bc^2} + \frac{3b(5bB-4Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}$$

output
```
3/8*b*(-4*A*c+5*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(7/2)-(-A*
c+B*b)*x^6/b/c/(c*x^4+b*x^2)^(1/2)-3/8*(-4*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/
c^3+1/4*(-4*A*c+5*B*b)*x^2*(c*x^4+b*x^2)^(1/2)/b/c^2
```

3.146.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^3\left(\sqrt{c}(b+cx^2)\left(-15b^2Bx+bcx(12A-5Bx^2)+2c^2x^3(2A+Bx^2)\right)+6b(5bB-4A\right.}{8c^{7/2}\left(x^2\left(b+cx^2\right)\right)^{3/2}}$$

input
```
Integrate[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x^3*(Sqrt[c]*(b + c*x^2)*(-15*b^2*B*x + b*c*x*(12*A - 5*B*x^2) + 2*c^2*x^
3*(2*A + B*x^2)) + 6*b*(5*b*B - 4*A*c)*(b + c*x^2)^(3/2)*ArcTanh[(Sqrt[c]*
x)/(-Sqrt[b] + Sqrt[b + c*x^2])]))/(8*c^(7/2)*(x^2*(b + c*x^2))^(3/2))
```

3.146.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1940, 1211, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 (A+Bx^2)}{(bx^2+cx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2} \int \frac{x^6 (Bx^2+A)}{(cx^4+bx^2)^{3/2}} dx^2$$

$$\Big\downarrow \text{1211}$$

$$\frac{1}{2} \left(\frac{\int \frac{Bc^2x^4-c(bB-Ac)x^2+b(bB-Ac)}{\sqrt{cx^4+bx^2}} dx^2}{c^3} - \frac{2bx^2(bB-Ac)}{c^3\sqrt{bx^2+cx^4}} \right)$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{2} \left(\frac{\frac{\int \frac{c\left(4b(bB-Ac)-c(7bB-4Ac)x^2\right)}{2\sqrt{cx^4+bx^2}} dx^2}{2c} + \frac{1}{2}Bcx^2\sqrt{bx^2+cx^4}}{c^3} - \frac{2bx^2(bB-Ac)}{c^3\sqrt{bx^2+cx^4}} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(\frac{\frac{1}{4} \int \frac{4b(bB-Ac)-c(7bB-4Ac)x^2}{\sqrt{cx^4+bx^2}} dx^2 + \frac{1}{2}Bcx^2\sqrt{bx^2+cx^4}}{c^3} - \frac{2bx^2(bB-Ac)}{c^3\sqrt{bx^2+cx^4}} \right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2} \left(\frac{\frac{1}{4}\left(\frac{3}{2}b(5bB-4Ac) \int \frac{1}{\sqrt{cx^4+bx^2}} dx^2 - \sqrt{bx^2+cx^4}(7bB-4Ac)\right) + \frac{1}{2}Bcx^2\sqrt{bx^2+cx^4}}{c^3} - \frac{2bx^2(bB-Ac)}{c^3\sqrt{bx^2+cx^4}} \right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2} \left(\frac{\frac{1}{4}\left(3b(5bB-4Ac) \int \frac{1}{1-cx^4} d\frac{x^2}{\sqrt{cx^4+bx^2}} - \sqrt{bx^2+cx^4}(7bB-4Ac)\right) + \frac{1}{2}Bcx^2\sqrt{bx^2+cx^4}}{c^3} - \frac{2bx^2(bB-Ac)}{c^3\sqrt{bx^2+cx^4}} \right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\frac{1}{4}\left(\frac{3b(5bB-4Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}-\sqrt{bx^2+cx^4}(7bB-4Ac)\right)+\frac{1}{2}Bcx^2\sqrt{bx^2+cx^4}}{c^3}-\frac{2bx^2(bB-Ac)}{c^3\sqrt{bx^2+cx^4}}\right)$$

input
```
Int[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
((-2*b*(b*B - A*c)*x^2)/(c^3*Sqrt[b*x^2 + c*x^4]) + ((B*c*x^2*Sqrt[b*x^2 +
 c*x^4])/2 + (-((7*b*B - 4*A*c)*Sqrt[b*x^2 + c*x^4]) + (3*b*(5*b*B - 4*A*c
)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/Sqrt[c])/4)/c^3)/2
```

3.146.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.146.4 Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95

method	result
default	$-\frac{x^{3} \left(c\,x^{2}+b\right) \left(-2 x^{5} B\,c^{\frac{7}{2}}-4 A\,c^{\frac{7}{2}} x^{3}+5 x^{3} B b\,c^{\frac{5}{2}}-12 A\,c^{\frac{5}{2}} b x+15 b^{2} B x\,c^{\frac{3}{2}}+12 A \sqrt{c\,x^{2}+b}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right) b\,c^{2}-15 B \sqrt{c\,x^{2}} \right.}{8 \left(x^{4} c+b\,x^{2}\right)^{\frac{3}{2}} c^{\frac{9}{2}}}$
risch	$\frac{x^{2} \left(2 B c\,x^{2}+4 A c-7 B b\right) \left(c\,x^{2}+b\right)}{8 c^{3} \sqrt{x^{2} \left(c\,x^{2}+b\right)}}-\frac{b \left(-\frac{7 B b x}{\sqrt{c\,x^{2}+b}}+\frac{4 A c x}{\sqrt{c\,x^{2}+b}}+\left(12 A\,c^{2}-15 B b c\right) \left(-\frac{x}{c \sqrt{c\,x^{2}+b}}+\frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)}{c^{\frac{3}{2}}}\right)\right) x \sqrt{c\,x^{2}}}{8 c^{3} \sqrt{x^{2} \left(c\,x^{2}+b\right)}}$
pseudoelliptic	$\frac{4 B\,c^{\frac{5}{2}} x^{6}+8 A\,c^{\frac{5}{2}} x^{4}-10 B b\,c^{\frac{3}{2}} x^{4}+24 A b\,c^{\frac{3}{2}} x^{2}-30 B\,b^{2} x^{2} \sqrt{c}+12 A \ln\left(2\right) b c \sqrt{x^{2} \left(c\,x^{2}+b\right)}-12 A \ln\left(\frac{2 c\,x^{2}+2 \sqrt{x^{2} \left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)}{16 c^{\frac{7}{2}} \sqrt{x^{2} \left(c\,x^{2}+b\right)}}$

input
```
int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*x^3*(c*x^2+b)*(-2*x^5*B*c^(7/2)-4*A*c^(7/2)*x^3+5*x^3*B*b*c^(5/2)-12*
A*c^(5/2)*b*x+15*b^2*B*x*c^(3/2)+12*A*(c*x^2+b)^(1/2)*ln(c^(1/2)*x+(c*x^2+
b)^(1/2))*b*c^2-15*B*(c*x^2+b)^(1/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^2*c)/
(c*x^4+b*x^2)^(3/2)/c^(9/2)
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.97

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \left[-\frac{3\,(5\,Bb^3-4\,Ab^2c+(5\,Bb^2c-4\,Abc^2)x^2)\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{16\,(c^5x^2+bc^4)}\right.$$

$$-\frac{3\,(5\,Bb^3-4\,Ab^2c+(5\,Bb^2c-4\,Abc^2)x^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-(2\,Bc^3x^4-15\,Bb^2c+12\,Abc^2-(5}{8\,(c^5x^2+bc^4)}$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(3*(5*B*b^3 - 4*A*b^2*c + (5*B*b^2*c - 4*A*b*c^2)*x^2)*sqrt(c)*log(
-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*(2*B*c^3*x^4 - 15*B*b^2*
c + 12*A*b*c^2 - (5*B*b*c^2 - 4*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/(c^5*x^2
+ b*c^4), -1/8*(3*(5*B*b^3 - 4*A*b^2*c + (5*B*b^2*c - 4*A*b*c^2)*x^2)*sqrt
(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (2*B*c^3*x^4 - 15*
B*b^2*c + 12*A*b*c^2 - (5*B*b*c^2 - 4*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/(c^
5*x^2 + b*c^4)]
```

3.146.6 Sympy [F]

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^7(A+Bx^2)}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**7*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.146.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.27

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{1}{4}\left(\frac{2\,x^4}{\sqrt{cx^4+bx^2}c} + \frac{6\,bx^2}{\sqrt{cx^4+bx^2}c^2} - \frac{3\,b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}}\right)A + \frac{1}{16}\left(\frac{4\,x^6}{\sqrt{cx^4+bx^2}c} - \frac{10\,bx^4}{\sqrt{cx^4+bx^2}c^2} - \frac{30\,b^2x^2}{\sqrt{cx^4+bx^2}c^3} + \frac{15\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{7}{2}}}\right)B$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(2*x^4/(sqrt(c*x^4 + b*x^2)*c) + 6*b*x^2/(sqrt(c*x^4 + b*x^2)*c^2) - 3
*b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2))*A + 1/16*(4*x
^6/(sqrt(c*x^4 + b*x^2)*c) - 10*b*x^4/(sqrt(c*x^4 + b*x^2)*c^2) - 30*b^2*x
^2/(sqrt(c*x^4 + b*x^2)*c^3) + 15*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x
^2)*sqrt(c))/c^(7/2))*B
```

3.146.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\left(x^2\left(\frac{2\,Bx^2}{c\mathrm{sgn}(x)} - \frac{5\,Bbc^3\mathrm{sgn}(x)-4\,Ac^4\mathrm{sgn}(x)}{c^5}\right) - \frac{3\,(5\,Bb^2c^2\mathrm{sgn}(x)-4\,Abc^3\mathrm{sgn}(x))}{c^5}\right)x}{8\,\sqrt{cx^2+b}} + \frac{3\,(5\,Bb^2\log\left(|b|\right)-4\,Abc\log\left(|b|\right))\mathrm{sgn}(x)}{16\,c^{\frac{7}{2}}} - \frac{3\,(5\,Bb^2-4\,Abc)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{8\,c^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*(x^2*(2*B*x^2/(c*sgn(x)) - (5*B*b*c^3*sgn(x) - 4*A*c^4*sgn(x))/c^5) -
3*(5*B*b^2*c^2*sgn(x) - 4*A*b*c^3*sgn(x))/c^5)*x/sqrt(c*x^2 + b) + 3/16*(5
*B*b^2*log(abs(b)) - 4*A*b*c*log(abs(b)))*sgn(x)/c^(7/2) - 3/8*(5*B*b^2 -
4*A*b*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(7/2)*sgn(x))
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^7\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.147 $\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.147.1 Optimal result

Integrand size = 26, antiderivative size = 112

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^4}{bc\sqrt{bx^2+cx^4}} + \frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{2bc^2} - \frac{(3bB-2Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}$$

output
```
-1/2*(-2*A*c+3*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(5/2)-(-A*c
+B*b)*x^4/b/c/(c*x^4+b*x^2)^(1/2)+1/2*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/b
/c^2
```

3.147.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^3\left(\sqrt{c}x(b+cx^2)(3bB-2Ac+Bcx^2)+2(-3bB+2Ac)(b+cx^2)^{3/2}\text{arctanh}\left(\frac{\sqrt{}}{-\sqrt{b}+}\right.\right.}{2c^{5/2}\left(x^2(b+cx^2)\right)^{3/2}}$$

input
```
Integrate[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x^3*(Sqrt[c]*x*(b + c*x^2)*(3*b*B - 2*A*c + B*c*x^2) + 2*(-3*b*B + 2*A*c)
*(b + c*x^2)^(3/2)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] + Sqrt[b + c*x^2])]))/(2*
c^(5/2)*(x^2*(b + c*x^2))^(3/2))
```

3.147.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1940, 1211, 25, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx^2)}{(bx^2+cx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{x^4 (Bx^2+A)}{(cx^4+bx^2)^{3/2}}dx^2$$

$$\Big\downarrow \text{1211}$$

$$\frac{1}{2}\left(\frac{\int -\frac{-Bcx^2+bB-Ac}{\sqrt{cx^4+bx^2}}dx^2}{c^2}+\frac{2x^2(bB-Ac)}{c^2\sqrt{bx^2+cx^4}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{2x^2(bB-Ac)}{c^2\sqrt{bx^2+cx^4}}-\frac{\int \frac{-Bcx^2+bB-Ac}{\sqrt{cx^4+bx^2}}dx^2}{c^2}\right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{2x^2(bB-Ac)}{c^2\sqrt{bx^2+cx^4}}-\frac{\frac{1}{2}(3bB-2Ac)\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2-B\sqrt{bx^2+cx^4}}{c^2}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{2x^2(bB-Ac)}{c^2\sqrt{bx^2+cx^4}}-\frac{(3bB-2Ac)\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}-B\sqrt{bx^2+cx^4}}{c^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{2x^2(bB-Ac)}{c^2\sqrt{bx^2+cx^4}}-\frac{\frac{(3bB-2Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}-B\sqrt{bx^2+cx^4}}{c^2}\right)$$

input
```
Int[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
((2*(b*B - A*c)*x^2)/(c^2*Sqrt[b*x^2 + c*x^4]) - (-(B*Sqrt[b*x^2 + c*x^4])
 + ((3*b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/Sqrt[c])/c
^2)/2
```

3.147.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.147. $\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.147.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02

method	result
default	$\frac{x^{3}\left(c\,x^{2}+b\right)\left(x^{3}B\,c^{\frac{5}{2}}-2A\,c^{\frac{5}{2}}x+3bBx\,c^{\frac{3}{2}}+2A\sqrt{c\,x^{2}+b}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)c^{2}-3B\sqrt{c\,x^{2}+b}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)bc\right)}{2\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}c^{\frac{7}{2}}}$
risch	$\frac{B\,x^{2}\left(c\,x^{2}+b\right)}{2c^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{\left(-\frac{Bbx}{\sqrt{c\,x^{2}+b}}+\left(2A\,c^{2}-3Bbc\right)\left(-\frac{x}{c\sqrt{c\,x^{2}+b}}+\frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)}{c^{\frac{3}{2}}}\right)\right)x\sqrt{c\,x^{2}+b}}{2c^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$
pseudoelliptic	$\frac{2B\,c^{\frac{3}{2}}x^{4}-4A\,c^{\frac{3}{2}}x^{2}+6Bb\,x^{2}\sqrt{c}-2A\ln(2)c\sqrt{x^{2}\left(c\,x^{2}+b\right)}+2A\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)c\sqrt{x^{2}\left(c\,x^{2}+b\right)}+3B\ln(2)b\sqrt{x^{2}}}{4c^{\frac{5}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$

input

```
int(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/2*x^3*(c*x^2+b)*(x^3*B*c^(5/2)-2*A*c^(5/2)*x+3*b*B*x*c^(3/2)+2*A*(c*x^2+
b)^(1/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*c^2-3*B*(c*x^2+b)^(1/2)*ln(c^(1/2)*
x+(c*x^2+b)^(1/2))*b*c)/(c*x^4+b*x^2)^(3/2)/c^(7/2)
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.05

$$\int \frac{x^5(A+Bx^2)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \left[-\frac{(3\,Bb^2-2\,Abc+(3\,Bbc-2\,Ac^2)x^2)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,(}{4\,(c^4x^2+bc^3)}\right.$$

input

```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output

```
[-1/4*((3*B*b^2 - 2*A*b*c + (3*B*b*c - 2*A*c^2)*x^2)*sqrt(c)*log(-2*c*x^2
- b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*(B*c^2*x^2 + 3*B*b*c - 2*A*c^2)*s
qrt(c*x^4 + b*x^2))/(c^4*x^2 + b*c^3), 1/2*((3*B*b^2 - 2*A*b*c + (3*B*b*c
- 2*A*c^2)*x^2)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b))
+ (B*c^2*x^2 + 3*B*b*c - 2*A*c^2)*sqrt(c*x^4 + b*x^2))/(c^4*x^2 + b*c^3)]
```

3.147.6 Sympy [F]

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\int \frac{x^5(A+Bx^2)}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**5*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.147.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.23

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\frac{1}{4}\left(\frac{2\,x^4}{\sqrt{cx^4+bx^2}c}+\frac{6\,bx^2}{\sqrt{cx^4+bx^2}c^2}-\frac{3\,b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}}\right)B-\frac{1}{2}\,A\left(\frac{2\,x^2}{\sqrt{cx^4+bx^2}c}-\frac{\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{3}{2}}}\right)$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(2*x^4/(sqrt(c*x^4 + b*x^2)*c) + 6*b*x^2/(sqrt(c*x^4 + b*x^2)*c^2) - 3
*b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2))*B - 1/2*A*(2*
x^2/(sqrt(c*x^4 + b*x^2)*c) - log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt
(c))/c^(3/2))
```

3.147.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.93

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\frac{x\left(\frac{Bx^2}{c\mathrm{sgn}(x)}+\frac{3\,Bbc\mathrm{sgn}(x)-2\,Ac^2\mathrm{sgn}(x)}{c^3}\right)}{2\,\sqrt{cx^2+b}}-\frac{(3\,Bb\log\left(|b|\right)-2\,Ac\log\left(|b|\right))\mathrm{sgn}(x)}{4\,c^{\frac{5}{2}}}+\frac{(3\,Bb-2\,Ac)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{2\,c^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*x*(B*x^2/(c*sgn(x)) + (3*B*b*c*sgn(x) - 2*A*c^2*sgn(x))/c^3)/sqrt(c*x^
2 + b) - 1/4*(3*B*b*log(abs(b)) - 2*A*c*log(abs(b)))*sgn(x)/c^(5/2) + 1/2*
(3*B*b - 2*A*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(5/2)*sgn(x))
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^5\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.148 $\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$

3.148.1 Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx = -\frac{(bB-Ac)x^2}{bc\sqrt{bx^2+cx^4}} + \frac{B\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}$$

output
```
B*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(3/2)-(-A*c+B*b)*x^2/b/c/(c*x
^4+b*x^2)^(1/2)
```

3.148.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx = -\frac{x\left(\sqrt{c}(bB-Ac)x + bB\sqrt{b+cx^2}\log\left(-\sqrt{c}x+\sqrt{b+cx^2}\right)\right)}{bc^{3/2}\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-((x*(Sqrt[c]*(b*B - A*c)*x + b*B*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[
b + c*x^2]]))/(b*c^(3/2)*Sqrt[x^2*(b + c*x^2)]))
```

3.148.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1940, 1211, 27, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (A+Bx^2)}{(bx^2+cx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2} \int \frac{x^2 (Bx^2+A)}{(cx^4+bx^2)^{3/2}} dx^2$$

$$\Big\downarrow \text{1211}$$

$$\frac{1}{2} \left(\frac{\int \frac{B}{\sqrt{cx^4+bx^2}} dx^2}{c} - \frac{2x^2(bB-Ac)}{bc\sqrt{bx^2+cx^4}} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(\frac{B \int \frac{1}{\sqrt{cx^4+bx^2}} dx^2}{c} - \frac{2x^2(bB-Ac)}{bc\sqrt{bx^2+cx^4}} \right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2} \left(\frac{2B \int \frac{1}{1-cx^4} d\frac{x^2}{\sqrt{cx^4+bx^2}}}{c} - \frac{2x^2(bB-Ac)}{bc\sqrt{bx^2+cx^4}} \right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2} \left(\frac{2B\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}} - \frac{2x^2(bB-Ac)}{bc\sqrt{bx^2+cx^4}} \right)$$

input
```
Int[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
((-2*(b*B - A*c)*x^2)/(b*c*Sqrt[b*x^2 + c*x^4]) + (2*B*ArcTanh[(Sqrt[c]*x^
2)/Sqrt[b*x^2 + c*x^4]])/c^(3/2))/2
```

3.148.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.148. $\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.148.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{x^{3} \left(c \,x^{2}+b \right) \left(A \,c^{\frac{5}{2}} x -b B x \,c^{\frac{3}{2}}+B \sqrt{c \,x^{2}+b}\, \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+b}\right) b c \right)}{\left(x^{4} c +b \,x^{2}\right)^{\frac{3}{2}} b \,c^{\frac{5}{2}}}$	75
pseudoelliptic	$\frac{2 A \,c^{\frac{3}{2}} x^{2}-2 B b \,x^{2} \sqrt{c}+B \ln \left(\frac{2 c \,x^{2}+2 \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, \sqrt{c}+b}{\sqrt{c}}\right) b \sqrt{x^{2} \left(c \,x^{2}+b \right)}-B \ln \left(2\right) b \sqrt{x^{2} \left(c \,x^{2}+b \right)}}{2 c^{\frac{3}{2}} \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, b}$	108

input
```
int(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x^3*(c*x^2+b)*(A*c^(5/2)*x-b*B*x*c^(3/2)+B*(c*x^2+b)^(1/2)*ln(c^(1/2)*x+(c
*x^2+b)^(1/2))*b*c)/(c*x^4+b*x^2)^(3/2)/b/c^(5/2)
```

3.148.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.81

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \left[\frac{(Bbcx^2+Bb^2)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}(Bbc-Ac^2)}{2\,(bc^3x^2+b^2c^2)}\right.$$

$$\left.-\frac{(Bbcx^2+Bb^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}(Bbc-Ac^2)}{bc^3x^2+b^2c^2}\right]$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((B*b*c*x^2 + B*b^2)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)
*sqrt(c)) - 2*sqrt(c*x^4 + b*x^2)*(B*b*c - A*c^2))/(b*c^3*x^2 + b^2*c^2),
-((B*b*c*x^2 + B*b^2)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2
+ b)) + sqrt(c*x^4 + b*x^2)*(B*b*c - A*c^2))/(b*c^3*x^2 + b^2*c^2)]
```

3.148.6 Sympy [F]

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\int \frac{x^3(A+Bx^2)}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**3*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.148.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.18

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{1}{2}\,B\left(\frac{2\,x^2}{\sqrt{cx^4+bx^2}c}-\frac{\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{3}{2}}}\right)+\frac{Ax^2}{\sqrt{cx^4+bx^2}b}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*B*(2*x^2/(sqrt(c*x^4 + b*x^2)*c) - log(2*c*x^2 + b + 2*sqrt(c*x^4 + b
*x^2)*sqrt(c))/c^(3/2)) + A*x^2/(sqrt(c*x^4 + b*x^2)*b)
```

3.148.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\frac{B\log(|b|)\,\mathrm{sgn}(x)}{2\,c^{\frac{3}{2}}}-\frac{(Bb\mathrm{sgn}(x)-Ac\mathrm{sgn}(x))x}{\sqrt{cx^2+b}bc}-\frac{B\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{c^{\frac{3}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*B*log(abs(b))*sgn(x)/c^(3/2) - (B*b*sgn(x) - A*c*sgn(x))*x/(sqrt(c*x^2
 + b)*b*c) - B*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(3/2)*sgn(x))
```

3.148.9 Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{B\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\,c^{3/2}}+\frac{A\,x^2}{b\,\sqrt{c\,x^4+b\,x^2}}-\frac{B\,x^2}{c\,\sqrt{c\,x^4+b\,x^2}}$$

input
```
int((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
(B*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(2*c^(3/2)) + (A*x^
2)/(b*(b*x^2 + c*x^4)^(1/2)) - (B*x^2)/(c*(b*x^2 + c*x^4)^(1/2))
```

3.149 $\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.149.1 Optimal result

Integrand size = 24, antiderivative size = 37

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{Ab-(bB-2Ac)x^2}{b^2\sqrt{bx^2+cx^4}}$$

output
```
(-A*b+(-2*A*c+B*b)*x^2)/b^2/(c*x^4+b*x^2)^(1/2)
```

3.149.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{bBx^2-A(b+2cx^2)}{b^2\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(x*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(b*B*x^2 - A*(b + 2*c*x^2))/(b^2*Sqrt[x^2*(b + c*x^2)])
```

3.149.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1940, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1940}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{(cx^4+bx^2)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 1158}$$

$$-\frac{Ab-x^2(bB-2Ac)}{b^2\sqrt{bx^2+cx^4}}$$

input
```
Int[(x*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-((A*b - (b*B - 2*A*c)*x^2)/(b^2*Sqrt[b*x^2 + c*x^4]))
```

3.149.3.1 Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.149.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

method	result	size
pseudoelliptic	$\frac{(-2c\,x^2-b)A+bB\,x^2}{\sqrt{x^2(c\,x^2+b)}\,b^2}$	37
gosper	$-\frac{x^2(c\,x^2+b)(2Ac\,x^2-bB\,x^2+Ab)}{b^2(x^4c+b\,x^2)^{\frac{3}{2}}}$	47
default	$-\frac{x^2(c\,x^2+b)(2Ac\,x^2-bB\,x^2+Ab)}{b^2(x^4c+b\,x^2)^{\frac{3}{2}}}$	47
trager	$-\frac{(2Ac\,x^2-bB\,x^2+Ab)\sqrt{x^4c+b\,x^2}}{(c\,x^2+b)b^2x^2}$	49
risch	$-\frac{A(c\,x^2+b)}{b^2\sqrt{x^2(c\,x^2+b)}}-\frac{x^2(Ac-Bb)}{b^2\sqrt{x^2(c\,x^2+b)}}$	57

input
```
int(x*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-2*c*x^2-b)*A+b*B*x^2)/(x^2*(c*x^2+b))^(1/2)/b^2
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\frac{\sqrt{cx^4+bx^2}((Bb-2\,Ac)x^2-Ab)}{b^2cx^4+b^3x^2}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(c*x^4 + b*x^2)*((B*b - 2*A*c)*x^2 - A*b)/(b^2*c*x^4 + b^3*x^2)
```

3.149.6 Sympy [F]

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\int \frac{x(A+Bx^2)}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.76

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -A\left(\frac{2\,cx^2}{\sqrt{cx^4+bx^2}b^2}+\frac{1}{\sqrt{cx^4+bx^2}b}\right)+\frac{Bx^2}{\sqrt{cx^4+bx^2}b}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-A*(2*c*x^2/(sqrt(c*x^4 + b*x^2)*b^2) + 1/(sqrt(c*x^4 + b*x^2)*b)) + B*x^2
/(sqrt(c*x^4 + b*x^2)*b)
```

3.149.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.76

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{2\,A\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)b\text{sgn}\,(x)}+\frac{(Bb-Ac)x}{\sqrt{cx^2+b}b^2\text{sgn}\,(x)}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
2*A*sqrt(c)/(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)*b*sgn(x)) + (B*b - A*c)
*x/(sqrt(c*x^2 + b)*b^2*sgn(x))
```

3.149.9 Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{\left(\frac{A}{b}-x^2\left(\frac{B}{b}-\frac{2\,A\,c}{b^2}\right)\right)\sqrt{c\,x^4+b\,x^2}}{x\,\left(c\,x^3+b\,x\right)}$$

input
```
int((x*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
-((A/b - x^2*(B/b - (2*A*c)/b^2))*(b*x^2 + c*x^4)^(1/2))/(x*(b*x + c*x^3))
```

3.150 $\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^{3/2}}\,dx$

3.150.1 Optimal result

Integrand size = 26, antiderivative size = 66

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{A}{3bx^2\sqrt{bx^2+cx^4}}-\frac{(3bB-4Ac)\left(b+2cx^2\right)}{3b^3\sqrt{bx^2+cx^4}}$$

output
```
-1/3*A/b/x^2/(c*x^4+b*x^2)^(1/2)-1/3*(-4*A*c+3*B*b)*(2*c*x^2+b)/b^3/(c*x^4
+b*x^2)^(1/2)
```

3.150.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^{3/2}}\,dx=\frac{-3bBx^2(b+2cx^2)+A(-b^2+4bcx^2+8c^2x^4)}{3b^3x^2\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[(A + B*x^2)/(x*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-3*b*B*x^2*(b + 2*c*x^2) + A*(-b^2 + 4*b*c*x^2 + 8*c^2*x^4))/(3*b^3*x^2*S
qrt[x^2*(b + c*x^2)])
```

3.150.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1940, 1220, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x(bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^2(cx^4+bx^2)^{3/2}}dx^2$$

$$\Big\downarrow \text{1220}$$

$$\frac{1}{2}\left(\frac{(3bB-4Ac)\int \frac{1}{(cx^4+bx^2)^{3/2}}dx^2}{3b}-\frac{2A}{3bx^2\sqrt{bx^2+cx^4}}\right)$$

$$\Big\downarrow \text{1088}$$

$$\frac{1}{2}\left(-\frac{2(b+2cx^2)(3bB-4Ac)}{3b^3\sqrt{bx^2+cx^4}}-\frac{2A}{3bx^2\sqrt{bx^2+cx^4}}\right)$$

input
```
Int[(A + B*x^2)/(x*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
((-2*A)/(3*b*x^2*Sqrt[b*x^2 + c*x^4]) - (2*(3*b*B - 4*A*c)*(b + 2*c*x^2))/
(3*b^3*Sqrt[b*x^2 + c*x^4]))/2
```

3.150.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.150.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

method	result	size
pseudoelliptic	$-\frac{\left(3 x^{2} B+A \right) b^{2}-4 \left(-\frac{3 x^{2} B}{2}+A \right) x^{2} c b -8 A \,c^{2} x^{4}}{3 \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, b^{3} x^{2}}$	59
gosper	$-\frac{\left(c \,x^{2}+b \right) \left(-8 A \,c^{2} x^{4}+6 x^{4} B b c -4 A b c \,x^{2}+3 b^{2} B \,x^{2}+b^{2} A \right)}{3 b^{3} \left(x^{4} c +b \,x^{2}\right)^{\frac{3}{2}}}$	66
default	$-\frac{\left(c \,x^{2}+b \right) \left(-8 A \,c^{2} x^{4}+6 x^{4} B b c -4 A b c \,x^{2}+3 b^{2} B \,x^{2}+b^{2} A \right)}{3 b^{3} \left(x^{4} c +b \,x^{2}\right)^{\frac{3}{2}}}$	66
trager	$-\frac{\left(-8 A \,c^{2} x^{4}+6 x^{4} B b c -4 A b c \,x^{2}+3 b^{2} B \,x^{2}+b^{2} A \right) \sqrt{x^{4} c +b \,x^{2}}}{3 \left(c \,x^{2}+b \right) b^{3} x^{4}}$	71
risch	$-\frac{\left(c \,x^{2}+b \right) \left(-5 A c \,x^{2}+3 b B \,x^{2}+A b \right)}{3 b^{3} x^{2} \sqrt{x^{2} \left(c \,x^{2}+b \right)}}+\frac{x^{2} \left(A c -B b \right) c}{b^{3} \sqrt{x^{2} \left(c \,x^{2}+b \right)}}$	77

input
```
int((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((3*B*x^2+A)*b^2-4*(-3/2*x^2*B+A)*x^2*c*b-8*A*c^2*x^4)/(x^2*(c*x^2+b)
)^(1/2)/b^3/x^2
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^2}{x(bx^2+cx^4)^{3/2}}\,dx = -\frac{\left(2\,(3\,Bbc-4\,Ac^2)x^4+Ab^2+(3\,Bb^2-4\,Abc)x^2\right)\sqrt{cx^4+bx^2}}{3\,(b^3cx^6+b^4x^4)}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(2*(3*B*b*c - 4*A*c^2)*x^4 + A*b^2 + (3*B*b^2 - 4*A*b*c)*x^2)*sqrt(c*
x^4 + b*x^2)/(b^3*c*x^6 + b^4*x^4)
```

3.150.6 Sympy [F]

$$\int \frac{A+Bx^2}{x(bx^2+cx^4)^{3/2}}\,dx = \int \frac{A+Bx^2}{x\,(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)/x/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral((A + B*x**2)/(x*(x**2*(b + c*x**2))**(3/2)), x)
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.70

$$\int \frac{A+Bx^2}{x(bx^2+cx^4)^{3/2}}\,dx = -B\left(\frac{2\,cx^2}{\sqrt{cx^4+bx^2}b^2}+\frac{1}{\sqrt{cx^4+bx^2}b}\right) + \frac{1}{3}\,A\left(\frac{8\,c^2x^2}{\sqrt{cx^4+bx^2}b^3}+\frac{4\,c}{\sqrt{cx^4+bx^2}b^2}-\frac{1}{\sqrt{cx^4+bx^2}bx^2}\right)$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-B*(2*c*x^2/(sqrt(c*x^4 + b*x^2)*b^2) + 1/(sqrt(c*x^4 + b*x^2)*b)) + 1/3*A
*(8*c^2*x^2/(sqrt(c*x^4 + b*x^2)*b^3) + 4*c/(sqrt(c*x^4 + b*x^2)*b^2) - 1/
(sqrt(c*x^4 + b*x^2)*b*x^2))
```

3.150.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(58) = 116.

Time = 0.50 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.86

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{(Bbc-Ac^2)x}{\sqrt{cx^2+b}b^3\text{sgn}(x)}+\frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Bb\sqrt{c}-3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Ac^{\frac{3}{2}}-6\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2Bb^2\sqrt{c}+12\left(\sqrt{c}x-\sqrt{\right.}\right.}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^3b^2\text{sgn}(x)}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
-(B*b*c - A*c^2)*x/(sqrt(c*x^2 + b)*b^3*sgn(x)) + 2/3*(3*(sqrt(c)*x - sqrt
(c*x^2 + b))^4*B*b*sqrt(c) - 3*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*c^(3/2) -
 6*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^2*sqrt(c) + 12*(sqrt(c)*x - sqrt(c*
x^2 + b))^2*A*b*c^(3/2) + 3*B*b^3*sqrt(c) - 5*A*b^2*c^(3/2))/(((sqrt(c)*x
- sqrt(c*x^2 + b))^2 - b)^3*b^2*sgn(x))
```

3.150.9 Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(3\,B\,b^2\,x^2+A\,b^2+6\,B\,b\,c\,x^4-4\,A\,b\,c\,x^2-8\,A\,c^2\,x^4\right)}{3\,b^3\,x^4\,\left(c\,x^2+b\right)}$$

input
```
int((A + B*x^2)/(x*(b*x^2 + c*x^4)^(3/2)),x)
```

output
```
-((b*x^2 + c*x^4)^(1/2)*(A*b^2 + 3*B*b^2*x^2 - 8*A*c^2*x^4 - 4*A*b*c*x^2 +
 6*B*b*c*x^4))/(3*b^3*x^4*(b + c*x^2))
```

3.151 $\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)^{3/2}}\,dx$

3.151.1 Optimal result

Integrand size = 26, antiderivative size = 101

$$\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{A}{5bx^4\sqrt{bx^2+cx^4}}-\frac{5bB-6Ac}{15b^2x^2\sqrt{bx^2+cx^4}}+\frac{4c(5bB-6Ac)\left(b+2cx^2\right)}{15b^4\sqrt{bx^2+cx^4}}$$

output
```
-1/5*A/b/x^4/(c*x^4+b*x^2)^(1/2)+1/15*(6*A*c-5*B*b)/b^2/x^2/(c*x^4+b*x^2)^
(1/2)+4/15*c*(-6*A*c+5*B*b)*(2*c*x^2+b)/b^4/(c*x^4+b*x^2)^(1/2)
```

3.151.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)^{3/2}}\,dx=\frac{-5bBx^2(b^2-4bcx^2-8c^2x^4)-3A(b^3-2b^2cx^2+8bc^2x^4+16c^3x^6)}{15b^4x^4\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[(A + B*x^2)/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-5*b*B*x^2*(b^2 - 4*b*c*x^2 - 8*c^2*x^4) - 3*A*(b^3 - 2*b^2*c*x^2 + 8*b*c
^2*x^4 + 16*c^3*x^6))/(15*b^4*x^4*Sqrt[x^2*(b + c*x^2)])
```

3.151.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1940, 1220, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^3 \left(bx^2+cx^4\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^4 \left(cx^4+bx^2\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{1220}$$

$$\frac{1}{2}\left(\frac{(5bB-6Ac)\int \frac{1}{x^2 \left(cx^4+bx^2\right)^{3/2}}dx^2}{5b}-\frac{2A}{5bx^4\sqrt{bx^2+cx^4}}\right)$$

$$\Big\downarrow \text{1129}$$

$$\frac{1}{2}\left(\frac{(5bB-6Ac)\left(-\frac{4c\int \frac{1}{\left(cx^4+bx^2\right)^{3/2}}dx^2}{3b}-\frac{2}{3bx^2\sqrt{bx^2+cx^4}}\right)}{5b}-\frac{2A}{5bx^4\sqrt{bx^2+cx^4}}\right)$$

$$\Big\downarrow \text{1088}$$

$$\frac{1}{2}\left(\frac{\left(\frac{8c\left(b+2cx^2\right)}{3b^3\sqrt{bx^2+cx^4}}-\frac{2}{3bx^2\sqrt{bx^2+cx^4}}\right)(5bB-6Ac)}{5b}-\frac{2A}{5bx^4\sqrt{bx^2+cx^4}}\right)$$

input
```
Int[(A + B*x^2)/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
((-2*A)/(5*b*x^4*Sqrt[b*x^2 + c*x^4]) + ((5*b*B - 6*A*c)*(-2/(3*b*x^2*Sqrt
[b*x^2 + c*x^4]) + (8*c*(b + 2*c*x^2))/(3*b^3*Sqrt[b*x^2 + c*x^4])))/(5*b)
)/2
```

3.151.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.151.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{(-5x^2B-3A)b^3+6x^2\left(\frac{10x^2B}{3}+A\right)c\,b^2-24x^4c^2\left(-\frac{5x^2B}{3}+A\right)b-48A\,c^3x^6}{15\sqrt{x^2(c\,x^2+b)}\,x^4b^4}$	80
gosper	$-\frac{(c\,x^2+b)(48A\,c^3x^6-40x^6Bb\,c^2+24Ab\,c^2x^4-20x^4B\,b^2c-6A\,b^2c\,x^2+5b^3B\,x^2+3b^3A)}{15x^2b^4(x^4c+b\,x^2)^{\frac{3}{2}}}$	94
default	$-\frac{(c\,x^2+b)(48A\,c^3x^6-40x^6Bb\,c^2+24Ab\,c^2x^4-20x^4B\,b^2c-6A\,b^2c\,x^2+5b^3B\,x^2+3b^3A)}{15x^2b^4(x^4c+b\,x^2)^{\frac{3}{2}}}$	94
trager	$-\frac{(48A\,c^3x^6-40x^6Bb\,c^2+24Ab\,c^2x^4-20x^4B\,b^2c-6A\,b^2c\,x^2+5b^3B\,x^2+3b^3A)\sqrt{x^4c+b\,x^2}}{15(c\,x^2+b)b^4x^6}$	96
risch	$-\frac{(c\,x^2+b)(33A\,c^2x^4-25x^4Bbc-9Abc\,x^2+5b^2B\,x^2+3b^2A)}{15b^4x^4\sqrt{x^2(c\,x^2+b)}}-\frac{x^2c^2(Ac-Bb)}{b^4\sqrt{x^2(c\,x^2+b)}}$	103

input
```
int((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*((-5*B*x^2-3*A)*b^3+6*x^2*(10/3*x^2*B+A)*c*b^2-24*x^4*c^2*(-5/3*x^2*B
+A)*b-48*A*c^3*x^6)/(x^2*(c*x^2+b))^(1/2)/x^4/b^4
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2}{x^3(bx^2+cx^4)^{3/2}}\,dx = \frac{(8(5\,Bbc^2-6\,Ac^3)x^6+4(5\,Bb^2c-6\,Abc^2)x^4-3\,Ab^3-(5\,Bb^3-6\,Ab^2c)x^2)\sqrt{cx^4}}{15(b^4cx^8+b^5x^6)}$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(8*(5*B*b*c^2 - 6*A*c^3)*x^6 + 4*(5*B*b^2*c - 6*A*b*c^2)*x^4 - 3*A*b^
3 - (5*B*b^3 - 6*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^4*c*x^8 + b^5*x^6)
```

3.151.6 Sympy [F]

$$\int \frac{A+Bx^2}{x^3(bx^2+cx^4)^{3/2}}\,dx = \int \frac{A+Bx^2}{x^3\left(x^2(b+cx^2)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)/x**3/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral((A + B*x**2)/(x**3*(x**2*(b + c*x**2))**(3/2)), x)
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.58

$$\int \frac{A+Bx^2}{x^3(bx^2+cx^4)^{3/2}}\,dx = \frac{1}{3}B\left(\frac{8\,c^2x^2}{\sqrt{cx^4+bx^2}b^3}+\frac{4\,c}{\sqrt{cx^4+bx^2}b^2}-\frac{1}{\sqrt{cx^4+bx^2}bx^2}\right) - \frac{1}{5}A\left(\frac{16\,c^3x^2}{\sqrt{cx^4+bx^2}b^4}+\frac{8\,c^2}{\sqrt{cx^4+bx^2}b^3}-\frac{2\,c}{\sqrt{cx^4+bx^2}b^2x^2}+\frac{1}{\sqrt{cx^4+bx^2}bx^4}\right)$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/3*B*(8*c^2*x^2/(sqrt(c*x^4 + b*x^2)*b^3) + 4*c/(sqrt(c*x^4 + b*x^2)*b^2)
 - 1/(sqrt(c*x^4 + b*x^2)*b*x^2)) - 1/5*A*(16*c^3*x^2/(sqrt(c*x^4 + b*x^2)
*b^4) + 8*c^2/(sqrt(c*x^4 + b*x^2)*b^3) - 2*c/(sqrt(c*x^4 + b*x^2)*b^2*x^2
) + 1/(sqrt(c*x^4 + b*x^2)*b*x^4))
```

3.151.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(89) = 178.

Time = 0.92 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.99

$$\int \frac{A+Bx^2}{x^3(bx^2+cx^4)^{3/2}}\,dx = \frac{(Bbc^2-Ac^3)x}{\sqrt{cx^2+b}b^4\mathrm{sgn}(x)} - \frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8Bbc^{\frac{3}{2}}-15\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8Ac^{\frac{5}{2}}-90\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6Bb^2c^{\frac{3}{2}}+90\left(\sqrt{c}x-\right.}{}$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
(B*b*c^2 - A*c^3)*x/(sqrt(c*x^2 + b)*b^4*sgn(x)) - 2/15*(15*(sqrt(c)*x - s
qrt(c*x^2 + b))^8*B*b*c^(3/2) - 15*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*c^(5/
2) - 90*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^2*c^(3/2) + 90*(sqrt(c)*x - sq
rt(c*x^2 + b))^6*A*b*c^(5/2) + 160*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^3*c
^(3/2) - 240*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^2*c^(5/2) - 110*(sqrt(c)*
x - sqrt(c*x^2 + b))^2*B*b^4*c^(3/2) + 150*(sqrt(c)*x - sqrt(c*x^2 + b))^2
*A*b^3*c^(5/2) + 25*B*b^5*c^(3/2) - 33*A*b^4*c^(5/2))/(((sqrt(c)*x - sqrt(
c*x^2 + b))^2 - b)^5*b^3*sgn(x))
```

3.151.9 Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{\sqrt{c\,x^4+b\,x^2}\,\left(5\,B\,b^3\,x^2+3\,A\,b^3-20\,B\,b^2\,c\,x^4-6\,A\,b^2\,c\,x^2-40\,B\,b\,c^2\,x^6+24\,A\,b\,c^2\,x^4+48\,A\,c^3\,x^6\right)}{15\,b^4\,x^6\,\left(c\,x^2+b\right)}$$

input
```
int((A + B*x^2)/(x^3*(b*x^2 + c*x^4)^(3/2)),x)
```

output
```
-((b*x^2 + c*x^4)^(1/2)*(3*A*b^3 + 5*B*b^3*x^2 + 48*A*c^3*x^6 - 6*A*b^2*c*
x^2 + 24*A*b*c^2*x^4 - 20*B*b^2*c*x^4 - 40*B*b*c^2*x^6))/(15*b^4*x^6*(b +
c*x^2))
```

3.152 $\int \frac{A+Bx^2}{x^5(bx^2+cx^4)^{3/2}}\,dx$

3.152.1 Optimal result

Integrand size = 26, antiderivative size = 138

$$\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{A}{7bx^6\sqrt{bx^2+cx^4}}-\frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}}+\frac{2c(7bB-8Ac)}{35b^3x^2\sqrt{bx^2+cx^4}}-\frac{8c^2(7bB-8Ac)\left(b+2cx^2\right)}{35b^5\sqrt{bx^2+cx^4}}$$

output
```
-1/7*A/b/x^6/(c*x^4+b*x^2)^(1/2)+1/35*(8*A*c-7*B*b)/b^2/x^4/(c*x^4+b*x^2)^
(1/2)+2/35*c*(-8*A*c+7*B*b)/b^3/x^2/(c*x^4+b*x^2)^(1/2)-8/35*c^2*(-8*A*c+7
*B*b)*(2*c*x^2+b)/b^5/(c*x^4+b*x^2)^(1/2)
```

3.152.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx=\frac{-7bBx^2(b^3-2b^2cx^2+8bc^2x^4+16c^3x^6)+A(-5b^4+8b^3cx^2-16b^2c^2x^4+64bc^3x^{\cdots}}{35b^5x^6\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[(A + B*x^2)/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-7*b*B*x^2*(b^3 - 2*b^2*c*x^2 + 8*b*c^2*x^4 + 16*c^3*x^6) + A*(-5*b^4 + 8
*b^3*c*x^2 - 16*b^2*c^2*x^4 + 64*b*c^3*x^6 + 128*c^4*x^8))/(35*b^5*x^6*Sqr
t[x^2*(b + c*x^2)])
```

3.152.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1940, 1220, 1129, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1940}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^6\left(cx^4+bx^2\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{1}{2}\left(\frac{(7bB-8Ac)\int \frac{1}{x^4(cx^4+bx^2)^{3/2}}dx^2}{7b}-\frac{2A}{7bx^6\sqrt{bx^2+cx^4}}\right)$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{1}{2}\left(\frac{(7bB-8Ac)\left(-\frac{6c\int \frac{1}{x^2\left(cx^4+bx^2\right)^{3/2}}dx^2}{5b}-\frac{2}{5bx^4\sqrt{bx^2+cx^4}}\right)}{7b}-\frac{2A}{7bx^6\sqrt{bx^2+cx^4}}\right)$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{1}{2}\left(\frac{(7bB-8Ac)\left(-\frac{6c\left(-\frac{4c\int \frac{1}{\left(cx^4+bx^2\right)^{3/2}}dx^2}{3b}-\frac{2}{3bx^2\sqrt{bx^2+cx^4}}\right)}{5b}-\frac{2}{5bx^4\sqrt{bx^2+cx^4}}\right)}{7b}-\frac{2A}{7bx^6\sqrt{bx^2+cx^4}}\right)$$

$$\Big\downarrow \text{ 1088}$$

$$\frac{1}{2}\left(\frac{\left(-\frac{6c\left(\frac{8c(b+2cx^2)}{3b^3\sqrt{bx^2+cx^4}}-\frac{2}{3bx^2\sqrt{bx^2+cx^4}}\right)}{5b}-\frac{2}{5bx^4\sqrt{bx^2+cx^4}}\right)(7bB-8Ac)}{7b}-\frac{2A}{7bx^6\sqrt{bx^2+cx^4}}\right)$$

input
```
Int[(A + B*x^2)/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
((-2*A)/(7*b*x^6*Sqrt[b*x^2 + c*x^4]) + ((7*b*B - 8*A*c)*(-2/(5*b*x^4*Sqrt
[b*x^2 + c*x^4]) - (6*c*(-2/(3*b*x^2*Sqrt[b*x^2 + c*x^4]) + (8*c*(b + 2*c*
x^2))/(3*b^3*Sqrt[b*x^2 + c*x^4])))/(5*b)))/(7*b))/2
```

3.152.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.152.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$-\frac{\left(\frac{7x^2B}{5}+A\right)b^4-\frac{8x^2\left(\frac{7x^2B}{4}+A\right)c\,b^3}{5}+\frac{16x^4\left(\frac{7x^2B}{2}+A\right)c^2b^2}{5}-\frac{64\left(-\frac{7x^2B}{4}+A\right)x^6c^3b}{5}-\frac{128A\,x^8c^4}{5}}{7\sqrt{x^2(c\,x^2+b)}\,x^6b^5}$
gosper	$-\frac{(c\,x^2+b)\,(-128A\,x^8c^4+112B\,x^8b\,c^3-64A\,x^6b\,c^3+56B\,x^6b^2c^2+16A\,b^2c^2x^4-14B\,b^3c\,x^4-8A\,x^2b^3c+7B\,x^2b^4+5A\,b^4)}{35x^4b^5(x^4c+b\,x^2)^{\frac{3}{2}}}$
default	$-\frac{(c\,x^2+b)\,(-128A\,x^8c^4+112B\,x^8b\,c^3-64A\,x^6b\,c^3+56B\,x^6b^2c^2+16A\,b^2c^2x^4-14B\,b^3c\,x^4-8A\,x^2b^3c+7B\,x^2b^4+5A\,b^4)}{35x^4b^5(x^4c+b\,x^2)^{\frac{3}{2}}}$
trager	$-\frac{(-128A\,x^8c^4+112B\,x^8b\,c^3-64A\,x^6b\,c^3+56B\,x^6b^2c^2+16A\,b^2c^2x^4-14B\,b^3c\,x^4-8A\,x^2b^3c+7B\,x^2b^4+5A\,b^4)\sqrt{x^4c+b\,x^2}}{35(c\,x^2+b)b^5x^8}$
risch	$-\frac{(c\,x^2+b)\,(-93A\,c^3x^6+77x^6Bb\,c^2+29Ab\,c^2x^4-21x^4B\,b^2c-13A\,b^2c\,x^2+7b^3B\,x^2+5b^3A)}{35b^5x^6\sqrt{x^2(c\,x^2+b)}}+\frac{x^2c^3(Ac-Bb)}{b^5\sqrt{x^2(c\,x^2+b)}}$

input
```
int((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7/(x^2*(c*x^2+b))^(1/2)*((7/5*x^2*B+A)*b^4-8/5*x^2*(7/4*x^2*B+A)*c*b^3+
16/5*x^4*(7/2*x^2*B+A)*c^2*b^2-64/5*(-7/4*x^2*B+A)*x^6*c^3*b-128/5*A*x^8*c
^4)/x^6/b^5
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx =$$
$$-\frac{(16\,(7\,Bbc^3-8\,Ac^4)x^8+8\,(7\,Bb^2c^2-8\,Abc^3)x^6+5\,Ab^4-2\,(7\,Bb^3c-8\,Ab^2c^2)x^4+(7\,Bb^4-8\,Ab^3c)x^2)\sqrt{}}{35\,(b^5cx^{10}+b^6x^8)}$$

input
```
integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/35*(16*(7*B*b*c^3 - 8*A*c^4)*x^8 + 8*(7*B*b^2*c^2 - 8*A*b*c^3)*x^6 + 5*
A*b^4 - 2*(7*B*b^3*c - 8*A*b^2*c^2)*x^4 + (7*B*b^4 - 8*A*b^3*c)*x^2)*sqrt(
c*x^4 + b*x^2)/(b^5*c*x^10 + b^6*x^8)
```

3.152.6 Sympy [F]

$$\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{A+Bx^2}{x^5\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)/x**5/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral((A + B*x**2)/(x**5*(x**2*(b + c*x**2))**(3/2)), x)
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.51

$$\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{1}{5}B\left(\frac{16\,c^3x^2}{\sqrt{cx^4+bx^2}b^4}+\frac{8\,c^2}{\sqrt{cx^4+bx^2}b^3}-\frac{2\,c}{\sqrt{cx^4+bx^2}b^2x^2}+\frac{1}{\sqrt{cx^4+bx^2}bx^4}\right) + \frac{1}{35}A\left(\frac{128\,c^4x^2}{\sqrt{cx^4+bx^2}b^5}+\frac{64\,c^3}{\sqrt{cx^4+bx^2}b^4}-\frac{16\,c^2}{\sqrt{cx^4+bx^2}b^3x^2}+\frac{8\,c}{\sqrt{cx^4+bx^2}b^2x^4}-\frac{5}{\sqrt{cx^4+bx^2}bx^6}\right)$$

input
```
integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/5*B*(16*c^3*x^2/(sqrt(c*x^4 + b*x^2)*b^4) + 8*c^2/(sqrt(c*x^4 + b*x^2)*
b^3) - 2*c/(sqrt(c*x^4 + b*x^2)*b^2*x^2) + 1/(sqrt(c*x^4 + b*x^2)*b*x^4))
+ 1/35*A*(128*c^4*x^2/(sqrt(c*x^4 + b*x^2)*b^5) + 64*c^3/(sqrt(c*x^4 + b*x
^2)*b^4) - 16*c^2/(sqrt(c*x^4 + b*x^2)*b^3*x^2) + 8*c/(sqrt(c*x^4 + b*x^2)
*b^2*x^4) - 5/(sqrt(c*x^4 + b*x^2)*b*x^6))
```

3.152.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(122) = 244$.

Time = 1.28 (sec) , antiderivative size = 415, normalized size of antiderivative = 3.01

$$\int \frac{A+Bx^2}{x^5 (bx^2+cx^4)^{3/2}}\,dx = -\frac{(Bbc^3-Ac^4)x}{\sqrt{cx^2+b}b^5\mathrm{sgn}\,(x)} + \frac{2\left(35\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Bbc^{\frac{5}{2}}-35\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Ac^{\frac{7}{2}}-280\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bb^2c^{\frac{5}{2}}+280\left(\sqrt{\right.}\right.}{}$$

input
```
integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
-(B*b*c^3 - A*c^4)*x/(sqrt(c*x^2 + b)*b^5*sgn(x)) + 2/35*(35*(sqrt(c)*x -
sqrt(c*x^2 + b))^12*B*b*c^(5/2) - 35*(sqrt(c)*x - sqrt(c*x^2 + b))^12*A*c^
(7/2) - 280*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*b^2*c^(5/2) + 280*(sqrt(c)*
x - sqrt(c*x^2 + b))^10*A*b*c^(7/2) + 1015*(sqrt(c)*x - sqrt(c*x^2 + b))^8
*B*b^3*c^(5/2) - 1015*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b^2*c^(7/2) - 1680
*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^4*c^(5/2) + 2240*(sqrt(c)*x - sqrt(c*
x^2 + b))^6*A*b^3*c^(7/2) + 1337*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^5*c^(
5/2) - 1673*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^4*c^(7/2) - 504*(sqrt(c)*x
 - sqrt(c*x^2 + b))^2*B*b^6*c^(5/2) + 616*(sqrt(c)*x - sqrt(c*x^2 + b))^2*
A*b^5*c^(7/2) + 77*B*b^7*c^(5/2) - 93*A*b^6*c^(7/2))/(((sqrt(c)*x - sqrt(c
*x^2 + b))^2 - b)^7*b^4*sgn(x))
```

3.152.9 Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx^2}{x^5 (bx^2+cx^4)^{3/2}}\,dx = -\frac{(7\,B\,b^2-13\,A\,b\,c)\,\sqrt{c\,x^4+b\,x^2}}{35\,b^4\,x^6} - \frac{\left(x^2\left(\frac{58\,A\,c^4-42\,B\,b\,c^3}{35\,b^5}-\frac{2\,c^3\,(93\,A\,c-77\,B\,b)}{35\,b^5}\right)-\frac{c^2\,(93\,A\,c-77\,B\,b)}{35\,b^4}\right)\sqrt{c\,x^4+b\,x^2}}{x^2\,(c\,x^2+b)} - \frac{A\,\sqrt{c\,x^4+b\,x^2}}{7\,b^2\,x^8} - \frac{c\,(29\,A\,c-21\,B\,b)\,\sqrt{c\,x^4+b\,x^2}}{35\,b^4\,x^4}$$

input
```
int((A + B*x^2)/(x^5*(b*x^2 + c*x^4)^(3/2)),x)
```

output
```
- ((7*B*b^2 - 13*A*b*c)*(b*x^2 + c*x^4)^(1/2))/(35*b^4*x^6) - ((x^2*((58*A
*c^4 - 42*B*b*c^3)/(35*b^5) - (2*c^3*(93*A*c - 77*B*b))/(35*b^5)) - (c^2*(
93*A*c - 77*B*b))/(35*b^4))*(b*x^2 + c*x^4)^(1/2))/(x^2*(b + c*x^2)) - (A*
(b*x^2 + c*x^4)^(1/2))/(7*b^2*x^8) - (c*(29*A*c - 21*B*b)*(b*x^2 + c*x^4)^
(1/2))/(35*b^4*x^4)
```

3.152. $\int \frac{A+Bx^2}{x^5(bx^2+cx^4)^{3/2}}\,dx$

3.153 $\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.153.1 Optimal result

Integrand size = 26, antiderivative size = 139

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^7}{bc\sqrt{bx^2+cx^4}} + \frac{8b(6bB-5Ac)\sqrt{bx^2+cx^4}}{15c^4x} - \frac{4(6bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^3} + \frac{(6bB-5Ac)x^3\sqrt{bx^2+cx^4}}{5bc^2}$$

output
```
-(-A*c+B*b)*x^7/b/c/(c*x^4+b*x^2)^(1/2)+8/15*b*(-5*A*c+6*B*b)*(c*x^4+b*x^2
)^(1/2)/c^4/x-4/15*(-5*A*c+6*B*b)*x*(c*x^4+b*x^2)^(1/2)/c^3+1/5*(-5*A*c+6*
B*b)*x^3*(c*x^4+b*x^2)^(1/2)/b/c^2
```

3.153.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.59

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x(48b^3B-8b^2c(5A-3Bx^2)+c^3x^4(5A+3Bx^2)-2bc^2x^2(10A+3Bx^2))}{15c^4\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(48*b^3*B - 8*b^2*c*(5*A - 3*B*x^2) + c^3*x^4*(5*A + 3*B*x^2) - 2*b*c^2
*x^2*(10*A + 3*B*x^2)))/(15*c^4*Sqrt[x^2*(b + c*x^2)])
```

3.153.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1943, 1421, 1421, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{(6bB-5Ac)\int \frac{x^6}{\sqrt{cx^4+bx^2}}dx}{bc}-\frac{x^7(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1421}$$

$$\frac{(6bB-5Ac)\left(\frac{x^3\sqrt{bx^2+cx^4}}{5c}-\frac{4b\int \frac{x^4}{\sqrt{cx^4+bx^2}}dx}{5c}\right)}{bc}-\frac{x^7(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1421}$$

$$\frac{(6bB-5Ac)\left(\frac{x^3\sqrt{bx^2+cx^4}}{5c}-\frac{4b\left(\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\int \frac{x^2}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{5c}\right)}{bc}-\frac{x^7(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1420}$$

$$\frac{\left(\frac{x^3\sqrt{bx^2+cx^4}}{5c}-\frac{4b\left(\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}\right)}{5c}\right)(6bB-5Ac)}{bc}-\frac{x^7(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

input
```
Int[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-(((b*B - A*c)*x^7)/(b*c*Sqrt[b*x^2 + c*x^4])) + ((6*b*B - 5*A*c)*((x^3*Sq
rt[b*x^2 + c*x^4])/(5*c) - (4*b*((-2*b*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (x
*Sqrt[b*x^2 + c*x^4])/(3*c)))/(5*c)))/(b*c)
```

3.153. $\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}}dx$

3.153.3.1 Defintions of rubi rules used

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

rule 1421
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && IGtQ[Simplify[(m
 + 2*p - 1)/2], 0] && NeQ[m + 4*p + 1, 0]
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

3.153.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{(c\,x^2+b)\left(-3B\,c^3x^6-5A\,c^3x^4+6Bb\,c^2x^4+20Ab\,c^2x^2-24B\,b^2c\,x^2+40b^2Ac-48B\,b^3\right)x^3}{15c^4(x^4c+b\,x^2)^{\frac{3}{2}}}$	91
default	$-\frac{(c\,x^2+b)\left(-3B\,c^3x^6-5A\,c^3x^4+6Bb\,c^2x^4+20Ab\,c^2x^2-24B\,b^2c\,x^2+40b^2Ac-48B\,b^3\right)x^3}{15c^4(x^4c+b\,x^2)^{\frac{3}{2}}}$	91
trager	$-\frac{\left(-3B\,c^3x^6-5A\,c^3x^4+6Bb\,c^2x^4+20Ab\,c^2x^2-24B\,b^2c\,x^2+40b^2Ac-48B\,b^3\right)\sqrt{x^4c+b\,x^2}}{15(c\,x^2+b)c^4x}$	93
risch	$-\frac{\left(-3B\,c^2x^4-5A\,c^2x^2+9Bbc\,x^2+25Abc-33B\,b^2\right)(c\,x^2+b)x}{15c^4\sqrt{x^2(c\,x^2+b)}}-\frac{b^2(Ac-Bb)x}{c^4\sqrt{x^2(c\,x^2+b)}}$	96

input
```
int(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(c*x^2+b)*(-3*B*c^3*x^6-5*A*c^3*x^4+6*B*b*c^2*x^4+20*A*b*c^2*x^2-24*
B*b^2*c*x^2+40*A*b^2*c-48*B*b^3)*x^3/c^4/(c*x^4+b*x^2)^(3/2)
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.67

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{(3\,Bc^3x^6-(6\,Bbc^2-5\,Ac^3)x^4+48\,Bb^3-40\,Ab^2c+4\,(6\,Bb^2c-5\,Abc^2)x^2)\sqrt{cx^4+b}}{15\,(c^5x^3+bc^4x)}$$

input
```
integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(3*B*c^3*x^6 - (6*B*b*c^2 - 5*A*c^3)*x^4 + 48*B*b^3 - 40*A*b^2*c + 4*
(6*B*b^2*c - 5*A*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^5*x^3 + b*c^4*x)
```

3.153.6 Sympy [F]

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^8(A+Bx^2)}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**8*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.59

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{(c^2x^4-4\,bcx^2-8\,b^2)A}{3\,\sqrt{cx^2+b}c^3} + \frac{(c^3x^6-2\,bc^2x^4+8\,b^2cx^2+16\,b^3)B}{5\,\sqrt{cx^2+b}c^4}$$

input
```
integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/3*(c^2*x^4 - 4*b*c*x^2 - 8*b^2)*A/(sqrt(c*x^2 + b)*c^3) + 1/5*(c^3*x^6 -
 2*b*c^2*x^4 + 8*b^2*c*x^2 + 16*b^3)*B/(sqrt(c*x^2 + b)*c^4)
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{8\,(6\,Bb^3-5\,Ab^2c)\mathrm{sgn}(x)}{15\,\sqrt{b}c^4}+\frac{Bb^3-Ab^2c}{\sqrt{cx^2+b}c^4\mathrm{sgn}\,(x)}$$
$$+\frac{3\,(cx^2+b)^{\frac{5}{2}}Bc^{16}-15\,(cx^2+b)^{\frac{3}{2}}Bbc^{16}+45\,\sqrt{cx^2+b}Bb^2c^{16}+5\,(cx^2+b)^{\frac{3}{2}}Ac^{17}-30\,\sqrt{cx^2+b}Abc^{17}}{15\,c^{20}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
-8/15*(6*B*b^3 - 5*A*b^2*c)*sgn(x)/(sqrt(b)*c^4) + (B*b^3 - A*b^2*c)/(sqrt
(c*x^2 + b)*c^4*sgn(x)) + 1/15*(3*(c*x^2 + b)^(5/2)*B*c^16 - 15*(c*x^2 + b
)^(3/2)*B*b*c^16 + 45*sqrt(c*x^2 + b)*B*b^2*c^16 + 5*(c*x^2 + b)^(3/2)*A*c
^17 - 30*sqrt(c*x^2 + b)*A*b*c^17)/(c^20*sgn(x))
```

3.153.9 Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.66

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}\,(48\,B\,b^3+24\,B\,b^2\,c\,x^2-40\,A\,b^2\,c-6\,B\,b\,c^2\,x^4-20\,A\,b\,c^2\,x^2+3\,B\,c^3\,x^6}{15\,c^4\,x\,\left(c\,x^2+b\right)}$$

input
```
int((x^8*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
((b*x^2 + c*x^4)^(1/2)*(48*B*b^3 + 5*A*c^3*x^4 + 3*B*c^3*x^6 - 40*A*b^2*c
- 20*A*b*c^2*x^2 + 24*B*b^2*c*x^2 - 6*B*b*c^2*x^4))/(15*c^4*x*(b + c*x^2))
```

3.154 $\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.154.1 Optimal result

Integrand size = 26, antiderivative size = 104

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=-\frac{(bB-Ac)x^5}{bc\sqrt{bx^2+cx^4}}-\frac{2(4bB-3Ac)\sqrt{bx^2+cx^4}}{3c^3x}+\frac{(4bB-3Ac)x\sqrt{bx^2+cx^4}}{3bc^2}$$

output
```
-(-A*c+B*b)*x^5/b/c/(c*x^4+b*x^2)^(1/2)-2/3*(-3*A*c+4*B*b)*(c*x^4+b*x^2)^(
1/2)/c^3/x+1/3*(-3*A*c+4*B*b)*x*(c*x^4+b*x^2)^(1/2)/b/c^2
```

3.154.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\frac{x(-8b^2B+c^2x^2(3A+Bx^2)+b(6Ac-4Bcx^2))}{3c^3\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(-8*b^2*B + c^2*x^2*(3*A + B*x^2) + b*(6*A*c - 4*B*c*x^2)))/(3*c^3*Sqrt
[x^2*(b + c*x^2)])
```

3.154.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1943, 1421, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (A+Bx^2)}{(bx^2+cx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{(4bB-3Ac)\int \frac{x^4}{\sqrt{cx^4+bx^2}}dx}{bc}-\frac{x^5(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1421}$$

$$\frac{(4bB-3Ac)\left(\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\int \frac{x^2}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{bc}-\frac{x^5(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1420}$$

$$\frac{\left(\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}\right)(4bB-3Ac)}{bc}-\frac{x^5(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

input
```
Int[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-(((b*B - A*c)*x^5)/(b*c*Sqrt[b*x^2 + c*x^4])) + ((4*b*B - 3*A*c)*((-2*b*S
qrt[b*x^2 + c*x^4])/(3*c^2*x) + (x*Sqrt[b*x^2 + c*x^4])/(3*c)))/(b*c)
```

3.154.3.1 Defintions of rubi rules used

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

rule 1421
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && IGtQ[Simplify[(m
 + 2*p - 1)/2], 0] && NeQ[m + 4*p + 1, 0]
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

3.154.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{(c\,x^2+b)\left(B\,c^2x^4+3A\,c^2x^2-4Bbc\,x^2+6Abc-8B\,b^2\right)x^3}{3c^3\left(x^4c+b\,x^2\right)^{\frac{3}{2}}}$	66
default	$\frac{(c\,x^2+b)\left(B\,c^2x^4+3A\,c^2x^2-4Bbc\,x^2+6Abc-8B\,b^2\right)x^3}{3c^3\left(x^4c+b\,x^2\right)^{\frac{3}{2}}}$	66
trager	$\frac{\left(B\,c^2x^4+3A\,c^2x^2-4Bbc\,x^2+6Abc-8B\,b^2\right)\sqrt{x^4c+b\,x^2}}{3(c\,x^2+b)c^3x}$	68
risch	$\frac{(Bc\,x^2+3Ac-5Bb)(c\,x^2+b)x}{3c^3\sqrt{x^2(c\,x^2+b)}}+\frac{b(Ac-Bb)x}{c^3\sqrt{x^2(c\,x^2+b)}}$	70

input
```
int(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(c*x^2+b)*(B*c^2*x^4+3*A*c^2*x^2-4*B*b*c*x^2+6*A*b*c-8*B*b^2)*x^3/c^3/
(c*x^4+b*x^2)^(3/2)
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.65

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{(Bc^2x^4-8\,Bb^2+6\,Abc-(4\,Bbc-3\,Ac^2)x^2)\sqrt{cx^4+bx^2}}{3\,(c^4x^3+bc^3x)}$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(B*c^2*x^4 - 8*B*b^2 + 6*A*b*c - (4*B*b*c - 3*A*c^2)*x^2)*sqrt(c*x^4 +
 b*x^2)/(c^4*x^3 + b*c^3*x)
```

3.154.6 Sympy [F]

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^6(A+Bx^2)}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**6*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.154.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.57

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{(cx^2+2\,b)A}{\sqrt{cx^2+b}c^2} + \frac{(c^2x^4-4\,bcx^2-8\,b^2)B}{3\,\sqrt{cx^2+b}c^3}$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
(c*x^2 + 2*b)*A/(sqrt(c*x^2 + b)*c^2) + 1/3*(c^2*x^4 - 4*b*c*x^2 - 8*b^2)*
B/(sqrt(c*x^2 + b)*c^3)
```

3.154.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{2\,(4\,Bb^2-3\,Abc)\mathrm{sgn}(x)}{3\,\sqrt{b}c^3} - \frac{Bb^2-Abc}{\sqrt{cx^2+b}c^3\mathrm{sgn}\,(x)} + \frac{(cx^2+b)^{\frac{3}{2}}Bc^6-6\,\sqrt{cx^2+b}Bbc^6+3\,\sqrt{cx^2+b}Ac^7}{3\,c^9\mathrm{sgn}\,(x)}$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
2/3*(4*B*b^2 - 3*A*b*c)*sgn(x)/(sqrt(b)*c^3) - (B*b^2 - A*b*c)/(sqrt(c*x^2
 + b)*c^3*sgn(x)) + 1/3*((c*x^2 + b)^(3/2)*B*c^6 - 6*sqrt(c*x^2 + b)*B*b*c
^6 + 3*sqrt(c*x^2 + b)*A*c^7)/(c^9*sgn(x))
```

3.154.9 Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}\,(-8\,B\,b^2-4\,B\,b\,c\,x^2+6\,A\,b\,c+B\,c^2\,x^4+3\,A\,c^2\,x^2)}{3\,c^3\,x\,\left(c\,x^2+b\right)}$$

input
```
int((x^6*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
((b*x^2 + c*x^4)^(1/2)*(3*A*c^2*x^2 - 8*B*b^2 + B*c^2*x^4 + 6*A*b*c - 4*B*
b*c*x^2))/(3*c^3*x*(b + c*x^2))
```

3.155 $\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.155.1 Optimal result

Integrand size = 26, antiderivative size = 69

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^3}{bc\sqrt{bx^2+cx^4}}+\frac{(2bB-Ac)\sqrt{bx^2+cx^4}}{bc^2x}$$

output
```
-(-A*c+B*b)*x^3/b/c/(c*x^4+b*x^2)^(1/2)+(-A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/b
/c^2/x
```

3.155.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x(2bB-Ac+Bcx^2)}{c^2\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(2*b*B - A*c + B*c*x^2))/(c^2*Sqrt[x^2*(b + c*x^2)])
```

3.155.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1943, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{(2bB-Ac)\int \frac{x^2}{\sqrt{cx^4+bx^2}}dx}{bc}-\frac{x^3(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1420}$$

$$\frac{\sqrt{bx^2+cx^4}(2bB-Ac)}{bc^2x}-\frac{x^3(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

input
```
Int[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-(((b*B - A*c)*x^3)/(b*c*Sqrt[b*x^2 + c*x^4])) + ((2*b*B - A*c)*Sqrt[b*x^2
 + c*x^4])/(b*c^2*x)
```

3.155.3.1 Defintions of rubi rules used

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

3.155.4 Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{(c x^2+b)(-Bc x^2+Ac-2Bb)x^3}{c^2(x^4c+b x^2)^{\frac{3}{2}}}$	44
default	$-\frac{(c x^2+b)(-Bc x^2+Ac-2Bb)x^3}{c^2(x^4c+b x^2)^{\frac{3}{2}}}$	44
trager	$-\frac{(-Bc x^2+Ac-2Bb)\sqrt{x^4c+b x^2}}{(c x^2+b)c^2 x}$	46
risch	$\frac{B(c x^2+b)x}{c^2\sqrt{x^2(c x^2+b)}}-\frac{(Ac-Bb)x}{c^2\sqrt{x^2(c x^2+b)}}$	55

input
```
int(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2+b)*(-B*c*x^2+A*c-2*B*b)*x^3/c^2/(c*x^4+b*x^2)^(3/2)
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\frac{\sqrt{cx^4+bx^2}(Bcx^2+2\,Bb-Ac)}{c^3x^3+bc^2x}$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(c*x^4 + b*x^2)*(B*c*x^2 + 2*B*b - A*c)/(c^3*x^3 + b*c^2*x)
```

3.155.6 Sympy [F]

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\int \frac{x^4(A+Bx^2)}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**4*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.155.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{(cx^2+2\,b)B}{\sqrt{cx^2+b}c^2} - \frac{A}{\sqrt{cx^2+b}c}$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
(c*x^2 + 2*b)*B/(sqrt(c*x^2 + b)*c^2) - A/(sqrt(c*x^2 + b)*c)
```

3.155.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(2\,Bb-Ac)\mathrm{sgn}(x)}{\sqrt{b}c^2} + \frac{\sqrt{cx^2+b}B}{c^2\mathrm{sgn}\,(x)} + \frac{Bb-Ac}{\sqrt{cx^2+b}c^2\mathrm{sgn}\,(x)}$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
-(2*B*b - A*c)*sgn(x)/(sqrt(b)*c^2) + sqrt(c*x^2 + b)*B/(c^2*sgn(x)) + (B*
b - A*c)/(sqrt(c*x^2 + b)*c^2*sgn(x))
```

3.155.9 Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}\,(B\,c\,x^2-A\,c+2\,B\,b)}{c^2\,x\,(c\,x^2+b)}$$

input
```
int((x^4*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
((b*x^2 + c*x^4)^(1/2)*(2*B*b - A*c + B*c*x^2))/(c^2*x*(b + c*x^2))
```

3.156 $\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.156.1 Optimal result

Integrand size = 26, antiderivative size = 64

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x}{bc\sqrt{bx^2+cx^4}} - \frac{A\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}$$

output
```
-A*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(3/2)-(-A*c+B*b)*x/b/c/(c*x^4+
b*x^2)^(1/2)
```

3.156.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{x\left(\sqrt{b}(bB-Ac)+Ac\sqrt{b+cx^2}\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{b^{3/2}c\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-((x*(Sqrt[b]*(b*B - A*c) + A*c*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sq
rt[b]]))/(b^(3/2)*c*Sqrt[x^2*(b + c*x^2)]))
```

3.156.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1943, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{A\int \frac{1}{\sqrt{cx^4+bx^2}}dx}{b}-\frac{x(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1400}$$

$$-\frac{A\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}}{b}-\frac{x(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{A\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}-\frac{x(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

input
```
Int[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-(((b*B - A*c)*x)/(b*c*Sqrt[b*x^2 + c*x^4])) - (A*ArcTanh[(Sqrt[b]*x)/Sqrt
[b*x^2 + c*x^4]])/b^(3/2)
```

3.156.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

3.156.4 Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.23

method	result	size
default	$-\frac{x^{3}\left(c\,x^{2}+b\right)\left(A\sqrt{c\,x^{2}+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)bc-A\,b^{\frac{3}{2}}c+B\,b^{\frac{5}{2}}\right)}{\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}c\,b^{\frac{5}{2}}}$	79

input
```
int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-x^3*(c*x^2+b)*(A*(c*x^2+b)^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b*c-
A*b^(3/2)*c+B*b^(5/2))/(c*x^4+b*x^2)^(3/2)/c/b^(5/2)
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 3.11

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \left[\frac{(Ac^2x^3+Abcx)\sqrt{b}\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}(Bb^2-Abc)}{2\,(b^2c^2x^3+b^3cx)}, \frac{(Ac^2x\right.$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((A*c^2*x^3 + A*b*c*x)*sqrt(b)*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b
*x^2)*sqrt(b))/x^3) - 2*sqrt(c*x^4 + b*x^2)*(B*b^2 - A*b*c))/(b^2*c^2*x^3
+ b^3*c*x), ((A*c^2*x^3 + A*b*c*x)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqr
t(-b)/(c*x^3 + b*x)) - sqrt(c*x^4 + b*x^2)*(B*b^2 - A*b*c))/(b^2*c^2*x^3 +
 b^3*c*x)]
```

3.156.6 Sympy [F]

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^2(A+Bx^2)}{(x^2(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**2*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.156.7 Maxima [F]

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^2}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^2/(c*x^4 + b*x^2)^(3/2), x)
```

3.156.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.70

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{A\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b\text{sgn}(x)} - \frac{\left(A\sqrt{b}c\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) - B\sqrt{-b}b + A\sqrt{-b}c\right)\text{sgn}(x)}{\sqrt{-b}b^{\frac{3}{2}}c} - \frac{Bb-Ac}{\sqrt{cx^2+b}bc\text{sgn}(x)}$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
A*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b*sgn(x)) - (A*sqrt(b)*c*arct
an(sqrt(b)/sqrt(-b)) - B*sqrt(-b)*b + A*sqrt(-b)*c)*sgn(x)/(sqrt(-b)*b^(3/
2)*c) - (B*b - A*c)/(sqrt(c*x^2 + b)*b*c*sgn(x))
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^2\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.157 $\int \frac{A+Bx^2}{(bx^2+cx^4)^{3/2}} dx$

3.157.1 Optimal result

Integrand size = 23, antiderivative size = 142

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^{3/2}} dx = -\frac{B}{3cx\sqrt{bx^2+cx^4}} - \frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}} + \frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{2b^2cx^3} - \frac{(2bB-3Ac)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}$$

output
```
-1/2*(-3*A*c+2*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(5/2)-1/3*B/c
/x/(c*x^4+b*x^2)^(1/2)+1/3*(3*A*c-2*B*b)/b/c/x/(c*x^4+b*x^2)^(1/2)+1/2*(-3
*A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/b^2/c/x^3
```

3.157.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^{3/2}} dx = \frac{\sqrt{b}(2bBx^2-A(b+3cx^2)) - (2bB-3Ac)x^2\sqrt{b+cx^2}\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{2b^{5/2}x\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(A + B*x^2)/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(Sqrt[b]*(2*b*B*x^2 - A*(b + 3*c*x^2)) - (2*b*B - 3*A*c)*x^2*Sqrt[b + c*x^
2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])/(2*b^(5/2)*x*Sqrt[x^2*(b + c*x^2)])
```

3.157.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1465, 1401, 1430, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1465}$$

$$-\frac{(2bB-3Ac)\int \frac{1}{(cx^4+bx^2)^{3/2}}dx}{3c}-\frac{B}{3cx\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1401}$$

$$-\frac{(2bB-3Ac)\left(\frac{3\int \frac{1}{x^2\sqrt{cx^4+bx^2}}dx}{b}+\frac{1}{bx\sqrt{bx^2+cx^4}}\right)}{3c}-\frac{B}{3cx\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1430}$$

$$-\frac{(2bB-3Ac)\left(\frac{3\left(-\frac{c\int \frac{1}{\sqrt{cx^4+bx^2}}dx}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{b}+\frac{1}{bx\sqrt{bx^2+cx^4}}\right)}{3c}-\frac{B}{3cx\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1400}$$

$$-\frac{(2bB-3Ac)\left(\frac{3\left(\frac{c\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{b}+\frac{1}{bx\sqrt{bx^2+cx^4}}\right)}{3c}-\frac{B}{3cx\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{(2bB-3Ac)\left(\frac{3\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{b}+\frac{1}{bx\sqrt{bx^2+cx^4}}\right)}{3c}-\frac{B}{3cx\sqrt{bx^2+cx^4}}$$

input
```
Int[(A + B*x^2)/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-1/3*B/(c*x*Sqrt[b*x^2 + c*x^4]) - ((2*b*B - 3*A*c)*(1/(b*x*Sqrt[b*x^2 + c
*x^4]) + (3*(-1/2*Sqrt[b*x^2 + c*x^4]/(b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqr
t[b*x^2 + c*x^4]])/(2*b^(3/2))))/b))/(3*c)
```

3.157.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1401
```
Int[((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[-(b*x^2 + c*x^4)^
(p + 1)/(2*b*(p + 1)*x), x] + Simp[(4*p + 3)/(2*b*(p + 1))   Int[(b*x^2 + c
*x^4)^(p + 1)/x^2, x], x] /; FreeQ[{b, c}, x] &&  !IntegerQ[p] && LtQ[p, -1
]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1465
```
Int[((d_) + (e_.)*(x_)^2)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
Simp[e*((b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 3)*x)), x] - Simp[(b*e*(2*p + 1)
- c*d*(4*p + 3))/(c*(4*p + 3))   Int[(b*x^2 + c*x^4)^p, x], x] /; FreeQ[{b,
 c, d, e, p}, x] &&  !IntegerQ[p] && NeQ[4*p + 3, 0] && NeQ[b*e*(2*p + 1) -
 c*d*(4*p + 3), 0]
```

3.157.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{A(c x^2+b)}{2b^2 x\sqrt{x^2(c x^2+b)}}-\frac{\left(-\frac{Ac}{\sqrt{c x^2+b}}+b(3Ac-2Bb)\left(\frac{1}{b\sqrt{c x^2+b}}-\frac{\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c x^2+b}}{x}\right)}{b^{\frac{3}{2}}}\right)\right)x\sqrt{c x^2+b}}{2b^2\sqrt{x^2(c x^2+b)}}$	126
default	$\frac{x(c x^2+b)\left(3A\sqrt{c x^2+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c x^2+b}}{x}\right)b c x^2-3A b^{\frac{3}{2}} c x^2-2B\sqrt{c x^2+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c x^2+b}}{x}\right)b^2 x^2+2B b^{\frac{5}{2}} x^2-A b^{\frac{5}{2}}\right)}{2(x^4 c+b x^2)^{\frac{3}{2}} b^{\frac{7}{2}}}$	130

input
```
int((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^2*A*(c*x^2+b)/x/(x^2*(c*x^2+b))^(1/2)-1/2/b^2*(-A*c/(c*x^2+b)^(1/2)
+b*(3*A*c-2*B*b)*(1/b/(c*x^2+b)^(1/2)-1/b^(3/2)*ln((2*b+2*b^(1/2)*(c*x^2+b
)^(1/2))/x)))*x/(x^2*(c*x^2+b))^(1/2)*(c*x^2+b)^(1/2)
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.83

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^{3/2}}\,dx = \left[-\frac{((2\,Bbc-3\,Ac^2)x^5+(2\,Bb^2-3\,Abc)x^3)\sqrt{b}\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx}}{4\,(b^3cx^5+b^4x^3)}\right.$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(((2*B*b*c - 3*A*c^2)*x^5 + (2*B*b^2 - 3*A*b*c)*x^3)*sqrt(b)*log(-(c
*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)
*(A*b^2 - (2*B*b^2 - 3*A*b*c)*x^2))/(b^3*c*x^5 + b^4*x^3), 1/2*(((2*B*b*c
- 3*A*c^2)*x^5 + (2*B*b^2 - 3*A*b*c)*x^3)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x
^2)*sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^4 + b*x^2)*(A*b^2 - (2*B*b^2 - 3*A*
b*c)*x^2))/(b^3*c*x^5 + b^4*x^3)]
```

3.157.6 Sympy [F]

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{A+Bx^2}{(x^2(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral((A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.157.7 Maxima [F]

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(c*x^4 + b*x^2)^(3/2), x)
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^{3/2}}\,dx = \frac{(2\,Bb-3\,Ac)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{2\,\sqrt{-b}b^2\mathrm{sgn}\left(x\right)} + \frac{2\,(cx^2+b)Bb-2\,Bb^2-3\,(cx^2+b)Ac+2\,Abc}{2\left((cx^2+b)^{\frac{3}{2}}-\sqrt{cx^2+b}b\right)b^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*(2*B*b - 3*A*c)*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^2*sgn(x))
 + 1/2*(2*(c*x^2 + b)*B*b - 2*B*b^2 - 3*(c*x^2 + b)*A*c + 2*A*b*c)/(((c*x^
2 + b)^(3/2) - sqrt(c*x^2 + b)*b)*b^2*sgn(x))
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{B\,x^2+A}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

input
```
int((A + B*x^2)/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((A + B*x^2)/(b*x^2 + c*x^4)^(3/2), x)
```

3.158 $\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx$

3.158.1 Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{A}{4bx^3\sqrt{bx^2+cx^4}}+\frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}}-\frac{3(4bB-5Ac)\sqrt{bx^2+cx^4}}{8b^3x^3}+\frac{3c(4bB-5Ac)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}$$

output
```
3/8*c*(-5*A*c+4*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(7/2)-1/4*A/
b/x^3/(c*x^4+b*x^2)^(1/2)+1/4*(-5*A*c+4*B*b)/b^2/x/(c*x^4+b*x^2)^(1/2)-3/8
*(-5*A*c+4*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^3
```

3.158.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx=\frac{\sqrt{b}\left(-4bBx^2\left(b+3cx^2\right)+A\left(-2b^2+5bcx^2+15c^2x^4\right)\right)+3c(4bB-5Ac)x^4\sqrt{b+cx^2}}{8b^{7/2}x^3\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(Sqrt[b]*(-4*b*B*x^2*(b + 3*c*x^2) + A*(-2*b^2 + 5*b*c*x^2 + 15*c^2*x^4))
+ 3*c*(4*b*B - 5*A*c)*x^4*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]
)/(8*b^(7/2)*x^3*Sqrt[x^2*(b + c*x^2)])
```

3.158.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1944, 1401, 1430, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^2 (bx^2+cx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(4bB-5Ac)\int \frac{1}{(cx^4+bx^2)^{3/2}}dx}{4b}-\frac{A}{4bx^3\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1401}$$

$$\frac{(4bB-5Ac)\left(\frac{3\int \frac{1}{x^2\sqrt{cx^4+bx^2}}dx}{b}+\frac{1}{bx\sqrt{bx^2+cx^4}}\right)}{4b}-\frac{A}{4bx^3\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{(4bB-5Ac)\left(\frac{3\left(-\frac{c\int \frac{1}{\sqrt{cx^4+bx^2}}dx}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{b}+\frac{1}{bx\sqrt{bx^2+cx^4}}\right)}{4b}-\frac{A}{4bx^3\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1400}$$

$$\frac{(4bB-5Ac)\left(\frac{3\left(\frac{c\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{b}+\frac{1}{bx\sqrt{bx^2+cx^4}}\right)}{4b}-\frac{A}{4bx^3\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(4bB-5Ac)\left(\frac{3\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{b}+\frac{1}{bx\sqrt{bx^2+cx^4}}\right)}{4b}-\frac{A}{4bx^3\sqrt{bx^2+cx^4}}$$

input
```
Int[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
-1/4*A/(b*x^3*Sqrt[b*x^2 + c*x^4]) + ((4*b*B - 5*A*c)*(1/(b*x*Sqrt[b*x^2 +
 c*x^4]) + (3*(-1/2*Sqrt[b*x^2 + c*x^4]/(b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/S
qrt[b*x^2 + c*x^4]])/(2*b^(3/2))))/b))/(4*b)
```

3.158.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1401
```
Int[((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[-(b*x^2 + c*x^4)^
(p + 1)/(2*b*(p + 1)*x), x] + Simp[(4*p + 3)/(2*b*(p + 1))   Int[(b*x^2 + c
*x^4)^(p + 1)/x^2, x], x] /; FreeQ[{b, c}, x] &&  !IntegerQ[p] && LtQ[p, -1
]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.158.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{(c\,x^2+b)\,(-7Ac\,x^2+4bB\,x^2+2Ab)}{8b^3x^3\sqrt{x^2(c\,x^2+b)}}+\frac{c\left(-\frac{7Ac-4Bb}{\sqrt{c\,x^2+b}}+3b(5Ac-4Bb)\left(\frac{1}{b\sqrt{c\,x^2+b}}-\frac{\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)}{b^{\frac{3}{2}}}\right)\right)x\sqrt{c\,x^2+b}}{8b^3\sqrt{x^2(c\,x^2+b)}}$
default	$-\frac{(c\,x^2+b)\left(15A\sqrt{c\,x^2+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)b\,c^2x^4-15A\,b^{\frac{3}{2}}\,c^2x^4-12B\sqrt{c\,x^2+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)b^2c\,x^4+12B\,b^{\frac{5}{2}}\,c\,x^4-5A\,b^{\frac{5}{2}}\right.}{8x(x^4c+b\,x^2)^{\frac{3}{2}}\,b^{\frac{9}{2}}}$

input
```
int((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(c*x^2+b)*(-7*A*c*x^2+4*B*b*x^2+2*A*b)/b^3/x^3/(x^2*(c*x^2+b))^(1/2)+
1/8*c/b^3*(-(7*A*c-4*B*b)/(c*x^2+b)^(1/2)+3*b*(5*A*c-4*B*b)*(1/b/(c*x^2+b)
^(1/2)-1/b^(3/2)*ln((2*b+2*b^(1/2)*(c*x^2+b)^(1/2))/x)))*x/(x^2*(c*x^2+b))
^(1/2)*(c*x^2+b)^(1/2)
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.30

$$\int \frac{A+Bx^2}{x^2\,(bx^2+cx^4)^{3/2}}\,dx = \left[-\frac{3\,((4\,Bbc^2-5\,Ac^3)x^7+(4\,Bb^2c-5\,Abc^2)x^5)\sqrt{b}\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+}{16\,(b^4cx^7+b^5}\right.$$

$$-\frac{3\,((4\,Bbc^2-5\,Ac^3)x^7+(4\,Bb^2c-5\,Abc^2)x^5)\sqrt{-b}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+(3\,(4\,Bb^2c-5\,Abc^2)x^4+2\,Ab^3}{8\,(b^4cx^7+b^5x^5)}$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(3*((4*B*b*c^2 - 5*A*c^3)*x^7 + (4*B*b^2*c - 5*A*b*c^2)*x^5)*sqrt(b
)*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3) + 2*(3*(4*B*b^
2*c - 5*A*b*c^2)*x^4 + 2*A*b^3 + (4*B*b^3 - 5*A*b^2*c)*x^2)*sqrt(c*x^4 + b
*x^2))/(b^4*c*x^7 + b^5*x^5), -1/8*(3*((4*B*b*c^2 - 5*A*c^3)*x^7 + (4*B*b^
2*c - 5*A*b*c^2)*x^5)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3
+ b*x)) + (3*(4*B*b^2*c - 5*A*b*c^2)*x^4 + 2*A*b^3 + (4*B*b^3 - 5*A*b^2*c)
*x^2)*sqrt(c*x^4 + b*x^2))/(b^4*c*x^7 + b^5*x^5)]
```

3.158.6 Sympy [F]

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{A+Bx^2}{x^2\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)/x**2/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral((A + B*x**2)/(x**2*(x**2*(b + c*x**2))**(3/2)), x)
```

3.158.7 Maxima [F]

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{Bx^2+A}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^2), x)
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{3\left(4\,Bbc-5\,Ac^2\right)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{8\sqrt{-b}b^3\mathrm{sgn}\left(x\right)}-\frac{Bbc-Ac^2}{\sqrt{cx^2+b}b^3\mathrm{sgn}\left(x\right)}-\frac{4\left(cx^2+b\right)^{\frac{3}{2}}Bbc-4\sqrt{cx^2+b}Bb^2c-7\left(cx^2+b\right)^{\frac{3}{2}}Ac^2+9\sqrt{cx^2+b}Abc^2}{8\,b^3c^2x^4\mathrm{sgn}\left(x\right)}$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
-3/8*(4*B*b*c - 5*A*c^2)*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^3*sg
n(x)) - (B*b*c - A*c^2)/(sqrt(c*x^2 + b)*b^3*sgn(x)) - 1/8*(4*(c*x^2 + b)^
(3/2)*B*b*c - 4*sqrt(c*x^2 + b)*B*b^2*c - 7*(c*x^2 + b)^(3/2)*A*c^2 + 9*sq
rt(c*x^2 + b)*A*b*c^2)/(b^3*c^2*x^4*sgn(x))
```

3.158.9 Mupad [B] (verification not implemented)

Time = 10.23 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{A\left(\frac{b}{c\,x^2}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{b}{c\,x^2}\right)}{7\,x\left(c\,x^4+b\,x^2\right)^{3/2}}-\frac{B\,x\left(\frac{b}{c\,x^2}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{b}{c\,x^2}\right)}{5\left(c\,x^4+b\,x^2\right)^{3/2}}$$

input
```
int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(3/2)),x)
```

output
```
- (A*(b/(c*x^2) + 1)^(3/2)*hypergeom([3/2, 7/2], 9/2, -b/(c*x^2)))/(7*x*(b
*x^2 + c*x^4)^(3/2)) - (B*x*(b/(c*x^2) + 1)^(3/2)*hypergeom([3/2, 5/2], 7/
2, -b/(c*x^2)))/(5*(b*x^2 + c*x^4)^(3/2))
```

3.159 $\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx$

3.159.1 Optimal result

Integrand size = 24, antiderivative size = 39

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{13}Abx^{13/2}+\frac{2}{17}(bB+Ac)x^{17/2}+\frac{2}{21}Bcx^{21/2}$$

output
```
2/13*A*b*x^(13/2)+2/17*(A*c+B*b)*x^(17/2)+2/21*B*c*x^(21/2)
```

3.159.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2x^{13/2}(357Ab+273bBx^2+273Acx^2+221Bcx^4)}{4641}$$

input
```
Integrate[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

output
```
(2*x^(13/2)*(357*A*b + 273*b*B*x^2 + 273*A*c*x^2 + 221*B*c*x^4))/4641
```

3.159.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx$$

$$\Big\downarrow \; 9$$

$$\int x^{11/2}\left(A+Bx^2\right)\left(b+cx^2\right)dx$$

$$\Big\downarrow \; 355$$

$$\int \left(x^{15/2}(Ac+bB)+Abx^{11/2}+Bcx^{19/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{17}x^{17/2}(Ac+bB)+\frac{2}{13}Abx^{13/2}+\frac{2}{21}Bcx^{21/2}$$

input
```
Int[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

output
```
(2*A*b*x^(13/2))/13 + (2*(b*B + A*c)*x^(17/2))/17 + (2*B*c*x^(21/2))/21
```

3.159.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.159.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2Ab\,x^{\frac{13}{2}}}{13}+\frac{2(Ac+Bb)x^{\frac{17}{2}}}{17}+\frac{2Bc\,x^{\frac{21}{2}}}{21}$	28
default	$\frac{2Ab\,x^{\frac{13}{2}}}{13}+\frac{2(Ac+Bb)x^{\frac{17}{2}}}{17}+\frac{2Bc\,x^{\frac{21}{2}}}{21}$	28
gosper	$\frac{2x^{\frac{13}{2}}\left(221Bc\,x^{4}+273Ac\,x^{2}+273bB\,x^{2}+357Ab\right)}{4641}$	32
trager	$\frac{2x^{\frac{13}{2}}\left(221Bc\,x^{4}+273Ac\,x^{2}+273bB\,x^{2}+357Ab\right)}{4641}$	32
risch	$\frac{2x^{\frac{13}{2}}\left(221Bc\,x^{4}+273Ac\,x^{2}+273bB\,x^{2}+357Ab\right)}{4641}$	32

input
```
int(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
2/13*A*b*x^(13/2)+2/17*(A*c+B*b)*x^(17/2)+2/21*B*c*x^(21/2)
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{2}{4641}\left(221\,Bcx^{10}+273\,(Bb+Ac)x^8+357\,Abx^6\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
2/4641*(221*B*c*x^10 + 273*(B*b + A*c)*x^8 + 357*A*b*x^6)*sqrt(x)
```

3.159.6 Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2Abx^{\frac{13}{2}}}{13}+\frac{2Acx^{\frac{17}{2}}}{17}+\frac{2Bbx^{\frac{17}{2}}}{17}+\frac{2Bcx^{\frac{21}{2}}}{21}$$

input
```
integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2),x)
```

output
```
2*A*b*x**(13/2)/13 + 2*A*c*x**(17/2)/17 + 2*B*b*x**(17/2)/17 + 2*B*c*x**(2
1/2)/21
```

3.159.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{21}\,Bcx^{\frac{21}{2}}+\frac{2}{17}\,(Bb+Ac)x^{\frac{17}{2}}+\frac{2}{13}\,Abx^{\frac{13}{2}}$$

input
```
integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
2/21*B*c*x^(21/2) + 2/17*(B*b + A*c)*x^(17/2) + 2/13*A*b*x^(13/2)
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{21}\,Bcx^{\frac{21}{2}}+\frac{2}{17}\,Bbx^{\frac{17}{2}}+\frac{2}{17}\,Acx^{\frac{17}{2}}+\frac{2}{13}\,Abx^{\frac{13}{2}}$$

input
```
integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
2/21*B*c*x^(21/2) + 2/17*B*b*x^(17/2) + 2/17*A*c*x^(17/2) + 2/13*A*b*x^(13
/2)
```

3.159.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2\,x^{13/2}\,\left(357\,A\,b+273\,A\,c\,x^2+273\,B\,b\,x^2+221\,B\,c\,x^4\right)}{4641}$$

input
```
int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4),x)
```

output
```
(2*x^(13/2)*(357*A*b + 273*A*c*x^2 + 273*B*b*x^2 + 221*B*c*x^4))/4641
```

3.160 $\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx$

3.160.1 Optimal result

Integrand size = 24, antiderivative size = 39

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{11}Abx^{11/2}+\frac{2}{15}(bB+Ac)x^{15/2}+\frac{2}{19}Bcx^{19/2}$$

output
```
2/11*A*b*x^(11/2)+2/15*(A*c+B*b)*x^(15/2)+2/19*B*c*x^(19/2)
```

3.160.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2x^{11/2}(285Ab+209bBx^2+209Acx^2+165Bcx^4)}{3135}$$

input
```
Integrate[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

output
```
(2*x^(11/2)*(285*A*b + 209*b*B*x^2 + 209*A*c*x^2 + 165*B*c*x^4))/3135
```

3.160.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx$$

$$\Big\downarrow 9$$

$$\int x^{9/2}\left(A+Bx^2\right)\left(b+cx^2\right)dx$$

$$\Big\downarrow 355$$

$$\int \left(x^{13/2}(Ac+bB)+Abx^{9/2}+Bcx^{17/2}\right)dx$$

$$\Big\downarrow 2009$$

$$\frac{2}{15}x^{15/2}(Ac+bB)+\frac{2}{11}Abx^{11/2}+\frac{2}{19}Bcx^{19/2}$$

input
```
Int[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

output
```
(2*A*b*x^(11/2))/11 + (2*(b*B + A*c)*x^(15/2))/15 + (2*B*c*x^(19/2))/19
```

3.160.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.160.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2Ab\,x^{\frac{11}{2}}}{11}+\frac{2(Ac+Bb)x^{\frac{15}{2}}}{15}+\frac{2Bc\,x^{\frac{19}{2}}}{19}$	28
default	$\frac{2Ab\,x^{\frac{11}{2}}}{11}+\frac{2(Ac+Bb)x^{\frac{15}{2}}}{15}+\frac{2Bc\,x^{\frac{19}{2}}}{19}$	28
gosper	$\frac{2x^{\frac{11}{2}}\left(165Bc\,x^{4}+209Ac\,x^{2}+209bB\,x^{2}+285Ab\right)}{3135}$	32
trager	$\frac{2x^{\frac{11}{2}}\left(165Bc\,x^{4}+209Ac\,x^{2}+209bB\,x^{2}+285Ab\right)}{3135}$	32
risch	$\frac{2x^{\frac{11}{2}}\left(165Bc\,x^{4}+209Ac\,x^{2}+209bB\,x^{2}+285Ab\right)}{3135}$	32

input
```
int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
2/11*A*b*x^(11/2)+2/15*(A*c+B*b)*x^(15/2)+2/19*B*c*x^(19/2)
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{2}{3135}\left(165\,Bcx^9+209\,(Bb+Ac)x^7+285\,Abx^5\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
2/3135*(165*B*c*x^9 + 209*(B*b + A*c)*x^7 + 285*A*b*x^5)*sqrt(x)
```

3.160.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2Abx^{\frac{11}{2}}}{11}+\frac{2Acx^{\frac{15}{2}}}{15}+\frac{2Bbx^{\frac{15}{2}}}{15}+\frac{2Bcx^{\frac{19}{2}}}{19}$$

input
```
integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2),x)
```

output
```
2*A*b*x**(11/2)/11 + 2*A*c*x**(15/2)/15 + 2*B*b*x**(15/2)/15 + 2*B*c*x**(1
9/2)/19
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{19}\,Bcx^{\frac{19}{2}}+\frac{2}{15}\,(Bb+Ac)x^{\frac{15}{2}}+\frac{2}{11}\,Abx^{\frac{11}{2}}$$

input
```
integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
2/19*B*c*x^(19/2) + 2/15*(B*b + A*c)*x^(15/2) + 2/11*A*b*x^(11/2)
```

3.160.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{19}\,Bcx^{\frac{19}{2}}+\frac{2}{15}\,Bbx^{\frac{15}{2}}+\frac{2}{15}\,Acx^{\frac{15}{2}}+\frac{2}{11}\,Abx^{\frac{11}{2}}$$

input
```
integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
2/19*B*c*x^(19/2) + 2/15*B*b*x^(15/2) + 2/15*A*c*x^(15/2) + 2/11*A*b*x^(11
/2)
```

3.160.9 Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2\,x^{11/2}\,\left(285\,A\,b+209\,A\,c\,x^2+209\,B\,b\,x^2+165\,B\,c\,x^4\right)}{3135}$$

input
```
int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4),x)
```

output
```
(2*x^(11/2)*(285*A*b + 209*A*c*x^2 + 209*B*b*x^2 + 165*B*c*x^4))/3135
```

3.161 $\int x^{3/2}(A+Bx^2)(bx^2+cx^4)\,dx$

3.161.1 Optimal result

Integrand size = 24, antiderivative size = 39

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{9}Abx^{9/2}+\frac{2}{13}(bB+Ac)x^{13/2}+\frac{2}{17}Bcx^{17/2}$$

output
```
2/9*A*b*x^(9/2)+2/13*(A*c+B*b)*x^(13/2)+2/17*B*c*x^(17/2)
```

3.161.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2\left(221Abx^{9/2}+153bBx^{13/2}+153Acx^{13/2}+117Bcx^{17/2}\right)}{1989}$$

input
```
Integrate[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

output
```
(2*(221*A*b*x^(9/2) + 153*b*B*x^(13/2) + 153*A*c*x^(13/2) + 117*B*c*x^(17/
2)))/1989
```

3.161.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx$$

$$\Big\downarrow \text{9}$$

$$\int x^{7/2}\left(A+Bx^2\right)\left(b+cx^2\right)dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(x^{11/2}(Ac+bB)+Abx^{7/2}+Bcx^{15/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{13}x^{13/2}(Ac+bB)+\frac{2}{9}Abx^{9/2}+\frac{2}{17}Bcx^{17/2}$$

input
```
Int[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

output
```
(2*A*b*x^(9/2))/9 + (2*(b*B + A*c)*x^(13/2))/13 + (2*B*c*x^(17/2))/17
```

3.161.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.161.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2Ab\,x^{\frac{9}{2}}}{9}+\frac{2(Ac+Bb)x^{\frac{13}{2}}}{13}+\frac{2Bc\,x^{\frac{17}{2}}}{17}$	28
default	$\frac{2Ab\,x^{\frac{9}{2}}}{9}+\frac{2(Ac+Bb)x^{\frac{13}{2}}}{13}+\frac{2Bc\,x^{\frac{17}{2}}}{17}$	28
gosper	$\frac{2x^{\frac{9}{2}}\left(117Bc\,x^{4}+153Ac\,x^{2}+153bB\,x^{2}+221Ab\right)}{1989}$	32
trager	$\frac{2x^{\frac{9}{2}}\left(117Bc\,x^{4}+153Ac\,x^{2}+153bB\,x^{2}+221Ab\right)}{1989}$	32
risch	$\frac{2x^{\frac{9}{2}}\left(117Bc\,x^{4}+153Ac\,x^{2}+153bB\,x^{2}+221Ab\right)}{1989}$	32

input
```
int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
2/9*A*b*x^(9/2)+2/13*(A*c+B*b)*x^(13/2)+2/17*B*c*x^(17/2)
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{1989}\left(117\,Bcx^8+153\,(Bb+Ac)x^6+221\,Abx^4\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
2/1989*(117*B*c*x^8 + 153*(B*b + A*c)*x^6 + 221*A*b*x^4)*sqrt(x)
```

3.161.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{2Abx^{\frac{9}{2}}}{9}+\frac{2Acx^{\frac{13}{2}}}{13}+\frac{2Bbx^{\frac{13}{2}}}{13}+\frac{2Bcx^{\frac{17}{2}}}{17}$$

input
```
integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2),x)
```

output
```
2*A*b*x**(9/2)/9 + 2*A*c*x**(13/2)/13 + 2*B*b*x**(13/2)/13 + 2*B*c*x**(17/
2)/17
```

3.161.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{2}{17}\,Bcx^{\frac{17}{2}}+\frac{2}{13}\,(Bb+Ac)x^{\frac{13}{2}}+\frac{2}{9}\,Abx^{\frac{9}{2}}$$

input
```
integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
2/17*B*c*x^(17/2) + 2/13*(B*b + A*c)*x^(13/2) + 2/9*A*b*x^(9/2)
```

3.161.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{2}{17}\,Bcx^{\frac{17}{2}}+\frac{2}{13}\,Bbx^{\frac{13}{2}}+\frac{2}{13}\,Acx^{\frac{13}{2}}+\frac{2}{9}\,Abx^{\frac{9}{2}}$$

input
```
integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
2/17*B*c*x^(17/2) + 2/13*B*b*x^(13/2) + 2/13*A*c*x^(13/2) + 2/9*A*b*x^(9/2
)
```

3.161.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2\,x^{9/2}\,(221\,A\,b+153\,A\,c\,x^2+153\,B\,b\,x^2+117\,B\,c\,x^4)}{1989}$$

input
```
int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4),x)
```

output
```
(2*x^(9/2)*(221*A*b + 153*A*c*x^2 + 153*B*b*x^2 + 117*B*c*x^4))/1989
```

3.162 $\int \sqrt{x}(A+Bx^2)(bx^2+cx^4)\,dx$

3.162.1 Optimal result

Integrand size = 24, antiderivative size = 39

$$\int \sqrt{x}(A+Bx^2)(bx^2+cx^4)\,dx=\frac{2}{7}Abx^{7/2}+\frac{2}{11}(bB+Ac)x^{11/2}+\frac{2}{15}Bcx^{15/2}$$

output
```
2/7*A*b*x^(7/2)+2/11*(A*c+B*b)*x^(11/2)+2/15*B*c*x^(15/2)
```

3.162.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \sqrt{x}(A+Bx^2)(bx^2+cx^4)\,dx=\frac{2x^{7/2}(165Ab+105bBx^2+105Acx^2+77Bcx^4)}{1155}$$

input
```
Integrate[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

output
```
(2*x^(7/2)*(165*A*b + 105*b*B*x^2 + 105*A*c*x^2 + 77*B*c*x^4))/1155
```

3.162.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)\,dx$$

$$\Big\downarrow \text{9}$$

$$\int x^{5/2}(A+Bx^2)\left(b+cx^2\right)dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(x^{9/2}(Ac+bB)+Abx^{5/2}+Bcx^{13/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{11}x^{11/2}(Ac+bB)+\frac{2}{7}Abx^{7/2}+\frac{2}{15}Bcx^{15/2}$$

input
```
Int[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

output
```
(2*A*b*x^(7/2))/7 + (2*(b*B + A*c)*x^(11/2))/11 + (2*B*c*x^(15/2))/15
```

3.162.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.162.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2Ab\,x^{\frac{7}{2}}}{7}+\frac{2(Ac+Bb)x^{\frac{11}{2}}}{11}+\frac{2Bc\,x^{\frac{15}{2}}}{15}$	28
default	$\frac{2Ab\,x^{\frac{7}{2}}}{7}+\frac{2(Ac+Bb)x^{\frac{11}{2}}}{11}+\frac{2Bc\,x^{\frac{15}{2}}}{15}$	28
gosper	$\frac{2x^{\frac{7}{2}}\left(77Bc\,x^{4}+105Ac\,x^{2}+105bB\,x^{2}+165Ab\right)}{1155}$	32
trager	$\frac{2x^{\frac{7}{2}}\left(77Bc\,x^{4}+105Ac\,x^{2}+105bB\,x^{2}+165Ab\right)}{1155}$	32
risch	$\frac{2x^{\frac{7}{2}}\left(77Bc\,x^{4}+105Ac\,x^{2}+105bB\,x^{2}+165Ab\right)}{1155}$	32

input
```
int((B*x^2+A)*(c*x^4+b*x^2)*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*A*b*x^(7/2)+2/11*(A*c+B*b)*x^(11/2)+2/15*B*c*x^(15/2)
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{1155}\left(77\,Bcx^7+105\,(Bb+Ac)x^5+165\,Abx^3\right)\sqrt{x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)*x^(1/2),x, algorithm="fricas")
```

output
```
2/1155*(77*B*c*x^7 + 105*(B*b + A*c)*x^5 + 165*A*b*x^3)*sqrt(x)
```

3.162.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2Abx^{\frac{7}{2}}}{7}+\frac{2Bcx^{\frac{15}{2}}}{15}+\frac{2x^{\frac{11}{2}}(Ac+Bb)}{11}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)*x**(1/2),x)
```

output
```
2*A*b*x**(7/2)/7 + 2*B*c*x**(15/2)/15 + 2*x**(11/2)*(A*c + B*b)/11
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{15}\,Bcx^{\frac{15}{2}}+\frac{2}{11}\,(Bb+Ac)x^{\frac{11}{2}}+\frac{2}{7}\,Abx^{\frac{7}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)*x^(1/2),x, algorithm="maxima")
```

output
```
2/15*B*c*x^(15/2) + 2/11*(B*b + A*c)*x^(11/2) + 2/7*A*b*x^(7/2)
```

3.162.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{15}\,Bcx^{\frac{15}{2}}+\frac{2}{11}\,Bbx^{\frac{11}{2}}+\frac{2}{11}\,Acx^{\frac{11}{2}}+\frac{2}{7}\,Abx^{\frac{7}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)*x^(1/2),x, algorithm="giac")
```

output
```
2/15*B*c*x^(15/2) + 2/11*B*b*x^(11/2) + 2/11*A*c*x^(11/2) + 2/7*A*b*x^(7/2
)
```

3.162.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2\,x^{7/2}\,(165\,A\,b+105\,A\,c\,x^2+105\,B\,b\,x^2+77\,B\,c\,x^4)}{1155}$$

input
```
int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4),x)
```

output
```
(2*x^(7/2)*(165*A*b + 105*A*c*x^2 + 105*B*b*x^2 + 77*B*c*x^4))/1155
```

3.163 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}}\,dx$

3.163.1 Optimal result

Integrand size = 24, antiderivative size = 39

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}}\,dx = \frac{2}{5}Abx^{5/2}+\frac{2}{9}(bB+Ac)x^{9/2}+\frac{2}{13}Bcx^{13/2}$$

output
```
2/5*A*b*x^(5/2)+2/9*(A*c+B*b)*x^(9/2)+2/13*B*c*x^(13/2)
```

3.163.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}}\,dx = \frac{2}{585}x^{5/2}\left(117Ab+65bBx^2+65Acx^2+45Bcx^4\right)$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/Sqrt[x],x]
```

output
```
(2*x^(5/2)*(117*A*b + 65*b*B*x^2 + 65*A*c*x^2 + 45*B*c*x^4))/585
```

3.163.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^{3/2}(A+Bx^2)(b+cx^2)dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(x^{7/2}(Ac+bB)+Abx^{3/2}+Bcx^{11/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{9}x^{9/2}(Ac+bB)+\frac{2}{5}Abx^{5/2}+\frac{2}{13}Bcx^{13/2}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4))/Sqrt[x],x]
```

output
```
(2*A*b*x^(5/2))/5 + (2*(b*B + A*c)*x^(9/2))/9 + (2*B*c*x^(13/2))/13
```

3.163.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.163.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2Ab\,x^{\frac{5}{2}}}{5}+\frac{2(Ac+Bb)x^{\frac{9}{2}}}{9}+\frac{2Bc\,x^{\frac{13}{2}}}{13}$	28
default	$\frac{2Ab\,x^{\frac{5}{2}}}{5}+\frac{2(Ac+Bb)x^{\frac{9}{2}}}{9}+\frac{2Bc\,x^{\frac{13}{2}}}{13}$	28
gosper	$\frac{2x^{\frac{5}{2}}\left(45Bc\,x^{4}+65Ac\,x^{2}+65bB\,x^{2}+117Ab\right)}{585}$	32
trager	$\frac{2x^{\frac{5}{2}}\left(45Bc\,x^{4}+65Ac\,x^{2}+65bB\,x^{2}+117Ab\right)}{585}$	32
risch	$\frac{2x^{\frac{5}{2}}\left(45Bc\,x^{4}+65Ac\,x^{2}+65bB\,x^{2}+117Ab\right)}{585}$	32

input
```
int((B*x^2+A)*(c*x^4+b*x^2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*A*b*x^(5/2)+2/9*(A*c+B*b)*x^(9/2)+2/13*B*c*x^(13/2)
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{\sqrt{x}}\,dx=\frac{2}{585}\left(45\,Bcx^6+65\,(Bb+Ac)x^4+117\,Abx^2\right)\sqrt{x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(1/2),x, algorithm="fricas")
```

output
```
2/585*(45*B*c*x^6 + 65*(B*b + A*c)*x^4 + 117*A*b*x^2)*sqrt(x)
```

3.163.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}}\,dx = \frac{2Abx^{\frac{5}{2}}}{5} + \frac{2Acx^{\frac{9}{2}}}{9} + \frac{2Bbx^{\frac{9}{2}}}{9} + \frac{2Bcx^{\frac{13}{2}}}{13}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(1/2),x)
```

output
```
2*A*b*x**(5/2)/5 + 2*A*c*x**(9/2)/9 + 2*B*b*x**(9/2)/9 + 2*B*c*x**(13/2)/1
3
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}}\,dx = \frac{2}{13}\,Bcx^{\frac{13}{2}} + \frac{2}{9}\,(Bb+Ac)x^{\frac{9}{2}} + \frac{2}{5}\,Abx^{\frac{5}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(1/2),x, algorithm="maxima")
```

output
```
2/13*B*c*x^(13/2) + 2/9*(B*b + A*c)*x^(9/2) + 2/5*A*b*x^(5/2)
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}}\,dx = \frac{2}{13}\,Bcx^{\frac{13}{2}} + \frac{2}{9}\,Bbx^{\frac{9}{2}} + \frac{2}{9}\,Acx^{\frac{9}{2}} + \frac{2}{5}\,Abx^{\frac{5}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(1/2),x, algorithm="giac")
```

output
```
2/13*B*c*x^(13/2) + 2/9*B*b*x^(9/2) + 2/9*A*c*x^(9/2) + 2/5*A*b*x^(5/2)
```

3.163.9 Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}}\,dx = \frac{2\,x^{5/2}\,(117\,A\,b+65\,A\,c\,x^2+65\,B\,b\,x^2+45\,B\,c\,x^4)}{585}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4))/x^(1/2),x)
```

output
```
(2*x^(5/2)*(117*A*b + 65*A*c*x^2 + 65*B*b*x^2 + 45*B*c*x^4))/585
```

3.164 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}} dx$

3.164.1 Optimal result

Integrand size = 24, antiderivative size = 39

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}} dx = \frac{2}{3}Abx^{3/2} + \frac{2}{7}(bB+Ac)x^{7/2} + \frac{2}{11}Bcx^{11/2}$$

output
```
2/3*A*b*x^(3/2)+2/7*(A*c+B*b)*x^(7/2)+2/11*B*c*x^(11/2)
```

3.164.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}} dx = \frac{2}{231}x^{3/2}(77Ab+33bBx^2+33Acx^2+21Bcx^4)$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^(3/2),x]
```

output
```
(2*x^(3/2)*(77*A*b + 33*b*B*x^2 + 33*A*c*x^2 + 21*B*c*x^4))/231
```

3.164.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \sqrt{x}(A+Bx^2)(b+cx^2)\,dx$$

$$\Big\downarrow \; 355$$

$$\int \left(x^{5/2}(Ac+bB)+Ab\sqrt{x}+Bcx^{9/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{7}x^{7/2}(Ac+bB)+\frac{2}{3}Abx^{3/2}+\frac{2}{11}Bcx^{11/2}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^(3/2),x]
```

output
```
(2*A*b*x^(3/2))/3 + (2*(b*B + A*c)*x^(7/2))/7 + (2*B*c*x^(11/2))/11
```

3.164.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.164.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2Ab\,x^{\frac{3}{2}}}{3}+\frac{2(Ac+Bb)x^{\frac{7}{2}}}{7}+\frac{2Bc\,x^{\frac{11}{2}}}{11}$	28
default	$\frac{2Ab\,x^{\frac{3}{2}}}{3}+\frac{2(Ac+Bb)x^{\frac{7}{2}}}{7}+\frac{2Bc\,x^{\frac{11}{2}}}{11}$	28
gosper	$\frac{2x^{\frac{3}{2}}\left(21Bc\,x^{4}+33Ac\,x^{2}+33bB\,x^{2}+77Ab\right)}{231}$	32
trager	$\frac{2x^{\frac{3}{2}}\left(21Bc\,x^{4}+33Ac\,x^{2}+33bB\,x^{2}+77Ab\right)}{231}$	32
risch	$\frac{2x^{\frac{3}{2}}\left(21Bc\,x^{4}+33Ac\,x^{2}+33bB\,x^{2}+77Ab\right)}{231}$	32

input
```
int((B*x^2+A)*(c*x^4+b*x^2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*A*b*x^(3/2)+2/7*(A*c+B*b)*x^(7/2)+2/11*B*c*x^(11/2)
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}}\,dx = \frac{2}{231}\left(21\,Bcx^5+33\,(Bb+Ac)x^3+77\,Abx\right)\sqrt{x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(3/2),x, algorithm="fricas")
```

output
```
2/231*(21*B*c*x^5 + 33*(B*b + A*c)*x^3 + 77*A*b*x)*sqrt(x)
```

3.164.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}}\,dx = \frac{2Abx^{\frac{3}{2}}}{3} + \frac{2Acx^{\frac{7}{2}}}{7} + \frac{2Bbx^{\frac{7}{2}}}{7} + \frac{2Bcx^{\frac{11}{2}}}{11}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(3/2),x)
```

output
```
2*A*b*x**(3/2)/3 + 2*A*c*x**(7/2)/7 + 2*B*b*x**(7/2)/7 + 2*B*c*x**(11/2)/1
1
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}}\,dx = \frac{2}{11}\,Bcx^{\frac{11}{2}} + \frac{2}{7}\,(Bb+Ac)x^{\frac{7}{2}} + \frac{2}{3}\,Abx^{\frac{3}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(3/2),x, algorithm="maxima")
```

output
```
2/11*B*c*x^(11/2) + 2/7*(B*b + A*c)*x^(7/2) + 2/3*A*b*x^(3/2)
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}}\,dx = \frac{2}{11}\,Bcx^{\frac{11}{2}} + \frac{2}{7}\,Bbx^{\frac{7}{2}} + \frac{2}{7}\,Acx^{\frac{7}{2}} + \frac{2}{3}\,Abx^{\frac{3}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(3/2),x, algorithm="giac")
```

output
```
2/11*B*c*x^(11/2) + 2/7*B*b*x^(7/2) + 2/7*A*c*x^(7/2) + 2/3*A*b*x^(3/2)
```

3.164.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}}\,dx = \frac{2\,x^{3/2}\,(77\,A\,b+33\,A\,c\,x^2+33\,B\,b\,x^2+21\,B\,c\,x^4)}{231}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4))/x^(3/2),x)
```

output
```
(2*x^(3/2)*(77*A*b + 33*A*c*x^2 + 33*B*b*x^2 + 21*B*c*x^4))/231
```

3.165 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx$

3.165.1 Optimal result

Integrand size = 24, antiderivative size = 37

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx = 2Ab\sqrt{x}+\frac{2}{5}(bB+Ac)x^{5/2}+\frac{2}{9}Bcx^{9/2}$$

output
```
2/5*(A*c+B*b)*x^(5/2)+2/9*B*c*x^(9/2)+2*A*b*x^(1/2)
```

3.165.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx = \frac{2}{45}\sqrt{x}\left(45Ab+9bBx^2+9Acx^2+5Bcx^4\right)$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^(5/2),x]
```

output
```
(2*Sqrt[x]*(45*A*b + 9*b*B*x^2 + 9*A*c*x^2 + 5*B*c*x^4))/45
```

3.165.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(A+Bx^2)(b+cx^2)}{\sqrt{x}}dx$$

$$\Big\downarrow \; 355$$

$$\int \left(x^{3/2}(Ac+bB)+\frac{Ab}{\sqrt{x}}+Bcx^{7/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{5}x^{5/2}(Ac+bB)+2Ab\sqrt{x}+\frac{2}{9}Bcx^{9/2}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^(5/2),x]
```

output
```
2*A*b*Sqrt[x] + (2*(b*B + A*c)*x^(5/2))/5 + (2*B*c*x^(9/2))/9
```

3.165.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.165.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{2(Ac+Bb)x^{\frac{5}{2}}}{5}+\frac{2Bc\,x^{\frac{9}{2}}}{9}+2Ab\sqrt{x}$	28
default	$\frac{2(Ac+Bb)x^{\frac{5}{2}}}{5}+\frac{2Bc\,x^{\frac{9}{2}}}{9}+2Ab\sqrt{x}$	28
trager	$\left(\frac{2}{9}Bc\,x^{4}+\frac{2}{5}Ac\,x^{2}+\frac{2}{5}bB\,x^{2}+2Ab\right)\sqrt{x}$	31
gosper	$\frac{2\sqrt{x}\,(5Bc\,x^{4}+9Ac\,x^{2}+9bB\,x^{2}+45Ab)}{45}$	32
risch	$\frac{2\sqrt{x}\,(5Bc\,x^{4}+9Ac\,x^{2}+9bB\,x^{2}+45Ab)}{45}$	32

input
```
int((B*x^2+A)*(c*x^4+b*x^2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(A*c+B*b)*x^(5/2)+2/9*B*c*x^(9/2)+2*A*b*x^(1/2)
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx=\frac{2}{45}\left(5\,Bcx^4+9\,(Bb+Ac)x^2+45\,Ab\right)\sqrt{x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(5/2),x, algorithm="fricas")
```

output
```
2/45*(5*B*c*x^4 + 9*(B*b + A*c)*x^2 + 45*A*b)*sqrt(x)
```

3.165.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx = 2Ab\sqrt{x} + \frac{2Acx^{\frac{5}{2}}}{5} + \frac{2Bbx^{\frac{5}{2}}}{5} + \frac{2Bcx^{\frac{9}{2}}}{9}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(5/2),x)
```

output
```
2*A*b*sqrt(x) + 2*A*c*x**(5/2)/5 + 2*B*b*x**(5/2)/5 + 2*B*c*x**(9/2)/9
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx = \frac{2}{9}\,Bcx^{\frac{9}{2}} + \frac{2}{5}\,(Bb+Ac)x^{\frac{5}{2}} + 2\,Ab\sqrt{x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(5/2),x, algorithm="maxima")
```

output
```
2/9*B*c*x^(9/2) + 2/5*(B*b + A*c)*x^(5/2) + 2*A*b*sqrt(x)
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx = \frac{2}{9}\,Bcx^{\frac{9}{2}} + \frac{2}{5}\,Bbx^{\frac{5}{2}} + \frac{2}{5}\,Acx^{\frac{5}{2}} + 2\,Ab\sqrt{x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(5/2),x, algorithm="giac")
```

output
```
2/9*B*c*x^(9/2) + 2/5*B*b*x^(5/2) + 2/5*A*c*x^(5/2) + 2*A*b*sqrt(x)
```

3.165.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx = \frac{2\sqrt{x}\,(45\,A\,b+9\,A\,c\,x^2+9\,B\,b\,x^2+5\,B\,c\,x^4)}{45}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4))/x^(5/2),x)
```

output
```
(2*x^(1/2)*(45*A*b + 9*A*c*x^2 + 9*B*b*x^2 + 5*B*c*x^4))/45
```

3.166 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}} dx$

3.166.1 Optimal result

Integrand size = 24, antiderivative size = 37

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}} dx = -\frac{2Ab}{\sqrt{x}} + \frac{2}{3}(bB+Ac)x^{3/2} + \frac{2}{7}Bcx^{7/2}$$

output
```
2/3*(A*c+B*b)*x^(3/2)+2/7*B*c*x^(7/2)-2*A*b/x^(1/2)
```

3.166.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}} dx = \frac{2(-21Ab+7bBx^2+7Acx^2+3Bcx^4)}{21\sqrt{x}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^(7/2),x]
```

output
```
(2*(-21*A*b + 7*b*B*x^2 + 7*A*c*x^2 + 3*B*c*x^4))/(21*Sqrt[x])
```

3.166.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(A+Bx^2)(b+cx^2)}{x^{3/2}}\,dx$$

$$\Big\downarrow \; 355$$

$$\int \left(\sqrt{x}(Ac+bB)+\frac{Ab}{x^{3/2}}+Bcx^{5/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{3}x^{3/2}(Ac+bB)-\frac{2Ab}{\sqrt{x}}+\frac{2}{7}Bcx^{7/2}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^(7/2),x]
```

output
```
(-2*A*b)/Sqrt[x] + (2*(b*B + A*c)*x^(3/2))/3 + (2*B*c*x^(7/2))/7
```

3.166.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.166.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2Bc\,x^{\frac{7}{2}}}{7}+\frac{2Ac\,x^{\frac{3}{2}}}{3}+\frac{2bB\,x^{\frac{3}{2}}}{3}-\frac{2Ab}{\sqrt{x}}$	30
default	$\frac{2Bc\,x^{\frac{7}{2}}}{7}+\frac{2Ac\,x^{\frac{3}{2}}}{3}+\frac{2bB\,x^{\frac{3}{2}}}{3}-\frac{2Ab}{\sqrt{x}}$	30
gosper	$-\frac{2(-3Bc\,x^{4}-7Ac\,x^{2}-7bB\,x^{2}+21Ab)}{21\sqrt{x}}$	32
trager	$-\frac{2(-3Bc\,x^{4}-7Ac\,x^{2}-7bB\,x^{2}+21Ab)}{21\sqrt{x}}$	32
risch	$-\frac{2(-3Bc\,x^{4}-7Ac\,x^{2}-7bB\,x^{2}+21Ab)}{21\sqrt{x}}$	32

input
```
int((B*x^2+A)*(c*x^4+b*x^2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*B*c*x^(7/2)+2/3*A*c*x^(3/2)+2/3*b*B*x^(3/2)-2*A*b/x^(1/2)
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}}\,dx=\frac{2\,(3\,Bcx^4+7\,(Bb+Ac)x^2-21\,Ab)}{21\,\sqrt{x}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(7/2),x, algorithm="fricas")
```

output
```
2/21*(3*B*c*x^4 + 7*(B*b + A*c)*x^2 - 21*A*b)/sqrt(x)
```

3.166.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}}\,dx = -\frac{2Ab}{\sqrt{x}} + \frac{2Acx^{\frac{3}{2}}}{3} + \frac{2Bbx^{\frac{3}{2}}}{3} + \frac{2Bcx^{\frac{7}{2}}}{7}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(7/2),x)
```

output
```
-2*A*b/sqrt(x) + 2*A*c*x**(3/2)/3 + 2*B*b*x**(3/2)/3 + 2*B*c*x**(7/2)/7
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}}\,dx = \frac{2}{7}\,Bcx^{\frac{7}{2}} + \frac{2}{3}\,(Bb+Ac)x^{\frac{3}{2}} - \frac{2\,Ab}{\sqrt{x}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(7/2),x, algorithm="maxima")
```

output
```
2/7*B*c*x^(7/2) + 2/3*(B*b + A*c)*x^(3/2) - 2*A*b/sqrt(x)
```

3.166.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}}\,dx = \frac{2}{7}\,Bcx^{\frac{7}{2}} + \frac{2}{3}\,Bbx^{\frac{3}{2}} + \frac{2}{3}\,Acx^{\frac{3}{2}} - \frac{2\,Ab}{\sqrt{x}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(7/2),x, algorithm="giac")
```

output
```
2/7*B*c*x^(7/2) + 2/3*B*b*x^(3/2) + 2/3*A*c*x^(3/2) - 2*A*b/sqrt(x)
```

3.166.9 Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}}\,dx = \frac{14\,A\,c\,x^2-42\,A\,b+14\,B\,b\,x^2+6\,B\,c\,x^4}{21\,\sqrt{x}}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4))/x^(7/2),x)
```

output
```
(14*A*c*x^2 - 42*A*b + 14*B*b*x^2 + 6*B*c*x^4)/(21*x^(1/2))
```

3.167 $\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$

3.167.1 Optimal result

Integrand size = 26, antiderivative size = 63

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{17}Ab^2x^{17/2}+\frac{2}{21}b(bB+2Ac)x^{21/2}+\frac{2}{25}c(2bB+Ac)x^{25/2}+\frac{2}{29}Bc^2x^{29/2}$$

output
```
2/17*A*b^2*x^(17/2)+2/21*b*(2*A*c+B*b)*x^(21/2)+2/25*c*(A*c+2*B*b)*x^(25/2
)+2/29*B*c^2*x^(29/2)
```

3.167.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2x^{17/2}(29A(525b^2+850bcx^2+357c^2x^4)+17Bx^2(725b^2+1218bcx^2+525c^2x^4))}{258825}$$

input
```
Integrate[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

output
```
(2*x^(17/2)*(29*A*(525*b^2 + 850*b*c*x^2 + 357*c^2*x^4) + 17*B*x^2*(725*b^
2 + 1218*b*c*x^2 + 525*c^2*x^4)))/258825
```

3.167.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$$

$$\Big\downarrow \; 9$$

$$\int x^{15/2}(A+Bx^2)\left(b+cx^2\right)^2dx$$

$$\Big\downarrow \; 355$$

$$\int \left(Ab^2x^{15/2}+cx^{23/2}(Ac+2bB)+bx^{19/2}(2Ac+bB)+Bc^2x^{27/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{17}Ab^2x^{17/2}+\frac{2}{25}cx^{25/2}(Ac+2bB)+\frac{2}{21}bx^{21/2}(2Ac+bB)+\frac{2}{29}Bc^2x^{29/2}$$

input
```
Int[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

output
```
(2*A*b^2*x^(17/2))/17 + (2*b*(b*B + 2*A*c)*x^(21/2))/21 + (2*c*(2*b*B + A*
c)*x^(25/2))/25 + (2*B*c^2*x^(29/2))/29
```

3.167.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.167.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2B\,c^{2}x^{\frac{29}{2}}}{29}+\frac{2(A\,c^{2}+2Bbc)x^{\frac{25}{2}}}{25}+\frac{2(2Abc+B\,b^{2})x^{\frac{21}{2}}}{21}+\frac{2A\,b^{2}x^{\frac{17}{2}}}{17}$	52
default	$\frac{2B\,c^{2}x^{\frac{29}{2}}}{29}+\frac{2(A\,c^{2}+2Bbc)x^{\frac{25}{2}}}{25}+\frac{2(2Abc+B\,b^{2})x^{\frac{21}{2}}}{21}+\frac{2A\,b^{2}x^{\frac{17}{2}}}{17}$	52
gosper	$\frac{2x^{\frac{17}{2}}(8925B\,c^{2}x^{6}+10353A\,c^{2}x^{4}+20706x^{4}Bbc+24650Abc\,x^{2}+12325b^{2}B\,x^{2}+15225b^{2}A)}{258825}$	56
trager	$\frac{2x^{\frac{17}{2}}(8925B\,c^{2}x^{6}+10353A\,c^{2}x^{4}+20706x^{4}Bbc+24650Abc\,x^{2}+12325b^{2}B\,x^{2}+15225b^{2}A)}{258825}$	56
risch	$\frac{2x^{\frac{17}{2}}(8925B\,c^{2}x^{6}+10353A\,c^{2}x^{4}+20706x^{4}Bbc+24650Abc\,x^{2}+12325b^{2}B\,x^{2}+15225b^{2}A)}{258825}$	56

input
```
int(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2/29*B*c^2*x^(29/2)+2/25*(A*c^2+2*B*b*c)*x^(25/2)+2/21*(2*A*b*c+B*b^2)*x^(
21/2)+2/17*A*b^2*x^(17/2)
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{258825}\left(8925\,Bc^2x^{14}+10353\left(2\,Bbc+Ac^2\right)x^{12}+15225\,Ab^2x^8+12325\left(Bb^2+2\,Abc\right)x^{10}\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
2/258825*(8925*B*c^2*x^14 + 10353*(2*B*b*c + A*c^2)*x^12 + 15225*A*b^2*x^8
 + 12325*(B*b^2 + 2*A*b*c)*x^10)*sqrt(x)
```

3.167.6 Sympy [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2Ab^2x^{\frac{17}{2}}}{17}+\frac{4Abcx^{\frac{21}{2}}}{21}+\frac{2Ac^2x^{\frac{25}{2}}}{25}+\frac{2Bb^2x^{\frac{21}{2}}}{21}+\frac{4Bbcx^{\frac{25}{2}}}{25}+\frac{2Bc^2x^{\frac{29}{2}}}{29}$$

input
```
integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

output
```
2*A*b**2*x**(17/2)/17 + 4*A*b*c*x**(21/2)/21 + 2*A*c**2*x**(25/2)/25 + 2*B
*b**2*x**(21/2)/21 + 4*B*b*c*x**(25/2)/25 + 2*B*c**2*x**(29/2)/29
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{29}\,Bc^2x^{\frac{29}{2}}+\frac{2}{25}\,(2\,Bbc+Ac^2)x^{\frac{25}{2}}+\frac{2}{17}\,Ab^2x^{\frac{17}{2}}+\frac{2}{21}\,(Bb^2+2\,Abc)x^{\frac{21}{2}}$$

input
```
integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
2/29*B*c^2*x^(29/2) + 2/25*(2*B*b*c + A*c^2)*x^(25/2) + 2/17*A*b^2*x^(17/2
) + 2/21*(B*b^2 + 2*A*b*c)*x^(21/2)
```

3.167.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{29}\,Bc^2x^{\frac{29}{2}}+\frac{4}{25}\,Bbcx^{\frac{25}{2}}+\frac{2}{25}\,Ac^2x^{\frac{25}{2}}+\frac{2}{21}\,Bb^2x^{\frac{21}{2}}+\frac{4}{21}\,Abcx^{\frac{21}{2}}+\frac{2}{17}\,Ab^2x^{\frac{17}{2}}$$

input
```
integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
2/29*B*c^2*x^(29/2) + 4/25*B*b*c*x^(25/2) + 2/25*A*c^2*x^(25/2) + 2/21*B*b
^2*x^(21/2) + 4/21*A*b*c*x^(21/2) + 2/17*A*b^2*x^(17/2)
```

3.167.9 Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx = x^{21/2}\left(\frac{2\,B\,b^2}{21}+\frac{4\,A\,c\,b}{21}\right) + x^{25/2}\left(\frac{2\,A\,c^2}{25}+\frac{4\,B\,b\,c}{25}\right)+\frac{2\,A\,b^2\,x^{17/2}}{17}+\frac{2\,B\,c^2\,x^{29/2}}{29}$$

input
```
int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

output
```
x^(21/2)*((2*B*b^2)/21 + (4*A*b*c)/21) + x^(25/2)*((2*A*c^2)/25 + (4*B*b*c
)/25) + (2*A*b^2*x^(17/2))/17 + (2*B*c^2*x^(29/2))/29
```

3.168 $\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$

3.168.1 Optimal result

Integrand size = 26, antiderivative size = 63

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{15}Ab^2x^{15/2}+\frac{2}{19}b(bB+2Ac)x^{19/2}+\frac{2}{23}c(2bB+Ac)x^{23/2}+\frac{2}{27}Bc^2x^{27/2}$$

output
```
2/15*A*b^2*x^(15/2)+2/19*b*(2*A*c+B*b)*x^(19/2)+2/23*c*(A*c+2*B*b)*x^(23/2
)+2/27*B*c^2*x^(27/2)
```

3.168.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2x^{15/2}(9A(437b^2+690bcx^2+285c^2x^4)+5Bx^2(621b^2+1026bcx^2+437c^2x^4))}{58995}$$

input
```
Integrate[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

output
```
(2*x^(15/2)*(9*A*(437*b^2 + 690*b*c*x^2 + 285*c^2*x^4) + 5*B*x^2*(621*b^2
+ 1026*b*c*x^2 + 437*c^2*x^4)))/58995
```

3.168.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$$

$$\Big\downarrow \; 9$$

$$\int x^{13/2}(A+Bx^2)\left(b+cx^2\right)^2\,dx$$

$$\Big\downarrow \; 355$$

$$\int \left(Ab^2x^{13/2}+cx^{21/2}(Ac+2bB)+bx^{17/2}(2Ac+bB)+Bc^2x^{25/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{15}Ab^2x^{15/2}+\frac{2}{23}cx^{23/2}(Ac+2bB)+\frac{2}{19}bx^{19/2}(2Ac+bB)+\frac{2}{27}Bc^2x^{27/2}$$

input
```
Int[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

output
```
(2*A*b^2*x^(15/2))/15 + (2*b*(b*B + 2*A*c)*x^(19/2))/19 + (2*c*(2*b*B + A*
c)*x^(23/2))/23 + (2*B*c^2*x^(27/2))/27
```

3.168.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.168.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2B\,c^2 x^{\frac{27}{2}}}{27}+\frac{2(A\,c^2+2Bbc)\,x^{\frac{23}{2}}}{23}+\frac{2(2Abc+B\,b^2)\,x^{\frac{19}{2}}}{19}+\frac{2A\,b^2 x^{\frac{15}{2}}}{15}$	52
default	$\frac{2B\,c^2 x^{\frac{27}{2}}}{27}+\frac{2(A\,c^2+2Bbc)\,x^{\frac{23}{2}}}{23}+\frac{2(2Abc+B\,b^2)\,x^{\frac{19}{2}}}{19}+\frac{2A\,b^2 x^{\frac{15}{2}}}{15}$	52
gosper	$\frac{2x^{\frac{15}{2}}(2185B\,c^2x^6+2565A\,c^2x^4+5130x^4Bbc+6210Abc\,x^2+3105b^2B\,x^2+3933b^2A)}{58995}$	56
trager	$\frac{2x^{\frac{15}{2}}(2185B\,c^2x^6+2565A\,c^2x^4+5130x^4Bbc+6210Abc\,x^2+3105b^2B\,x^2+3933b^2A)}{58995}$	56
risch	$\frac{2x^{\frac{15}{2}}(2185B\,c^2x^6+2565A\,c^2x^4+5130x^4Bbc+6210Abc\,x^2+3105b^2B\,x^2+3933b^2A)}{58995}$	56

input
```
int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2/27*B*c^2*x^(27/2)+2/23*(A*c^2+2*B*b*c)*x^(23/2)+2/19*(2*A*b*c+B*b^2)*x^(
19/2)+2/15*A*b^2*x^(15/2)
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{58995}\left(2185\,Bc^2x^{13}+2565\left(2\,Bbc+Ac^2\right)x^{11}+3933\,Ab^2x^7+3105\left(Bb^2+2\,Abc\right)x^9\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
2/58995*(2185*B*c^2*x^13 + 2565*(2*B*b*c + A*c^2)*x^11 + 3933*A*b^2*x^7 +
3105*(B*b^2 + 2*A*b*c)*x^9)*sqrt(x)
```

3.168.6 Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2Ab^2x^{\frac{15}{2}}}{15}+\frac{4Abcx^{\frac{19}{2}}}{19}+\frac{2Ac^2x^{\frac{23}{2}}}{23}+\frac{2Bb^2x^{\frac{19}{2}}}{19}+\frac{4Bbcx^{\frac{23}{2}}}{23}+\frac{2Bc^2x^{\frac{27}{2}}}{27}$$

input
```
integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

output
```
2*A*b**2*x**(15/2)/15 + 4*A*b*c*x**(19/2)/19 + 2*A*c**2*x**(23/2)/23 + 2*B
*b**2*x**(19/2)/19 + 4*B*b*c*x**(23/2)/23 + 2*B*c**2*x**(27/2)/27
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{27}Bc^2x^{\frac{27}{2}}+\frac{2}{23}\left(2\,Bbc+Ac^2\right)x^{\frac{23}{2}}+\frac{2}{15}Ab^2x^{\frac{15}{2}}+\frac{2}{19}\left(Bb^2+2\,Abc\right)x^{\frac{19}{2}}$$

input
```
integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
2/27*B*c^2*x^(27/2) + 2/23*(2*B*b*c + A*c^2)*x^(23/2) + 2/15*A*b^2*x^(15/2
) + 2/19*(B*b^2 + 2*A*b*c)*x^(19/2)
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{27}Bc^2x^{\frac{27}{2}}+\frac{4}{23}Bbcx^{\frac{23}{2}}+\frac{2}{23}Ac^2x^{\frac{23}{2}}+\frac{2}{19}Bb^2x^{\frac{19}{2}}+\frac{4}{19}Abcx^{\frac{19}{2}}+\frac{2}{15}Ab^2x^{\frac{15}{2}}$$

input
```
integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
2/27*B*c^2*x^(27/2) + 4/23*B*b*c*x^(23/2) + 2/23*A*c^2*x^(23/2) + 2/19*B*b
^2*x^(19/2) + 4/19*A*b*c*x^(19/2) + 2/15*A*b^2*x^(15/2)
```

3.168.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=x^{19/2}\left(\frac{2\,B\,b^2}{19}+\frac{4\,A\,c\,b}{19}\right)+x^{23/2}\left(\frac{2\,A\,c^2}{23}+\frac{4\,B\,b\,c}{23}\right)+\frac{2\,A\,b^2\,x^{15/2}}{15}+\frac{2\,B\,c^2\,x^{27/2}}{27}$$

input
```
int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

output
```
x^(19/2)*((2*B*b^2)/19 + (4*A*b*c)/19) + x^(23/2)*((2*A*c^2)/23 + (4*B*b*c
)/23) + (2*A*b^2*x^(15/2))/15 + (2*B*c^2*x^(27/2))/27
```

3.169 $\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$

3.169.1 Optimal result

Integrand size = 26, antiderivative size = 63

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{13}Ab^2x^{13/2}+\frac{2}{17}b(bB+2Ac)x^{17/2}+\frac{2}{21}c(2bB+Ac)x^{21/2}+\frac{2}{25}Bc^2x^{25/2}$$

output
```
2/13*A*b^2*x^(13/2)+2/17*b*(2*A*c+B*b)*x^(17/2)+2/21*c*(A*c+2*B*b)*x^(21/2
)+2/25*B*c^2*x^(25/2)
```

3.169.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2x^{13/2}(25A(357b^2+546bcx^2+221c^2x^4)+13Bx^2(525b^2+850bcx^2+357c^2x^4))}{116025}$$

input
```
Integrate[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

output
```
(2*x^(13/2)*(25*A*(357*b^2 + 546*b*c*x^2 + 221*c^2*x^4) + 13*B*x^2*(525*b^
2 + 850*b*c*x^2 + 357*c^2*x^4)))/116025
```

3.169.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx$$
$$\Big\downarrow \ 9$$
$$\int x^{11/2}\left(A+Bx^2\right)\left(b+cx^2\right)^2dx$$
$$\Big\downarrow \ 355$$
$$\int \left(Ab^2x^{11/2}+cx^{19/2}(Ac+2bB)+bx^{15/2}(2Ac+bB)+Bc^2x^{23/2}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{2}{13}Ab^2x^{13/2}+\frac{2}{21}cx^{21/2}(Ac+2bB)+\frac{2}{17}bx^{17/2}(2Ac+bB)+\frac{2}{25}Bc^2x^{25/2}$$

input
```
Int[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

output
```
(2*A*b^2*x^(13/2))/13 + (2*b*(b*B + 2*A*c)*x^(17/2))/17 + (2*c*(2*b*B + A*
c)*x^(21/2))/21 + (2*B*c^2*x^(25/2))/25
```

3.169.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.169.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2Bc^2x^{\frac{25}{2}}}{25}+\frac{2(Ac^2+2Bbc)x^{\frac{21}{2}}}{21}+\frac{2(2Abc+Bb^2)x^{\frac{17}{2}}}{17}+\frac{2Ab^2x^{\frac{13}{2}}}{13}$	52
default	$\frac{2Bc^2x^{\frac{25}{2}}}{25}+\frac{2(Ac^2+2Bbc)x^{\frac{21}{2}}}{21}+\frac{2(2Abc+Bb^2)x^{\frac{17}{2}}}{17}+\frac{2Ab^2x^{\frac{13}{2}}}{13}$	52
gosper	$\frac{2x^{\frac{13}{2}}(4641Bc^2x^6+5525Ac^2x^4+11050x^4Bbc+13650Abcx^2+6825b^2Bx^2+8925b^2A)}{116025}$	56
trager	$\frac{2x^{\frac{13}{2}}(4641Bc^2x^6+5525Ac^2x^4+11050x^4Bbc+13650Abcx^2+6825b^2Bx^2+8925b^2A)}{116025}$	56
risch	$\frac{2x^{\frac{13}{2}}(4641Bc^2x^6+5525Ac^2x^4+11050x^4Bbc+13650Abcx^2+6825b^2Bx^2+8925b^2A)}{116025}$	56

input
```
int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2/25*B*c^2*x^(25/2)+2/21*(A*c^2+2*B*b*c)*x^(21/2)+2/17*(2*A*b*c+B*b^2)*x^(
17/2)+2/13*A*b^2*x^(13/2)
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{116025}\left(4641\,Bc^2x^{12}+5525\left(2\,Bbc+Ac^2\right)x^{10}+8925\,Ab^2x^6+6825\left(Bb^2+2\,Abc\right)x^8\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
2/116025*(4641*B*c^2*x^12 + 5525*(2*B*b*c + A*c^2)*x^10 + 8925*A*b^2*x^6 +
 6825*(B*b^2 + 2*A*b*c)*x^8)*sqrt(x)
```

3.169.6 Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2Ab^2x^{\frac{13}{2}}}{13}+\frac{4Abcx^{\frac{17}{2}}}{17}+\frac{2Ac^2x^{\frac{21}{2}}}{21}+\frac{2Bb^2x^{\frac{17}{2}}}{17}+\frac{4Bbcx^{\frac{21}{2}}}{21}+\frac{2Bc^2x^{\frac{25}{2}}}{25}$$

input
```
integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

output
```
2*A*b**2*x**(13/2)/13 + 4*A*b*c*x**(17/2)/17 + 2*A*c**2*x**(21/2)/21 + 2*B
*b**2*x**(17/2)/17 + 4*B*b*c*x**(21/2)/21 + 2*B*c**2*x**(25/2)/25
```

3.169.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{25}\,Bc^2x^{\frac{25}{2}}+\frac{2}{21}\,(2\,Bbc+Ac^2)x^{\frac{21}{2}}+\frac{2}{13}\,Ab^2x^{\frac{13}{2}}+\frac{2}{17}\,(Bb^2+2\,Abc)x^{\frac{17}{2}}$$

input
```
integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
2/25*B*c^2*x^(25/2) + 2/21*(2*B*b*c + A*c^2)*x^(21/2) + 2/13*A*b^2*x^(13/2
) + 2/17*(B*b^2 + 2*A*b*c)*x^(17/2)
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{25}\,Bc^2x^{\frac{25}{2}}+\frac{4}{21}\,Bbcx^{\frac{21}{2}}+\frac{2}{21}\,Ac^2x^{\frac{21}{2}}+\frac{2}{17}\,Bb^2x^{\frac{17}{2}}+\frac{4}{17}\,Abcx^{\frac{17}{2}}+\frac{2}{13}\,Ab^2x^{\frac{13}{2}}$$

input
```
integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
2/25*B*c^2*x^(25/2) + 4/21*B*b*c*x^(21/2) + 2/21*A*c^2*x^(21/2) + 2/17*B*b
^2*x^(17/2) + 4/17*A*b*c*x^(17/2) + 2/13*A*b^2*x^(13/2)
```

3.169.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=x^{17/2}\left(\frac{2\,B\,b^2}{17}+\frac{4\,A\,c\,b}{17}\right)+x^{21/2}\left(\frac{2\,A\,c^2}{21}+\frac{4\,B\,b\,c}{21}\right)+\frac{2\,A\,b^2\,x^{13/2}}{13}+\frac{2\,B\,c^2\,x^{25/2}}{25}$$

input
```
int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

output
```
x^(17/2)*((2*B*b^2)/17 + (4*A*b*c)/17) + x^(21/2)*((2*A*c^2)/21 + (4*B*b*c
)/21) + (2*A*b^2*x^(13/2))/13 + (2*B*c^2*x^(25/2))/25
```

3.170 $\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$

3.170.1 Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{11}Ab^2x^{11/2}+\frac{2}{15}b(bB+2Ac)x^{15/2}+\frac{2}{19}c(2bB+Ac)x^{19/2}+\frac{2}{23}Bc^2x^{23/2}$$

output
```
2/11*A*b^2*x^(11/2)+2/15*b*(2*A*c+B*b)*x^(15/2)+2/19*c*(A*c+2*B*b)*x^(19/2
)+2/23*B*c^2*x^(23/2)
```

3.170.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2x^{11/2}(23A(285b^2+418bcx^2+165c^2x^4)+11Bx^2(437b^2+690bcx^2+285c^2x^4))}{72105}$$

input
```
Integrate[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

output
```
(2*x^(11/2)*(23*A*(285*b^2 + 418*b*c*x^2 + 165*c^2*x^4) + 11*B*x^2*(437*b^
2 + 690*b*c*x^2 + 285*c^2*x^4)))/72105
```

3.170.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^2 dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^{9/2}(A+Bx^2)\left(b+cx^2\right)^2 dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(Ab^2x^{9/2}+cx^{17/2}(Ac+2bB)+bx^{13/2}(2Ac+bB)+Bc^2x^{21/2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{11}Ab^2x^{11/2}+\frac{2}{19}cx^{19/2}(Ac+2bB)+\frac{2}{15}bx^{15/2}(2Ac+bB)+\frac{2}{23}Bc^2x^{23/2}$$

input
```
Int[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

output
```
(2*A*b^2*x^(11/2))/11 + (2*b*(b*B + 2*A*c)*x^(15/2))/15 + (2*c*(2*b*B + A*
c)*x^(19/2))/19 + (2*B*c^2*x^(23/2))/23
```

3.170.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.170.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2Bc^2x^{\frac{23}{2}}}{23}+\frac{2(Ac^2+2Bbc)x^{\frac{19}{2}}}{19}+\frac{2(2Abc+Bb^2)x^{\frac{15}{2}}}{15}+\frac{2Ab^2x^{\frac{11}{2}}}{11}$	52
default	$\frac{2Bc^2x^{\frac{23}{2}}}{23}+\frac{2(Ac^2+2Bbc)x^{\frac{19}{2}}}{19}+\frac{2(2Abc+Bb^2)x^{\frac{15}{2}}}{15}+\frac{2Ab^2x^{\frac{11}{2}}}{11}$	52
gosper	$\frac{2x^{\frac{11}{2}}(3135Bc^2x^6+3795Ac^2x^4+7590x^4Bbc+9614Abcx^2+4807b^2Bx^2+6555b^2A)}{72105}$	56
trager	$\frac{2x^{\frac{11}{2}}(3135Bc^2x^6+3795Ac^2x^4+7590x^4Bbc+9614Abcx^2+4807b^2Bx^2+6555b^2A)}{72105}$	56
risch	$\frac{2x^{\frac{11}{2}}(3135Bc^2x^6+3795Ac^2x^4+7590x^4Bbc+9614Abcx^2+4807b^2Bx^2+6555b^2A)}{72105}$	56

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^2*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/23*B*c^2*x^(23/2)+2/19*(A*c^2+2*B*b*c)*x^(19/2)+2/15*(2*A*b*c+B*b^2)*x^(
15/2)+2/11*A*b^2*x^(11/2)
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx = \frac{2}{72105}\left(3135\,Bc^2x^{11}+3795\left(2\,Bbc+Ac^2\right)x^9+6555\,Ab^2x^5+4807\left(Bb^2+2\,Abc\right)x^7\right)\sqrt{x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2*x^(1/2),x, algorithm="fricas")
```

output
```
2/72105*(3135*B*c^2*x^11 + 3795*(2*B*b*c + A*c^2)*x^9 + 6555*A*b^2*x^5 + 4
807*(B*b^2 + 2*A*b*c)*x^7)*sqrt(x)
```

3.170.6 Sympy [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx = \frac{2Ab^2x^{\frac{11}{2}}}{11}+\frac{2Bc^2x^{\frac{23}{2}}}{23}+\frac{2x^{\frac{19}{2}}(Ac^2+2Bbc)}{19}+\frac{2x^{\frac{15}{2}}\cdot(2Abc+Bb^2)}{15}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**2*x**(1/2),x)
```

output
```
2*A*b**2*x**(11/2)/11 + 2*B*c**2*x**(23/2)/23 + 2*x**(19/2)*(A*c**2 + 2*B*
b*c)/19 + 2*x**(15/2)*(2*A*b*c + B*b**2)/15
```

3.170.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx = \frac{2}{23}\,Bc^2x^{\frac{23}{2}}+\frac{2}{19}\left(2\,Bbc+Ac^2\right)x^{\frac{19}{2}}+\frac{2}{11}\,Ab^2x^{\frac{11}{2}}+\frac{2}{15}\left(Bb^2+2\,Abc\right)x^{\frac{15}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2*x^(1/2),x, algorithm="maxima")
```

output
```
2/23*B*c^2*x^(23/2) + 2/19*(2*B*b*c + A*c^2)*x^(19/2) + 2/11*A*b^2*x^(11/2
) + 2/15*(B*b^2 + 2*A*b*c)*x^(15/2)
```

3.170.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx = \frac{2}{23}\,Bc^2x^{\frac{23}{2}}+\frac{4}{19}\,Bbcx^{\frac{19}{2}}+\frac{2}{19}\,Ac^2x^{\frac{19}{2}}+\frac{2}{15}\,Bb^2x^{\frac{15}{2}}+\frac{4}{15}\,Abcx^{\frac{15}{2}}+\frac{2}{11}\,Ab^2x^{\frac{11}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2*x^(1/2),x, algorithm="giac")
```

output
```
2/23*B*c^2*x^(23/2) + 4/19*B*b*c*x^(19/2) + 2/19*A*c^2*x^(19/2) + 2/15*B*b
^2*x^(15/2) + 4/15*A*b*c*x^(15/2) + 2/11*A*b^2*x^(11/2)
```

3.170.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx = x^{15/2}\left(\frac{2\,B\,b^2}{15}+\frac{4\,A\,c\,b}{15}\right) + x^{19/2}\left(\frac{2\,A\,c^2}{19}+\frac{4\,B\,b\,c}{19}\right)+\frac{2\,A\,b^2\,x^{11/2}}{11}+\frac{2\,B\,c^2\,x^{23/2}}{23}$$

input
```
int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

output
```
x^(15/2)*((2*B*b^2)/15 + (4*A*b*c)/15) + x^(19/2)*((2*A*c^2)/19 + (4*B*b*c
)/19) + (2*A*b^2*x^(11/2))/11 + (2*B*c^2*x^(23/2))/23
```

3.171 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}}\,dx$

3.171.1 Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}}\,dx = \frac{2}{9}Ab^2x^{9/2} + \frac{2}{13}b(bB+2Ac)x^{13/2} + \frac{2}{17}c(2bB+Ac)x^{17/2} + \frac{2}{21}Bc^2x^{21/2}$$

output
```
2/9*A*b^2*x^(9/2)+2/13*b*(2*A*c+B*b)*x^(13/2)+2/17*c*(A*c+2*B*b)*x^(17/2)+
2/21*B*c^2*x^(21/2)
```

3.171.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}}\,dx = \frac{2x^{9/2}(7A(221b^2+306bcx^2+117c^2x^4)+3Bx^2(357b^2+546bcx^2+221c^2x^4))}{13923}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/Sqrt[x],x]
```

output
```
(2*x^(9/2)*(7*A*(221*b^2 + 306*b*c*x^2 + 117*c^2*x^4) + 3*B*x^2*(357*b^2 +
 546*b*c*x^2 + 221*c^2*x^4)))/13923
```

3.171.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}}dx$$

$$\Big\downarrow \text{9}$$

$$\int x^{7/2}(A+Bx^2)(b+cx^2)^2dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(Ab^2x^{7/2}+cx^{15/2}(Ac+2bB)+bx^{11/2}(2Ac+bB)+Bc^2x^{19/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9}Ab^2x^{9/2}+\frac{2}{17}cx^{17/2}(Ac+2bB)+\frac{2}{13}bx^{13/2}(2Ac+bB)+\frac{2}{21}Bc^2x^{21/2}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/Sqrt[x],x]
```

output
```
(2*A*b^2*x^(9/2))/9 + (2*b*(b*B + 2*A*c)*x^(13/2))/13 + (2*c*(2*b*B + A*c)
*x^(17/2))/17 + (2*B*c^2*x^(21/2))/21
```

3.171.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.171.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2B\,c^{2}x^{\frac{21}{2}}}{21}+\frac{2(A\,c^{2}+2Bbc)x^{\frac{17}{2}}}{17}+\frac{2(2Abc+B\,b^{2})x^{\frac{13}{2}}}{13}+\frac{2A\,b^{2}x^{\frac{9}{2}}}{9}$	52
default	$\frac{2B\,c^{2}x^{\frac{21}{2}}}{21}+\frac{2(A\,c^{2}+2Bbc)x^{\frac{17}{2}}}{17}+\frac{2(2Abc+B\,b^{2})x^{\frac{13}{2}}}{13}+\frac{2A\,b^{2}x^{\frac{9}{2}}}{9}$	52
gosper	$\frac{2x^{\frac{9}{2}}(663B\,c^{2}x^{6}+819A\,c^{2}x^{4}+1638x^{4}Bbc+2142Abc\,x^{2}+1071b^{2}B\,x^{2}+1547b^{2}A)}{13923}$	56
trager	$\frac{2x^{\frac{9}{2}}(663B\,c^{2}x^{6}+819A\,c^{2}x^{4}+1638x^{4}Bbc+2142Abc\,x^{2}+1071b^{2}B\,x^{2}+1547b^{2}A)}{13923}$	56
risch	$\frac{2x^{\frac{9}{2}}(663B\,c^{2}x^{6}+819A\,c^{2}x^{4}+1638x^{4}Bbc+2142Abc\,x^{2}+1071b^{2}B\,x^{2}+1547b^{2}A)}{13923}$	56

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*B*c^2*x^(21/2)+2/17*(A*c^2+2*B*b*c)*x^(17/2)+2/13*(2*A*b*c+B*b^2)*x^(
13/2)+2/9*A*b^2*x^(9/2)
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{\sqrt{x}}\,dx$$
$$=\frac{2}{13923}\left(663\,Bc^2x^{10}+819\,\left(2\,Bbc+Ac^2\right)x^8+1547\,Ab^2x^4+1071\,\left(Bb^2+2\,Abc\right)x^6\right)\sqrt{x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="fricas")
```

output
```
2/13923*(663*B*c^2*x^10 + 819*(2*B*b*c + A*c^2)*x^8 + 1547*A*b^2*x^4 + 107
1*(B*b^2 + 2*A*b*c)*x^6)*sqrt(x)
```

3.171.6 Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}}\,dx = \frac{2Ab^2x^{\frac{9}{2}}}{9} + \frac{4Abcx^{\frac{13}{2}}}{13} + \frac{2Ac^2x^{\frac{17}{2}}}{17} + \frac{2Bb^2x^{\frac{13}{2}}}{13} + \frac{4Bbcx^{\frac{17}{2}}}{17} + \frac{2Bc^2x^{\frac{21}{2}}}{21}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(1/2),x)
```

output
```
2*A*b**2*x**(9/2)/9 + 4*A*b*c*x**(13/2)/13 + 2*A*c**2*x**(17/2)/17 + 2*B*b
**2*x**(13/2)/13 + 4*B*b*c*x**(17/2)/17 + 2*B*c**2*x**(21/2)/21
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}}\,dx = \frac{2}{21}Bc^2x^{\frac{21}{2}} + \frac{2}{17}(2Bbc+Ac^2)x^{\frac{17}{2}} + \frac{2}{9}Ab^2x^{\frac{9}{2}} + \frac{2}{13}(Bb^2+2Abc)x^{\frac{13}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="maxima")
```

output
```
2/21*B*c^2*x^(21/2) + 2/17*(2*B*b*c + A*c^2)*x^(17/2) + 2/9*A*b^2*x^(9/2)
+ 2/13*(B*b^2 + 2*A*b*c)*x^(13/2)
```

3.171.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}}\,dx = \frac{2}{21}Bc^2x^{\frac{21}{2}} + \frac{4}{17}Bbcx^{\frac{17}{2}} + \frac{2}{17}Ac^2x^{\frac{17}{2}} + \frac{2}{13}Bb^2x^{\frac{13}{2}} + \frac{4}{13}Abcx^{\frac{13}{2}} + \frac{2}{9}Ab^2x^{\frac{9}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="giac")
```

output
```
2/21*B*c^2*x^(21/2) + 4/17*B*b*c*x^(17/2) + 2/17*A*c^2*x^(17/2) + 2/13*B*b
^2*x^(13/2) + 4/13*A*b*c*x^(13/2) + 2/9*A*b^2*x^(9/2)
```

3.171.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}}\,dx = x^{13/2}\left(\frac{2\,B\,b^2}{13}+\frac{4\,A\,c\,b}{13}\right) + x^{17/2}\left(\frac{2\,A\,c^2}{17}+\frac{4\,B\,b\,c}{17}\right)+\frac{2\,A\,b^2\,x^{9/2}}{9}+\frac{2\,B\,c^2\,x^{21/2}}{21}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(1/2),x)
```

output
```
x^(13/2)*((2*B*b^2)/13 + (4*A*b*c)/13) + x^(17/2)*((2*A*c^2)/17 + (4*B*b*c
)/17) + (2*A*b^2*x^(9/2))/9 + (2*B*c^2*x^(21/2))/21
```

3.172 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{3/2}}\,dx$

3.172.1 Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{3/2}}\,dx = \frac{2}{7}Ab^2x^{7/2} + \frac{2}{11}b(bB+2Ac)x^{11/2} + \frac{2}{15}c(2bB+Ac)x^{15/2} + \frac{2}{19}Bc^2x^{19/2}$$

output
```
2/7*A*b^2*x^(7/2)+2/11*b*(2*A*c+B*b)*x^(11/2)+2/15*c*(A*c+2*B*b)*x^(15/2)+
2/19*B*c^2*x^(19/2)
```

3.172.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{3/2}}\,dx = \frac{2x^{7/2}(19A(165b^2+210bcx^2+77c^2x^4)+7Bx^2(285b^2+418bcx^2+165c^2x^4)}{21945}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(3/2),x]
```

output
```
(2*x^(7/2)*(19*A*(165*b^2 + 210*b*c*x^2 + 77*c^2*x^4) + 7*B*x^2*(285*b^2 +
 418*b*c*x^2 + 165*c^2*x^4)))/21945
```

3.172.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{3/2}}dx$$

$$\Big\downarrow \; 9$$

$$\int x^{5/2}(A+Bx^2)(b+cx^2)^2dx$$

$$\Big\downarrow \; 355$$

$$\int \left(Ab^2x^{5/2}+cx^{13/2}(Ac+2bB)+bx^{9/2}(2Ac+bB)+Bc^2x^{17/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{7}Ab^2x^{7/2}+\frac{2}{15}cx^{15/2}(Ac+2bB)+\frac{2}{11}bx^{11/2}(2Ac+bB)+\frac{2}{19}Bc^2x^{19/2}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(3/2),x]
```

output
```
(2*A*b^2*x^(7/2))/7 + (2*b*(b*B + 2*A*c)*x^(11/2))/11 + (2*c*(2*b*B + A*c)
*x^(15/2))/15 + (2*B*c^2*x^(19/2))/19
```

3.172.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.172.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2B\,c^2x^{\frac{19}{2}}}{19}+\frac{2(A\,c^2+2Bbc)x^{\frac{15}{2}}}{15}+\frac{2(2Abc+B\,b^2)x^{\frac{11}{2}}}{11}+\frac{2A\,b^2x^{\frac{7}{2}}}{7}$	52
default	$\frac{2B\,c^2x^{\frac{19}{2}}}{19}+\frac{2(A\,c^2+2Bbc)x^{\frac{15}{2}}}{15}+\frac{2(2Abc+B\,b^2)x^{\frac{11}{2}}}{11}+\frac{2A\,b^2x^{\frac{7}{2}}}{7}$	52
gosper	$\frac{2x^{\frac{7}{2}}(1155B\,c^2x^6+1463A\,c^2x^4+2926x^4Bbc+3990Abc\,x^2+1995b^2B\,x^2+3135b^2A)}{21945}$	56
trager	$\frac{2x^{\frac{7}{2}}(1155B\,c^2x^6+1463A\,c^2x^4+2926x^4Bbc+3990Abc\,x^2+1995b^2B\,x^2+3135b^2A)}{21945}$	56
risch	$\frac{2x^{\frac{7}{2}}(1155B\,c^2x^6+1463A\,c^2x^4+2926x^4Bbc+3990Abc\,x^2+1995b^2B\,x^2+3135b^2A)}{21945}$	56

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^2/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/19*B*c^2*x^(19/2)+2/15*(A*c^2+2*B*b*c)*x^(15/2)+2/11*(2*A*b*c+B*b^2)*x^(
11/2)+2/7*A*b^2*x^(7/2)
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{3/2}}\,dx=\frac{2}{21945}\left(1155\,Bc^2x^9+1463\left(2\,Bbc+Ac^2\right)x^7+3135\,Ab^2x^3+1995\left(Bb^2+2\right.\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(3/2),x, algorithm="fricas")
```

output
```
2/21945*(1155*B*c^2*x^9 + 1463*(2*B*b*c + A*c^2)*x^7 + 3135*A*b^2*x^3 + 19
95*(B*b^2 + 2*A*b*c)*x^5)*sqrt(x)
```

3.172.6 Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{3/2}}\,dx = \frac{2Ab^2x^{\frac{7}{2}}}{7} + \frac{4Abcx^{\frac{11}{2}}}{11} + \frac{2Ac^2x^{\frac{15}{2}}}{15} + \frac{2Bb^2x^{\frac{11}{2}}}{11} + \frac{4Bbcx^{\frac{15}{2}}}{15} + \frac{2Bc^2x^{\frac{19}{2}}}{19}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(3/2),x)
```

output
```
2*A*b**2*x**(7/2)/7 + 4*A*b*c*x**(11/2)/11 + 2*A*c**2*x**(15/2)/15 + 2*B*b
**2*x**(11/2)/11 + 4*B*b*c*x**(15/2)/15 + 2*B*c**2*x**(19/2)/19
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{3/2}}\,dx = \frac{2}{19}\,Bc^2x^{\frac{19}{2}} + \frac{2}{15}\,(2\,Bbc + Ac^2)x^{\frac{15}{2}} + \frac{2}{7}\,Ab^2x^{\frac{7}{2}} + \frac{2}{11}\,(Bb^2 + 2\,Abc)x^{\frac{11}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(3/2),x, algorithm="maxima")
```

output
```
2/19*B*c^2*x^(19/2) + 2/15*(2*B*b*c + A*c^2)*x^(15/2) + 2/7*A*b^2*x^(7/2)
+ 2/11*(B*b^2 + 2*A*b*c)*x^(11/2)
```

3.172.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{3/2}}\,dx = \frac{2}{19}\,Bc^2x^{\frac{19}{2}} + \frac{4}{15}\,Bbcx^{\frac{15}{2}} + \frac{2}{15}\,Ac^2x^{\frac{15}{2}} + \frac{2}{11}\,Bb^2x^{\frac{11}{2}} + \frac{4}{11}\,Abcx^{\frac{11}{2}} + \frac{2}{7}\,Ab^2x^{\frac{7}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(3/2),x, algorithm="giac")
```

output
```
2/19*B*c^2*x^(19/2) + 4/15*B*b*c*x^(15/2) + 2/15*A*c^2*x^(15/2) + 2/11*B*b
^2*x^(11/2) + 4/11*A*b*c*x^(11/2) + 2/7*A*b^2*x^(7/2)
```

3.172.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{3/2}}\,dx = x^{11/2}\left(\frac{2\,B\,b^2}{11}+\frac{4\,A\,c\,b}{11}\right) + x^{15/2}\left(\frac{2\,A\,c^2}{15}+\frac{4\,B\,b\,c}{15}\right)+\frac{2\,A\,b^2\,x^{7/2}}{7}+\frac{2\,B\,c^2\,x^{19/2}}{19}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(3/2),x)
```

output
```
x^(11/2)*((2*B*b^2)/11 + (4*A*b*c)/11) + x^(15/2)*((2*A*c^2)/15 + (4*B*b*c
)/15) + (2*A*b^2*x^(7/2))/7 + (2*B*c^2*x^(19/2))/19
```

3.173 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{5/2}}\,dx$

3.173.1 Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{5/2}}\,dx = \frac{2}{5}Ab^2x^{5/2} + \frac{2}{9}b(bB+2Ac)x^{9/2} + \frac{2}{13}c(2bB+Ac)x^{13/2} + \frac{2}{17}Bc^2x^{17/2}$$

output
```
2/5*A*b^2*x^(5/2)+2/9*b*(2*A*c+B*b)*x^(9/2)+2/13*c*(A*c+2*B*b)*x^(13/2)+2/
17*B*c^2*x^(17/2)
```

3.173.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{5/2}}\,dx = \frac{2x^{5/2}(17A(117b^2+130bcx^2+45c^2x^4)+5Bx^2(221b^2+306bcx^2+117c^2x^4)}{9945}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(5/2),x]
```

output
```
(2*x^(5/2)*(17*A*(117*b^2 + 130*b*c*x^2 + 45*c^2*x^4) + 5*B*x^2*(221*b^2 +
 306*b*c*x^2 + 117*c^2*x^4)))/9945
```

3.173.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{5/2}}\,dx$$

$$\Big\downarrow \; 9$$

$$\int x^{3/2}(A+Bx^2)(b+cx^2)^2\,dx$$

$$\Big\downarrow \; 355$$

$$\int \left(Ab^2x^{3/2}+cx^{11/2}(Ac+2bB)+bx^{7/2}(2Ac+bB)+Bc^2x^{15/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{5}Ab^2x^{5/2}+\frac{2}{13}cx^{13/2}(Ac+2bB)+\frac{2}{9}bx^{9/2}(2Ac+bB)+\frac{2}{17}Bc^2x^{17/2}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(5/2),x]
```

output
```
(2*A*b^2*x^(5/2))/5 + (2*b*(b*B + 2*A*c)*x^(9/2))/9 + (2*c*(2*b*B + A*c)*x
^(13/2))/13 + (2*B*c^2*x^(17/2))/17
```

3.173.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.173.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2B\,c^2 x^{\frac{17}{2}}}{17}+\frac{2(A\,c^2+2Bbc)\,x^{\frac{13}{2}}}{13}+\frac{2(2Abc+B\,b^2)\,x^{\frac{9}{2}}}{9}+\frac{2A\,b^2 x^{\frac{5}{2}}}{5}$	52
default	$\frac{2B\,c^2 x^{\frac{17}{2}}}{17}+\frac{2(A\,c^2+2Bbc)\,x^{\frac{13}{2}}}{13}+\frac{2(2Abc+B\,b^2)\,x^{\frac{9}{2}}}{9}+\frac{2A\,b^2 x^{\frac{5}{2}}}{5}$	52
gosper	$\frac{2x^{\frac{5}{2}}(585B\,c^2x^6+765A\,c^2x^4+1530x^4Bbc+2210Abc\,x^2+1105b^2B\,x^2+1989b^2A)}{9945}$	56
trager	$\frac{2x^{\frac{5}{2}}(585B\,c^2x^6+765A\,c^2x^4+1530x^4Bbc+2210Abc\,x^2+1105b^2B\,x^2+1989b^2A)}{9945}$	56
risch	$\frac{2x^{\frac{5}{2}}(585B\,c^2x^6+765A\,c^2x^4+1530x^4Bbc+2210Abc\,x^2+1105b^2B\,x^2+1989b^2A)}{9945}$	56

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^2/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/17*B*c^2*x^(17/2)+2/13*(A*c^2+2*B*b*c)*x^(13/2)+2/9*(2*A*b*c+B*b^2)*x^(9
/2)+2/5*A*b^2*x^(5/2)
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^{5/2}}\,dx=\frac{2}{9945}\left(585\,Bc^2x^8+765\,(2\,Bbc+Ac^2)x^6+1989\,Ab^2x^2+1105\,(Bb^2+2\,Abc$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(5/2),x, algorithm="fricas")
```

output
```
2/9945*(585*B*c^2*x^8 + 765*(2*B*b*c + A*c^2)*x^6 + 1989*A*b^2*x^2 + 1105*
(B*b^2 + 2*A*b*c)*x^4)*sqrt(x)
```

3.173.6 Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^{5/2}}\,dx = \frac{2Ab^2x^{\frac{5}{2}}}{5} + \frac{4Abcx^{\frac{9}{2}}}{9} + \frac{2Ac^2x^{\frac{13}{2}}}{13} + \frac{2Bb^2x^{\frac{9}{2}}}{9} + \frac{4Bbcx^{\frac{13}{2}}}{13} + \frac{2Bc^2x^{\frac{17}{2}}}{17}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(5/2),x)
```

output
```
2*A*b**2*x**(5/2)/5 + 4*A*b*c*x**(9/2)/9 + 2*A*c**2*x**(13/2)/13 + 2*B*b**
2*x**(9/2)/9 + 4*B*b*c*x**(13/2)/13 + 2*B*c**2*x**(17/2)/17
```

3.173.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^{5/2}}\,dx = \frac{2}{17}\,Bc^2x^{\frac{17}{2}} + \frac{2}{13}\,(2\,Bbc+Ac^2)x^{\frac{13}{2}} + \frac{2}{5}\,Ab^2x^{\frac{5}{2}} + \frac{2}{9}\,(Bb^2+2\,Abc)x^{\frac{9}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(5/2),x, algorithm="maxima")
```

output
```
2/17*B*c^2*x^(17/2) + 2/13*(2*B*b*c + A*c^2)*x^(13/2) + 2/5*A*b^2*x^(5/2)
+ 2/9*(B*b^2 + 2*A*b*c)*x^(9/2)
```

3.173.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^{5/2}}\,dx = \frac{2}{17}\,Bc^2x^{\frac{17}{2}} + \frac{4}{13}\,Bbcx^{\frac{13}{2}} + \frac{2}{13}\,Ac^2x^{\frac{13}{2}} + \frac{2}{9}\,Bb^2x^{\frac{9}{2}} + \frac{4}{9}\,Abcx^{\frac{9}{2}} + \frac{2}{5}\,Ab^2x^{\frac{5}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(5/2),x, algorithm="giac")
```

output
```
2/17*B*c^2*x^(17/2) + 4/13*B*b*c*x^(13/2) + 2/13*A*c^2*x^(13/2) + 2/9*B*b^
2*x^(9/2) + 4/9*A*b*c*x^(9/2) + 2/5*A*b^2*x^(5/2)
```

3.173.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{5/2}}\,dx = x^{9/2}\left(\frac{2\,B\,b^2}{9}+\frac{4\,A\,c\,b}{9}\right) + x^{13/2}\left(\frac{2\,A\,c^2}{13}+\frac{4\,B\,b\,c}{13}\right)+\frac{2\,A\,b^2\,x^{5/2}}{5}+\frac{2\,B\,c^2\,x^{17/2}}{17}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(5/2),x)
```

output
```
x^(9/2)*((2*B*b^2)/9 + (4*A*b*c)/9) + x^(13/2)*((2*A*c^2)/13 + (4*B*b*c)/1
3) + (2*A*b^2*x^(5/2))/5 + (2*B*c^2*x^(17/2))/17
```

3.174 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{7/2}}\,dx$

3.174.1 Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{7/2}}\,dx = \frac{2}{3}Ab^2x^{3/2} + \frac{2}{7}b(bB+2Ac)x^{7/2} + \frac{2}{11}c(2bB+Ac)x^{11/2} + \frac{2}{15}Bc^2x^{15/2}$$

output
```
2/3*A*b^2*x^(3/2)+2/7*b*(2*A*c+B*b)*x^(7/2)+2/11*c*(A*c+2*B*b)*x^(11/2)+2/
15*B*c^2*x^(15/2)
```

3.174.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{7/2}}\,dx = \frac{2x^{3/2}(5A(77b^2+66bcx^2+21c^2x^4)+Bx^2(165b^2+210bcx^2+77c^2x^4))}{1155}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(7/2),x]
```

output
```
(2*x^(3/2)*(5*A*(77*b^2 + 66*b*c*x^2 + 21*c^2*x^4) + B*x^2*(165*b^2 + 210*
b*c*x^2 + 77*c^2*x^4)))/1155
```

3.174.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{7/2}}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \sqrt{x}(A+Bx^2)(b+cx^2)^2dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(Ab^2\sqrt{x}+cx^{9/2}(Ac+2bB)+bx^{5/2}(2Ac+bB)+Bc^2x^{13/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{3}Ab^2x^{3/2}+\frac{2}{11}cx^{11/2}(Ac+2bB)+\frac{2}{7}bx^{7/2}(2Ac+bB)+\frac{2}{15}Bc^2x^{15/2}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(7/2),x]
```

output
```
(2*A*b^2*x^(3/2))/3 + (2*b*(b*B + 2*A*c)*x^(7/2))/7 + (2*c*(2*b*B + A*c)*x
^(11/2))/11 + (2*B*c^2*x^(15/2))/15
```

3.174.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.174.4 Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2B\,c^{2}x^{\frac{15}{2}}}{15}+\frac{2(A\,c^{2}+2Bbc)x^{\frac{11}{2}}}{11}+\frac{2(2Abc+B\,b^{2})x^{\frac{7}{2}}}{7}+\frac{2A\,b^{2}x^{\frac{3}{2}}}{3}$	52
default	$\frac{2B\,c^{2}x^{\frac{15}{2}}}{15}+\frac{2(A\,c^{2}+2Bbc)x^{\frac{11}{2}}}{11}+\frac{2(2Abc+B\,b^{2})x^{\frac{7}{2}}}{7}+\frac{2A\,b^{2}x^{\frac{3}{2}}}{3}$	52
gosper	$\frac{2x^{\frac{3}{2}}(77B\,c^{2}x^{6}+105A\,c^{2}x^{4}+210x^{4}Bbc+330Abc\,x^{2}+165b^{2}B\,x^{2}+385b^{2}A)}{1155}$	56
trager	$\frac{2x^{\frac{3}{2}}(77B\,c^{2}x^{6}+105A\,c^{2}x^{4}+210x^{4}Bbc+330Abc\,x^{2}+165b^{2}B\,x^{2}+385b^{2}A)}{1155}$	56
risch	$\frac{2x^{\frac{3}{2}}(77B\,c^{2}x^{6}+105A\,c^{2}x^{4}+210x^{4}Bbc+330Abc\,x^{2}+165b^{2}B\,x^{2}+385b^{2}A)}{1155}$	56

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^2/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*B*c^2*x^(15/2)+2/11*(A*c^2+2*B*b*c)*x^(11/2)+2/7*(2*A*b*c+B*b^2)*x^(7
/2)+2/3*A*b^2*x^(3/2)
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{7/2}}\,dx=\frac{2}{1155}\left(77\,Bc^2x^7+105\,(2\,Bbc+Ac^2)x^5+385\,Ab^2x+165\,(Bb^2+2\,Abc)x^3\right)$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(7/2),x, algorithm="fricas")
```

output
```
2/1155*(77*B*c^2*x^7 + 105*(2*B*b*c + A*c^2)*x^5 + 385*A*b^2*x + 165*(B*b^
2 + 2*A*b*c)*x^3)*sqrt(x)
```

3.174.6 Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^{7/2}}\,dx = \frac{2Ab^2x^{\frac{3}{2}}}{3}+\frac{4Abcx^{\frac{7}{2}}}{7}+\frac{2Ac^2x^{\frac{11}{2}}}{11}+\frac{2Bb^2x^{\frac{7}{2}}}{7}+\frac{4Bbcx^{\frac{11}{2}}}{11}+\frac{2Bc^2x^{\frac{15}{2}}}{15}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(7/2),x)
```

output
```
2*A*b**2*x**(3/2)/3 + 4*A*b*c*x**(7/2)/7 + 2*A*c**2*x**(11/2)/11 + 2*B*b**
2*x**(7/2)/7 + 4*B*b*c*x**(11/2)/11 + 2*B*c**2*x**(15/2)/15
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^{7/2}}\,dx = \frac{2}{15}\,Bc^2x^{\frac{15}{2}}+\frac{2}{11}\left(2\,Bbc+Ac^2\right)x^{\frac{11}{2}}+\frac{2}{3}\,Ab^2x^{\frac{3}{2}}+\frac{2}{7}\left(Bb^2+2\,Abc\right)x^{\frac{7}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(7/2),x, algorithm="maxima")
```

output
```
2/15*B*c^2*x^(15/2) + 2/11*(2*B*b*c + A*c^2)*x^(11/2) + 2/3*A*b^2*x^(3/2)
+ 2/7*(B*b^2 + 2*A*b*c)*x^(7/2)
```

3.174.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^{7/2}}\,dx = \frac{2}{15}\,Bc^2x^{\frac{15}{2}}+\frac{4}{11}\,Bbcx^{\frac{11}{2}}+\frac{2}{11}\,Ac^2x^{\frac{11}{2}}+\frac{2}{7}\,Bb^2x^{\frac{7}{2}}+\frac{4}{7}\,Abcx^{\frac{7}{2}}+\frac{2}{3}\,Ab^2x^{\frac{3}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(7/2),x, algorithm="giac")
```

output
```
2/15*B*c^2*x^(15/2) + 4/11*B*b*c*x^(11/2) + 2/11*A*c^2*x^(11/2) + 2/7*B*b^
2*x^(7/2) + 4/7*A*b*c*x^(7/2) + 2/3*A*b^2*x^(3/2)
```

3.174.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^{7/2}}\,dx = x^{7/2}\left(\frac{2\,B\,b^2}{7}+\frac{4\,A\,c\,b}{7}\right) + x^{11/2}\left(\frac{2\,A\,c^2}{11}+\frac{4\,B\,b\,c}{11}\right)+\frac{2\,A\,b^2\,x^{3/2}}{3}+\frac{2\,B\,c^2\,x^{15/2}}{15}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(7/2),x)
```

output
```
x^(7/2)*((2*B*b^2)/7 + (4*A*b*c)/7) + x^(11/2)*((2*A*c^2)/11 + (4*B*b*c)/1
1) + (2*A*b^2*x^(3/2))/3 + (2*B*c^2*x^(15/2))/15
```

3.175 $\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx$

3.175.1 Optimal result

Integrand size = 26, antiderivative size = 85

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{21}Ab^3x^{21/2}+\frac{2}{25}b^2(bB+3Ac)x^{25/2}+\frac{6}{29}bc(bB+Ac)x^{29/2}+\frac{2}{33}c^2(3bB+Ac)x^{33/2}+\frac{2}{37}Bc^3x^{37/2}$$

output
```
2/21*A*b^3*x^(21/2)+2/25*b^2*(3*A*c+B*b)*x^(25/2)+6/29*b*c*(A*c+B*b)*x^(29
/2)+2/33*c^2*(A*c+3*B*b)*x^(33/2)+2/37*B*c^3*x^(37/2)
```

3.175.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{21}Ab^3x^{21/2}+\frac{2}{25}b^2(bB+3Ac)x^{25/2}+\frac{6}{29}bc(bB+Ac)x^{29/2}+\frac{2}{33}c^2(3bB+Ac)x^{33/2}+\frac{2}{37}Bc^3x^{37/2}$$

input
```
Integrate[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

output
```
(2*A*b^3*x^(21/2))/21 + (2*b^2*(b*B + 3*A*c)*x^(25/2))/25 + (6*b*c*(b*B +
A*c)*x^(29/2))/29 + (2*c^2*(3*b*B + A*c)*x^(33/2))/33 + (2*B*c^3*x^(37/2))
/37
```

3.175.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx$$

$$\Big\downarrow 9$$

$$\int x^{19/2}\left(A+Bx^2\right)\left(b+cx^2\right)^3dx$$

$$\Big\downarrow 355$$

$$\int \left(Ab^3x^{19/2}+b^2x^{23/2}(3Ac+bB)+c^2x^{31/2}(Ac+3bB)+3bcx^{27/2}(Ac+bB)+Bc^3x^{35/2}\right)dx$$

$$\Big\downarrow 2009$$

$$\frac{2}{21}Ab^3x^{21/2}+\frac{2}{25}b^2x^{25/2}(3Ac+bB)+\frac{2}{33}c^2x^{33/2}(Ac+3bB)+\frac{6}{29}bcx^{29/2}(Ac+bB)+\frac{2}{37}Bc^3x^{37/2}$$

input
```
Int[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

output
```
(2*A*b^3*x^(21/2))/21 + (2*b^2*(b*B + 3*A*c)*x^(25/2))/25 + (6*b*c*(b*B +
A*c)*x^(29/2))/29 + (2*c^2*(3*b*B + A*c)*x^(33/2))/33 + (2*B*c^3*x^(37/2))
/37
```

3.175.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.175.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2B\,c^{3}x^{\frac{37}{2}}}{37}+\frac{2\left(A\,c^{3}+3Bb\,c^{2}\right)x^{\frac{33}{2}}}{33}+\frac{2\left(3Ab\,c^{2}+3B\,b^{2}c\right)x^{\frac{29}{2}}}{29}+\frac{2\left(3b^{2}Ac+B\,b^{3}\right)x^{\frac{25}{2}}}{25}+\frac{2A\,b^{3}x^{\frac{21}{2}}}{21}$
default	$\frac{2B\,c^{3}x^{\frac{37}{2}}}{37}+\frac{2\left(A\,c^{3}+3Bb\,c^{2}\right)x^{\frac{33}{2}}}{33}+\frac{2\left(3Ab\,c^{2}+3B\,b^{2}c\right)x^{\frac{29}{2}}}{29}+\frac{2\left(3b^{2}Ac+B\,b^{3}\right)x^{\frac{25}{2}}}{25}+\frac{2A\,b^{3}x^{\frac{21}{2}}}{21}$
gosper	$\frac{2x^{\frac{21}{2}}\left(167475B\,c^{3}x^{8}+187775A\,c^{3}x^{6}+563325x^{6}Bb\,c^{2}+641025Ab\,c^{2}x^{4}+641025x^{4}B\,b^{2}c+743589A\,b^{2}c\,x^{2}+247863b^{3}B\,x^{2}\right.}{6196575}$
trager	$\frac{2x^{\frac{21}{2}}\left(167475B\,c^{3}x^{8}+187775A\,c^{3}x^{6}+563325x^{6}Bb\,c^{2}+641025Ab\,c^{2}x^{4}+641025x^{4}B\,b^{2}c+743589A\,b^{2}c\,x^{2}+247863b^{3}B\,x^{2}\right.}{6196575}$
risch	$\frac{2x^{\frac{21}{2}}\left(167475B\,c^{3}x^{8}+187775A\,c^{3}x^{6}+563325x^{6}Bb\,c^{2}+641025Ab\,c^{2}x^{4}+641025x^{4}B\,b^{2}c+743589A\,b^{2}c\,x^{2}+247863b^{3}B\,x^{2}\right.}{6196575}$

input
```
int(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2/37*B*c^3*x^(37/2)+2/33*(A*c^3+3*B*b*c^2)*x^(33/2)+2/29*(3*A*b*c^2+3*B*b^
2*c)*x^(29/2)+2/25*(3*A*b^2*c+B*b^3)*x^(25/2)+2/21*A*b^3*x^(21/2)
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{7/2}\left(A+Bx^{2}\right)\left(bx^{2}+cx^{4}\right)^{3}\,dx=\frac{2}{6196575}\left(167475\,Bc^{3}x^{18}+187775\left(3\,Bbc^{2}+Ac^{3}\right)x^{16}+641025\left(Bb^{2}c+Abc^{2}\right)x^{14}+295075\,Ab\right.$$

input
```
integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
2/6196575*(167475*B*c^3*x^18 + 187775*(3*B*b*c^2 + A*c^3)*x^16 + 641025*(B
*b^2*c + A*b*c^2)*x^14 + 295075*A*b^3*x^10 + 247863*(B*b^3 + 3*A*b^2*c)*x^
12)*sqrt(x)
```

3.175.6 Sympy [A] (verification not implemented)

Time = 3.36 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2Ab^3x^{\frac{21}{2}}}{21}+\frac{6Ab^2cx^{\frac{25}{2}}}{25}+\frac{6Abc^2x^{\frac{29}{2}}}{29}+\frac{2Ac^3x^{\frac{33}{2}}}{33}+\frac{2Bb^3x^{\frac{25}{2}}}{25}+\frac{6Bb^2cx^{\frac{29}{2}}}{29}+\frac{2Bbc^2x^{\frac{33}{2}}}{11}+\frac{2Bc^3x^{\frac{37}{2}}}{37}$$

input
```
integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2)**3,x)
```

output
```
2*A*b**3*x**(21/2)/21 + 6*A*b**2*c*x**(25/2)/25 + 6*A*b*c**2*x**(29/2)/29
+ 2*A*c**3*x**(33/2)/33 + 2*B*b**3*x**(25/2)/25 + 6*B*b**2*c*x**(29/2)/29
+ 2*B*b*c**2*x**(33/2)/11 + 2*B*c**3*x**(37/2)/37
```

3.175.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{37}\,Bc^3x^{\frac{37}{2}}+\frac{2}{33}\,(3\,Bbc^2+Ac^3)x^{\frac{33}{2}}+\frac{6}{29}\,(Bb^2c+Abc^2)x^{\frac{29}{2}}+\frac{2}{21}\,Ab^3x^{\frac{21}{2}}+\frac{2}{25}\,(Bb^3+3\,Ab^2c)x^{\frac{25}{2}}$$

input
```
integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
2/37*B*c^3*x^(37/2) + 2/33*(3*B*b*c^2 + A*c^3)*x^(33/2) + 6/29*(B*b^2*c +
A*b*c^2)*x^(29/2) + 2/21*A*b^3*x^(21/2) + 2/25*(B*b^3 + 3*A*b^2*c)*x^(25/2
)
```

3.175.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{37}\,Bc^3x^{\frac{37}{2}}+\frac{2}{11}\,Bbc^2x^{\frac{33}{2}}+\frac{2}{33}\,Ac^3x^{\frac{33}{2}}+\frac{6}{29}\,Bb^2cx^{\frac{29}{2}}+\frac{6}{29}\,Abc^2x^{\frac{29}{2}}+\frac{2}{25}\,Bb^3x^{\frac{25}{2}}+\frac{6}{25}\,Ab^2cx^{\frac{25}{2}}+\frac{2}{21}\,Ab^3x^{\frac{21}{2}}$$

input
```
integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
2/37*B*c^3*x^(37/2) + 2/11*B*b*c^2*x^(33/2) + 2/33*A*c^3*x^(33/2) + 6/29*B
*b^2*c*x^(29/2) + 6/29*A*b*c^2*x^(29/2) + 2/25*B*b^3*x^(25/2) + 6/25*A*b^2
*c*x^(25/2) + 2/21*A*b^3*x^(21/2)
```

3.175.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=x^{25/2}\left(\frac{2\,B\,b^3}{25}+\frac{6\,A\,c\,b^2}{25}\right)+x^{33/2}\left(\frac{2\,A\,c^3}{33}+\frac{2\,B\,b\,c^2}{11}\right)+\frac{2\,A\,b^3\,x^{21/2}}{21}+\frac{2\,B\,c^3\,x^{37/2}}{37}+\frac{6\,b\,c\,x^{29/2}\,(A\,c+B\,b)}{29}$$

input
```
int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)
```

output
```
x^(25/2)*((2*B*b^3)/25 + (6*A*b^2*c)/25) + x^(33/2)*((2*A*c^3)/33 + (2*B*b
*c^2)/11) + (2*A*b^3*x^(21/2))/21 + (2*B*c^3*x^(37/2))/37 + (6*b*c*x^(29/2
)*(A*c + B*b))/29
```

3.176 $\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx$

3.176.1 Optimal result

Integrand size = 26, antiderivative size = 85

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{19}Ab^3x^{19/2}+\frac{2}{23}b^2(bB+3Ac)x^{23/2}+\frac{2}{9}bc(bB+Ac)x^{27/2}+\frac{2}{31}c^2(3bB+Ac)x^{31/2}+\frac{2}{35}Bc^3x^{35/2}$$

output
```
2/19*A*b^3*x^(19/2)+2/23*b^2*(3*A*c+B*b)*x^(23/2)+2/9*b*c*(A*c+B*b)*x^(27/
2)+2/31*c^2*(A*c+3*B*b)*x^(31/2)+2/35*B*c^3*x^(35/2)
```

3.176.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{19}Ab^3x^{19/2}+\frac{2}{23}b^2(bB+3Ac)x^{23/2}+\frac{2}{9}bc(bB+Ac)x^{27/2}+\frac{2}{31}c^2(3bB+Ac)x^{31/2}+\frac{2}{35}Bc^3x^{35/2}$$

input
```
Integrate[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

output
```
(2*A*b^3*x^(19/2))/19 + (2*b^2*(b*B + 3*A*c)*x^(23/2))/23 + (2*b*c*(b*B +
A*c)*x^(27/2))/9 + (2*c^2*(3*b*B + A*c)*x^(31/2))/31 + (2*B*c^3*x^(35/2))/
35
```

3.176.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3dx$$

$$\Big\downarrow \text{9}$$

$$\int x^{17/2}\left(A+Bx^2\right)\left(b+cx^2\right)^3dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(Ab^3x^{17/2}+b^2x^{21/2}(3Ac+bB)+c^2x^{29/2}(Ac+3bB)+3bcx^{25/2}(Ac+bB)+Bc^3x^{33/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{19}Ab^3x^{19/2}+\frac{2}{23}b^2x^{23/2}(3Ac+bB)+\frac{2}{31}c^2x^{31/2}(Ac+3bB)+\frac{2}{9}bcx^{27/2}(Ac+bB)+\frac{2}{35}Bc^3x^{35/2}$$

input
```
Int[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

output
```
(2*A*b^3*x^(19/2))/19 + (2*b^2*(b*B + 3*A*c)*x^(23/2))/23 + (2*b*c*(b*B +
A*c)*x^(27/2))/9 + (2*c^2*(3*b*B + A*c)*x^(31/2))/31 + (2*B*c^3*x^(35/2))/
35
```

3.176.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.176.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2Bc^3x^{\frac{35}{2}}}{35}+\frac{2(Ac^3+3Bbc^2)x^{\frac{31}{2}}}{31}+\frac{2(3Abc^2+3Bb^2c)x^{\frac{27}{2}}}{27}+\frac{2(3b^2Ac+Bb^3)x^{\frac{23}{2}}}{23}+\frac{2Ab^3x^{\frac{19}{2}}}{19}$
default	$\frac{2Bc^3x^{\frac{35}{2}}}{35}+\frac{2(Ac^3+3Bbc^2)x^{\frac{31}{2}}}{31}+\frac{2(3Abc^2+3Bb^2c)x^{\frac{27}{2}}}{27}+\frac{2(3b^2Ac+Bb^3)x^{\frac{23}{2}}}{23}+\frac{2Ab^3x^{\frac{19}{2}}}{19}$
gosper	$\frac{2x^{\frac{19}{2}}(121923Bc^3x^8+137655Ac^3x^6+412965x^6Bbc^2+474145Abc^2x^4+474145x^4Bb^2c+556605Ab^2cx^2+185535b^3Bx^2$
trager	$\frac{2x^{\frac{19}{2}}(121923Bc^3x^8+137655Ac^3x^6+412965x^6Bbc^2+474145Abc^2x^4+474145x^4Bb^2c+556605Ab^2cx^2+185535b^3Bx^2$
risch	$\frac{2x^{\frac{19}{2}}(121923Bc^3x^8+137655Ac^3x^6+412965x^6Bbc^2+474145Abc^2x^4+474145x^4Bb^2c+556605Ab^2cx^2+185535b^3Bx^2$

input
```
int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2/35*B*c^3*x^(35/2)+2/31*(A*c^3+3*B*b*c^2)*x^(31/2)+2/27*(3*A*b*c^2+3*B*b^
2*c)*x^(27/2)+2/23*(3*A*b^2*c+B*b^3)*x^(23/2)+2/19*A*b^3*x^(19/2)
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{4267305}\left(121923\,Bc^3x^{17}+137655\left(3\,Bbc^2+Ac^3\right)x^{15}+474145\left(Bb^2c+Abc^2\right)x^{13}+224595\,Ab\right.$$

input
```
integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
2/4267305*(121923*B*c^3*x^17 + 137655*(3*B*b*c^2 + A*c^3)*x^15 + 474145*(B
*b^2*c + A*b*c^2)*x^13 + 224595*A*b^3*x^9 + 185535*(B*b^3 + 3*A*b^2*c)*x^1
1)*sqrt(x)
```

3.176.6 Sympy [A] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2Ab^3x^{\frac{19}{2}}}{19}+\frac{6Ab^2cx^{\frac{23}{2}}}{23}+\frac{2Abc^2x^{\frac{27}{2}}}{9}+\frac{2Ac^3x^{\frac{31}{2}}}{31}+\frac{2Bb^3x^{\frac{23}{2}}}{23}+\frac{2Bb^2cx^{\frac{27}{2}}}{9}+\frac{6Bbc^2x^{\frac{31}{2}}}{31}+\frac{2Bc^3x^{\frac{35}{2}}}{35}$$

input
```
integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**3,x)
```

output
```
2*A*b**3*x**(19/2)/19 + 6*A*b**2*c*x**(23/2)/23 + 2*A*b*c**2*x**(27/2)/9 +
 2*A*c**3*x**(31/2)/31 + 2*B*b**3*x**(23/2)/23 + 2*B*b**2*c*x**(27/2)/9 +
6*B*b*c**2*x**(31/2)/31 + 2*B*c**3*x**(35/2)/35
```

3.176.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{35}\,Bc^3x^{\frac{35}{2}}+\frac{2}{31}\,(3\,Bbc^2+Ac^3)x^{\frac{31}{2}}+\frac{2}{9}\,(Bb^2c+Abc^2)x^{\frac{27}{2}}+\frac{2}{19}\,Ab^3x^{\frac{19}{2}}+\frac{2}{23}\,(Bb^3+3\,Ab^2c)x^{\frac{23}{2}}$$

input
```
integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
2/35*B*c^3*x^(35/2) + 2/31*(3*B*b*c^2 + A*c^3)*x^(31/2) + 2/9*(B*b^2*c + A
*b*c^2)*x^(27/2) + 2/19*A*b^3*x^(19/2) + 2/23*(B*b^3 + 3*A*b^2*c)*x^(23/2)
```

3.176.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{35}\,Bc^3x^{\frac{35}{2}}+\frac{6}{31}\,Bbc^2x^{\frac{31}{2}}+\frac{2}{31}\,Ac^3x^{\frac{31}{2}}+\frac{2}{9}\,Bb^2cx^{\frac{27}{2}}+\frac{2}{9}\,Abc^2x^{\frac{27}{2}}+\frac{2}{23}\,Bb^3x^{\frac{23}{2}}+\frac{6}{23}\,Ab^2cx^{\frac{23}{2}}+\frac{2}{19}\,Ab^3x^{\frac{19}{2}}$$

input
```
integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
2/35*B*c^3*x^(35/2) + 6/31*B*b*c^2*x^(31/2) + 2/31*A*c^3*x^(31/2) + 2/9*B*
b^2*c*x^(27/2) + 2/9*A*b*c^2*x^(27/2) + 2/23*B*b^3*x^(23/2) + 6/23*A*b^2*c
*x^(23/2) + 2/19*A*b^3*x^(19/2)
```

3.176.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx = x^{23/2}\left(\frac{2\,B\,b^3}{23}+\frac{6\,A\,c\,b^2}{23}\right) + x^{31/2}\left(\frac{2\,A\,c^3}{31}+\frac{6\,B\,b\,c^2}{31}\right)+\frac{2\,A\,b^3\,x^{19/2}}{19}+\frac{2\,B\,c^3\,x^{35/2}}{35}+\frac{2\,b\,c\,x^{27/2}\,(A\,c+B\,b)}{9}$$

input
```
int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)
```

output
```
x^(23/2)*((2*B*b^3)/23 + (6*A*b^2*c)/23) + x^(31/2)*((2*A*c^3)/31 + (6*B*b
*c^2)/31) + (2*A*b^3*x^(19/2))/19 + (2*B*c^3*x^(35/2))/35 + (2*b*c*x^(27/2
)*(A*c + B*b))/9
```

3.177 $\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx$

3.177.1 Optimal result

Integrand size = 26, antiderivative size = 85

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{17}Ab^3x^{17/2}+\frac{2}{21}b^2(bB+3Ac)x^{21/2}+\frac{6}{25}bc(bB+Ac)x^{25/2}+\frac{2}{29}c^2(3bB+Ac)x^{29/2}+\frac{2}{33}Bc^3x^{33/2}$$

output
```
2/17*A*b^3*x^(17/2)+2/21*b^2*(3*A*c+B*b)*x^(21/2)+6/25*b*c*(A*c+B*b)*x^(25
/2)+2/29*c^2*(A*c+3*B*b)*x^(29/2)+2/33*B*c^3*x^(33/2)
```

3.177.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.14

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2\left(167475Ab^3x^{17/2}+135575b^3Bx^{21/2}+406725Ab^2cx^{21/2}+341649b^2Bcx^{25/2}+341649Abc^2x^{25/}\right.}{2847075}$$

input
```
Integrate[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

output
```
(2*(167475*A*b^3*x^(17/2) + 135575*b^3*B*x^(21/2) + 406725*A*b^2*c*x^(21/2
) + 341649*b^2*B*c*x^(25/2) + 341649*A*b*c^2*x^(25/2) + 294525*b*B*c^2*x^(
29/2) + 98175*A*c^3*x^(29/2) + 86275*B*c^3*x^(33/2)))/2847075
```

3.177.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx$$

$$\Big\downarrow \; 9$$

$$\int x^{15/2}\left(A+Bx^2\right)\left(b+cx^2\right)^3dx$$

$$\Big\downarrow \; 355$$

$$\int \left(Ab^3x^{15/2}+b^2x^{19/2}(3Ac+bB)+c^2x^{27/2}(Ac+3bB)+3bcx^{23/2}(Ac+bB)+Bc^3x^{31/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{17}Ab^3x^{17/2}+\frac{2}{21}b^2x^{21/2}(3Ac+bB)+\frac{2}{29}c^2x^{29/2}(Ac+3bB)+\frac{6}{25}bcx^{25/2}(Ac+bB)+\frac{2}{33}Bc^3x^{33/2}$$

input
```
Int[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

output
```
(2*A*b^3*x^(17/2))/17 + (2*b^2*(b*B + 3*A*c)*x^(21/2))/21 + (6*b*c*(b*B +
A*c)*x^(25/2))/25 + (2*c^2*(3*b*B + A*c)*x^(29/2))/29 + (2*B*c^3*x^(33/2))
/33
```

3.177.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.177.4 Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2B\,c^{3}x^{\frac{33}{2}}}{33}+\frac{2(A\,c^{3}+3Bb\,c^{2})x^{\frac{29}{2}}}{29}+\frac{2(3Ab\,c^{2}+3B\,b^{2}c)x^{\frac{25}{2}}}{25}+\frac{2(3b^{2}Ac+B\,b^{3})x^{\frac{21}{2}}}{21}+\frac{2A\,b^{3}x^{\frac{17}{2}}}{17}$
default	$\frac{2B\,c^{3}x^{\frac{33}{2}}}{33}+\frac{2(A\,c^{3}+3Bb\,c^{2})x^{\frac{29}{2}}}{29}+\frac{2(3Ab\,c^{2}+3B\,b^{2}c)x^{\frac{25}{2}}}{25}+\frac{2(3b^{2}Ac+B\,b^{3})x^{\frac{21}{2}}}{21}+\frac{2A\,b^{3}x^{\frac{17}{2}}}{17}$
gosper	$\frac{2x^{\frac{17}{2}}(86275B\,c^{3}x^{8}+98175A\,c^{3}x^{6}+294525x^{6}Bb\,c^{2}+341649Ab\,c^{2}x^{4}+341649x^{4}B\,b^{2}c+406725A\,b^{2}c\,x^{2}+135575b^{3}B\,x^{2}+}{2847075}$
trager	$\frac{2x^{\frac{17}{2}}(86275B\,c^{3}x^{8}+98175A\,c^{3}x^{6}+294525x^{6}Bb\,c^{2}+341649Ab\,c^{2}x^{4}+341649x^{4}B\,b^{2}c+406725A\,b^{2}c\,x^{2}+135575b^{3}B\,x^{2}+}{2847075}$
risch	$\frac{2x^{\frac{17}{2}}(86275B\,c^{3}x^{8}+98175A\,c^{3}x^{6}+294525x^{6}Bb\,c^{2}+341649Ab\,c^{2}x^{4}+341649x^{4}B\,b^{2}c+406725A\,b^{2}c\,x^{2}+135575b^{3}B\,x^{2}+}{2847075}$

input
```
int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2/33*B*c^3*x^(33/2)+2/29*(A*c^3+3*B*b*c^2)*x^(29/2)+2/25*(3*A*b*c^2+3*B*b^
2*c)*x^(25/2)+2/21*(3*A*b^2*c+B*b^3)*x^(21/2)+2/17*A*b^3*x^(17/2)
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{2847075}\left(86275\,Bc^3x^{16}+98175\,\left(3\,Bbc^2+Ac^3\right)x^{14}+341649\,\left(Bb^2c+Abc^2\right)x^{12}+167475\,Ab^3x\right.$$

input
```
integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
2/2847075*(86275*B*c^3*x^16 + 98175*(3*B*b*c^2 + A*c^3)*x^14 + 341649*(B*b
^2*c + A*b*c^2)*x^12 + 167475*A*b^3*x^8 + 135575*(B*b^3 + 3*A*b^2*c)*x^10)
*sqrt(x)
```

3.177.6 Sympy [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2Ab^3x^{\frac{17}{2}}}{17}+\frac{2Ab^2cx^{\frac{21}{2}}}{7}+\frac{6Abc^2x^{\frac{25}{2}}}{25}+\frac{2Ac^3x^{\frac{29}{2}}}{29}+\frac{2Bb^3x^{\frac{21}{2}}}{21}+\frac{6Bb^2cx^{\frac{25}{2}}}{25}+\frac{6Bbc^2x^{\frac{29}{2}}}{29}+\frac{2Bc^3x^{\frac{33}{2}}}{33}$$

input
```
integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**3,x)
```

output
```
2*A*b**3*x**(17/2)/17 + 2*A*b**2*c*x**(21/2)/7 + 6*A*b*c**2*x**(25/2)/25 +
 2*A*c**3*x**(29/2)/29 + 2*B*b**3*x**(21/2)/21 + 6*B*b**2*c*x**(25/2)/25 +
 6*B*b*c**2*x**(29/2)/29 + 2*B*c**3*x**(33/2)/33
```

3.177.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{33}\,Bc^3x^{\frac{33}{2}}+\frac{2}{29}\,(3\,Bbc^2+Ac^3)x^{\frac{29}{2}}+\frac{6}{25}\,(Bb^2c+Abc^2)x^{\frac{25}{2}}+\frac{2}{17}\,Ab^3x^{\frac{17}{2}}+\frac{2}{21}\,(Bb^3+3\,Ab^2c)x^{\frac{21}{2}}$$

input
```
integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
2/33*B*c^3*x^(33/2) + 2/29*(3*B*b*c^2 + A*c^3)*x^(29/2) + 6/25*(B*b^2*c +
A*b*c^2)*x^(25/2) + 2/17*A*b^3*x^(17/2) + 2/21*(B*b^3 + 3*A*b^2*c)*x^(21/2
)
```

3.177.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{33}\,Bc^3x^{\frac{33}{2}}+\frac{6}{29}\,Bbc^2x^{\frac{29}{2}}+\frac{2}{29}\,Ac^3x^{\frac{29}{2}}+\frac{6}{25}\,Bb^2cx^{\frac{25}{2}}+\frac{6}{25}\,Abc^2x^{\frac{25}{2}}+\frac{2}{21}\,Bb^3x^{\frac{21}{2}}+\frac{2}{7}\,Ab^2cx^{\frac{21}{2}}+\frac{2}{17}\,Ab^3x^{\frac{17}{2}}$$

input
```
integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
2/33*B*c^3*x^(33/2) + 6/29*B*b*c^2*x^(29/2) + 2/29*A*c^3*x^(29/2) + 6/25*B
*b^2*c*x^(25/2) + 6/25*A*b*c^2*x^(25/2) + 2/21*B*b^3*x^(21/2) + 2/7*A*b^2*
c*x^(21/2) + 2/17*A*b^3*x^(17/2)
```

3.177.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx = x^{21/2}\left(\frac{2\,B\,b^3}{21}+\frac{2\,A\,c\,b^2}{7}\right) + x^{29/2}\left(\frac{2\,A\,c^3}{29}+\frac{6\,B\,b\,c^2}{29}\right)+\frac{2\,A\,b^3\,x^{17/2}}{17}+\frac{2\,B\,c^3\,x^{33/2}}{33}+\frac{6\,b\,c\,x^{25/2}\,(A\,c+B\,b)}{25}$$

input
```
int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)
```

output
```
x^(21/2)*((2*B*b^3)/21 + (2*A*b^2*c)/7) + x^(29/2)*((2*A*c^3)/29 + (6*B*b*
c^2)/29) + (2*A*b^3*x^(17/2))/17 + (2*B*c^3*x^(33/2))/33 + (6*b*c*x^(25/2)
*(A*c + B*b))/25
```

3.178 $\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx$

3.178.1 Optimal result

Integrand size = 26, antiderivative size = 85

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{15}Ab^3x^{15/2}+\frac{2}{19}b^2(bB+3Ac)x^{19/2}+\frac{6}{23}bc(bB+Ac)x^{23/2}+\frac{2}{27}c^2(3bB+Ac)x^{27/2}+\frac{2}{31}Bc^3x^{31/2}$$

output
```
2/15*A*b^3*x^(15/2)+2/19*b^2*(3*A*c+B*b)*x^(19/2)+6/23*b*c*(A*c+B*b)*x^(23
/2)+2/27*c^2*(A*c+3*B*b)*x^(27/2)+2/31*B*c^3*x^(31/2)
```

3.178.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2x^{15/2}(31A(3933b^3+9315b^2cx^2+7695bc^2x^4+2185c^3x^6)+15Bx^2(6417b^3+15903b^2cx^2+13547bc^2x^4+}{1828845}$$

input
```
Integrate[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

output
```
(2*x^(15/2)*(31*A*(3933*b^3 + 9315*b^2*c*x^2 + 7695*b*c^2*x^4 + 2185*c^3*x
^6) + 15*B*x^2*(6417*b^3 + 15903*b^2*c*x^2 + 13547*b*c^2*x^4 + 3933*c^3*x^
6)))/1828845
```

3.178.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx$$

$$\Big\downarrow \ 9$$

$$\int x^{13/2}(A+Bx^2)\left(b+cx^2\right)^3dx$$

$$\Big\downarrow \ 355$$

$$\int \left(Ab^3x^{13/2}+b^2x^{17/2}(3Ac+bB)+c^2x^{25/2}(Ac+3bB)+3bcx^{21/2}(Ac+bB)+Bc^3x^{29/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2}{15}Ab^3x^{15/2}+\frac{2}{19}b^2x^{19/2}(3Ac+bB)+\frac{2}{27}c^2x^{27/2}(Ac+3bB)+\frac{6}{23}bcx^{23/2}(Ac+bB)+\frac{2}{31}Bc^3x^{31/2}$$

input
```
Int[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

output
```
(2*A*b^3*x^(15/2))/15 + (2*b^2*(b*B + 3*A*c)*x^(19/2))/19 + (6*b*c*(b*B +
A*c)*x^(23/2))/23 + (2*c^2*(3*b*B + A*c)*x^(27/2))/27 + (2*B*c^3*x^(31/2))
/31
```

3.178.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.178.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2B\,c^3x^{\frac{31}{2}}}{31}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{27}{2}}}{27}+\frac{2(3Ab\,c^2+3B\,b^2c)x^{\frac{23}{2}}}{23}+\frac{2(3b^2Ac+B\,b^3)x^{\frac{19}{2}}}{19}+\frac{2A\,b^3x^{\frac{15}{2}}}{15}$
default	$\frac{2B\,c^3x^{\frac{31}{2}}}{31}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{27}{2}}}{27}+\frac{2(3Ab\,c^2+3B\,b^2c)x^{\frac{23}{2}}}{23}+\frac{2(3b^2Ac+B\,b^3)x^{\frac{19}{2}}}{19}+\frac{2A\,b^3x^{\frac{15}{2}}}{15}$
gosper	$\frac{2x^{\frac{15}{2}}(58995B\,c^3x^8+67735A\,c^3x^6+203205x^6Bb\,c^2+238545Ab\,c^2x^4+238545x^4B\,b^2c+288765A\,b^2c\,x^2+96255b^3B\,x^2+1}{1828845}$
trager	$\frac{2x^{\frac{15}{2}}(58995B\,c^3x^8+67735A\,c^3x^6+203205x^6Bb\,c^2+238545Ab\,c^2x^4+238545x^4B\,b^2c+288765A\,b^2c\,x^2+96255b^3B\,x^2+1}{1828845}$
risch	$\frac{2x^{\frac{15}{2}}(58995B\,c^3x^8+67735A\,c^3x^6+203205x^6Bb\,c^2+238545Ab\,c^2x^4+238545x^4B\,b^2c+288765A\,b^2c\,x^2+96255b^3B\,x^2+1}{1828845}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/31*B*c^3*x^(31/2)+2/27*(A*c^3+3*B*b*c^2)*x^(27/2)+2/23*(3*A*b*c^2+3*B*b^
2*c)*x^(23/2)+2/19*(3*A*b^2*c+B*b^3)*x^(19/2)+2/15*A*b^3*x^(15/2)
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx$$
$$=\frac{2}{1828845}\left(58995\,Bc^3x^{15}+67735\left(3\,Bbc^2+Ac^3\right)x^{13}+238545\left(Bb^2c+Abc^2\right)x^{11}+121923\,Ab^3x^7+96255\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3*x^(1/2),x, algorithm="fricas")
```

output
```
2/1828845*(58995*B*c^3*x^15 + 67735*(3*B*b*c^2 + A*c^3)*x^13 + 238545*(B*b
^2*c + A*b*c^2)*x^11 + 121923*A*b^3*x^7 + 96255*(B*b^3 + 3*A*b^2*c)*x^9)*s
qrt(x)
```

3.178.6 Sympy [A] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx = \frac{2Ab^3x^{\frac{15}{2}}}{15}+\frac{2Bc^3x^{\frac{31}{2}}}{31}+\frac{2x^{\frac{27}{2}}(Ac^3+3Bbc^2)}{27}+\frac{2x^{\frac{23}{2}}\cdot(3Abc^2+3Bb^2c)}{23}+\frac{2x^{\frac{19}{2}}\cdot(3Ab^2c+Bb^3)}{19}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3*x**(1/2),x)
```

output
```
2*A*b**3*x**(15/2)/15 + 2*B*c**3*x**(31/2)/31 + 2*x**(27/2)*(A*c**3 + 3*B*
b*c**2)/27 + 2*x**(23/2)*(3*A*b*c**2 + 3*B*b**2*c)/23 + 2*x**(19/2)*(3*A*b
**2*c + B*b**3)/19
```

3.178.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx = \frac{2}{31}\,Bc^3x^{\frac{31}{2}}+\frac{2}{27}\,(3\,Bbc^2+Ac^3)x^{\frac{27}{2}}+\frac{6}{23}\,(Bb^2c+Abc^2)x^{\frac{23}{2}}+\frac{2}{15}\,Ab^3x^{\frac{15}{2}}+\frac{2}{19}\,(Bb^3+3\,Ab^2c)x^{\frac{19}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3*x^(1/2),x, algorithm="maxima")
```

output
```
2/31*B*c^3*x^(31/2) + 2/27*(3*B*b*c^2 + A*c^3)*x^(27/2) + 6/23*(B*b^2*c +
A*b*c^2)*x^(23/2) + 2/15*A*b^3*x^(15/2) + 2/19*(B*b^3 + 3*A*b^2*c)*x^(19/2
)
```

3.178.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx = \frac{2}{31}\,Bc^3x^{\frac{31}{2}}+\frac{2}{9}\,Bbc^2x^{\frac{27}{2}}+\frac{2}{27}\,Ac^3x^{\frac{27}{2}}+\frac{6}{23}\,Bb^2cx^{\frac{23}{2}}+\frac{6}{23}\,Abc^2x^{\frac{23}{2}}+\frac{2}{19}\,Bb^3x^{\frac{19}{2}}+\frac{6}{19}\,Ab^2cx^{\frac{19}{2}}+\frac{2}{15}\,Ab^3x^{\frac{15}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3*x^(1/2),x, algorithm="giac")
```

output
```
2/31*B*c^3*x^(31/2) + 2/9*B*b*c^2*x^(27/2) + 2/27*A*c^3*x^(27/2) + 6/23*B*
b^2*c*x^(23/2) + 6/23*A*b*c^2*x^(23/2) + 2/19*B*b^3*x^(19/2) + 6/19*A*b^2*
c*x^(19/2) + 2/15*A*b^3*x^(15/2)
```

3.178.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx = x^{19/2}\left(\frac{2\,B\,b^3}{19}+\frac{6\,A\,c\,b^2}{19}\right)+x^{27/2}\left(\frac{2\,A\,c^3}{27}+\frac{2\,B\,b\,c^2}{9}\right)+\frac{2\,A\,b^3\,x^{15/2}}{15}+\frac{2\,B\,c^3\,x^{31/2}}{31}+\frac{6\,b\,c\,x^{23/2}\,(A\,c+B\,b)}{23}$$

input
```
int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)
```

output
```
x^(19/2)*((2*B*b^3)/19 + (6*A*b^2*c)/19) + x^(27/2)*((2*A*c^3)/27 + (2*B*b
*c^2)/9) + (2*A*b^3*x^(15/2))/15 + (2*B*c^3*x^(31/2))/31 + (6*b*c*x^(23/2)
*(A*c + B*b))/23
```

3.179 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{\sqrt{x}}\,dx$

3.179.1 Optimal result

Integrand size = 26, antiderivative size = 85

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{\sqrt{x}}\,dx = \frac{2}{13}Ab^3x^{13/2}+\frac{2}{17}b^2(bB+3Ac)x^{17/2} + \frac{2}{7}bc(bB+Ac)x^{21/2}+\frac{2}{25}c^2(3bB+Ac)x^{25/2}+\frac{2}{29}Bc^3x^{29/2}$$

output
```
2/13*A*b^3*x^(13/2)+2/17*b^2*(3*A*c+B*b)*x^(17/2)+2/7*b*c*(A*c+B*b)*x^(21/
2)+2/25*c^2*(A*c+3*B*b)*x^(25/2)+2/29*B*c^3*x^(29/2)
```

3.179.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{\sqrt{x}}\,dx = \frac{2x^{13/2}(29A(2975b^3+6825b^2cx^2+5525bc^2x^4+1547c^3x^6)+13Bx^2(5075b^3+12325b^2cx^2+10353bc^2x^4+\ldots}{1121575}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/Sqrt[x],x]
```

output
```
(2*x^(13/2)*(29*A*(2975*b^3 + 6825*b^2*c*x^2 + 5525*b*c^2*x^4 + 1547*c^3*x
^6) + 13*B*x^2*(5075*b^3 + 12325*b^2*c*x^2 + 10353*b*c^2*x^4 + 2975*c^3*x^
6)))/1121575
```

3.179.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{9}$$

$$\int x^{11/2}(A+Bx^2)\left(b+cx^2\right)^3dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(Ab^3x^{11/2}+b^2x^{15/2}(3Ac+bB)+c^2x^{23/2}(Ac+3bB)+3bcx^{19/2}(Ac+bB)+Bc^3x^{27/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{13}Ab^3x^{13/2}+\frac{2}{17}b^2x^{17/2}(3Ac+bB)+\frac{2}{25}c^2x^{25/2}(Ac+3bB)+\frac{2}{7}bcx^{21/2}(Ac+bB)+\frac{2}{29}Bc^3x^{29/2}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/Sqrt[x],x]
```

output
```
(2*A*b^3*x^(13/2))/13 + (2*b^2*(b*B + 3*A*c)*x^(17/2))/17 + (2*b*c*(b*B +
A*c)*x^(21/2))/7 + (2*c^2*(3*b*B + A*c)*x^(25/2))/25 + (2*B*c^3*x^(29/2))/
29
```

3.179.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.179.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2B c^3 x^{\frac{29}{2}}}{29}+\frac{2(A c^3+3Bb c^2)x^{\frac{25}{2}}}{25}+\frac{2(3Ab c^2+3B b^2 c)x^{\frac{21}{2}}}{21}+\frac{2(3b^2Ac+B b^3)x^{\frac{17}{2}}}{17}+\frac{2A b^3 x^{\frac{13}{2}}}{13}$
default	$\frac{2B c^3 x^{\frac{29}{2}}}{29}+\frac{2(A c^3+3Bb c^2)x^{\frac{25}{2}}}{25}+\frac{2(3Ab c^2+3B b^2 c)x^{\frac{21}{2}}}{21}+\frac{2(3b^2Ac+B b^3)x^{\frac{17}{2}}}{17}+\frac{2A b^3 x^{\frac{13}{2}}}{13}$
gosper	$\frac{2x^{\frac{13}{2}}(38675B c^3x^8+44863A c^3x^6+134589x^6 Bb c^2+160225Ab c^2x^4+160225x^4 B b^2c+197925A b^2c x^2+65975b^3 B x^2+8$
trager	$\frac{2x^{\frac{13}{2}}(38675B c^3x^8+44863A c^3x^6+134589x^6 Bb c^2+160225Ab c^2x^4+160225x^4 B b^2c+197925A b^2c x^2+65975b^3 B x^2+8$
risch	$\frac{2x^{\frac{13}{2}}(38675B c^3x^8+44863A c^3x^6+134589x^6 Bb c^2+160225Ab c^2x^4+160225x^4 B b^2c+197925A b^2c x^2+65975b^3 B x^2+8$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/29*B*c^3*x^(29/2)+2/25*(A*c^3+3*B*b*c^2)*x^(25/2)+2/21*(3*A*b*c^2+3*B*b^
2*c)*x^(21/2)+2/17*(3*A*b^2*c+B*b^3)*x^(17/2)+2/13*A*b^3*x^(13/2)
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{\sqrt{x}}\,dx$$
$$=\frac{2}{1121575}\left(38675\,Bc^3x^{14}+44863\left(3\,Bbc^2+Ac^3\right)x^{12}+160225\left(Bb^2c+Abc^2\right)x^{10}+86275\,Ab^3x^6+65975\,(\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="fricas")
```

output
```
2/1121575*(38675*B*c^3*x^14 + 44863*(3*B*b*c^2 + A*c^3)*x^12 + 160225*(B*b
^2*c + A*b*c^2)*x^10 + 86275*A*b^3*x^6 + 65975*(B*b^3 + 3*A*b^2*c)*x^8)*sq
rt(x)
```

3.179.6 Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{\sqrt{x}}\,dx = \frac{2Ab^3x^{\frac{13}{2}}}{13} + \frac{6Ab^2cx^{\frac{17}{2}}}{17} + \frac{2Abc^2x^{\frac{21}{2}}}{7} + \frac{2Ac^3x^{\frac{25}{2}}}{25} + \frac{2Bb^3x^{\frac{17}{2}}}{17} + \frac{2Bb^2cx^{\frac{21}{2}}}{7} + \frac{6Bbc^2x^{\frac{25}{2}}}{25} + \frac{2Bc^3x^{\frac{29}{2}}}{29}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(1/2),x)
```

output
```
2*A*b**3*x**(13/2)/13 + 6*A*b**2*c*x**(17/2)/17 + 2*A*b*c**2*x**(21/2)/7 +
 2*A*c**3*x**(25/2)/25 + 2*B*b**3*x**(17/2)/17 + 2*B*b**2*c*x**(21/2)/7 +
6*B*b*c**2*x**(25/2)/25 + 2*B*c**3*x**(29/2)/29
```

3.179.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{\sqrt{x}}\,dx = \frac{2}{29}\,Bc^3x^{\frac{29}{2}} + \frac{2}{25}\,(3\,Bbc^2 + Ac^3)x^{\frac{25}{2}} + \frac{2}{7}\,(Bb^2c + Abc^2)x^{\frac{21}{2}} + \frac{2}{13}\,Ab^3x^{\frac{13}{2}} + \frac{2}{17}\,(Bb^3 + 3\,Ab^2c)x^{\frac{17}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="maxima")
```

output
```
2/29*B*c^3*x^(29/2) + 2/25*(3*B*b*c^2 + A*c^3)*x^(25/2) + 2/7*(B*b^2*c + A
*b*c^2)*x^(21/2) + 2/13*A*b^3*x^(13/2) + 2/17*(B*b^3 + 3*A*b^2*c)*x^(17/2)
```

3.179.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{\sqrt{x}}\,dx = \frac{2}{29}Bc^3x^{\frac{29}{2}} + \frac{6}{25}Bbc^2x^{\frac{25}{2}} + \frac{2}{25}Ac^3x^{\frac{25}{2}} + \frac{2}{7}Bb^2cx^{\frac{21}{2}} + \frac{2}{7}Abc^2x^{\frac{21}{2}} + \frac{2}{17}Bb^3x^{\frac{17}{2}} + \frac{6}{17}Ab^2cx^{\frac{17}{2}} + \frac{2}{13}Ab^3x^{\frac{13}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="giac")
```

output
```
2/29*B*c^3*x^(29/2) + 6/25*B*b*c^2*x^(25/2) + 2/25*A*c^3*x^(25/2) + 2/7*B*
b^2*c*x^(21/2) + 2/7*A*b*c^2*x^(21/2) + 2/17*B*b^3*x^(17/2) + 6/17*A*b^2*c
*x^(17/2) + 2/13*A*b^3*x^(13/2)
```

3.179.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{\sqrt{x}}\,dx = x^{17/2}\left(\frac{2\,B\,b^3}{17}+\frac{6\,A\,c\,b^2}{17}\right)+x^{25/2}\left(\frac{2\,A\,c^3}{25}+\frac{6\,B\,b\,c^2}{25}\right) + \frac{2\,A\,b^3\,x^{13/2}}{13}+\frac{2\,B\,c^3\,x^{29/2}}{29}+\frac{2\,b\,c\,x^{21/2}\,(A\,c+B\,b)}{7}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(1/2),x)
```

output
```
x^(17/2)*((2*B*b^3)/17 + (6*A*b^2*c)/17) + x^(25/2)*((2*A*c^3)/25 + (6*B*b
*c^2)/25) + (2*A*b^3*x^(13/2))/13 + (2*B*c^3*x^(29/2))/29 + (2*b*c*x^(21/2
)*(A*c + B*b))/7
```

3.180 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}\,dx$

3.180.1 Optimal result

Integrand size = 26, antiderivative size = 85

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}\,dx = \frac{2}{11}Ab^3x^{11/2} + \frac{2}{15}b^2(bB+3Ac)x^{15/2} + \frac{6}{19}bc(bB+Ac)x^{19/2} + \frac{2}{23}c^2(3bB+Ac)x^{23/2} + \frac{2}{27}Bc^3x^{27/2}$$

output
```
2/11*A*b^3*x^(11/2)+2/15*b^2*(3*A*c+B*b)*x^(15/2)+6/19*b*c*(A*c+B*b)*x^(19
/2)+2/23*c^2*(A*c+3*B*b)*x^(23/2)+2/27*B*c^3*x^(27/2)
```

3.180.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}\,dx = \frac{2x^{11/2}(27A(2185b^3+4807b^2cx^2+3795bc^2x^4+1045c^3x^6)+11Bx^2(3933b^3}{648945}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(3/2),x]
```

output
```
(2*x^(11/2)*(27*A*(2185*b^3 + 4807*b^2*c*x^2 + 3795*b*c^2*x^4 + 1045*c^3*x
^6) + 11*B*x^2*(3933*b^3 + 9315*b^2*c*x^2 + 7695*b*c^2*x^4 + 2185*c^3*x^6)
))/648945
```

3.180.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^{9/2}(A+Bx^2)(b+cx^2)^3dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(Ab^3x^{9/2}+b^2x^{13/2}(3Ac+bB)+c^2x^{21/2}(Ac+3bB)+3bcx^{17/2}(Ac+bB)+Bc^3x^{25/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{11}Ab^3x^{11/2}+\frac{2}{15}b^2x^{15/2}(3Ac+bB)+\frac{2}{23}c^2x^{23/2}(Ac+3bB)+\frac{6}{19}bcx^{19/2}(Ac+bB)+\frac{2}{27}Bc^3x^{27/2}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(3/2),x]
```

output
```
(2*A*b^3*x^(11/2))/11 + (2*b^2*(b*B + 3*A*c)*x^(15/2))/15 + (6*b*c*(b*B +
A*c)*x^(19/2))/19 + (2*c^2*(3*b*B + A*c)*x^(23/2))/23 + (2*B*c^3*x^(27/2))
/27
```

3.180.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.180.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2Bc^3x^{\frac{27}{2}}}{27}+\frac{2(Ac^3+3Bbc^2)x^{\frac{23}{2}}}{23}+\frac{2(3Abc^2+3Bb^2c)x^{\frac{19}{2}}}{19}+\frac{2(3b^2Ac+Bb^3)x^{\frac{15}{2}}}{15}+\frac{2Ab^3x^{\frac{11}{2}}}{11}$
default	$\frac{2Bc^3x^{\frac{27}{2}}}{27}+\frac{2(Ac^3+3Bbc^2)x^{\frac{23}{2}}}{23}+\frac{2(3Abc^2+3Bb^2c)x^{\frac{19}{2}}}{19}+\frac{2(3b^2Ac+Bb^3)x^{\frac{15}{2}}}{15}+\frac{2Ab^3x^{\frac{11}{2}}}{11}$
gosper	$\frac{2x^{\frac{11}{2}}(24035Bc^3x^8+28215Ac^3x^6+84645x^6Bbc^2+102465Abc^2x^4+102465x^4Bb^2c+129789Ab^2cx^2+43263b^3Bx^2+58}{648945}$
trager	$\frac{2x^{\frac{11}{2}}(24035Bc^3x^8+28215Ac^3x^6+84645x^6Bbc^2+102465Abc^2x^4+102465x^4Bb^2c+129789Ab^2cx^2+43263b^3Bx^2+58}{648945}$
risch	$\frac{2x^{\frac{11}{2}}(24035Bc^3x^8+28215Ac^3x^6+84645x^6Bbc^2+102465Abc^2x^4+102465x^4Bb^2c+129789Ab^2cx^2+43263b^3Bx^2+58}{648945}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/27*B*c^3*x^(27/2)+2/23*(A*c^3+3*B*b*c^2)*x^(23/2)+2/19*(3*A*b*c^2+3*B*b^
2*c)*x^(19/2)+2/15*(3*A*b^2*c+B*b^3)*x^(15/2)+2/11*A*b^3*x^(11/2)
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int\frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}\,dx=\frac{2}{648945}\left(24035\,Bc^3x^{13}+28215\left(3\,Bbc^2+Ac^3\right)x^{11}+102465\left(Bb^2c+Abc^2\right)\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(3/2),x, algorithm="fricas")
```

output
```
2/648945*(24035*B*c^3*x^13 + 28215*(3*B*b*c^2 + A*c^3)*x^11 + 102465*(B*b^
2*c + A*b*c^2)*x^9 + 58995*A*b^3*x^5 + 43263*(B*b^3 + 3*A*b^2*c)*x^7)*sqrt
(x)
```

3.180.6 Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}\,dx = \frac{2Ab^3x^{\frac{11}{2}}}{11} + \frac{2Ab^2cx^{\frac{15}{2}}}{5} + \frac{6Abc^2x^{\frac{19}{2}}}{19} + \frac{2Ac^3x^{\frac{23}{2}}}{23} + \frac{2Bb^3x^{\frac{15}{2}}}{15} + \frac{6Bb^2cx^{\frac{19}{2}}}{19} + \frac{6Bbc^2x^{\frac{23}{2}}}{23} + \frac{2Bc^3x^{\frac{27}{2}}}{27}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(3/2),x)
```

output
```
2*A*b**3*x**(11/2)/11 + 2*A*b**2*c*x**(15/2)/5 + 6*A*b*c**2*x**(19/2)/19 +
 2*A*c**3*x**(23/2)/23 + 2*B*b**3*x**(15/2)/15 + 6*B*b**2*c*x**(19/2)/19 +
 6*B*b*c**2*x**(23/2)/23 + 2*B*c**3*x**(27/2)/27
```

3.180.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}\,dx = \frac{2}{27}Bc^3x^{\frac{27}{2}} + \frac{2}{23}\left(3\,Bbc^2+Ac^3\right)x^{\frac{23}{2}} + \frac{6}{19}\left(Bb^2c+Abc^2\right)x^{\frac{19}{2}} + \frac{2}{11}Ab^3x^{\frac{11}{2}} + \frac{2}{15}\left(Bb^3+3\,Ab^2c\right)x^{\frac{15}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(3/2),x, algorithm="maxima")
```

output
```
2/27*B*c^3*x^(27/2) + 2/23*(3*B*b*c^2 + A*c^3)*x^(23/2) + 6/19*(B*b^2*c +
A*b*c^2)*x^(19/2) + 2/11*A*b^3*x^(11/2) + 2/15*(B*b^3 + 3*A*b^2*c)*x^(15/2
)
```

3.180.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}\,dx = \frac{2}{27}\,Bc^3x^{\frac{27}{2}} + \frac{6}{23}\,Bbc^2x^{\frac{23}{2}} + \frac{2}{23}\,Ac^3x^{\frac{23}{2}} + \frac{6}{19}\,Bb^2cx^{\frac{19}{2}} + \frac{6}{19}\,Abc^2x^{\frac{19}{2}} + \frac{2}{15}\,Bb^3x^{\frac{15}{2}} + \frac{2}{5}\,Ab^2cx^{\frac{15}{2}} + \frac{2}{11}\,Ab^3x^{\frac{11}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(3/2),x, algorithm="giac")
```

output
```
2/27*B*c^3*x^(27/2) + 6/23*B*b*c^2*x^(23/2) + 2/23*A*c^3*x^(23/2) + 6/19*B
*b^2*c*x^(19/2) + 6/19*A*b*c^2*x^(19/2) + 2/15*B*b^3*x^(15/2) + 2/5*A*b^2*
c*x^(15/2) + 2/11*A*b^3*x^(11/2)
```

3.180.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}\,dx = x^{15/2}\left(\frac{2\,B\,b^3}{15}+\frac{2\,A\,c\,b^2}{5}\right) + x^{23/2}\left(\frac{2\,A\,c^3}{23}+\frac{6\,B\,b\,c^2}{23}\right) + \frac{2\,A\,b^3\,x^{11/2}}{11} + \frac{2\,B\,c^3\,x^{27/2}}{27} + \frac{6\,b\,c\,x^{19/2}\,(A\,c+B\,b)}{19}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(3/2),x)
```

output
```
x^(15/2)*((2*B*b^3)/15 + (2*A*b^2*c)/5) + x^(23/2)*((2*A*c^3)/23 + (6*B*b*
c^2)/23) + (2*A*b^3*x^(11/2))/11 + (2*B*c^3*x^(27/2))/27 + (6*b*c*x^(19/2)
*(A*c + B*b))/19
```

3.181 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{5/2}}\,dx$

3.181.1 Optimal result

Integrand size = 26, antiderivative size = 85

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{5/2}}\,dx = \frac{2}{9}Ab^3x^{9/2} + \frac{2}{13}b^2(bB+3Ac)x^{13/2} + \frac{6}{17}bc(bB+Ac)x^{17/2} + \frac{2}{21}c^2(3bB+Ac)x^{21/2} + \frac{2}{25}Bc^3x^{25/2}$$

output
```
2/9*A*b^3*x^(9/2)+2/13*b^2*(3*A*c+B*b)*x^(13/2)+6/17*b*c*(A*c+B*b)*x^(17/2
)+2/21*c^2*(A*c+3*B*b)*x^(21/2)+2/25*B*c^3*x^(25/2)
```

3.181.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{5/2}}\,dx = \frac{2x^{9/2}(25A(1547b^3+3213b^2cx^2+2457bc^2x^4+663c^3x^6)+9Bx^2(2975b^3+6}{348075}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(5/2),x]
```

output
```
(2*x^(9/2)*(25*A*(1547*b^3 + 3213*b^2*c*x^2 + 2457*b*c^2*x^4 + 663*c^3*x^6
) + 9*B*x^2*(2975*b^3 + 6825*b^2*c*x^2 + 5525*b*c^2*x^4 + 1547*c^3*x^6)))/
348075
```

3.181.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^{7/2}(A+Bx^2)(b+cx^2)^3\,dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(Ab^3x^{7/2}+b^2x^{11/2}(3Ac+bB)+c^2x^{19/2}(Ac+3bB)+3bcx^{15/2}(Ac+bB)+Bc^3x^{23/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{9}Ab^3x^{9/2}+\frac{2}{13}b^2x^{13/2}(3Ac+bB)+\frac{2}{21}c^2x^{21/2}(Ac+3bB)+\frac{6}{17}bcx^{17/2}(Ac+bB)+\frac{2}{25}Bc^3x^{25/2}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(5/2),x]
```

output
```
(2*A*b^3*x^(9/2))/9 + (2*b^2*(b*B + 3*A*c)*x^(13/2))/13 + (6*b*c*(b*B + A*
c)*x^(17/2))/17 + (2*c^2*(3*b*B + A*c)*x^(21/2))/21 + (2*B*c^3*x^(25/2))/2
5
```

3.181.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.181.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2Bc^3x^{\frac{25}{2}}}{25}+\frac{2(Ac^3+3Bbc^2)x^{\frac{21}{2}}}{21}+\frac{2(3Abc^2+3Bb^2c)x^{\frac{17}{2}}}{17}+\frac{2(3b^2Ac+Bb^3)x^{\frac{13}{2}}}{13}+\frac{2Ab^3x^{\frac{9}{2}}}{9}$
default	$\frac{2Bc^3x^{\frac{25}{2}}}{25}+\frac{2(Ac^3+3Bbc^2)x^{\frac{21}{2}}}{21}+\frac{2(3Abc^2+3Bb^2c)x^{\frac{17}{2}}}{17}+\frac{2(3b^2Ac+Bb^3)x^{\frac{13}{2}}}{13}+\frac{2Ab^3x^{\frac{9}{2}}}{9}$
gosper	$\frac{2x^{\frac{9}{2}}(13923Bc^3x^8+16575Ac^3x^6+49725x^6Bbc^2+61425Abc^2x^4+61425x^4Bb^2c+80325Ab^2cx^2+26775b^3Bx^2+38675b}{348075}$
trager	$\frac{2x^{\frac{9}{2}}(13923Bc^3x^8+16575Ac^3x^6+49725x^6Bbc^2+61425Abc^2x^4+61425x^4Bb^2c+80325Ab^2cx^2+26775b^3Bx^2+38675b}{348075}$
risch	$\frac{2x^{\frac{9}{2}}(13923Bc^3x^8+16575Ac^3x^6+49725x^6Bbc^2+61425Abc^2x^4+61425x^4Bb^2c+80325Ab^2cx^2+26775b^3Bx^2+38675b}{348075}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/25*B*c^3*x^(25/2)+2/21*(A*c^3+3*B*b*c^2)*x^(21/2)+2/17*(3*A*b*c^2+3*B*b^
2*c)*x^(17/2)+2/13*(3*A*b^2*c+B*b^3)*x^(13/2)+2/9*A*b^3*x^(9/2)
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{5/2}}\,dx = \frac{2}{348075}\left(13923\,Bc^3x^{12}+16575\left(3\,Bbc^2+Ac^3\right)x^{10}+61425\left(Bb^2c+Abc^2\right)x\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(5/2),x, algorithm="fricas")
```

output
```
2/348075*(13923*B*c^3*x^12 + 16575*(3*B*b*c^2 + A*c^3)*x^10 + 61425*(B*b^2
*c + A*b*c^2)*x^8 + 38675*A*b^3*x^4 + 26775*(B*b^3 + 3*A*b^2*c)*x^6)*sqrt(
x)
```

3.181.6 Sympy [A] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{5/2}}\,dx = \frac{2Ab^3x^{\frac{9}{2}}}{9} + \frac{6Ab^2cx^{\frac{13}{2}}}{13} + \frac{6Abc^2x^{\frac{17}{2}}}{17} + \frac{2Ac^3x^{\frac{21}{2}}}{21} + \frac{2Bb^3x^{\frac{13}{2}}}{13} + \frac{6Bb^2cx^{\frac{17}{2}}}{17} + \frac{2Bbc^2x^{\frac{21}{2}}}{7} + \frac{2Bc^3x^{\frac{25}{2}}}{25}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(5/2),x)
```

output
```
2*A*b**3*x**(9/2)/9 + 6*A*b**2*c*x**(13/2)/13 + 6*A*b*c**2*x**(17/2)/17 +
2*A*c**3*x**(21/2)/21 + 2*B*b**3*x**(13/2)/13 + 6*B*b**2*c*x**(17/2)/17 +
2*B*b*c**2*x**(21/2)/7 + 2*B*c**3*x**(25/2)/25
```

3.181.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{5/2}}\,dx = \frac{2}{25}\,Bc^3x^{\frac{25}{2}} + \frac{2}{21}\,(3\,Bbc^2+Ac^3)x^{\frac{21}{2}} + \frac{6}{17}\,(Bb^2c+Abc^2)x^{\frac{17}{2}} + \frac{2}{9}\,Ab^3x^{\frac{9}{2}} + \frac{2}{13}\,(Bb^3+3\,Ab^2c)x^{\frac{13}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(5/2),x, algorithm="maxima")
```

output
```
2/25*B*c^3*x^(25/2) + 2/21*(3*B*b*c^2 + A*c^3)*x^(21/2) + 6/17*(B*b^2*c +
A*b*c^2)*x^(17/2) + 2/9*A*b^3*x^(9/2) + 2/13*(B*b^3 + 3*A*b^2*c)*x^(13/2)
```

3.181.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{5/2}}\,dx = \frac{2}{25}\,Bc^3x^{\frac{25}{2}} + \frac{2}{7}\,Bbc^2x^{\frac{21}{2}} + \frac{2}{21}\,Ac^3x^{\frac{21}{2}} + \frac{6}{17}\,Bb^2cx^{\frac{17}{2}} + \frac{6}{17}\,Abc^2x^{\frac{17}{2}} + \frac{2}{13}\,Bb^3x^{\frac{13}{2}} + \frac{6}{13}\,Ab^2cx^{\frac{13}{2}} + \frac{2}{9}\,Ab^3x^{\frac{9}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(5/2),x, algorithm="giac")
```

output
```
2/25*B*c^3*x^(25/2) + 2/7*B*b*c^2*x^(21/2) + 2/21*A*c^3*x^(21/2) + 6/17*B*
b^2*c*x^(17/2) + 6/17*A*b*c^2*x^(17/2) + 2/13*B*b^3*x^(13/2) + 6/13*A*b^2*
c*x^(13/2) + 2/9*A*b^3*x^(9/2)
```

3.181.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{5/2}}\,dx = x^{13/2}\left(\frac{2\,B\,b^3}{13}+\frac{6\,A\,c\,b^2}{13}\right) + x^{21/2}\left(\frac{2\,A\,c^3}{21}+\frac{2\,B\,b\,c^2}{7}\right) + \frac{2\,A\,b^3\,x^{9/2}}{9} + \frac{2\,B\,c^3\,x^{25/2}}{25} + \frac{6\,b\,c\,x^{17/2}\,(A\,c+B\,b)}{17}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(5/2),x)
```

output
```
x^(13/2)*((2*B*b^3)/13 + (6*A*b^2*c)/13) + x^(21/2)*((2*A*c^3)/21 + (2*B*b
*c^2)/7) + (2*A*b^3*x^(9/2))/9 + (2*B*c^3*x^(25/2))/25 + (6*b*c*x^(17/2)*(
A*c + B*b))/17
```

3.182 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}}\,dx$

3.182.1 Optimal result

Integrand size = 26, antiderivative size = 85

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2}{7}Ab^3x^{7/2} + \frac{2}{11}b^2(bB+3Ac)x^{11/2} + \frac{2}{5}bc(bB+Ac)x^{15/2} + \frac{2}{19}c^2(3bB+Ac)x^{19/2} + \frac{2}{23}Bc^3x^{23/2}$$

output
```
2/7*A*b^3*x^(7/2)+2/11*b^2*(3*A*c+B*b)*x^(11/2)+2/5*b*c*(A*c+B*b)*x^(15/2)
+2/19*c^2*(A*c+3*B*b)*x^(19/2)+2/23*B*c^3*x^(23/2)
```

3.182.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2x^{7/2}(23A(1045b^3+1995b^2cx^2+1463bc^2x^4+385c^3x^6)+7Bx^2(2185b^3+4}{168245}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(7/2),x]
```

output
```
(2*x^(7/2)*(23*A*(1045*b^3 + 1995*b^2*c*x^2 + 1463*b*c^2*x^4 + 385*c^3*x^6
) + 7*B*x^2*(2185*b^3 + 4807*b^2*c*x^2 + 3795*b*c^2*x^4 + 1045*c^3*x^6)))/
168245
```

3.182.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{9}$$

$$\int x^{5/2}(A+Bx^2)(b+cx^2)^3\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(Ab^3x^{5/2}+b^2x^{9/2}(3Ac+bB)+c^2x^{17/2}(Ac+3bB)+3bcx^{13/2}(Ac+bB)+Bc^3x^{21/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}Ab^3x^{7/2}+\frac{2}{11}b^2x^{11/2}(3Ac+bB)+\frac{2}{19}c^2x^{19/2}(Ac+3bB)+\frac{2}{5}bcx^{15/2}(Ac+bB)+\frac{2}{23}Bc^3x^{23/2}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(7/2),x]
```

output
```
(2*A*b^3*x^(7/2))/7 + (2*b^2*(b*B + 3*A*c)*x^(11/2))/11 + (2*b*c*(b*B + A*
c)*x^(15/2))/5 + (2*c^2*(3*b*B + A*c)*x^(19/2))/19 + (2*B*c^3*x^(23/2))/23
```

3.182.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.182.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2Bc^3x^{\frac{23}{2}}}{23}+\frac{2(Ac^3+3Bbc^2)x^{\frac{19}{2}}}{19}+\frac{2(3Abc^2+3Bb^2c)x^{\frac{15}{2}}}{15}+\frac{2(3b^2Ac+Bb^3)x^{\frac{11}{2}}}{11}+\frac{2Ab^3x^{\frac{7}{2}}}{7}$
default	$\frac{2Bc^3x^{\frac{23}{2}}}{23}+\frac{2(Ac^3+3Bbc^2)x^{\frac{19}{2}}}{19}+\frac{2(3Abc^2+3Bb^2c)x^{\frac{15}{2}}}{15}+\frac{2(3b^2Ac+Bb^3)x^{\frac{11}{2}}}{11}+\frac{2Ab^3x^{\frac{7}{2}}}{7}$
gosper	$\frac{2x^{\frac{7}{2}}(7315Bc^3x^8+8855Ac^3x^6+26565x^6Bbc^2+33649Abc^2x^4+33649x^4Bb^2c+45885Ab^2cx^2+15295b^3Bx^2+24035b^3A}{168245}$
trager	$\frac{2x^{\frac{7}{2}}(7315Bc^3x^8+8855Ac^3x^6+26565x^6Bbc^2+33649Abc^2x^4+33649x^4Bb^2c+45885Ab^2cx^2+15295b^3Bx^2+24035b^3A}{168245}$
risch	$\frac{2x^{\frac{7}{2}}(7315Bc^3x^8+8855Ac^3x^6+26565x^6Bbc^2+33649Abc^2x^4+33649x^4Bb^2c+45885Ab^2cx^2+15295b^3Bx^2+24035b^3A}{168245}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^3/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/23*B*c^3*x^(23/2)+2/19*(A*c^3+3*B*b*c^2)*x^(19/2)+2/15*(3*A*b*c^2+3*B*b^
2*c)*x^(15/2)+2/11*(3*A*b^2*c+B*b^3)*x^(11/2)+2/7*A*b^3*x^(7/2)
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2}{168245}\left(7315\,Bc^3x^{11}+8855\left(3\,Bbc^2+Ac^3\right)x^9+33649\left(Bb^2c+Abc^2\right)x^7+\right.$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(7/2),x, algorithm="fricas")
```

output
```
2/168245*(7315*B*c^3*x^11 + 8855*(3*B*b*c^2 + A*c^3)*x^9 + 33649*(B*b^2*c
+ A*b*c^2)*x^7 + 24035*A*b^3*x^3 + 15295*(B*b^3 + 3*A*b^2*c)*x^5)*sqrt(x)
```

3.182.6 Sympy [A] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2Ab^3x^{\frac{7}{2}}}{7} + \frac{6Ab^2cx^{\frac{11}{2}}}{11} + \frac{2Abc^2x^{\frac{15}{2}}}{5} + \frac{2Ac^3x^{\frac{19}{2}}}{19} + \frac{2Bb^3x^{\frac{11}{2}}}{11} + \frac{2Bb^2cx^{\frac{15}{2}}}{5} + \frac{6Bbc^2x^{\frac{19}{2}}}{19} + \frac{2Bc^3x^{\frac{23}{2}}}{23}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(7/2),x)
```

output
```
2*A*b**3*x**(7/2)/7 + 6*A*b**2*c*x**(11/2)/11 + 2*A*b*c**2*x**(15/2)/5 + 2
*A*c**3*x**(19/2)/19 + 2*B*b**3*x**(11/2)/11 + 2*B*b**2*c*x**(15/2)/5 + 6*
B*b*c**2*x**(19/2)/19 + 2*B*c**3*x**(23/2)/23
```

3.182.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2}{23}\,Bc^3x^{\frac{23}{2}} + \frac{2}{19}\left(3\,Bbc^2 + Ac^3\right)x^{\frac{19}{2}} + \frac{2}{5}\left(Bb^2c + Abc^2\right)x^{\frac{15}{2}} + \frac{2}{7}\,Ab^3x^{\frac{7}{2}} + \frac{2}{11}\left(Bb^3 + 3\,Ab^2c\right)x^{\frac{11}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(7/2),x, algorithm="maxima")
```

output
```
2/23*B*c^3*x^(23/2) + 2/19*(3*B*b*c^2 + A*c^3)*x^(19/2) + 2/5*(B*b^2*c + A
*b*c^2)*x^(15/2) + 2/7*A*b^3*x^(7/2) + 2/11*(B*b^3 + 3*A*b^2*c)*x^(11/2)
```

3.182.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2}{23}\,Bc^3x^{\frac{23}{2}} + \frac{6}{19}\,Bbc^2x^{\frac{19}{2}} + \frac{2}{19}\,Ac^3x^{\frac{19}{2}} + \frac{2}{5}\,Bb^2cx^{\frac{15}{2}} + \frac{2}{5}\,Abc^2x^{\frac{15}{2}} + \frac{2}{11}\,Bb^3x^{\frac{11}{2}} + \frac{6}{11}\,Ab^2cx^{\frac{11}{2}} + \frac{2}{7}\,Ab^3x^{\frac{7}{2}}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(7/2),x, algorithm="giac")
```

output
```
2/23*B*c^3*x^(23/2) + 6/19*B*b*c^2*x^(19/2) + 2/19*A*c^3*x^(19/2) + 2/5*B*
b^2*c*x^(15/2) + 2/5*A*b*c^2*x^(15/2) + 2/11*B*b^3*x^(11/2) + 6/11*A*b^2*c
*x^(11/2) + 2/7*A*b^3*x^(7/2)
```

3.182.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}}\,dx = x^{11/2}\left(\frac{2\,B\,b^3}{11}+\frac{6\,A\,c\,b^2}{11}\right) + x^{19/2}\left(\frac{2\,A\,c^3}{19}+\frac{6\,B\,b\,c^2}{19}\right)+\frac{2\,A\,b^3\,x^{7/2}}{7}+\frac{2\,B\,c^3\,x^{23/2}}{23}+\frac{2\,b\,c\,x^{15/2}\,(A\,c+B\,b)}{5}$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(7/2),x)
```

output
```
x^(11/2)*((2*B*b^3)/11 + (6*A*b^2*c)/11) + x^(19/2)*((2*A*c^3)/19 + (6*B*b
*c^2)/19) + (2*A*b^3*x^(7/2))/7 + (2*B*c^3*x^(23/2))/23 + (2*b*c*x^(15/2)*
(A*c + B*b))/5
```

3.183 $\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

3.183.1 Optimal result

Integrand size = 26, antiderivative size = 278

$$\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} + \frac{b^{7/4}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{15/4}} - \frac{b^{7/4}(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{15/4}} - \frac{b^{7/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{15/4}} + \frac{b^{7/4}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{15/4}}$$

output
```
2/3*b*(-A*c+B*b)*x^(3/2)/c^3-2/7*(-A*c+B*b)*x^(7/2)/c^2+2/11*B*x^(11/2)/c+
1/2*b^(7/4)*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(15/4)*
2^(1/2)-1/2*b^(7/4)*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c
^(15/4)*2^(1/2)-1/4*b^(7/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4
)*2^(1/2)*x^(1/2))/c^(15/4)*2^(1/2)+1/4*b^(7/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^
(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(15/4)*2^(1/2)
```

3.183.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.62

$$\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2x^{3/2}(77b^2B-77Abc-33bBcx^2+33Ac^2x^2+21Bc^2x^4)}{231c^3} + \frac{b^{7/4}(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}c^{15/4}} + \frac{b^{7/4}(bB-Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}c^{15/4}}$$

input
```
Integrate[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(2*x^(3/2)*(77*b^2*B - 77*A*b*c - 33*b*B*c*x^2 + 33*A*c^2*x^2 + 21*B*c^2*x
^4))/(231*c^3) + (b^(7/4)*(b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*c^(15/4)) + (b^(7/4)*(b*B - A*c)*Arc
Tanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*c^
(15/4))
```

3.183.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {9, 363, 262, 262, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^{9/2}(A+Bx^2)}{b+cx^2}dx$$

$$\Big\downarrow \; 363$$

$$\frac{2Bx^{11/2}}{11c} - \frac{(bB-Ac)\int \frac{x^{9/2}}{cx^2+b}dx}{c}$$

$$\Big\downarrow \; 262$$

$$\frac{2Bx^{11/2}}{11c}-\frac{(bB-Ac)\left(\frac{2x^{7/2}}{7c}-\frac{b\int\frac{x^{5/2}}{cx^2+b}dx}{c}\right)}{c}$$

$\Big\downarrow$ 262

$$\frac{2Bx^{11/2}}{11c}-\frac{(bB-Ac)\left(\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{b\int\frac{\sqrt{x}}{cx^2+b}dx}{c}\right)}{c}\right)}{c}$$

$\Big\downarrow$ 266

$$\frac{2Bx^{11/2}}{11c}-\frac{(bB-Ac)\left(\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\int\frac{x}{cx^2+b}d\sqrt{x}}{c}\right)}{c}\right)}{c}$$

$\Big\downarrow$ 826

$$\frac{2Bx^{11/2}}{11c}-\frac{(bB-Ac)\left(\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{c}\right)}{c}$$

$\Big\downarrow$ 1476

3.183. $\int\frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{2Bx^{11/2}}{11c}-\frac{(bB-Ac)\left(\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{c}\right)}{c}$$

$\downarrow$ 1082

3.183. $\int\frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{2Bx^{11/2}}{11c}-\frac{(bB-Ac)\left(\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{c}\right)}{c}$$

$$\Big\downarrow \; 217$$

3.183. $\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{2Bx^{11/2}}{11c}-\frac{(bB-Ac)\left(\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{c}\right)}{c}$$

$\downarrow$ 1479

3.183. $\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{2Bx^{11/2}}{11c}-\frac{(bB-Ac)\left(\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\cdots\right.}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}\right.}\cdots}{2\sqrt{2}\sqrt[4]{\cdots}}}{2\sqrt{c}}\cdots\right)}{c}\cdots\right)}{c}\cdots\right)}{c}\cdots$$

$\downarrow$ 25

3.183. $\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{2Bx^{11/2}}{11c} - \frac{(bB-Ac)\left(\frac{2x^{7/2}}{7c} - \frac{b\left(\frac{2x^{3/2}}{3c} - \frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}} - \frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}} + \frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\cdots\right.}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}\cdots\right.}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{\cdots}}}{2\sqrt{c}}\cdots\right)}{c}\right)}{c}\right)}{c}$$

$$\Big\downarrow \; 27$$

3.183. $\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{2Bx^{11/2}}{11c}-\frac{(bB-Ac)\left(\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{c}\right)}{c}\right)}{c}$$

$\Big\downarrow$ 1103

3.183. $\int \frac{x^{13/2}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx$

$$\frac{2Bx^{11/2}}{11c}-\frac{(bB-Ac)\left(\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{}\right.}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right.}{c}}{c}\right.}{c}$$

input
```
Int[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(2*B*x^(11/2))/(11*c) - ((b*B - A*c)*((2*x^(7/2))/(7*c) - (b*((2*x^(3/2))/
(3*c) - (2*b*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^
(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b
^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)
*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]
)))/c))/c))/c
```

3.183. $\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

3.183.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.183. $\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.183.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.59

method	result
risch	$-\frac{2x^{\frac{3}{2}}\left(-21B\,c^{2}x^{4}-33A\,c^{2}x^{2}+33Bbc\,x^{2}+77Abc-77B\,b^{2}\right)}{231c^{3}}+\frac{b^{2}(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\right.\right.}{4c^{4}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
derivativedivides	$-\frac{2\left(-\frac{B\,x^{\frac{11}{2}}c^{2}}{11}+\frac{\left(-A\,c^{2}+Bbc\right)x^{\frac{7}{2}}}{7}+\frac{\left(Abc-B\,b^{2}\right)x^{\frac{3}{2}}}{3}\right)}{c^{3}}+\frac{b^{2}(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{4c^{4}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$-\frac{2\left(-\frac{B\,x^{\frac{11}{2}}c^{2}}{11}+\frac{\left(-A\,c^{2}+Bbc\right)x^{\frac{7}{2}}}{7}+\frac{\left(Abc-B\,b^{2}\right)x^{\frac{3}{2}}}{3}\right)}{c^{3}}+\frac{b^{2}(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{4c^{4}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

input
```
int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-2/231*x^(3/2)*(-21*B*c^2*x^4-33*A*c^2*x^2+33*B*b*c*x^2+77*A*b*c-77*B*b^2)
/c^3+1/4*b^2*(A*c-B*b)/c^4/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1
/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))
)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)
*x^(1/2)-1))
```

3.183.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.77

$$\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{231\,c^3\left(-\frac{B^4b^{11}-4\,AB^3b^{10}c+6\,A^2B^2b^9c^2-4\,A^3Bb^8c^3+A^4b^7c^4}{c^{15}}\right)^{\frac{1}{4}}\log\left(c^{11}\left(-\frac{B^4b^{11}-4\,AB^3b^{10}c+6\,A^2B}{c}\right.\right.}{}$$

input
```
integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/462*(231*c^3*(-(B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*
b^8*c^3 + A^4*b^7*c^4)/c^15)^(1/4)*log(c^11*(-(B^4*b^11 - 4*A*B^3*b^10*c +
 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^15)^(3/4) - (B^3*b^8
 - 3*A*B^2*b^7*c + 3*A^2*B*b^6*c^2 - A^3*b^5*c^3)*sqrt(x)) - 231*I*c^3*(-(
B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*
c^4)/c^15)^(1/4)*log(I*c^11*(-(B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c
^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^15)^(3/4) - (B^3*b^8 - 3*A*B^2*b^7*c
 + 3*A^2*B*b^6*c^2 - A^3*b^5*c^3)*sqrt(x)) + 231*I*c^3*(-(B^4*b^11 - 4*A*B
^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^15)^(1/4)
*log(-I*c^11*(-(B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^
8*c^3 + A^4*b^7*c^4)/c^15)^(3/4) - (B^3*b^8 - 3*A*B^2*b^7*c + 3*A^2*B*b^6*
c^2 - A^3*b^5*c^3)*sqrt(x)) - 231*c^3*(-(B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2
*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^15)^(1/4)*log(-c^11*(-(B^4
*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4
)/c^15)^(3/4) - (B^3*b^8 - 3*A*B^2*b^7*c + 3*A^2*B*b^6*c^2 - A^3*b^5*c^3)*
sqrt(x)) + 4*(21*B*c^2*x^5 - 33*(B*b*c - A*c^2)*x^3 + 77*(B*b^2 - A*b*c)*x
)*sqrt(x))/c^3
```

3.183.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

output
```
Timed out
```

3.183.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.85

$$\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx =$$

$$-\frac{(Bb^3-Ab^2c)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\right.}{4\,c^3}$$

$$+\frac{2\left(21\,Bc^2x^{\frac{11}{2}}-33\,(Bbc-Ac^2)x^{\frac{7}{2}}+77\,(Bb^2-Abc)x^{\frac{3}{2}}\right)}{231\,c^3}$$

input

```
integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output

```
-1/4*(B*b^3 - A*b^2*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1
/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqr
t(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)
*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)
*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/
4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/
(b^(1/4)*c^(3/4)))/c^3 + 2/231*(21*B*c^2*x^(11/2) - 33*(B*b*c - A*c^2)*x^(
7/2) + 77*(B*b^2 - A*b*c)*x^(3/2))/c^3
```

3.183.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.07

$$\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx =$$
$$-\frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb^2-(bc^3)^{\frac{3}{4}}Abc\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^6}$$
$$-\frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb^2-(bc^3)^{\frac{3}{4}}Abc\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^6}$$
$$+\frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb^2-(bc^3)^{\frac{3}{4}}Abc\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^6}$$
$$-\frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb^2-(bc^3)^{\frac{3}{4}}Abc\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^6}$$
$$+\frac{2\left(21\,Bc^{10}x^{\frac{11}{2}}-33\,Bbc^9x^{\frac{7}{2}}+33\,Ac^{10}x^{\frac{7}{2}}+77\,Bb^2c^8x^{\frac{3}{2}}-77\,Abc^9x^{\frac{3}{2}}\right)}{231\,c^{11}}$$

input
```
integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*((b*c^3)^(3/4)*B*b^2 - (b*c^3)^(3/4)*A*b*c)*arctan(1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^6 - 1/2*sqrt(2)*((b*c^3
)^(3/4)*B*b^2 - (b*c^3)^(3/4)*A*b*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) - 2*sqrt(x))/(b/c)^(1/4))/c^6 + 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b^2 - (b*
c^3)^(3/4)*A*b*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^6 - 1
/4*sqrt(2)*((b*c^3)^(3/4)*B*b^2 - (b*c^3)^(3/4)*A*b*c)*log(-sqrt(2)*sqrt(x
)*(b/c)^(1/4) + x + sqrt(b/c))/c^6 + 2/231*(21*B*c^10*x^(11/2) - 33*B*b*c^
9*x^(7/2) + 33*A*c^10*x^(7/2) + 77*B*b^2*c^8*x^(3/2) - 77*A*b*c^9*x^(3/2))
/c^11
```

3.183.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.41

$$\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx = x^{7/2}\left(\frac{2\,A}{7\,c}-\frac{2\,B\,b}{7\,c^2}\right) + \frac{2\,B\,x^{11/2}}{11\,c} + \frac{(-b)^{7/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(A\,c-B\,b)}{c^{15/4}} - \frac{b\,x^{3/2}\,\left(\frac{2\,A}{c}-\frac{2\,B\,b}{c^2}\right)}{3\,c} + \frac{(-b)^{7/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right)\,(A\,c-B\,b)\,1\mathrm{i}}{c^{15/4}}$$

input
```
int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

output
```
x^(7/2)*((2*A)/(7*c) - (2*B*b)/(7*c^2)) + (2*B*x^(11/2))/(11*c) + ((-b)^(7
/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/c^(15/4) + ((-b)^(7/4)
*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*(A*c - B*b)*1i)/c^(15/4) - (b*x^(3/
2)*((2*A)/c - (2*B*b)/c^2))/(3*c)
```

3.184 $\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

3.184.1 Optimal result

Integrand size = 26, antiderivative size = 276

$$\begin{aligned}\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx &= \frac{2b(bB-Ac)\sqrt{x}}{c^3}-\frac{2(bB-Ac)x^{5/2}}{5c^2}+\frac{2Bx^{9/2}}{9c}\\&+\frac{b^{5/4}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{13/4}}-\frac{b^{5/4}(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{13/4}}\\&+\frac{b^{5/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{13/4}}\\&-\frac{b^{5/4}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{13/4}}\end{aligned}$$

output
```
-2/5*(-A*c+B*b)*x^(5/2)/c^2+2/9*B*x^(9/2)/c+1/2*b^(5/4)*(-A*c+B*b)*arctan(
1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(13/4)*2^(1/2)-1/2*b^(5/4)*(-A*c+B*b)
*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(13/4)*2^(1/2)+1/4*b^(5/4)*(-
A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)*2^
(1/2)-1/4*b^(5/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*
x^(1/2))/c^(13/4)*2^(1/2)+2*b*(-A*c+B*b)*x^(1/2)/c^3
```

3.184.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.63

$$\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2\sqrt{x}(45b^2B-45Abc-9bBcx^2+9Ac^2x^2+5Bc^2x^4)}{45c^3} + \frac{b^{5/4}(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}c^{13/4}} - \frac{b^{5/4}(bB-Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}c^{13/4}}$$

input
```
Integrate[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(2*Sqrt[x]*(45*b^2*B - 45*A*b*c - 9*b*B*c*x^2 + 9*A*c^2*x^2 + 5*B*c^2*x^4)
)/(45*c^3) + (b^(5/4)*(b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*c^(13/4)) - (b^(5/4)*(b*B - A*c)*ArcTanh
[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*c^(13/
4))
```

3.184.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {9, 363, 262, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^{7/2}(A+Bx^2)}{b+cx^2}dx$$

$$\Big\downarrow \ 363$$

$$\frac{2Bx^{9/2}}{9c} - \frac{(bB-Ac)\int \frac{x^{7/2}}{cx^2+b}dx}{c}$$

$$\Big\downarrow \ 262$$

$$\frac{2Bx^{9/2}}{9c}-\frac{(bB-Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\int\frac{x^{3/2}}{cx^2+b}dx}{c}\right)}{c}$$

$\Big\downarrow$ 262

$$\frac{2Bx^{9/2}}{9c}-\frac{(bB-Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{b\int\frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{c}\right)}{c}\right)}{c}$$

$\Big\downarrow$ 266

$$\frac{2Bx^{9/2}}{9c}-\frac{(bB-Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\int\frac{1}{cx^2+b}d\sqrt{x}}{c}\right)}{c}\right)}{c}$$

$\Big\downarrow$ 755

$$\frac{2Bx^{9/2}}{9c}-\frac{(bB-Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{c}$$

$\Big\downarrow$ 1476

3.184. $\int\frac{x^{11/2}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx$

$$\frac{2Bx^{9/2}}{9c}-\frac{(bB-Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{c}$$

$$\Big\downarrow \text{1082}$$

3.184. $\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{\frac{2Bx^{9/2}}{9c}-(bB-Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{c}$$

$\Big\downarrow$ 217

3.184. $\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{2Bx^{9/2}}{9c}-\frac{(bB-Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{c}$$

$\downarrow$ 1479

3.184. $\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{2Bx^{9/2}}{9c}-\frac{(bB-Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(\cdots\right)}{\sqrt{2}\cdots}\cdots}{2\sqrt{b}}\cdots\right)}{c}\cdots\right)}{c}\cdots\right)}{c}\cdots$$

$\downarrow$ 25

3.184. $\int \frac{x^{11/2}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx$

$$\frac{2Bx^{9/2}}{9c} - \frac{(bB-Ac)\left(\frac{2x^{5/2}}{5c} - \frac{b\left(\frac{2\sqrt{x}}{c} - \frac{2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}} + \frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}} + \frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}} - \frac{\arctan\left(1-\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt{2}\sqrt[4]{b}\cdots}}{2\sqrt{b}} \cdots\right)}{c}\right)}{c}\right)}{c}$$

$\Big\downarrow$ 27

3.184. $\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{2Bx^{9/2}}{9c}-\frac{(bB-Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{c}$$

$\big\downarrow$ 1103

3.184. $\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{2Bx^{9/2}}{9c} - \frac{(bB-Ac)\left(\frac{2x^{5/2}}{5c} - \frac{b\left(\frac{2\sqrt{x}}{c} - \frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}} + \frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}} - \frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\cdots\right.}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{c}$$

input
```
Int[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(2*B*x^(9/2))/(9*c) - ((b*B - A*c)*((2*x^(5/2))/(5*c) - (b*((2*Sqrt[x])/c
- (2*b*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*
c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)
*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*
c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/c)
)/c))/c
```

3.184. $\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

3.184.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.184. $\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.184.4 Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.59

method	result
risch	$-\frac{2(-5Bc^2x^4-9Ac^2x^2+9Bbcx^2+45Abc-45Bb^2)\sqrt{x}}{45c^3}+\frac{b(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\right.}{4c^3}$
derivativedivides	$-\frac{2\left(-\frac{Bx^{\frac{9}{2}}c^2}{9}-\frac{Ac^2x^{\frac{5}{2}}}{5}+\frac{Bbcx^{\frac{5}{2}}}{5}+Abc\sqrt{x}-Bb^2\sqrt{x}\right)}{c^3}+\frac{b(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\right.\right.}{4c^3}$
default	$-\frac{2\left(-\frac{Bx^{\frac{9}{2}}c^2}{9}-\frac{Ac^2x^{\frac{5}{2}}}{5}+\frac{Bbcx^{\frac{5}{2}}}{5}+Abc\sqrt{x}-Bb^2\sqrt{x}\right)}{c^3}+\frac{b(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\right.\right.}{4c^3}$

input
```
int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*(-5*B*c^2*x^4-9*A*c^2*x^2+9*B*b*c*x^2+45*A*b*c-45*B*b^2)*x^(1/2)/c^3
+1/4*b*(A*c-B*b)/c^3*(1/c*b)^(1/4)*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^
(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*ar
ctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/
2)-1))
```

3.184.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 647, normalized size of antiderivative = 2.34

$$\int\frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{45\,c^3\left(-\frac{B^4b^9-4\,AB^3b^8c+6\,A^2B^2b^7c^2-4\,A^3Bb^6c^3+A^4b^5c^4}{c^{13}}\right)^{\frac{1}{4}}\log\left(c^3\left(-\frac{B^4b^9-4\,AB^3b^8c+6\,A^2B^2b^7c^2-}{c^{13}}\right.\right.}{}$$

input
```
integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/90*(45*c^3*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*
c^3 + A^4*b^5*c^4)/c^13)^(1/4)*log(c^3*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*
B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(1/4) - (B*b^2 - A*b*c)
*sqrt(x)) + 45*I*c^3*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2 - 4*A^
3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(1/4)*log(I*c^3*(-(B^4*b^9 - 4*A*B^3*b^8*
c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(1/4) - (B*b^
2 - A*b*c)*sqrt(x)) - 45*I*c^3*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*
c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(1/4)*log(-I*c^3*(-(B^4*b^9 - 4
*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(1
/4) - (B*b^2 - A*b*c)*sqrt(x)) - 45*c^3*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2
*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(1/4)*log(-c^3*(-(B^4*
b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c
^13)^(1/4) - (B*b^2 - A*b*c)*sqrt(x)) + 4*(5*B*c^2*x^4 + 45*B*b^2 - 45*A*b
*c - 9*(B*b*c - A*c^2)*x^2)*sqrt(x))/c^3
```

3.184.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

output
```
Timed out
```

3.184.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.94

$$\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{\left(\frac{2\sqrt{2}(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}(Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}(Bb-Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x \cdots\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}} \cdots\right)}{4c^3} + \frac{2\left(5\,Bc^2x^{\frac{9}{2}} - 9\,(Bbc-Ac^2)x^{\frac{5}{2}} + 45\,(Bb^2-Abc)\sqrt{x}\right)}{45\,c^3}$$

input
```
integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*(B*b - A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) +
2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c)))
+ 2*sqrt(2)*(B*b - A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*s
qrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + s
qrt(2)*(B*b - A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(
b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(B*b - A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*s
qrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))*b^2/c^3 + 2/45*(5*B*c^2*x
^(9/2) - 9*(B*b*c - A*c^2)*x^(5/2) + 45*(B*b^2 - A*b*c)*sqrt(x))/c^3
```

3.184.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.08

$$\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx =$$

$$-\frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb^2-(bc^3)^{\frac{1}{4}}Abc\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^4}$$
$$-\frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb^2-(bc^3)^{\frac{1}{4}}Abc\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^4}$$
$$-\frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb^2-(bc^3)^{\frac{1}{4}}Abc\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^4}$$
$$+\frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb^2-(bc^3)^{\frac{1}{4}}Abc\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^4}$$
$$+\frac{2\left(5Bc^8x^{\frac{9}{2}}-9Bbc^7x^{\frac{5}{2}}+9Ac^8x^{\frac{5}{2}}+45Bb^2c^6\sqrt{x}-45Abc^7\sqrt{x}\right)}{45c^9}$$

input
```
integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*((b*c^3)^(1/4)*B*b^2 - (b*c^3)^(1/4)*A*b*c)*arctan(1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^4 - 1/2*sqrt(2)*((b*c^3
)^(1/4)*B*b^2 - (b*c^3)^(1/4)*A*b*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) - 2*sqrt(x))/(b/c)^(1/4))/c^4 - 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b^2 - (b*
c^3)^(1/4)*A*b*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 + 1
/4*sqrt(2)*((b*c^3)^(1/4)*B*b^2 - (b*c^3)^(1/4)*A*b*c)*log(-sqrt(2)*sqrt(x
)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 + 2/45*(5*B*c^8*x^(9/2) - 9*B*b*c^7*x^(
5/2) + 9*A*c^8*x^(5/2) + 45*B*b^2*c^6*sqrt(x) - 45*A*b*c^7*sqrt(x))/c^9
```

3.184.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 788, normalized size of antiderivative = 2.86

$$\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx = x^{5/2}\left(\frac{2\,A}{5\,c}-\frac{2\,B\,b}{5\,c^2}\right)+\frac{2\,B\,x^{9/2}}{9\,c}$$

$$-\frac{(-b)^{5/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{5/4}\left(\frac{16\,\sqrt{x}\,\left(A^2\,b^4\,c^2-2\,A\,B\,b^5\,c+B^2\,b^6\right)}{c^3}-\frac{(-b)^{5/4}\,(A\,c-B\,b)\,\left(32\,B\,b^4-32\,A\,b^3\,c\right)\,1\mathrm{i}}{2\,c^{13/4}}\right)(A\,c-B\,b)}{2\,c^{13/4}}+\frac{(-b)^{5/4}\left(\frac{16\,\sqrt{x}\,\left(A^2\,b^4\,c^2-2\,A\right.}{c^3}\right.}{}}{\frac{(-b)^{5/4}\left(\frac{16\,\sqrt{x}\,\left(A^2\,b^4\,c^2-2\,A\,B\,b^5\,c+B^2\,b^6\right)}{c^3}-\frac{(-b)^{5/4}\,(A\,c-B\,b)\,\left(32\,B\,b^4-32\,A\,b^3\,c\right)\,1\mathrm{i}}{2\,c^{13/4}}\right)(A\,c-B\,b)\,1\mathrm{i}}{2\,c^{13/4}}-\frac{(-b)^{5/4}\left(\frac{16\,\sqrt{x}\,\left(A^2\,b^4\,c^2-2\,A\right.}{c^3}\right.}{}}\right)}{c^{13/4}}$$

$$-\frac{b\,\sqrt{x}\,\left(\frac{2\,A}{c}-\frac{2\,B\,b}{c^2}\right)}{c}$$

$$-\frac{(-b)^{5/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{5/4}\left(\frac{16\,\sqrt{x}\,\left(A^2\,b^4\,c^2-2\,A\,B\,b^5\,c+B^2\,b^6\right)}{c^3}-\frac{(-b)^{5/4}\,(A\,c-B\,b)\,\left(32\,B\,b^4-32\,A\,b^3\,c\right)}{2\,c^{13/4}}\right)(A\,c-B\,b)\,1\mathrm{i}}{2\,c^{13/4}}+\frac{(-b)^{5/4}\left(\frac{16\,\sqrt{x}\,\left(A^2\,b^4\,c^2-2\,A\,B\right.}{c^3}\right.}{}}{\frac{(-b)^{5/4}\left(\frac{16\,\sqrt{x}\,\left(A^2\,b^4\,c^2-2\,A\,B\,b^5\,c+B^2\,b^6\right)}{c^3}-\frac{(-b)^{5/4}\,(A\,c-B\,b)\,\left(32\,B\,b^4-32\,A\,b^3\,c\right)}{2\,c^{13/4}}\right)(A\,c-B\,b)}{2\,c^{13/4}}-\frac{(-b)^{5/4}\left(\frac{16\,\sqrt{x}\,\left(A^2\,b^4\,c^2-2\,A\,B\right.}{c^3}\right.}{}}\right)}{c^{13/4}}$$

input
```
int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

output
```
x^(5/2)*((2*A)/(5*c) - (2*B*b)/(5*c^2)) + (2*B*x^(9/2))/(9*c) - ((-b)^(5/4
)*atan((((-b)^(5/4)*((16*x^(1/2)*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^
3 - ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c))/(2*c^(13/4)))*(A*c -
B*b)*1i)/(2*c^(13/4)) + ((-b)^(5/4)*((16*x^(1/2)*(B^2*b^6 + A^2*b^4*c^2 -
2*A*B*b^5*c))/c^3 + ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c))/(2*c^
(13/4)))*(A*c - B*b)*1i)/(2*c^(13/4)))/(((-b)^(5/4)*((16*x^(1/2)*(B^2*b^6
+ A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 - ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4 - 32
*A*b^3*c))/(2*c^(13/4)))*(A*c - B*b))/(2*c^(13/4)) - ((-b)^(5/4)*((16*x^(1
/2)*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 + ((-b)^(5/4)*(A*c - B*b)*(
32*B*b^4 - 32*A*b^3*c))/(2*c^(13/4)))*(A*c - B*b))/(2*c^(13/4))))*(A*c - B
*b)*1i)/c^(13/4) - ((-b)^(5/4)*atan((((-b)^(5/4)*((16*x^(1/2)*(B^2*b^6 + A
^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 - ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4 - 32*A*
b^3*c)*1i)/(2*c^(13/4)))*(A*c - B*b))/(2*c^(13/4)) + ((-b)^(5/4)*((16*x^(1
/2)*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 + ((-b)^(5/4)*(A*c - B*b)*(
32*B*b^4 - 32*A*b^3*c)*1i)/(2*c^(13/4)))*(A*c - B*b))/(2*c^(13/4)))/(((-b)
^(5/4)*((16*x^(1/2)*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 - ((-b)^(5/
4)*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c)*1i)/(2*c^(13/4)))*(A*c - B*b)*1i)/(
2*c^(13/4)) - ((-b)^(5/4)*((16*x^(1/2)*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*
c))/c^3 + ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c)*1i)/(2*c^(13/4))
)*(A*c - B*b)*1i)/(2*c^(13/4))))*(A*c - B*b))/c^(13/4) - (b*x^(1/2)*((2...
```

3.184. $\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

3.185 $\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

3.185.1 Optimal result

Integrand size = 26, antiderivative size = 257

$$\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{2(bB-Ac)x^{3/2}}{3c^2}+\frac{2Bx^{7/2}}{7c} - \frac{b^{3/4}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}} + \frac{b^{3/4}(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}} + \frac{b^{3/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}} - \frac{b^{3/4}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}$$

output
```
-2/3*(-A*c+B*b)*x^(3/2)/c^2+2/7*B*x^(7/2)/c-1/2*b^(3/4)*(-A*c+B*b)*arctan(
1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(11/4)*2^(1/2)+1/2*b^(3/4)*(-A*c+B*b)
*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(11/4)*2^(1/2)+1/4*b^(3/4)*(-
A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)*2^
(1/2)-1/4*b^(3/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*
x^(1/2))/c^(11/4)*2^(1/2)
```

3.185.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.59

$$\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2x^{3/2}(-7bB+7Ac+3Bcx^2)}{21c^2} - \frac{b^{3/4}(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}c^{11/4}} - \frac{b^{3/4}(bB-Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}c^{11/4}}$$

input
```
Integrate[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(2*x^(3/2)*(-7*b*B + 7*A*c + 3*B*c*x^2))/(21*c^2) - (b^(3/4)*(b*B - A*c)*A
rcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*c
^(11/4)) - (b^(3/4)*(b*B - A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/
(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*c^(11/4))
```

3.185.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {9, 363, 262, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^{5/2}(A+Bx^2)}{b+cx^2}dx$$

$$\Big\downarrow \ 363$$

$$\frac{2Bx^{7/2}}{7c} - \frac{(bB-Ac)\int \frac{x^{5/2}}{cx^2+b}dx}{c}$$

$$\Big\downarrow \ 262$$

$$\frac{2Bx^{7/2}}{7c} - \frac{(bB-Ac)\left(\frac{2x^{3/2}}{3c} - \frac{b\int \frac{\sqrt{x}}{cx^2+b}dx}{c}\right)}{c}$$

$$\Big\downarrow \ 266$$

$$\frac{2Bx^{7/2}}{7c}-\frac{(bB-Ac)\left(\frac{2x^{3/2}}{3c}-\frac{2b\int \frac{x}{cx^2+b}d\sqrt{x}}{c}\right)}{c}$$

$$\Big\downarrow \ 826$$

$$\frac{2Bx^{7/2}}{7c}-\frac{(bB-Ac)\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{c}$$

$$\Big\downarrow \ 1476$$

$$\frac{2Bx^{7/2}}{7c}-\frac{(bB-Ac)\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{c}$$

$$\Big\downarrow \ 1082$$

$$\frac{2Bx^{7/2}}{7c}-\frac{(bB-Ac)\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{c}$$

$$\Big\downarrow \ 217$$

3.185. $\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{2Bx^{7/2}}{7c}-\frac{(bB-Ac)\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{c}$$

$\Big\downarrow$ 1479

$$\frac{2Bx^{7/2}}{7c}-\frac{(bB-Ac)\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}\cdots}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\cdots\right)}{c}\right)}{c}$$

$\Big\downarrow$ 25

3.185. $\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{2Bx^{7/2}}{7c} - \frac{(bB-Ac)\left(\frac{2x^{3/2}}{3c} - \frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}} - \frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}} + \frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{\cdots}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}} \cdots}{c}\right)}{c}$$

$\downarrow$ 27

$$\frac{2Bx^{7/2}}{7c} - \frac{(bB-Ac)\left(\frac{2x^{3/2}}{3c} - \frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}} - \frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}} + \frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{c}\right)}{c}$$

$\downarrow$ 1103

3.185. $\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{2Bx^{7/2}}{7c}-\frac{(bB-Ac)\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}\right)}{c}$$

input
```
Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(2*B*x^(7/2))/(7*c) - ((b*B - A*c)*((2*x^(3/2))/(3*c) - (2*b*((-(ArcTan[1
- (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1
 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c
]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqr
t[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sq
rt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/c))/c
```

3.185.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.185. $\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.185.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.54

method	result
risch	$\frac{2x^{\frac{3}{2}}(3Bc\,x^2+7Ac-7Bb)}{21c^2}-\frac{b(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^3\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
derivativedivides	$\frac{\frac{2Bc\,x^{\frac{7}{2}}}{7}+\frac{2(Ac-Bb)x^{\frac{3}{2}}}{3}}{c^2}-\frac{b(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^3\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$\frac{\frac{2Bc\,x^{\frac{7}{2}}}{7}+\frac{2(Ac-Bb)x^{\frac{3}{2}}}{3}}{c^2}-\frac{b(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^3\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

input
```
int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
2/21*x^(3/2)*(3*B*c*x^2+7*A*c-7*B*b)/c^2-1/4*b*(A*c-B*b)/c^3/(1/c*b)^(1/4)
*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1
/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)
+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

3.185.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 749, normalized size of antiderivative = 2.91

$$\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{21\,c^2\left(-\frac{B^4b^7-4\,AB^3b^6c+6\,A^2B^2b^5c^2-4\,A^3Bb^4c^3+A^4b^3c^4}{c^{11}}\right)^{\frac{1}{4}}\log\left(c^8\left(-\frac{B^4b^7-4\,AB^3b^6c+6\,A^2B^2b^5c^2-4\,A^3Bb^4c^3+A^4b^3c^4}{c^{11}}\right)^{\frac{3}{4}}-(B$$

input
```
integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
-1/42*(21*c^2*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4
*c^3 + A^4*b^3*c^4)/c^11)^(1/4)*log(c^8*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2
*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11)^(3/4) - (B^3*b^5 - 3*A
*B^2*b^4*c + 3*A^2*B*b^3*c^2 - A^3*b^2*c^3)*sqrt(x)) - 21*I*c^2*(-(B^4*b^7
 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11
)^(1/4)*log(I*c^8*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B
*b^4*c^3 + A^4*b^3*c^4)/c^11)^(3/4) - (B^3*b^5 - 3*A*B^2*b^4*c + 3*A^2*B*b
^3*c^2 - A^3*b^2*c^3)*sqrt(x)) + 21*I*c^2*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A
^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11)^(1/4)*log(-I*c^8*(-(
B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^
4)/c^11)^(3/4) - (B^3*b^5 - 3*A*B^2*b^4*c + 3*A^2*B*b^3*c^2 - A^3*b^2*c^3)
*sqrt(x)) - 21*c^2*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*
B*b^4*c^3 + A^4*b^3*c^4)/c^11)^(1/4)*log(-c^8*(-(B^4*b^7 - 4*A*B^3*b^6*c +
 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11)^(3/4) - (B^3*b^5
 - 3*A*B^2*b^4*c + 3*A^2*B*b^3*c^2 - A^3*b^2*c^3)*sqrt(x)) - 4*(3*B*c*x^3
- 7*(B*b - A*c)*x)*sqrt(x))/c^2
```

3.185.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

output
```
Timed out
```

3.185.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.83

$$\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{(Bb^2-Abc)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log}{}\right.}{4c^2} + \frac{2\left(3Bcx^{\frac{7}{2}}-7(Bb-Ac)x^{\frac{3}{2}}\right)}{21c^2}$$

input
```
integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/4*(B*b^2 - A*b*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4)
 + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c
)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sq
rt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*lo
g(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4))
 + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^
(1/4)*c^(3/4)))/c^2 + 2/21*(3*B*c*x^(7/2) - 7*(B*b - A*c)*x^(3/2))/c^2
```

3.185.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.03

$$\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^5} + \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^5} - \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^5} + \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^5} + \frac{2\left(3\,Bc^6x^{\frac{7}{2}}-7\,Bbc^5x^{\frac{3}{2}}+7\,Ac^6x^{\frac{3}{2}}\right)}{21\,c^7}$$

input
```
integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^5 + 1/2*sqrt(2)*((b*c^3)^(3/
4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*s
qrt(x))/(b/c)^(1/4))/c^5 - 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*
A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 1/4*sqrt(2)*((
b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x
 + sqrt(b/c))/c^5 + 2/21*(3*B*c^6*x^(7/2) - 7*B*b*c^5*x^(3/2) + 7*A*c^6*x^
(3/2))/c^7
```

3.185.9 Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.36

$$\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx = x^{3/2}\left(\frac{2\,A}{3\,c}-\frac{2\,B\,b}{3\,c^2}\right)+\frac{2\,B\,x^{7/2}}{7\,c} + \frac{(-b)^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(A\,c-B\,b)}{c^{11/4}} + \frac{(-b)^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right)\,(A\,c-B\,b)\,1\mathrm{i}}{c^{11/4}}$$

input
```
int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

output
```
x^(3/2)*((2*A)/(3*c) - (2*B*b)/(3*c^2)) + (2*B*x^(7/2))/(7*c) + ((-b)^(3/4
)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/c^(11/4) + ((-b)^(3/4)*a
tan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*(A*c - B*b)*1i)/c^(11/4)
```

3.186 $\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

3.186.1 Optimal result

Integrand size = 26, antiderivative size = 255

$$\begin{aligned}\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx &= -\frac{2(bB-Ac)\sqrt{x}}{c^2}+\frac{2Bx^{5/2}}{5c}\\ &-\frac{\sqrt[4]{b}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}}+\frac{\sqrt[4]{b}(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}}\\ &-\frac{\sqrt[4]{b}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}}\\ &+\frac{\sqrt[4]{b}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}}\end{aligned}$$

output
```
2/5*B*x^(5/2)/c-1/2*b^(1/4)*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^
(1/4))/c^(9/4)*2^(1/2)+1/2*b^(1/4)*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(
1/2)/b^(1/4))/c^(9/4)*2^(1/2)-1/4*b^(1/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-
b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)*2^(1/2)+1/4*b^(1/4)*(-A*c+B*b)*ln
(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)*2^(1/2)-2*(-A*
c+B*b)*x^(1/2)/c^2
```

3.186.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.59

$$\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{4\sqrt[4]{c}\sqrt{x}(-5bB+5Ac+Bcx^2)-5\sqrt{2}\sqrt[4]{b}(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+5\sqrt{2}\sqrt[4]{\cdots}}{10c^{9/4}}$$

input
```
Integrate[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(4*c^(1/4)*Sqrt[x]*(-5*b*B + 5*A*c + B*c*x^2) - 5*Sqrt[2]*b^(1/4)*(b*B - A
*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 5*Sq
rt[2]*b^(1/4)*(b*B - A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[
b] + Sqrt[c]*x)])/(10*c^(9/4))
```

3.186.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {9, 363, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^{3/2}(A+Bx^2)}{b+cx^2}dx$$

$$\Big\downarrow \; 363$$

$$\frac{2Bx^{5/2}}{5c}-\frac{(bB-Ac)\int \frac{x^{3/2}}{cx^2+b}dx}{c}$$

$$\Big\downarrow \; 262$$

$$\frac{2Bx^{5/2}}{5c}-\frac{(bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{b\int \frac{1}{\sqrt{x}(cx^2+b)}dx}{c}\right)}{c}$$

$$\Big\downarrow \; 266$$

$$\frac{2Bx^{5/2}}{5c}-\frac{(bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2b\int\frac{1}{cx^2+b}d\sqrt{x}}{c}\right)}{c}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{2Bx^{5/2}}{5c}-\frac{(bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{c}\right)}{c}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{2Bx^{5/2}}{5c}-\frac{(bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{2Bx^{5/2}}{5c}-\frac{(bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}$$

$$\Big\downarrow \text{ 217}$$

3.186. $\int\frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{2Bx^{5/2}}{5c}-\frac{(bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}$$

$\downarrow$ 1479

$$\frac{2Bx^{5/2}}{5c}-\frac{(bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}$$

$\downarrow$ 25

3.186. $\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{2Bx^{5/2}}{5c}-\frac{(bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}$$

$\downarrow$ 27

$$\frac{2Bx^{5/2}}{5c}-\frac{(bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}$$

$\downarrow$ 1103

3.186. $\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{2Bx^{5/2}}{5c}-\frac{(bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}$$

input
```
Int[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(2*B*x^(5/2))/(5*c) - ((b*B - A*c)*((2*Sqrt[x])/c - (2*b*((-(ArcTan[1 - (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (
Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) +
 (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]
*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/c))/c
```

3.186.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.186. $\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.186.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.54

method	result
risch	$\frac{2(Bc\,x^2+5Ac-5Bb)\sqrt{x}}{5c^2}-\frac{(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{4c^2}$
derivativedivides	$\frac{\frac{2Bc\,x^{\frac{5}{2}}}{5}+2Ac\sqrt{x}-2bB\sqrt{x}}{c^2}-\frac{(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{4c^2}$
default	$\frac{\frac{2Bc\,x^{\frac{5}{2}}}{5}+2Ac\sqrt{x}-2bB\sqrt{x}}{c^2}-\frac{(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{4c^2}$

input
```
int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(B*c*x^2+5*A*c-5*B*b)*x^(1/2)/c^2-1/4*(A*c-B*b)/c^2*(1/c*b)^(1/4)*2^(1
/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x
^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2
*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

3.186.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 597, normalized size of antiderivative = 2.34

$$\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx =$$

$$-\frac{5\,c^2\left(-\frac{B^4b^5-4\,AB^3b^4c+6\,A^2B^2b^3c^2-4\,A^3Bb^2c^3+A^4bc^4}{c^9}\right)^{\frac{1}{4}}\log\left(c^2\left(-\frac{B^4b^5-4\,AB^3b^4c+6\,A^2B^2b^3c^2-4\,A^3Bb^2c^3+A^4bc^4}{c^9}\right)^{\frac{1}{4}}-(Bb\right.}{}$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
-1/10*(5*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*
c^3 + A^4*b*c^4)/c^9)^(1/4)*log(c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2
*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^(1/4) - (B*b - A*c)*sqrt(x))
+ 5*I*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3
 + A^4*b*c^4)/c^9)^(1/4)*log(I*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*
b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^(1/4) - (B*b - A*c)*sqrt(x)) -
 5*I*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3
+ A^4*b*c^4)/c^9)^(1/4)*log(-I*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*
b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^(1/4) - (B*b - A*c)*sqrt(x)) -
 5*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 +
A^4*b*c^4)/c^9)^(1/4)*log(-c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*
c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^(1/4) - (B*b - A*c)*sqrt(x)) - 4*(
B*c*x^2 - 5*B*b + 5*A*c)*sqrt(x))/c^2
```

3.186.6 Sympy [A] (verification not implemented)

Time = 98.31 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.08

$$\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \begin{cases} \tilde{\infty}\left(2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}\right) \\ \frac{\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{9}{2}}}{9}}{b} \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}}{c} \\ \frac{2A\sqrt{x}}{c}+\frac{A\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2c}-\frac{A\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2c}-\frac{A\sqrt[4]{-\frac{b}{c}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{c}- \end{cases}$$

input
```
integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

output
```
Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(5/2)/5), Eq(b, 0) & Eq(c, 0)), ((2*A
*x**(5/2)/5 + 2*B*x**(9/2)/9)/b, Eq(c, 0)), ((2*A*sqrt(x) + 2*B*x**(5/2)/5
)/c, Eq(b, 0)), (2*A*sqrt(x)/c + A*(-b/c)**(1/4)*log(sqrt(x) - (-b/c)**(1/
4))/(2*c) - A*(-b/c)**(1/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*c) - A*(-b/c)*
*(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/c - 2*B*b*sqrt(x)/c**2 - B*b*(-b/c)**(1
/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*c**2) + B*b*(-b/c)**(1/4)*log(sqrt(x)
+ (-b/c)**(1/4))/(2*c**2) + B*b*(-b/c)**(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/
c**2 + 2*B*x**(5/2)/(5*c), True))
```

3.186.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.92

$$\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{\left(\frac{2\sqrt{2}(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(\cdots}{\cdots}\cdots}{4\,c^2\cdots} + \frac{2\left(Bcx^{\frac{5}{2}}-5\,(Bb-Ac)\sqrt{x}\right)}{5\,c^2}$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*(B*b - A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2
*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) +
 2*sqrt(2)*(B*b - A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sq
rt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sq
rt(2)*(B*b - A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b
))/(b^(3/4)*c^(1/4)) - sqrt(2)*(B*b - A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sq
rt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))*b/c^2 + 2/5*(B*c*x^(5/2) -
 5*(B*b - A*c)*sqrt(x))/c^2
```

3.186.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.03

$$\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb-(bc^3)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^3} + \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb-(bc^3)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^3} + \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb-(bc^3)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^3} - \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb-(bc^3)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^3} + \frac{2\left(Bc^4x^{\frac{5}{2}}-5Bbc^3\sqrt{x}+5Ac^4\sqrt{x}\right)}{5c^5}$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^3 + 1/2*sqrt(2)*((b*c^3)^(1/
4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*s
qrt(x))/(b/c)^(1/4))/c^3 + 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*
A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 - 1/4*sqrt(2)*((
b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x
 + sqrt(b/c))/c^3 + 2/5*(B*c^4*x^(5/2) - 5*B*b*c^3*sqrt(x) + 5*A*c^4*sqrt(
x))/c^5
```

3.186.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 789, normalized size of antiderivative = 3.09

$$\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \sqrt{x}\left(\frac{2\,A}{c}-\frac{2\,B\,b}{c^2}\right)+\frac{2\,B\,x^{5/2}}{5\,c}$$
$$-\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{1/4}\,(A\,c-B\,b)\left(\frac{16\,\sqrt{x}\,\left(A^2\,b^2\,c^2-2\,A\,B\,b^3\,c+B^2\,b^4\right)}{c}-\frac{(-b)^{1/4}\,\left(32\,A\,b^2\,c^2-32\,B\,b^3\,c\right)\,(A\,c-B\,b)}{2\,c^{9/4}}\right)\,1\mathrm{i}}{2\,c^{9/4}}+\frac{(-b)^{1/4}\,(A\,c-B\,b)\left(\frac{16\,\sqrt{x}\,\left(A\cdots\right.}{\cdots}\right.}{\cdots}}{\frac{(-b)^{1/4}\,(A\,c-B\,b)\left(\frac{16\,\sqrt{x}\,\left(A^2\,b^2\,c^2-2\,A\,B\,b^3\,c+B^2\,b^4\right)}{c}-\frac{(-b)^{1/4}\,\left(32\,A\,b^2\,c^2-32\,B\,b^3\,c\right)\,(A\,c-B\,b)}{2\,c^{9/4}}\right)}{2\,c^{9/4}}-\frac{(-b)^{1/4}\,(A\,c-B\,b)\left(\frac{16\,\sqrt{x}\,\left(A\cdots\right.}{\cdots}\right.}{\cdots}}\right.\cdots}{c^{9/4}}$$
$$-\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{1/4}\,(A\,c-B\,b)\left(\frac{16\,\sqrt{x}\,\left(A^2\,b^2\,c^2-2\,A\,B\,b^3\,c+B^2\,b^4\right)}{c}-\frac{(-b)^{1/4}\,\left(32\,A\,b^2\,c^2-32\,B\,b^3\,c\right)\,(A\,c-B\,b)\,1\mathrm{i}}{2\,c^{9/4}}\right)}{2\,c^{9/4}}+\frac{(-b)^{1/4}\,(A\,c-B\,b)\left(\frac{16\,\sqrt{x}\,\left(\cdots\right.}{\cdots}\right.}{\cdots}}{\frac{(-b)^{1/4}\,(A\,c-B\,b)\left(\frac{16\,\sqrt{x}\,\left(A^2\,b^2\,c^2-2\,A\,B\,b^3\,c+B^2\,b^4\right)}{c}-\frac{(-b)^{1/4}\,\left(32\,A\,b^2\,c^2-32\,B\,b^3\,c\right)\,(A\,c-B\,b)\,1\mathrm{i}}{2\,c^{9/4}}\right)\,1\mathrm{i}}{2\,c^{9/4}}-\frac{(-b)^{1/4}\,(A\,c-B\,b)\left(\frac{16\,\sqrt{x}\,\left(\cdots\right.}{\cdots}\right.}{\cdots}}\right.\cdots}{c^{9/4}}$$

input
```
int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

output
```
x^(1/2)*((2*A)/c - (2*B*b)/c^2) + (2*B*x^(5/2))/(5*c) - ((-b)^(1/4)*atan((
((-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*(B^2*b^4 + A^2*b^2*c^2 - 2*A*B*b^3*c)
)/c - ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B*b^3*c)*(A*c - B*b))/(2*c^(9/4)))*1i
)/(2*c^(9/4)) + ((-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*(B^2*b^4 + A^2*b^2*c^
2 - 2*A*B*b^3*c))/c + ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B*b^3*c)*(A*c - B*b))
/(2*c^(9/4)))*1i)/(2*c^(9/4)))/(((-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*(B^2*
b^4 + A^2*b^2*c^2 - 2*A*B*b^3*c))/c - ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B*b^3
*c)*(A*c - B*b))/(2*c^(9/4))))/(2*c^(9/4)) - ((-b)^(1/4)*(A*c - B*b)*((16*
x^(1/2)*(B^2*b^4 + A^2*b^2*c^2 - 2*A*B*b^3*c))/c + ((-b)^(1/4)*(32*A*b^2*c
^2 - 32*B*b^3*c)*(A*c - B*b))/(2*c^(9/4))))/(2*c^(9/4))))*(A*c - B*b)*1i)/
c^(9/4) - ((-b)^(1/4)*atan((((-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*(B^2*b^4
+ A^2*b^2*c^2 - 2*A*B*b^3*c))/c - ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B*b^3*c)*
(A*c - B*b)*1i)/(2*c^(9/4))))/(2*c^(9/4)) + ((-b)^(1/4)*(A*c - B*b)*((16*x
^(1/2)*(B^2*b^4 + A^2*b^2*c^2 - 2*A*B*b^3*c))/c + ((-b)^(1/4)*(32*A*b^2*c^
2 - 32*B*b^3*c)*(A*c - B*b)*1i)/(2*c^(9/4))))/(2*c^(9/4)))/(((-b)^(1/4)*(A
*c - B*b)*((16*x^(1/2)*(B^2*b^4 + A^2*b^2*c^2 - 2*A*B*b^3*c))/c - ((-b)^(1
/4)*(32*A*b^2*c^2 - 32*B*b^3*c)*(A*c - B*b)*1i)/(2*c^(9/4)))*1i)/(2*c^(9/4
)) - ((-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*(B^2*b^4 + A^2*b^2*c^2 - 2*A*B*b
^3*c))/c + ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B*b^3*c)*(A*c - B*b)*1i)/(2*c^(9
/4)))*1i)/(2*c^(9/4))))*(A*c - B*b))/c^(9/4)
```

3.187 $\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

3.187.1 Optimal result

Integrand size = 26, antiderivative size = 237

$$\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2Bx^{3/2}}{3c} + \frac{(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}} - \frac{(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}} - \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}} + \frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}}$$

output
```
2/3*B*x^(3/2)/c+1/2*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b
^(1/4)/c^(7/4)*2^(1/2)-1/2*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(
1/4))/b^(1/4)/c^(7/4)*2^(1/2)-1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*
c^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(7/4)*2^(1/2)+1/4*(-A*c+B*b)*ln(b^(1/2)
+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(7/4)*2^(1/2)
```

3.187.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.57

$$\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2Bx^{3/2}}{3c} + \frac{(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}} + \frac{(bB-Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}}$$

input
```
Integrate[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(2*B*x^(3/2))/(3*c) + ((b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(1/4)*c^(7/4)) + ((b*B - A*c)*ArcTanh
[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(1/4
)*c^(7/4))
```

3.187.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {9, 363, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{\sqrt{x}(A+Bx^2)}{b+cx^2}dx$$

$$\Big\downarrow \text{363}$$

$$\frac{2Bx^{3/2}}{3c} - \frac{(bB-Ac)\int \frac{\sqrt{x}}{cx^2+b}dx}{c}$$

$$\Big\downarrow \text{266}$$

$$\frac{2Bx^{3/2}}{3c} - \frac{2(bB-Ac)\int \frac{x}{cx^2+b}d\sqrt{x}}{c}$$

$\downarrow$ 826

$$\frac{2Bx^{3/2}}{3c}-\frac{2(bB-Ac)\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}$$

$\downarrow$ 1476

$$\frac{2Bx^{3/2}}{3c}-\frac{2(bB-Ac)\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}$$

$\downarrow$ 1082

$$\frac{2Bx^{3/2}}{3c}-\frac{2(bB-Ac)\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}$$

$\downarrow$ 217

$$\frac{2Bx^{3/2}}{3c}-\frac{2(bB-Ac)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}$$

$\downarrow$ 1479

$$\frac{2Bx^{3/2}}{3c}-\frac{2(bB-Ac)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}$$

$\downarrow$ 25

3.187. $\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{\frac{2Bx^{3/2}}{3c}-2(bB-Ac)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}$$

$\Big\downarrow$ 27

$$\frac{\frac{2Bx^{3/2}}{3c}-2(bB-Ac)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{c}$$

$\Big\downarrow$ 1103

$$\frac{\frac{2Bx^{3/2}}{3c}-2(bB-Ac)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}$$

input
```
Int[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(2*B*x^(3/2))/(3*c) - (2*(b*B - A*c)*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[
x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt
[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[b] -
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) +
Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1
/4)*c^(1/4)))/(2*Sqrt[c])))/c
```

3.187. $\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

3.187.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.187. $\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.187.4 Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.52

method	result	size
derivativedivides	$\frac{2Bx^{\frac{3}{2}}}{3c}+\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	124
default	$\frac{2Bx^{\frac{3}{2}}}{3c}+\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	124
risch	$\frac{2Bx^{\frac{3}{2}}}{3c}+\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	124

input
```
int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
2/3*B*x^(3/2)/c+1/4*(A*c-B*b)/c^2/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/
4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)
^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b
)^(1/4)*x^(1/2)-1))
```

3.187.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 691, normalized size of antiderivative = 2.92

$$\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{4\,Bx^{\frac{3}{2}}+3\,c\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{bc^7}\right)^{\frac{1}{4}}\log\left(bc^5\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B}{bc}\right.\right.}{}$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/6*(4*B*x^(3/2) + 3*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*
A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(1/4)*log(b*c^5*(-(B^4*b^4 - 4*A*B^3*b^3*c
 + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(3/4) - (B^3*b^3
- 3*A*B^2*b^2*c + 3*A^2*B*b*c^2 - A^3*c^3)*sqrt(x)) - 3*I*c*(-(B^4*b^4 - 4
*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(1/4)
*log(I*b*c^5*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^
3 + A^4*c^4)/(b*c^7))^(3/4) - (B^3*b^3 - 3*A*B^2*b^2*c + 3*A^2*B*b*c^2 - A
^3*c^3)*sqrt(x)) + 3*I*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 -
4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(1/4)*log(-I*b*c^5*(-(B^4*b^4 - 4*A*B^3*
b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(3/4) - (B^3
*b^3 - 3*A*B^2*b^2*c + 3*A^2*B*b*c^2 - A^3*c^3)*sqrt(x)) - 3*c*(-(B^4*b^4
- 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(1
/4)*log(-b*c^5*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*
c^3 + A^4*c^4)/(b*c^7))^(3/4) - (B^3*b^3 - 3*A*B^2*b^2*c + 3*A^2*B*b*c^2 -
 A^3*c^3)*sqrt(x)))/c
```

3.187.6 Sympy [A] (verification not implemented)

Time = 37.34 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.28

$$\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{3}{2}}}{3}\right) \\ \frac{-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{3}{2}}}{3}}{c} \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{7}{2}}}{7}}{b} \\ \frac{2A\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{c\sqrt[4]{-\frac{b}{c}}} - \frac{A\left(-\frac{b}{c}\right)^{\frac{3}{4}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b} + \frac{A\left(-\frac{b}{c}\right)^{\frac{3}{4}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b} + \frac{A\left(-\frac{b}{c}\right)^{\frac{3}{4}}\operatorname{atan}\left(\right.}{b} \end{cases}$$

input
```
integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

output
```
Piecewise((zoo*(-2*A/sqrt(x) + 2*B*x**(3/2)/3), Eq(b, 0) & Eq(c, 0)), ((-2
*A/sqrt(x) + 2*B*x**(3/2)/3)/c, Eq(b, 0)), ((2*A*x**(3/2)/3 + 2*B*x**(7/2)
/7)/b, Eq(c, 0)), (2*A*atan(sqrt(x)/(-b/c)**(1/4))/(c*(-b/c)**(1/4)) - A*(
-b/c)**(3/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b) + A*(-b/c)**(3/4)*log(sqrt
(x) + (-b/c)**(1/4))/(2*b) + A*(-b/c)**(3/4)*atan(sqrt(x)/(-b/c)**(1/4))/b
 - 2*B*b*atan(sqrt(x)/(-b/c)**(1/4))/(c**2*(-b/c)**(1/4)) + 2*B*x**(3/2)/(
3*c) + B*(-b/c)**(3/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*c) - B*(-b/c)**(3/4
)*log(sqrt(x) + (-b/c)**(1/4))/(2*c) - B*(-b/c)**(3/4)*atan(sqrt(x)/(-b/c)
**(1/4))/c, True))
```

3.187.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.82

$$\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,c} - \frac{(Bb-Ac)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \right.}{4\,c}$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
2/3*B*x^(3/2)/c - 1/4*(B*b - A*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b
^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*s
qrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4)
- 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)
) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^
(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x
+ sqrt(b))/(b^(1/4)*c^(3/4)))/c
```

3.187.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.06

$$\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,c} - \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}\,Bb-(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^4} - \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}\,Bb-(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^4} + \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}\,Bb-(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc^4} - \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}\,Bb-(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc^4}$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
2/3*B*x^(3/2)/c - 1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arct
an(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) - 1/
2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) + 1/4*sqrt(2)*((b*c^3)^
(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(
b/c))/(b*c^4) - 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-s
qrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^4)
```

3.187.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.30

$$\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2\,B\,x^{3/2}}{3\,c} + \frac{\operatorname{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(A\,c-B\,b)}{(-b)^{1/4}\,c^{7/4}} - \frac{\operatorname{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(A\,c-B\,b)}{(-b)^{1/4}\,c^{7/4}}$$

input
```
int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

output
```
(2*B*x^(3/2))/(3*c) + (atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/((-
b)^(1/4)*c^(7/4)) - (atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/((-b
)^(1/4)*c^(7/4))
```

3.188 $\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

3.188.1 Optimal result

Integrand size = 26, antiderivative size = 235

$$\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2B\sqrt{x}}{c} + \frac{(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}c^{5/4}} - \frac{(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}c^{5/4}} + \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}c^{5/4}} - \frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}c^{5/4}}$$

output
```
1/2*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(5/4)*2
^(1/2)-1/2*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^
(5/4)*2^(1/2)+1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*
x^(1/2))/b^(3/4)/c^(5/4)*2^(1/2)-1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/
4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(5/4)*2^(1/2)+2*B*x^(1/2)/c
```

3.188.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.57

$$\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2B\sqrt{x}}{c} + \frac{(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}b^{3/4}c^{5/4}} - \frac{(bB-Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{3/4}c^{5/4}}$$

input
```
Integrate[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(2*B*Sqrt[x])/c + ((b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(3/4)*c^(5/4)) - ((b*B - A*c)*ArcTanh[(Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(3/4)*c^
(5/4))
```

3.188.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {9, 363, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{A+Bx^2}{\sqrt{x}(b+cx^2)}dx$$

$$\Big\downarrow \; 363$$

$$\frac{2B\sqrt{x}}{c} - \frac{(bB-Ac)\int \frac{1}{\sqrt{x}(cx^2+b)}dx}{c}$$

$$\Big\downarrow \; 266$$

$$\frac{2B\sqrt{x}}{c} - \frac{2(bB-Ac)\int \frac{1}{cx^2+b}d\sqrt{x}}{c}$$

$$\Big\downarrow 755$$

$$\frac{2B\sqrt{x}}{c}-\frac{2(bB-Ac)\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{c}$$

$$\Big\downarrow 1476$$

$$\frac{2B\sqrt{x}}{c}-\frac{2(bB-Ac)\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{c}$$

$$\Big\downarrow 1082$$

$$\frac{2B\sqrt{x}}{c}-\frac{2(bB-Ac)\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}$$

$$\Big\downarrow 217$$

$$\frac{2B\sqrt{x}}{c}-\frac{2(bB-Ac)\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}$$

$$\Big\downarrow 1479$$

$$\frac{2B\sqrt{x}}{c}-\frac{2(bB-Ac)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}$$

$$\Big\downarrow 25$$

3.188. $\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{2B\sqrt{x}}{c}-\frac{2(bB-Ac)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}$$

$$\Big\downarrow \; 27$$

$$\frac{2B\sqrt{x}}{c}-\frac{2(bB-Ac)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}$$

$$\Big\downarrow \; 1103$$

$$\frac{2B\sqrt{x}}{c}-\frac{2(bB-Ac)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}$$

input
```
Int[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(2*B*Sqrt[x])/c - (2*(b*B - A*c)*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/
b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])
/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqr
t[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[
Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*
c^(1/4)))/(2*Sqrt[b])))/c
```

3.188. $\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

3.188.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.188.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.54

method	result	size
derivativedivides	$\frac{2B\sqrt{x}}{c}+\frac{(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4cb}$	127
default	$\frac{2B\sqrt{x}}{c}+\frac{(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4cb}$	127
risch	$\frac{2B\sqrt{x}}{c}+\frac{(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4cb}$	127

input
```
int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
2*B*x^(1/2)/c+1/4*(A*c-B*b)/c*(1/c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)
*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(
1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^
(1/4)*x^(1/2)-1))
```

3.188.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.43

$$\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{c\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^3c^5}\right)^{\frac{1}{4}}\log\left(bc\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3}{b^3c^5}\right.\right.}{}$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/2*(c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^
4*c^4)/(b^3*c^5))^(1/4)*log(b*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2
*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5))^(1/4) - (B*b - A*c)*sqrt(x)) +
I*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c
^4)/(b^3*c^5))^(1/4)*log(I*b*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*
c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5))^(1/4) - (B*b - A*c)*sqrt(x)) - I
*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^
4)/(b^3*c^5))^(1/4)*log(-I*b*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*
c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5))^(1/4) - (B*b - A*c)*sqrt(x)) - c
*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)
/(b^3*c^5))^(1/4)*log(-b*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2
- 4*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5))^(1/4) - (B*b - A*c)*sqrt(x)) + 4*B*s
qrt(x))/c
```

3.188.6 Sympy [A] (verification not implemented)

Time = 15.75 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.01

$$\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}\right) \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}}{c} \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}}{b} \\ -\frac{A\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b}+\frac{A\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b}+\frac{A\sqrt[4]{-\frac{b}{c}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b}+\frac{2B\sqrt{x}}{c} \end{cases}$$

input
```
integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

output
```
Piecewise((zoo*(-2*A/(3*x**(3/2)) + 2*B*sqrt(x)), Eq(b, 0) & Eq(c, 0)), ((
-2*A/(3*x**(3/2)) + 2*B*sqrt(x))/c, Eq(b, 0)), ((2*A*sqrt(x) + 2*B*x**(5/2
)/5)/b, Eq(c, 0)), (-A*(-b/c)**(1/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b) +
A*(-b/c)**(1/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*b) + A*(-b/c)**(1/4)*atan(
sqrt(x)/(-b/c)**(1/4))/b + 2*B*sqrt(x)/c + B*(-b/c)**(1/4)*log(sqrt(x) - (
-b/c)**(1/4))/(2*c) - B*(-b/c)**(1/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*c) -
 B*(-b/c)**(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/c, True))
```

3.188.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.93

$$\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2\,B\sqrt{x}}{c} - \frac{\frac{2\,\sqrt{2}(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\,\sqrt{2}(Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(Bb-Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{4\,c}$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
2*B*sqrt(x)/c - 1/4*(2*sqrt(2)*(B*b - A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(
1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqr
t(b)*sqrt(c))) + 2*sqrt(2)*(B*b - A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4
)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b
)*sqrt(c))) + sqrt(2)*(B*b - A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sq
rt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(B*b - A*c)*log(-sqrt(2)*b^
(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/c
```

3.188.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.07

$$\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2B\sqrt{x}}{c} - \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb-(bc^3)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2bc^2} - \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb-(bc^3)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2bc^2} - \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb-(bc^3)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4bc^2} + \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb-(bc^3)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4bc^2}$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
2*B*sqrt(x)/c - 1/2*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan
(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^2) - 1/2*
sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(
2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^2) - 1/4*sqrt(2)*((b*c^3)^(1
/4)*B*b - (b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/
c))/(b*c^2) + 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(-sqr
t(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^2)
```

3.188.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 739, normalized size of antiderivative = 3.14

$$\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2\,B\,\sqrt{x}}{c}$$

$$-\frac{\operatorname{atan}\left(\frac{\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,c^3-32\,A\,B\,b\,c^2+16\,B^2\,b^2\,c\right)-\frac{\left(32\,B\,b^2\,c^2-32\,A\,b\,c^3\right)(A\,c-B\,b)}{2\,(-b)^{3/4}\,c^{5/4}}\right)1\mathrm{i}}{2\,(-b)^{3/4}\,c^{5/4}}+\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,c^3-32\,A\,B\,b\,c^2+16\,B^2\,b^2\,c\right)+\frac{(32\,\cdots}{\cdots}\right.}{2\,(-b)^{3/4}\,c^{5/4}}}{\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,c^3-32\,A\,B\,b\,c^2+16\,B^2\,b^2\,c\right)-\frac{\left(32\,B\,b^2\,c^2-32\,A\,b\,c^3\right)(A\,c-B\,b)}{2\,(-b)^{3/4}\,c^{5/4}}\right)}{2\,(-b)^{3/4}\,c^{5/4}}-\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,c^3-32\,A\,B\,b\,c^2+16\,B^2\,b^2\,c\right)+\frac{(32\,\cdots}{\cdots}\right.}{2\,(-b)^{3/4}\,c^{5/4}}}\right.}{(-b)^{3/4}\,c^{5/4}}$$

$$-\frac{\operatorname{atan}\left(\frac{\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,c^3-32\,A\,B\,b\,c^2+16\,B^2\,b^2\,c\right)-\frac{\left(32\,B\,b^2\,c^2-32\,A\,b\,c^3\right)(A\,c-B\,b)\,1\mathrm{i}}{2\,(-b)^{3/4}\,c^{5/4}}\right)}{2\,(-b)^{3/4}\,c^{5/4}}+\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,c^3-32\,A\,B\,b\,c^2+16\,B^2\,b^2\,c\right)+\frac{(3\cdots}{\cdots}\right.}{2\,(-b)^{3/4}\,c^{5/4}}}{\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,c^3-32\,A\,B\,b\,c^2+16\,B^2\,b^2\,c\right)-\frac{\left(32\,B\,b^2\,c^2-32\,A\,b\,c^3\right)(A\,c-B\,b)\,1\mathrm{i}}{2\,(-b)^{3/4}\,c^{5/4}}\right)1\mathrm{i}}{2\,(-b)^{3/4}\,c^{5/4}}-\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,c^3-32\,A\,B\,b\,c^2+16\,B^2\,b^2\,c\right)+\frac{(3\cdots}{\cdots}\right.}{2\,(-b)^{3/4}\,c^{5/4}}}\right.}{(-b)^{3/4}\,c^{5/4}}$$

input
```
int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

output
```
(2*B*x^(1/2))/c - (atan((((A*c - B*b)*(x^(1/2)*(16*A^2*c^3 + 16*B^2*b^2*c
- 32*A*B*b*c^2) - ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c - B*b))/(2*(-b)^(3/4)*
c^(5/4)))*1i)/(2*(-b)^(3/4)*c^(5/4)) + ((A*c - B*b)*(x^(1/2)*(16*A^2*c^3 +
 16*B^2*b^2*c - 32*A*B*b*c^2) + ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c - B*b))/
(2*(-b)^(3/4)*c^(5/4)))*1i)/(2*(-b)^(3/4)*c^(5/4)))/(((A*c - B*b)*(x^(1/2)
*(16*A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) - ((32*B*b^2*c^2 - 32*A*b*c^3)
*(A*c - B*b))/(2*(-b)^(3/4)*c^(5/4))))/(2*(-b)^(3/4)*c^(5/4)) - ((A*c - B*
b)*(x^(1/2)*(16*A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) + ((32*B*b^2*c^2 -
32*A*b*c^3)*(A*c - B*b))/(2*(-b)^(3/4)*c^(5/4))))/(2*(-b)^(3/4)*c^(5/4))))
*(A*c - B*b)*1i)/((-b)^(3/4)*c^(5/4)) - (atan((((A*c - B*b)*(x^(1/2)*(16*A
^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) - ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c
- B*b)*1i)/(2*(-b)^(3/4)*c^(5/4))))/(2*(-b)^(3/4)*c^(5/4)) + ((A*c - B*b)*
(x^(1/2)*(16*A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) + ((32*B*b^2*c^2 - 32*
A*b*c^3)*(A*c - B*b)*1i)/(2*(-b)^(3/4)*c^(5/4))))/(2*(-b)^(3/4)*c^(5/4)))/
(((A*c - B*b)*(x^(1/2)*(16*A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) - ((32*B
*b^2*c^2 - 32*A*b*c^3)*(A*c - B*b)*1i)/(2*(-b)^(3/4)*c^(5/4)))*1i)/(2*(-b)
^(3/4)*c^(5/4)) - ((A*c - B*b)*(x^(1/2)*(16*A^2*c^3 + 16*B^2*b^2*c - 32*A*
B*b*c^2) + ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c - B*b)*1i)/(2*(-b)^(3/4)*c^(5
/4)))*1i)/(2*(-b)^(3/4)*c^(5/4))))*(A*c - B*b))/((-b)^(3/4)*c^(5/4))
```

3.189 $\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx$

3.189.1 Optimal result

Integrand size = 26, antiderivative size = 235

$$\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{2A}{b\sqrt{x}} - \frac{(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}c^{3/4}} + \frac{(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}c^{3/4}} + \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}} - \frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}}$$

output
```
-1/2*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^(3/4)*
2^(1/2)+1/2*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c
^(3/4)*2^(1/2)+1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)
*x^(1/2))/b^(5/4)/c^(3/4)*2^(1/2)-1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1
/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/c^(3/4)*2^(1/2)-2*A/b/x^(1/2)
```

3.189.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{2A}{b\sqrt{x}} - \frac{(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}b^{5/4}c^{3/4}} - \frac{(bB-Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{5/4}c^{3/4}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(-2*A)/(b*Sqrt[x]) - ((b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(5/4)*c^(3/4)) - ((b*B - A*c)*ArcTanh[
(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(5/4)
*c^(3/4))
```

3.189.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {9, 359, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{A+Bx^2}{x^{3/2}(b+cx^2)}dx$$

$$\Big\downarrow \text{ 359}$$

$$\frac{(bB-Ac)\int \frac{\sqrt{x}}{cx^2+b}dx}{b} - \frac{2A}{b\sqrt{x}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2(bB-Ac)\int \frac{x}{cx^2+b}d\sqrt{x}}{b}-\frac{2A}{b\sqrt{x}}$$

$$\Big\downarrow \text{826}$$

$$\frac{2(bB-Ac)\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2A}{b\sqrt{x}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2(bB-Ac)\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2A}{b\sqrt{x}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2(bB-Ac)\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2A}{b\sqrt{x}}$$

$$\Big\downarrow \text{217}$$

$$\frac{2(bB-Ac)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2A}{b\sqrt{x}}$$

$$\Big\downarrow \text{1479}$$

3.189. $\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{2(bB-Ac)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2A}{b\sqrt{x}}$$

$\downarrow$ 25

$$\frac{2(bB-Ac)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2A}{b\sqrt{x}}$$

$\downarrow$ 27

$$\frac{2(bB-Ac)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{b}-\frac{2A}{b\sqrt{x}}$$

$\downarrow$ 1103

3.189. $\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx$

$$\frac{2(bB-Ac)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2A}{b\sqrt{x}}$$

input
```
Int[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(-2*A)/(b*Sqrt[x]) + (2*(b*B - A*c)*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x
])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[
x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[b] -
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + L
og[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/
4)*c^(1/4)))/(2*Sqrt[c])))/b
```

3.189.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.189. $\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx$

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.189. $\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx$

3.189.4 Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.54

method	result	size
derivativedivides	$-\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4bc\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{2A}{b\sqrt{x}}$	127
default	$-\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4bc\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{2A}{b\sqrt{x}}$	127
risch	$-\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4bc\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{2A}{b\sqrt{x}}$	127

input
```
int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(A*c-B*b)/b/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1
/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arct
an(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)
-1))-2*A/b/x^(1/2)
```

3.189.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.00

$$\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx =$$

$$-\frac{bx\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^5c^3}\right)^{\frac{1}{4}}\log\left(b^4c^2\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^5c^3}\right)^{\frac{3}{4}}-(B^3\right.}{}$$

input
```
integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
-1/2*(b*x*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 +
 A^4*c^4)/(b^5*c^3))^(1/4)*log(b^4*c^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*
B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(3/4) - (B^3*b^3 - 3*A*B
^2*b^2*c + 3*A^2*B*b*c^2 - A^3*c^3)*sqrt(x)) - I*b*x*(-(B^4*b^4 - 4*A*B^3*
b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(1/4)*log(
I*b^4*c^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 +
 A^4*c^4)/(b^5*c^3))^(3/4) - (B^3*b^3 - 3*A*B^2*b^2*c + 3*A^2*B*b*c^2 - A^
3*c^3)*sqrt(x)) + I*b*x*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4
*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(1/4)*log(-I*b^4*c^2*(-(B^4*b^4 - 4*A*B
^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(3/4) -
 (B^3*b^3 - 3*A*B^2*b^2*c + 3*A^2*B*b*c^2 - A^3*c^3)*sqrt(x)) - b*x*(-(B^4
*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c
^3))^(1/4)*log(-b^4*c^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4
*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(3/4) - (B^3*b^3 - 3*A*B^2*b^2*c + 3*A^
2*B*b*c^2 - A^3*c^3)*sqrt(x)) + 4*A*sqrt(x))/(b*x)
```

3.189.6 Sympy [A] (verification not implemented)

Time = 8.34 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx$$

$$= \begin{cases} \tilde{\infty}\left(-\frac{2A}{5x^{\frac{5}{2}}}-\frac{2B}{\sqrt{x}}\right) \\ \frac{-\frac{2A}{5x^{\frac{5}{2}}}-\frac{2B}{\sqrt{x}}}{c} \\ \frac{-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{3}{2}}}{3}}{b} \\ -\frac{2A\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b\sqrt[4]{-\frac{b}{c}}}-\frac{2A}{b\sqrt{x}}+\frac{Ac\left(-\frac{b}{c}\right)^{\frac{3}{4}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b^2}-\frac{Ac\left(-\frac{b}{c}\right)^{\frac{3}{4}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b^2}-\frac{Ac\left(-\frac{b}{c}\right)^{\frac{3}{4}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b^2}+ \end{cases}$$

input
```
integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2),x)
```

output
```
Piecewise((zoo*(-2*A/(5*x**(5/2)) - 2*B/sqrt(x)), Eq(b, 0) & Eq(c, 0)), ((
-2*A/(5*x**(5/2)) - 2*B/sqrt(x))/c, Eq(b, 0)), ((-2*A/sqrt(x) + 2*B*x**(3/
2)/3)/b, Eq(c, 0)), (-2*A*atan(sqrt(x)/(-b/c)**(1/4))/(b*(-b/c)**(1/4)) -
2*A/(b*sqrt(x)) + A*c*(-b/c)**(3/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b**2)
- A*c*(-b/c)**(3/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*b**2) - A*c*(-b/c)**(3
/4)*atan(sqrt(x)/(-b/c)**(1/4))/b**2 + 2*B*atan(sqrt(x)/(-b/c)**(1/4))/(c*
(-b/c)**(1/4)) - B*(-b/c)**(3/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b) + B*(-
b/c)**(3/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*b) + B*(-b/c)**(3/4)*atan(sqrt
(x)/(-b/c)**(1/4))/b, True))
```

3.189.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx$$
$$= \frac{(Bb-Ac)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\cdots\right.}{4\,b}$$
$$-\frac{2\,A}{b\sqrt{x}}$$

input
```
integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/4*(B*b - A*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2
*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) +
 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x
))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + s
qrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4
)*c^(3/4)))/b - 2*A/(b*sqrt(x))
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{2A}{b\sqrt{x}} + \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^2c^3} + \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^2c^3} - \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4b^2c^3} + \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4b^2c^3}$$

input
```
integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-2*A/(b*sqrt(x)) + 1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arc
tan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) +
 1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) - 1/4*sqrt(2)*((b*
c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
sqrt(b/c))/(b^2*c^3) + 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)
*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^3)
```

3.189.9 Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{\mathrm{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(A\,c-B\,b)}{(-b)^{5/4}\,c^{3/4}} - \frac{2A}{b\sqrt{x}} - \frac{\mathrm{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(A\,c-B\,b)}{(-b)^{5/4}\,c^{3/4}}$$

input
```
int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

output
```
(atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/((-b)^(5/4)*c^(3/4)) - (2
*A)/(b*x^(1/2)) - (atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/((-b)^
(5/4)*c^(3/4))
```

3.189. $\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx$

3.190 $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)} dx$

3.190.1 Optimal result

Integrand size = 26, antiderivative size = 237

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)}\,dx = -\frac{2A}{3bx^{3/2}} - \frac{(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}\sqrt[4]{c}} + \frac{(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}\sqrt[4]{c}} - \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}} + \frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}}$$

output
```
-2/3*A/b/x^(3/2)-1/2*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/
b^(7/4)/c^(1/4)*2^(1/2)+1/2*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^
(1/4))/b^(7/4)/c^(1/4)*2^(1/2)-1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)
*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(1/4)*2^(1/2)+1/4*(-A*c+B*b)*ln(b^(1/2
)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(1/4)*2^(1/2)
```

3.190.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.57

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)}\,dx = -\frac{2A}{3bx^{3/2}} - \frac{(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}b^{7/4}\sqrt[4]{c}} + \frac{(bB-Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{7/4}\sqrt[4]{c}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)),x]
```

output
```
(-2*A)/(3*b*x^(3/2)) - ((b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*
b^(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(7/4)*c^(1/4)) + ((b*B - A*c)*ArcTan
h[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(7/
4)*c^(1/4))
```

3.190.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {9, 359, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{A+Bx^2}{x^{5/2}\,(b+cx^2)}dx$$

$$\Big\downarrow \text{ 359}$$

$$\frac{(bB-Ac)\int \frac{1}{\sqrt{x}(cx^2+b)}dx}{b} - \frac{2A}{3bx^{3/2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2(bB-Ac)\int \frac{1}{cx^2+b}d\sqrt{x}}{b} - \frac{2A}{3bx^{3/2}}$$

$$\Big\downarrow \text{755}$$

$$\frac{2(bB-Ac)\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{b}-\frac{2A}{3bx^{3/2}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2(bB-Ac)\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{b}-\frac{2A}{3bx^{3/2}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2(bB-Ac)\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2A}{3bx^{3/2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{2(bB-Ac)\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2A}{3bx^{3/2}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{2(bB-Ac)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2A}{3bx^{3/2}}$$

3.190. $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)}\,dx$

$\downarrow$ 25

$$\frac{2(bB-Ac)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2A}{3bx^{3/2}}$$

$\downarrow$ 27

$$\frac{2(bB-Ac)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2A}{3bx^{3/2}}$$

$\downarrow$ 1103

$$\frac{2(bB-Ac)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2A}{3bx^{3/2}}$$

input
```
Int[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)),x]
```

3.190. $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)}\,dx$

output
```
(-2*A)/(3*b*x^(3/2)) + (2*(b*B - A*c)*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt
[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b]
- Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) +
 Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(
1/4)*c^(1/4)))/(2*Sqrt[b])))/b
```

3.190.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.190.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.52

method	result	size
derivativedivides	$\frac{(-Ac+Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}}-\frac{2A}{3b\,x^{\frac{3}{2}}}$	124
default	$\frac{(-Ac+Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}}-\frac{2A}{3b\,x^{\frac{3}{2}}}$	124
risch	$-\frac{2A}{3b\,x^{\frac{3}{2}}}-\frac{(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}}$	124

input
```
int((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-A*c+B*b)/b^2*(1/c*b)^(1/4)*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1
/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arct
an(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)
-1))-2/3*A/b/x^(3/2)
```

3.190.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.49

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)}\,dx =$$

$$-\,3\,bx^2\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^7c}\right)^{\frac{1}{4}}\log\left(b^2\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^7c}\right)^{\frac{1}{4}}-(Bb\right.$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(3*b*x^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c
^3 + A^4*c^4)/(b^7*c))^(1/4)*log(b^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^
2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4) - (B*b - A*c)*sqrt(x))
 + 3*I*b*x^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^
3 + A^4*c^4)/(b^7*c))^(1/4)*log(I*b^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B
^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4) - (B*b - A*c)*sqrt(x)
) - 3*I*b*x^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c
^3 + A^4*c^4)/(b^7*c))^(1/4)*log(-I*b^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2
*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4) - (B*b - A*c)*sqrt(
x)) - 3*b*x^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c
^3 + A^4*c^4)/(b^7*c))^(1/4)*log(-b^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B
^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4) - (B*b - A*c)*sqrt(x)
) + 4*A*sqrt(x))/(b*x^2)
```

3.190.6 Sympy [A] (verification not implemented)

Time = 13.82 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}\right) \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{c} \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}}{b} \\ -\frac{2A}{3bx^{\frac{3}{2}}}+\frac{Ac\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b^2}-\frac{Ac\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b^2}-\frac{Ac\sqrt[4]{-\frac{b}{c}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b^2}-\frac{B\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}-\sqrt[4]{\ldots}\right.}{2b} \end{cases}$$

input
```
integrate((B*x**2+A)/(c*x**4+b*x**2)/x**(1/2),x)
```

output
```
Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2))), Eq(b, 0) & Eq(c, 0)
), ((-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2)))/c, Eq(b, 0)), ((-2*A/(3*x**(3/2
)) + 2*B*sqrt(x))/b, Eq(c, 0)), (-2*A/(3*b*x**(3/2)) + A*c*(-b/c)**(1/4)*l
og(sqrt(x) - (-b/c)**(1/4))/(2*b**2) - A*c*(-b/c)**(1/4)*log(sqrt(x) + (-b
/c)**(1/4))/(2*b**2) - A*c*(-b/c)**(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/b**2
- B*(-b/c)**(1/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b) + B*(-b/c)**(1/4)*log
(sqrt(x) + (-b/c)**(1/4))/(2*b) + B*(-b/c)**(1/4)*atan(sqrt(x)/(-b/c)**(1/
4))/b, True))
```

3.190.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)}\,dx$$

$$= \frac{\frac{2\sqrt{2}(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(Bb-Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{4\,b}$$
$$-\frac{2\,A}{3\,bx^{\frac{3}{2}}}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*(B*b - A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2
*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) +
 2*sqrt(2)*(B*b - A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sq
rt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sq
rt(2)*(B*b - A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b
))/(b^(3/4)*c^(1/4)) - sqrt(2)*(B*b - A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sq
rt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b - 2/3*A/(b*x^(3/2))
```

3.190.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)}\,dx = \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb-(bc^3)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2c} + \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb-(bc^3)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2c} + \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb-(bc^3)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^2c} - \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb-(bc^3)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^2c} - \frac{2\,A}{3\,bx^{\frac{3}{2}}}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c) + 1/2*sqrt(2)*((b*c^3)
^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) -
 2*sqrt(x))/(b/c)^(1/4))/(b^2*c) + 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3
)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c) - 1/
4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/
c)^(1/4) + x + sqrt(b/c))/(b^2*c) - 2/3*A/(b*x^(3/2))
```

3.190.9 Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 811, normalized size of antiderivative = 3.42

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)}\,dx=-\frac{2\,A}{3\,b\,x^{3/2}}$$
$$-\frac{\mathrm{atan}\left(\frac{\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,b^3\,c^5-32\,A\,B\,b^4\,c^4+16\,B^2\,b^5\,c^3\right)-\frac{(A\,c-B\,b)\left(32\,A\,b^5\,c^4-32\,B\,b^6\,c^3\right)}{2\,(-b)^{7/4}\,c^{1/4}}\right)1\mathrm{i}}{2\,(-b)^{7/4}\,c^{1/4}}+\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,b^3\,c^5-32\,A\,B\,b^4\,c^4+16\,B\right.\right.}{2\,(-b)}}{\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,b^3\,c^5-32\,A\,B\,b^4\,c^4+16\,B^2\,b^5\,c^3\right)-\frac{(A\,c-B\,b)\left(32\,A\,b^5\,c^4-32\,B\,b^6\,c^3\right)}{2\,(-b)^{7/4}\,c^{1/4}}\right)}{2\,(-b)^{7/4}\,c^{1/4}}-\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,b^3\,c^5-32\,A\,B\,b^4\,c^4+16\,B\right.\right.}{2\,(-b)^7}}\right)}{(-b)^{7/4}\,c^{1/4}}$$
$$-\frac{\mathrm{atan}\left(\frac{\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,b^3\,c^5-32\,A\,B\,b^4\,c^4+16\,B^2\,b^5\,c^3\right)-\frac{(A\,c-B\,b)\left(32\,A\,b^5\,c^4-32\,B\,b^6\,c^3\right)1\mathrm{i}}{2\,(-b)^{7/4}\,c^{1/4}}\right)}{2\,(-b)^{7/4}\,c^{1/4}}+\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,b^3\,c^5-32\,A\,B\,b^4\,c^4+1\right.\right.}{2\,(-}}{\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,b^3\,c^5-32\,A\,B\,b^4\,c^4+16\,B^2\,b^5\,c^3\right)-\frac{(A\,c-B\,b)\left(32\,A\,b^5\,c^4-32\,B\,b^6\,c^3\right)1\mathrm{i}}{2\,(-b)^{7/4}\,c^{1/4}}\right)1\mathrm{i}}{2\,(-b)^{7/4}\,c^{1/4}}-\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,b^3\,c^5-32\,A\,B\,b^4\,c^4+1\right.\right.}{2\,(}}\right)}{(-b)^{7/4}\,c^{1/4}}$$

input
```
int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)),x)
```

output
```
- (2*A)/(3*b*x^(3/2)) - (atan((((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*
B^2*b^5*c^3 - 32*A*B*b^4*c^4) - ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3)
)/(2*(-b)^(7/4)*c^(1/4)))*1i)/(2*(-b)^(7/4)*c^(1/4)) + ((A*c - B*b)*(x^(1/
2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^4*c^4) + ((A*c - B*b)*(32*A
*b^5*c^4 - 32*B*b^6*c^3))/(2*(-b)^(7/4)*c^(1/4)))*1i)/(2*(-b)^(7/4)*c^(1/4
)))/(((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^4*c
^4) - ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3))/(2*(-b)^(7/4)*c^(1/4))))
/(2*(-b)^(7/4)*c^(1/4)) - ((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b
^5*c^3 - 32*A*B*b^4*c^4) + ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3))/(2*
(-b)^(7/4)*c^(1/4))))/(2*(-b)^(7/4)*c^(1/4))))*(A*c - B*b)*1i)/((-b)^(7/4)
*c^(1/4)) - (atan((((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3
- 32*A*B*b^4*c^4) - ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3)*1i)/(2*(-b)
^(7/4)*c^(1/4))))/(2*(-b)^(7/4)*c^(1/4)) + ((A*c - B*b)*(x^(1/2)*(16*A^2*b
^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^4*c^4) + ((A*c - B*b)*(32*A*b^5*c^4 - 3
2*B*b^6*c^3)*1i)/(2*(-b)^(7/4)*c^(1/4))))/(2*(-b)^(7/4)*c^(1/4)))/(((A*c -
 B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^4*c^4) - ((A*c
- B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3)*1i)/(2*(-b)^(7/4)*c^(1/4)))*1i)/(2*(-
b)^(7/4)*c^(1/4)) - ((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3
 - 32*A*B*b^4*c^4) + ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3)*1i)/(2*(-b
)^(7/4)*c^(1/4)))*1i)/(2*(-b)^(7/4)*c^(1/4))))*(A*c - B*b))/((-b)^(7/4)...
```

3.191 $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)}\,dx$

3.191.1 Optimal result

Integrand size = 26, antiderivative size = 255

$$\begin{aligned}\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)}\,dx = & -\frac{2A}{5bx^{5/2}}-\frac{2(bB-Ac)}{b^2\sqrt{x}} \\ & +\frac{\sqrt[4]{c}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{9/4}}-\frac{\sqrt[4]{c}(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{9/4}} \\ & -\frac{\sqrt[4]{c}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}} \\ & +\frac{\sqrt[4]{c}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}}\end{aligned}$$

output
```
-2/5*A/b/x^(5/2)+1/2*c^(1/4)*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b
^(1/4))/b^(9/4)*2^(1/2)-1/2*c^(1/4)*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^
(1/2)/b^(1/4))/b^(9/4)*2^(1/2)-1/4*c^(1/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)
-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)*2^(1/2)+1/4*c^(1/4)*(-A*c+B*b)*l
n(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)*2^(1/2)-2*(-A
*c+B*b)/b^2/x^(1/2)
```

3.191.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)}\,dx = -\frac{2(Ab+5bBx^2-5Acx^2)}{5b^2x^{5/2}} + \frac{\sqrt[4]{c}(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}b^{9/4}} + \frac{\sqrt[4]{c}(bB-Ac)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{9/4}}$$

input
```
Integrate[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

output
```
(-2*(A*b + 5*b*B*x^2 - 5*A*c*x^2))/(5*b^2*x^(5/2)) + (c^(1/4)*(b*B - A*c)*
ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*
b^(9/4)) + (c^(1/4)*(b*B - A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/
(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(9/4))
```

3.191.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {9, 359, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)}\,dx$$

$$\Big\downarrow 9$$

$$\int \frac{A+Bx^2}{x^{7/2}(b+cx^2)}dx$$

$$\Big\downarrow 359$$

$$\frac{(bB-Ac)\int \frac{1}{x^{3/2}(cx^2+b)}dx}{b} - \frac{2A}{5bx^{5/2}}$$

$$\Big\downarrow 264$$

$$\frac{(bB-Ac)\left(-\frac{c\int \frac{\sqrt{x}}{cx^2+b}dx}{b} - \frac{2}{b\sqrt{x}}\right)}{b} - \frac{2A}{5bx^{5/2}}$$

$$\Big\downarrow 266$$

$$\frac{(bB-Ac)\left(-\frac{2c\int \frac{x}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2A}{5bx^{5/2}}$$

$$\Big\downarrow 826$$

$$\frac{(bB-Ac)\left(-\frac{2c\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2A}{5bx^{5/2}}$$

$$\Big\downarrow 1476$$

$$\frac{(bB-Ac)\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2A}{5bx^{5/2}}$$

$$\Big\downarrow 1082$$

$$\frac{(bB-Ac)\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2A}{5bx^{5/2}}$$

$$\Big\downarrow 217$$

3.191. $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)}\,dx$

$$\frac{(bB-Ac)\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2A}{5bx^{5/2}}$$

$\downarrow$ 1479

$$\frac{(bB-Ac)\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}}{b}$$

$$\frac{2A}{5bx^{5/2}}$$

$\downarrow$ 25

3.191. $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)}\,dx$

$$\frac{(bB-Ac)\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\right.}{b}$$

$$\frac{2A}{5bx^{5/2}}$$

$\downarrow$ 27

$$\frac{(bB-Ac)\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-$$

$$\frac{2A}{5bx^{5/2}}$$

$\downarrow$ 1103

3.191. $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)}\,dx$

$$\frac{(bB-Ac)\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{\ldots}}\right)}{b}$$

$$\frac{2A}{5bx^{5/2}}$$

input
```
Int[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

output
```
(-2*A)/(5*b*x^(5/2)) + ((b*B - A*c)*(-2/(b*Sqrt[x]) - (2*c*((-(ArcTan[1 -
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 +
 (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])
 - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[
2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt
[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/b))/b
```

3.191.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.191. $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.191.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.55

method	result
derivativedivides	$-\frac{2A}{5b\,x^{\frac{5}{2}}}-\frac{2(-Ac+Bb)}{b^{2}\sqrt{x}}+\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$-\frac{2A}{5b\,x^{\frac{5}{2}}}-\frac{2(-Ac+Bb)}{b^{2}\sqrt{x}}+\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
risch	$-\frac{2(-5Ac\,x^{2}+5bB\,x^{2}+Ab)}{5b^{2}x^{\frac{5}{2}}}+\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

input
```
int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*A/b/x^(5/2)-2*(-A*c+B*b)/b^2/x^(1/2)+1/4*(A*c-B*b)/b^2/(1/c*b)^(1/4)*
2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/
4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+
1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

3.191.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 738, normalized size of antiderivative = 2.89

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)}\,dx = \frac{5\,b^2x^3\left(-\frac{B^4b^4c-4\,AB^3b^3c^2+6\,A^2B^2b^2c^3-4\,A^3Bbc^4+A^4c^5}{b^9}\right)^{\frac{1}{4}}\log\left(b^7\left(-\frac{B^4b^4c-4\,AB^3b^3c^2+6\,A^2B^2b^2c}{b^9}\right.\right.}{}$$

input
```
integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/10*(5*b^2*x^3*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3
*B*b*c^4 + A^4*c^5)/b^9)^(1/4)*log(b^7*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*
A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(3/4) - (B^3*b^3*c - 3*A*B
^2*b^2*c^2 + 3*A^2*B*b*c^3 - A^3*c^4)*sqrt(x)) - 5*I*b^2*x^3*(-(B^4*b^4*c
 - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(1/4
)*log(I*b^7*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b
*c^4 + A^4*c^5)/b^9)^(3/4) - (B^3*b^3*c - 3*A*B^2*b^2*c^2 + 3*A^2*B*b*c^3
 - A^3*c^4)*sqrt(x)) + 5*I*b^2*x^3*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B
^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(1/4)*log(-I*b^7*(-(B^4*b^4*c -
 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(3/4)
 - (B^3*b^3*c - 3*A*B^2*b^2*c^2 + 3*A^2*B*b*c^3 - A^3*c^4)*sqrt(x)) - 5*b^
2*x^3*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 +
 A^4*c^5)/b^9)^(1/4)*log(-b^7*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b
^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(3/4) - (B^3*b^3*c - 3*A*B^2*b^2*c^
2 + 3*A^2*B*b*c^3 - A^3*c^4)*sqrt(x)) - 4*(5*(B*b - A*c)*x^2 + A*b)*sqrt(x
))/(b^2*x^3)
```

3.191.6 Sympy [A] (verification not implemented)

Time = 73.16 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)}\,dx = A\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for } b = \\ -\frac{2}{9cx^{\frac{9}{2}}} & \text{for } b = \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } c = \\ -\frac{2}{5bx^{\frac{5}{2}}} + \frac{c\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b^2\sqrt[4]{-\frac{b}{c}}} - \frac{c\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b^2\sqrt[4]{-\frac{b}{c}}} + \frac{c\,\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b^2\sqrt[4]{-\frac{b}{c}}} + \frac{2c}{b^2\sqrt{x}} & \text{other} \end{cases}\right.$$

$$+B\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } b = 0 \wedge c = 0 \\ -\frac{2}{5cx^{\frac{5}{2}}} & \text{for } b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for } c = 0 \\ -\frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b\sqrt[4]{-\frac{b}{c}}} + \frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b\sqrt[4]{-\frac{b}{c}}} - \frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b\sqrt[4]{-\frac{b}{c}}} - \frac{2}{b\sqrt{x}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((B*x**2+A)/x**(3/2)/(c*x**4+b*x**2),x)
```

output
```
A*Piecewise((zoo/x**(9/2), Eq(b, 0) & Eq(c, 0)), (-2/(9*c*x**(9/2)), Eq(b,
 0)), (-2/(5*b*x**(5/2)), Eq(c, 0)), (-2/(5*b*x**(5/2)) + c*log(sqrt(x) -
(-b/c)**(1/4))/(2*b**2*(-b/c)**(1/4)) - c*log(sqrt(x) + (-b/c)**(1/4))/(2*
b**2*(-b/c)**(1/4)) + c*atan(sqrt(x)/(-b/c)**(1/4))/(b**2*(-b/c)**(1/4)) +
 2*c/(b**2*sqrt(x)), True)) + B*Piecewise((zoo/x**(5/2), Eq(b, 0) & Eq(c,
0)), (-2/(5*c*x**(5/2)), Eq(b, 0)), (-2/(b*sqrt(x)), Eq(c, 0)), (-log(sqrt
(x) - (-b/c)**(1/4))/(2*b*(-b/c)**(1/4)) + log(sqrt(x) + (-b/c)**(1/4))/(2
*b*(-b/c)**(1/4)) - atan(sqrt(x)/(-b/c)**(1/4))/(b*(-b/c)**(1/4)) - 2/(b*s
qrt(x)), True))
```

3.191.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)}\,dx =$$

$$-\frac{(Bbc-Ac^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\right.}{4\,b^2}$$

$$-\frac{2\left(5\left(Bb-Ac\right)x^2+Ab\right)}{5\,b^2x^{\frac{5}{2}}}$$

input
```
integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-1/4*(B*b*c - A*c^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4
) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(
c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*s
qrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*l
og(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)
) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b
^(1/4)*c^(3/4)))/b^2 - 2/5*(5*(B*b - A*c)*x^2 + A*b)/(b^2*x^(5/2))
```

3.191.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2}{x^{3/2}\,(bx^2+cx^4)}\,dx = -\frac{\sqrt{2}\Big(\left(bc^3\right)^{\frac{3}{4}} Bb - \left(bc^3\right)^{\frac{3}{4}} Ac\Big)\arctan\left(\frac{\sqrt{2}\Big(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\Big)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3c^2} - \frac{\sqrt{2}\Big(\left(bc^3\right)^{\frac{3}{4}} Bb - \left(bc^3\right)^{\frac{3}{4}} Ac\Big)\arctan\left(-\frac{\sqrt{2}\Big(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\Big)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3c^2} + \frac{\sqrt{2}\Big(\left(bc^3\right)^{\frac{3}{4}} Bb - \left(bc^3\right)^{\frac{3}{4}} Ac\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^3c^2} - \frac{\sqrt{2}\Big(\left(bc^3\right)^{\frac{3}{4}} Bb - \left(bc^3\right)^{\frac{3}{4}} Ac\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^3c^2} - \frac{2\,(5\,Bbx^2-5\,Acx^2+Ab)}{5\,b^2x^{\frac{5}{2}}}$$

input
```
integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) - 1/2*sqrt(2)*((b*c
^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4
) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (
b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^
2) - 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt
(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) - 2/5*(5*B*b*x^2 - 5*A*c*x^2 +
A*b)/(b^2*x^(5/2))
```

3.191.9 Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.35

$$\int \frac{A+Bx^2}{x^{3/2}\,(bx^2+cx^4)}\,dx = \frac{{(-c)}^{1/4}\,\mathrm{atan}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,\left(A\,c-B\,b\right)}{b^{9/4}} - \frac{\frac{2\,A}{5\,b}-\frac{2\,x^2\,\left(A\,c-B\,b\right)}{b^2}}{x^{5/2}} - \frac{{(-c)}^{1/4}\,\mathrm{atanh}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,\left(A\,c-B\,b\right)}{b^{9/4}}$$

input
```
int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)),x)
```

output
```
((-c)^(1/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4))*(A*c - B*b))/b^(9/4) - ((2*
A)/(5*b) - (2*x^2*(A*c - B*b))/b^2)/x^(5/2) - ((-c)^(1/4)*atanh(((-c)^(1/4
)*x^(1/2))/b^(1/4))*(A*c - B*b))/b^(9/4)
```

3.191. $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)}\,dx$

3.192 $\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)}\,dx$

3.192.1 Optimal result

Integrand size = 26, antiderivative size = 257

$$\int \frac{A+Bx^2}{x^{5/2}\,(bx^2+cx^4)}\,dx = -\frac{2A}{7bx^{7/2}}-\frac{2(bB-Ac)}{3b^2x^{3/2}} + \frac{c^{3/4}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{11/4}} - \frac{c^{3/4}(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{11/4}} + \frac{c^{3/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}} - \frac{c^{3/4}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}}$$

output
```
-2/7*A/b/x^(7/2)-2/3*(-A*c+B*b)/b^2/x^(3/2)+1/2*c^(3/4)*(-A*c+B*b)*arctan(
1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)*2^(1/2)-1/2*c^(3/4)*(-A*c+B*b)
*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)*2^(1/2)+1/4*c^(3/4)*(-
A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)*2^
(1/2)-1/4*c^(3/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*
x^(1/2))/b^(11/4)*2^(1/2)
```

3.192.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)}\,dx = -\frac{2(3Ab+7bBx^2-7Acx^2)}{21b^2x^{7/2}} + \frac{c^{3/4}(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}b^{11/4}} - \frac{c^{3/4}(bB-Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{11/4}}$$

input
```
Integrate[(A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

output
```
(-2*(3*A*b + 7*b*B*x^2 - 7*A*c*x^2))/(21*b^2*x^(7/2)) + (c^(3/4)*(b*B - A*
c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[
2]*b^(11/4)) - (c^(3/4)*(b*B - A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(11/4))
```

3.192.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {9, 359, 264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{A+Bx^2}{x^{9/2}\left(b+cx^2\right)}dx$$

$$\Big\downarrow \; 359$$

$$\frac{(bB-Ac)\int \frac{1}{x^{5/2}(cx^2+b)}dx}{b} - \frac{2A}{7bx^{7/2}}$$

$$\Big\downarrow \; 264$$

$$\frac{(bB-Ac)\left(-\frac{c\int \frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{b} - \frac{2}{3bx^{3/2}}\right)}{b} - \frac{2A}{7bx^{7/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(bB-Ac)\left(-\frac{2c\int \frac{1}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2A}{7bx^{7/2}}$$

$$\Big\downarrow \text{755}$$

$$\frac{(bB-Ac)\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2A}{7bx^{7/2}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{(bB-Ac)\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2A}{7bx^{7/2}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(bB-Ac)\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2A}{7bx^{7/2}}$$

$$\Big\downarrow \text{217}$$

3.192. $\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)}\,dx$

$$\frac{(bB-Ac)\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2A}{7bx^{7/2}}$$

$\downarrow$ 1479

$$\frac{(bB-Ac)\left(-\frac{2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}\right)}{b}$$

$$\frac{2A}{7bx^{7/2}}$$

$\downarrow$ 25

3.192. $\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)}\,dx$

$$\frac{(bB-Ac)\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\right.}{b}-\frac{2A}{7bx^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{(bB-Ac)\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2A}{7bx^{7/2}}$$

$$\Big\downarrow \; 1103$$

3.192. $\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)}\,dx$

$$\frac{(bB-Ac)\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx\cdots}\right)}{b}$$

$$\frac{2A}{7bx^{7/2}}$$

input
```
Int[(A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

output
```
(-2*A)/(7*b*x^(7/2)) + ((b*B - A*c)*(-2/(3*b*x^(3/2)) - (2*c*((-(ArcTan[1
- (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1
 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b
]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqr
t[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sq
rt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/b))/b
```

3.192.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.192. $\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.192.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.55

method	result
derivativedivides	$-\frac{2A}{7b\,x^{\frac{7}{2}}}-\frac{2(-Ac+Bb)}{3b^{2}x^{\frac{3}{2}}}+\frac{c(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{4b^{3}}$
default	$-\frac{2A}{7b\,x^{\frac{7}{2}}}-\frac{2(-Ac+Bb)}{3b^{2}x^{\frac{3}{2}}}+\frac{c(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{4b^{3}}$
risch	$-\frac{2(-7Ac\,x^{2}+7bB\,x^{2}+3Ab)}{21b^{2}x^{\frac{7}{2}}}+\frac{c(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{4b^{3}}$

input
```
int((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*A/b/x^(7/2)-2/3*(-A*c+B*b)/b^2/x^(3/2)+1/4*c*(A*c-B*b)/b^3*(1/c*b)^(1
/4)*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)
^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1
/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

3.192.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 643, normalized size of antiderivative = 2.50

$$\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)}\,dx = \frac{21\,b^2x^4\left(-\frac{B^4b^4c^3-4\,AB^3b^3c^4+6\,A^2B^2b^2c^5-4\,A^3Bbc^6+A^4c^7}{b^{11}}\right)^{\frac{1}{4}}\log\left(b^3\left(-\frac{B^4b^4c^3-4\,AB^3b^3c^4+6\,A^2B}{b^{11}}\right.\right.}{}$$

input
```
integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/42*(21*b^2*x^4*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*
A^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4)*log(b^3*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^
4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4) - (B*b*c - A*
c^2)*sqrt(x)) + 21*I*b^2*x^4*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*
b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4)*log(I*b^3*(-(B^4*b^4*c^3 -
4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4)
 - (B*b*c - A*c^2)*sqrt(x)) - 21*I*b^2*x^4*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^
4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4)*log(-I*b^3*(-
(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c
^7)/b^11)^(1/4) - (B*b*c - A*c^2)*sqrt(x)) - 21*b^2*x^4*(-(B^4*b^4*c^3 - 4
*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4)*
log(-b^3*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*
c^6 + A^4*c^7)/b^11)^(1/4) - (B*b*c - A*c^2)*sqrt(x)) - 4*(7*(B*b - A*c)*x
^2 + 3*A*b)*sqrt(x))/(b^2*x^4)
```

3.192.6 Sympy [A] (verification not implemented)

Time = 67.26 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{11x^{\frac{11}{2}}}-\frac{2B}{7x^{\frac{7}{2}}}\right) \\ \frac{-\frac{2A}{11x^{\frac{11}{2}}}-\frac{2B}{7x^{\frac{7}{2}}}}{c} \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{b} \\ -\frac{2A}{7bx^{\frac{7}{2}}}+\frac{2Ac}{3b^2x^{\frac{3}{2}}}-\frac{Ac^2\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b^3}+\frac{Ac^2\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b^3}+\frac{Ac^2\sqrt[4]{-\frac{b}{c}}}{} \end{cases}$$

input
```
integrate((B*x**2+A)/x**(5/2)/(c*x**4+b*x**2),x)
```

output
```
Piecewise((zoo*(-2*A/(11*x**(11/2)) - 2*B/(7*x**(7/2))), Eq(b, 0) & Eq(c,
0)), ((-2*A/(11*x**(11/2)) - 2*B/(7*x**(7/2)))/c, Eq(b, 0)), ((-2*A/(7*x**
(7/2)) - 2*B/(3*x**(3/2)))/b, Eq(c, 0)), (-2*A/(7*b*x**(7/2)) + 2*A*c/(3*b
**2*x**(3/2)) - A*c**2*(-b/c)**(1/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b**3)
 + A*c**2*(-b/c)**(1/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*b**3) + A*c**2*(-b
/c)**(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/b**3 - 2*B/(3*b*x**(3/2)) + B*c*(-b
/c)**(1/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b**2) - B*c*(-b/c)**(1/4)*log(s
qrt(x) + (-b/c)**(1/4))/(2*b**2) - B*c*(-b/c)**(1/4)*atan(sqrt(x)/(-b/c)**
(1/4))/b**2, True))
```

3.192.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)}\,dx =$$

$$-\frac{\frac{2\sqrt{2}(Bbc-Ac^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(Bbc-Ac^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(Bbc-Ac^2)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{4\,b^2}$$

$$-\frac{2\,(7\,(Bb-Ac)x^2+3\,Ab)}{21\,b^2x^{\frac{7}{2}}}$$

input
```
integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*(B*b*c - A*c^2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4
) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c
))) + 2*sqrt(2)*(B*b*c - A*c^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/
4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(
c))) + sqrt(2)*(B*b*c - A*c^2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(
c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(B*b*c - A*c^2)*log(-sqrt(2)*b
^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^2 - 2/2
1*(7*(B*b - A*c)*x^2 + 3*A*b)/(b^2*x^(7/2))
```

3.192.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)}\,dx = -\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb-\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^3} - \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb-\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^3} - \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb-\left(bc^3\right)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4b^3} + \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb-\left(bc^3\right)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4b^3} - \frac{2\left(7Bbx^2-7Acx^2+3Ab\right)}{21b^2x^{\frac{7}{2}}}$$

input
```
integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^3 - 1/2*sqrt(2)*((b*c^3)^(1
/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*
sqrt(x))/(b/c)^(1/4))/b^3 - 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)
*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 + 1/4*sqrt(2)*(
(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) +
x + sqrt(b/c))/b^3 - 2/21*(7*B*b*x^2 - 7*A*c*x^2 + 3*A*b)/(b^2*x^(7/2))
```

3.192.9 Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.16

$$\int \frac{A+Bx^2}{x^{5/2}\,(bx^2+cx^4)}\,dx = -\frac{\frac{2\,A}{7\,b}-\frac{2\,x^2\,(A\,c-B\,b)}{3\,b^2}}{x^{7/2}}$$

$$+\frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{\frac{(-c)^{3/4}\,(A\,c-B\,b)\,\left(\sqrt{x}\,\left(16\,A^2\,b^6\,c^7-32\,A\,B\,b^7\,c^6+16\,B^2\,b^8\,c^5\right)-\frac{(-c)^{3/4}\,(A\,c-B\,b)\,\left(32\,A\,b^9\,c^5-32\,B\,b^{10}\,c^4\right)\,1\mathrm{i}}{2\,b^{11/4}}\right)}{2\,b^{11/4}}+\frac{(-c)^{3/4}\,(A\,c-B\,b)}{}}{\frac{(-c)^{3/4}\,(A\,c-B\,b)\,\left(\sqrt{x}\,\left(16\,A^2\,b^6\,c^7-32\,A\,B\,b^7\,c^6+16\,B^2\,b^8\,c^5\right)-\frac{(-c)^{3/4}\,(A\,c-B\,b)\,\left(32\,A\,b^9\,c^5-32\,B\,b^{10}\,c^4\right)\,1\mathrm{i}}{2\,b^{11/4}}\right)\,1\mathrm{i}}{2\,b^{11/4}}-\frac{(-c)^{3/4}\,(A\,c-B\,b)}{}}\right.}{b^{11/4}}$$

$$-\frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{A^3\,c^8\,\sqrt{x}\,1\mathrm{i}-B^3\,b^3\,c^5\,\sqrt{x}\,1\mathrm{i}-A^2\,B\,b\,c^7\,\sqrt{x}\,3\mathrm{i}+A\,B^2\,b^2\,c^6\,\sqrt{x}\,3\mathrm{i}}{b^{1/4}\,(-c)^{19/4}\,(c\,(c\,(A^3\,c-3\,A^2\,B\,b)+3\,A\,B^2\,b^2)-B^3\,b^3)}\right)\,(A\,c-B\,b)\,1\mathrm{i}}{b^{11/4}}$$

input
```
int((A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)),x)
```

output
```
((-c)^(3/4)*atan((((-c)^(3/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^6*c^7 + 16*B^
2*b^8*c^5 - 32*A*B*b^7*c^6) - ((-c)^(3/4)*(A*c - B*b)*(32*A*b^9*c^5 - 32*B
*b^10*c^4)*1i)/(2*b^(11/4))))/(2*b^(11/4)) + ((-c)^(3/4)*(A*c - B*b)*(x^(1
/2)*(16*A^2*b^6*c^7 + 16*B^2*b^8*c^5 - 32*A*B*b^7*c^6) + ((-c)^(3/4)*(A*c
- B*b)*(32*A*b^9*c^5 - 32*B*b^10*c^4)*1i)/(2*b^(11/4))))/(2*b^(11/4)))/(((
-c)^(3/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^6*c^7 + 16*B^2*b^8*c^5 - 32*A*B*b
^7*c^6) - ((-c)^(3/4)*(A*c - B*b)*(32*A*b^9*c^5 - 32*B*b^10*c^4)*1i)/(2*b^
(11/4)))*1i)/(2*b^(11/4)) - ((-c)^(3/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^6*c
^7 + 16*B^2*b^8*c^5 - 32*A*B*b^7*c^6) + ((-c)^(3/4)*(A*c - B*b)*(32*A*b^9*
c^5 - 32*B*b^10*c^4)*1i)/(2*b^(11/4)))*1i)/(2*b^(11/4))))*(A*c - B*b))/b^(
11/4) - ((2*A)/(7*b) - (2*x^2*(A*c - B*b))/(3*b^2))/x^(7/2) - ((-c)^(3/4)*
atan((A^3*c^8*x^(1/2)*1i - B^3*b^3*c^5*x^(1/2)*1i - A^2*B*b*c^7*x^(1/2)*3i
 + A*B^2*b^2*c^6*x^(1/2)*3i)/(b^(1/4)*(-c)^(19/4)*(c*(c*(A^3*c - 3*A^2*B*b
) + 3*A*B^2*b^2) - B^3*b^3)))*(A*c - B*b)*1i)/b^(11/4)
```

3.193 $\int \frac{A+Bx^2}{x^{7/2}(bx^2+cx^4)}\,dx$

3.193.1 Optimal result

Integrand size = 26, antiderivative size = 276

$$\int \frac{A+Bx^2}{x^{7/2}\left(bx^2+cx^4\right)}\,dx = -\frac{2A}{9bx^{9/2}}-\frac{2(bB-Ac)}{5b^2x^{5/2}}+\frac{2c(bB-Ac)}{b^3\sqrt{x}} - \frac{c^{5/4}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{13/4}} + \frac{c^{5/4}(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{13/4}} + \frac{c^{5/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}} - \frac{c^{5/4}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}}$$

output
```
-2/9*A/b/x^(9/2)-2/5*(-A*c+B*b)/b^2/x^(5/2)-1/2*c^(5/4)*(-A*c+B*b)*arctan(
1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(13/4)*2^(1/2)+1/2*c^(5/4)*(-A*c+B*b)
*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(13/4)*2^(1/2)+1/4*c^(5/4)*(-
A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^
(1/2)-1/4*c^(5/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*
x^(1/2))/b^(13/4)*2^(1/2)+2*c*(-A*c+B*b)/b^3/x^(1/2)
```

3.193.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx^2}{x^{7/2}(bx^2+cx^4)}\,dx = \frac{-\frac{4\sqrt[4]{b}(9bBx^2(b-5cx^2)+A(5b^2-9bcx^2+45c^2x^4))}{x^{9/2}}+45\sqrt{2}c^{5/4}(-bB+Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{}}\right.}{90b^{13/4}}$$

input
```
Integrate[(A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)),x]
```

output
```
((-4*b^(1/4)*(9*b*B*x^2*(b - 5*c*x^2) + A*(5*b^2 - 9*b*c*x^2 + 45*c^2*x^4)
))/x^(9/2) + 45*Sqrt[2]*c^(5/4)*(-(b*B) + A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x
)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 45*Sqrt[2]*c^(5/4)*(-(b*B) + A*c)*A
rcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(90*b^(13
/4))
```

3.193.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {9, 359, 264, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{7/2}(bx^2+cx^4)}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{A+Bx^2}{x^{11/2}(b+cx^2)}dx$$

$$\Big\downarrow \text{ 359}$$

$$\frac{(bB-Ac)\int \frac{1}{x^{7/2}(cx^2+b)}dx}{b}-\frac{2A}{9bx^{9/2}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{(bB-Ac)\left(-\frac{c\int \frac{1}{x^{3/2}(cx^2+b)}dx}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2A}{9bx^{9/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{c\int \frac{\sqrt{x}}{cx^2+b}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2A}{9bx^{9/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{2c\int \frac{x}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2A}{9bx^{9/2}}$$

$$\Big\downarrow \text{826}$$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2A}{9bx^{9/2}}$$

$$\Big\downarrow \text{1476}$$

3.193. $\int \frac{A+Bx^2}{x^{7/2}(bx^2+cx^4)}\,dx$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2A}{9bx^{9/2}}$$

$$\Big\downarrow \; 1082$$

3.193. $\int \frac{A+Bx^2}{x^{7/2}(bx^2+cx^4)}\,dx$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2A}{9bx^{9/2}}$$

$$\Big\downarrow \; 217$$

3.193. $\int \frac{A+Bx^2}{x^{7/2}(bx^2+cx^4)}\,dx$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2A}{9bx^{9/2}}$$

$$\Big\downarrow \; 1479$$

3.193. $\int \frac{A+Bx^2}{x^{7/2}(bx^2+cx^4)}\,dx$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\cdots}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{b}$$

$$\frac{2A}{9bx^{9/2}}$$

$$\Big\downarrow \; 25$$

3.193. $\int \frac{A+Bx^2}{x^{7/2}(bx^2+cx^4)}\, dx$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}\right)}{b}\right)}{b}$$

$$\frac{2A}{9bx^{9/2}}$$

$\downarrow$ 27

3.193. $\int \frac{A+Bx^2}{x^{7/2}(bx^2+cx^4)}\,dx$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-5b\right.}{b}$$

$$\frac{2A}{9bx^{9/2}}$$

$\downarrow$ 1103

3.193. $\int \frac{A+Bx^2}{x^{7/2}(bx^2+cx^4)}\,dx$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{}{b}\right)}{b}\right)}{b}$$

$$\frac{2A}{9bx^{9/2}}$$

input
```
Int[(A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)),x]
```

output
```
(-2*A)/(9*b*x^(9/2)) + ((b*B - A*c)*(-2/(5*b*x^(5/2)) - (c*(-2/(b*Sqrt[x])
 - (2*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)
*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4
)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)
*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/b
))/b))/b
```

3.193. $\int \frac{A+Bx^2}{x^{7/2}(bx^2+cx^4)}\,dx$

3.193.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.193. $\int \frac{A+Bx^2}{x^{7/2}(bx^2+cx^4)}\,dx$

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.193.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.57

method	result
derivativedivides	$-\frac{2A}{9b\,x^{\frac{9}{2}}}-\frac{2(-Ac+Bb)}{5b^{2}x^{\frac{5}{2}}}-\frac{2c(Ac-Bb)}{b^{3}\sqrt{x}}-\frac{c(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arct}}{4b^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$-\frac{2A}{9b\,x^{\frac{9}{2}}}-\frac{2(-Ac+Bb)}{5b^{2}x^{\frac{5}{2}}}-\frac{2c(Ac-Bb)}{b^{3}\sqrt{x}}-\frac{c(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arct}}{4b^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
risch	$-\frac{2(45A\,c^{2}x^{4}-45x^{4}Bbc-9Abc\,x^{2}+9b^{2}B\,x^{2}+5b^{2}A)}{45b^{3}x^{\frac{9}{2}}}-\frac{c(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+\right.}{4b^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

input
```
int((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*A/b/x^(9/2)-2/5*(-A*c+B*b)/b^2/x^(5/2)-2*c*(A*c-B*b)/b^3/x^(1/2)-1/4*
c*(A*c-B*b)/b^3/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)
+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(
2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)
)
```

3.193.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 784, normalized size of antiderivative = 2.84

$$\int \frac{A+Bx^2}{x^{7/2}\,(bx^2+cx^4)}\,dx =$$

$$-\frac{45\,b^3x^5\left(-\frac{B^4b^4c^5-4\,AB^3b^3c^6+6\,A^2B^2b^2c^7-4\,A^3Bbc^8+A^4c^9}{b^{13}}\right)^{\frac{1}{4}}\log\left(b^{10}\left(-\frac{B^4b^4c^5-4\,AB^3b^3c^6+6\,A^2B^2b^2c^7-4\,A^3Bbc^8+A^4c^9}{b^{13}}\right)^{\frac{3}{4}}-\right.}{}$$

input
```
integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
-1/90*(45*b^3*x^5*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4
*A^3*B*b*c^8 + A^4*c^9)/b^13)^(1/4)*log(b^10*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*
c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(3/4) - (B^3*b^3*
c^4 - 3*A*B^2*b^2*c^5 + 3*A^2*B*b*c^6 - A^3*c^7)*sqrt(x)) - 45*I*b^3*x^5*(
-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*
c^9)/b^13)^(1/4)*log(I*b^10*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b
^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(3/4) - (B^3*b^3*c^4 - 3*A*B^2*b^2
*c^5 + 3*A^2*B*b*c^6 - A^3*c^7)*sqrt(x)) + 45*I*b^3*x^5*(-(B^4*b^4*c^5 - 4
*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(1/4)*
log(-I*b^10*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B
*b*c^8 + A^4*c^9)/b^13)^(3/4) - (B^3*b^3*c^4 - 3*A*B^2*b^2*c^5 + 3*A^2*B*b
*c^6 - A^3*c^7)*sqrt(x)) - 45*b^3*x^5*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6
*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(1/4)*log(-b^10*(-(B^4*b
^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^
13)^(3/4) - (B^3*b^3*c^4 - 3*A*B^2*b^2*c^5 + 3*A^2*B*b*c^6 - A^3*c^7)*sqrt
(x)) - 4*(45*(B*b*c - A*c^2)*x^4 - 5*A*b^2 - 9*(B*b^2 - A*b*c)*x^2)*sqrt(x
))/(b^3*x^5)
```

3.193.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{7/2}\left(bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate((B*x**2+A)/x**(7/2)/(c*x**4+b*x**2),x)
```

output
```
Timed out
```

3.193.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^2}{x^{7/2}\left(bx^2+cx^4\right)}\,dx=\frac{\left(Bbc^2-Ac^3\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac14}c^{\frac14}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac14}c^{\frac14}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}}{}\right.}{4b^3}+\frac{2\left(45\left(Bbc-Ac^2\right)x^4-5Ab^2-9\left(Bb^2-Abc\right)x^2\right)}{45b^3x^{\frac92}}$$

input
```
integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/4*(B*b*c^2 - A*c^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/
4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt
(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*
sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*
log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4
)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(
b^(1/4)*c^(3/4)))/b^3 + 2/45*(45*(B*b*c - A*c^2)*x^4 - 5*A*b^2 - 9*(B*b^2
- A*b*c)*x^2)/(b^3*x^(9/2))
```

3.193.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2}{x^{7/2}\left(bx^2+cx^4\right)}\,dx = \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^4c} + \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^4c} - \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-\left(bc^3\right)^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^4c} + \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-\left(bc^3\right)^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^4c} + \frac{2\left(45\,Bbcx^4-45\,Ac^2x^4-9\,Bb^2x^2+9\,Abcx^2-5\,Ab^2\right)}{45\,b^3x^{\frac{9}{2}}}$$

input
```
integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) + 1/2*sqrt(2)*((b*c^3)
^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) -
 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) - 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3
)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c) + 1/
4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/
c)^(1/4) + x + sqrt(b/c))/(b^4*c) + 2/45*(45*B*b*c*x^4 - 45*A*c^2*x^4 - 9*
B*b^2*x^2 + 9*A*b*c*x^2 - 5*A*b^2)/(b^3*x^(9/2))
```

3.193.9 Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx^2}{x^{7/2}\,(bx^2+cx^4)}\,dx = \frac{(-c)^{5/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(A\,c-B\,b)}{b^{13/4}} - \frac{\frac{2\,A}{9\,b}-\frac{2\,x^2\,(A\,c-B\,b)}{5\,b^2}+\frac{2\,c\,x^4\,(A\,c-B\,b)}{b^3}}{x^{9/2}} - \frac{(-c)^{5/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(A\,c-B\,b)}{b^{13/4}}$$

input
```
int((A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)),x)
```

output
```
((-c)^(5/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4))*(A*c - B*b))/b^(13/4) - ((2
*A)/(9*b) - (2*x^2*(A*c - B*b))/(5*b^2) + (2*c*x^4*(A*c - B*b))/b^3)/x^(9/
2) - ((-c)^(5/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4))*(A*c - B*b))/b^(13/4)
```

3.194 $\int \frac{A+Bx^2}{x^{9/2}(bx^2+cx^4)}\,dx$

3.194.1 Optimal result

Integrand size = 26, antiderivative size = 278

$$\int \frac{A+Bx^2}{x^{9/2}(bx^2+cx^4)}\,dx = -\frac{2A}{11bx^{11/2}}-\frac{2(bB-Ac)}{7b^2x^{7/2}}+\frac{2c(bB-Ac)}{3b^3x^{3/2}} - \frac{c^{7/4}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{15/4}}+\frac{c^{7/4}(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{15/4}} - \frac{c^{7/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{15/4}} + \frac{c^{7/4}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{15/4}}$$

output
```
-2/11*A/b/x^(11/2)-2/7*(-A*c+B*b)/b^2/x^(7/2)+2/3*c*(-A*c+B*b)/b^3/x^(3/2)
-1/2*c^(7/4)*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)
*2^(1/2)+1/2*c^(7/4)*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/
b^(15/4)*2^(1/2)-1/4*c^(7/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/
4)*2^(1/2)*x^(1/2))/b^(15/4)*2^(1/2)+1/4*c^(7/4)*(-A*c+B*b)*ln(b^(1/2)+x*c
^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(15/4)*2^(1/2)
```

3.194.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx^2}{x^{9/2}\left(bx^2+cx^4\right)}\,dx=-\frac{2(21Ab^2+33b^2Bx^2-33Abcx^2-77bBcx^4+77Ac^2x^4)}{231b^3x^{11/2}}-\frac{c^{7/4}(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}b^{15/4}}+\frac{c^{7/4}(bB-Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{15/4}}$$

input
```
Integrate[(A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)),x]
```

output
```
(-2*(21*A*b^2 + 33*b^2*B*x^2 - 33*A*b*c*x^2 - 77*b*B*c*x^4 + 77*A*c^2*x^4)
)/(231*b^3*x^(11/2)) - (c^(7/4)*(b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(15/4)) + (c^(7/4)*(b*B - A*
c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt
[2]*b^(15/4))
```

3.194.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {9, 359, 264, 264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{9/2}\left(bx^2+cx^4\right)}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{A+Bx^2}{x^{13/2}\left(b+cx^2\right)}dx$$

$$\Big\downarrow \; 359$$

$$\frac{(bB-Ac)\int \frac{1}{x^{9/2}(cx^2+b)}dx}{b}-\frac{2A}{11bx^{11/2}}$$

$$\Big\downarrow \; 264$$

$$\frac{(bB-Ac)\left(-\frac{c\int \frac{1}{x^{5/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2A}{11bx^{11/2}}$$

$$\Big\downarrow \; 264$$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{c\int \frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2A}{11bx^{11/2}}$$

$$\Big\downarrow \; 266$$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{2c\int \frac{1}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2A}{11bx^{11/2}}$$

$$\Big\downarrow \; 755$$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2A}{11bx^{11/2}}$$

$$\Big\downarrow \; 1476$$

3.194. $\int \frac{A+Bx^2}{x^{9/2}(bx^2+cx^4)}\,dx$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2A}{11bx^{11/2}}$$

$\downarrow$ 1082

3.194. $\int \frac{A+Bx^2}{x^{9/2}(bx^2+cx^4)}\,dx$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2A}{11bx^{11/2}}$$

$$\Big\downarrow \text{ 217}$$

3.194. $\int \frac{A+Bx^2}{x^{9/2}(bx^2+cx^4)}\,dx$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}\,d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2A}{11bx^{11/2}}$$

$\Big\downarrow$ 1479

3.194. $\int \frac{A+Bx^2}{x^{9/2}(bx^2+cx^4)}\,dx$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}\cdots}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{b}\cdots$$

$$\frac{2A}{11bx^{11/2}}$$

$\downarrow$ 25

3.194. $\int \frac{A+Bx^2}{x^{9/2}(bx^2+cx^4)}\,dx$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}\right)}{b}\right)}{b}$$

$$\frac{2A}{11bx^{11/2}}$$

$\downarrow$ 27

3.194. $\int \frac{A+Bx^2}{x^{9/2}(bx^2+cx^4)}\,dx$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\right.}{b}$$

$$\frac{2A}{11bx^{11/2}}$$

$\downarrow$ 1103

3.194. $\int \frac{A+Bx^2}{x^{9/2}(bx^2+cx^4)}\,dx$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{}{3\ldots}\right)}{b}\ldots\right)}{b}$$

$$\frac{2A}{11bx^{11/2}}$$

input
```
Int[(A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)),x]
```

output
```
(-2*A)/(11*b*x^(11/2)) + ((b*B - A*c)*(-2/(7*b*x^(7/2)) - (c*(-2/(3*b*x^(3
/2)) - (2*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(
1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^
(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*
Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])
))/b))/b))/b
```

3.194. $\int \frac{A+Bx^2}{x^{9/2}(bx^2+cx^4)}\,dx$

3.194.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.194.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.58

method	result
derivativedivides	$-\frac{2A}{11b\,x^{\frac{11}{2}}}-\frac{2(-Ac+Bb)}{7b^2x^{\frac{7}{2}}}-\frac{2c(Ac-Bb)}{3b^3x^{\frac{3}{2}}}-\frac{c^2(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right.\right.}{4b^4}$
default	$-\frac{2A}{11b\,x^{\frac{11}{2}}}-\frac{2(-Ac+Bb)}{7b^2x^{\frac{7}{2}}}-\frac{2c(Ac-Bb)}{3b^3x^{\frac{3}{2}}}-\frac{c^2(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right.\right.}{4b^4}$
risch	$-\frac{2(77A\,c^2x^4-77x^4Bbc-33Abc\,x^2+33b^2B\,x^2+21b^2A)}{231b^3x^{\frac{11}{2}}}-\frac{c^2(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\right.}{4b^4}$

input
```
int((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-2/11*A/b/x^(11/2)-2/7*(-A*c+B*b)/b^2/x^(7/2)-2/3*c*(A*c-B*b)/b^3/x^(3/2)-
1/4*c^2*(A*c-B*b)/b^4*(1/c*b)^(1/4)*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2
^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*a
rctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1
/2)-1))
```

3.194.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.42

$$\int \frac{A+Bx^2}{x^{9/2}\,(bx^2+cx^4)}\,dx =$$

$$-\frac{231\,b^3x^6\left(-\frac{B^4b^4c^7-4\,AB^3b^3c^8+6\,A^2B^2b^2c^9-4\,A^3Bbc^{10}+A^4c^{11}}{b^{15}}\right)^{\frac{1}{4}}\log\left(b^4\left(-\frac{B^4b^4c^7-4\,AB^3b^3c^8+6\,A^2B^2b^2c^9-4\,A^3Bbc^{10}+A^4c^{11}}{b^{15}}\right)\right.}{}$$

input
```
integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
-1/462*(231*b^3*x^6*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 -
 4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(1/4)*log(b^4*(-(B^4*b^4*c^7 - 4*A*B^3*b
^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(1/4) - (B*b
*c^2 - A*c^3)*sqrt(x)) + 231*I*b^3*x^6*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 +
6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(1/4)*log(I*b^4*(-(B^
4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^1
1)/b^15)^(1/4) - (B*b*c^2 - A*c^3)*sqrt(x)) - 231*I*b^3*x^6*(-(B^4*b^4*c^7
 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^15)^
(1/4)*log(-I*b^4*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*
A^3*B*b*c^10 + A^4*c^11)/b^15)^(1/4) - (B*b*c^2 - A*c^3)*sqrt(x)) - 231*b^
3*x^6*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^1
0 + A^4*c^11)/b^15)^(1/4)*log(-b^4*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^
2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(1/4) - (B*b*c^2 - A*c^3)
*sqrt(x)) - 4*(77*(B*b*c - A*c^2)*x^4 - 21*A*b^2 - 33*(B*b^2 - A*b*c)*x^2)
*sqrt(x))/(b^3*x^6)
```

3.194.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{9/2}(bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)/x**(9/2)/(c*x**4+b*x**2),x)
```

output
```
Timed out
```

3.194.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2}{x^{9/2}(bx^2+cx^4)}\,dx = \frac{\frac{2\sqrt{2}(Bbc^2-Ac^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(Bbc^2-Ac^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+}{4\,b^3} + \frac{2\,(77\,(Bbc-Ac^2)x^4-21\,Ab^2-33\,(Bb^2-Abc)x^2)}{231\,b^3x^{\frac{11}{2}}}$$

input
```
integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*(B*b*c^2 - A*c^3)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/
4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(
c))) + 2*sqrt(2)*(B*b*c^2 - A*c^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^
(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sq
rt(c))) + sqrt(2)*(B*b*c^2 - A*c^3)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) +
sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(B*b*c^2 - A*c^3)*log(-sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^
3 + 2/231*(77*(B*b*c - A*c^2)*x^4 - 21*A*b^2 - 33*(B*b^2 - A*b*c)*x^2)/(b^
3*x^(11/2))
```

3.194.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2}{x^{9/2}(bx^2+cx^4)}\,dx = \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bbc-\left(bc^3\right)^{\frac{1}{4}}Ac^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^4} + \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bbc-\left(bc^3\right)^{\frac{1}{4}}Ac^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^4} + \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bbc-\left(bc^3\right)^{\frac{1}{4}}Ac^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^4} - \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bbc-\left(bc^3\right)^{\frac{1}{4}}Ac^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^4} + \frac{2\left(77\,Bbcx^4-77\,Ac^2x^4-33\,Bb^2x^2+33\,Abcx^2-21\,Ab^2\right)}{231\,b^3x^{\frac{11}{2}}}$$

input
```
integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*((b*c^3)^(1/4)*B*b*c - (b*c^3)^(1/4)*A*c^2)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^4 + 1/2*sqrt(2)*((b*c^3)
^(1/4)*B*b*c - (b*c^3)^(1/4)*A*c^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/
4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 + 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b*c - (b*c
^3)^(1/4)*A*c^2)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 1/
4*sqrt(2)*((b*c^3)^(1/4)*B*b*c - (b*c^3)^(1/4)*A*c^2)*log(-sqrt(2)*sqrt(x)
*(b/c)^(1/4) + x + sqrt(b/c))/b^4 + 2/231*(77*B*b*c*x^4 - 77*A*c^2*x^4 - 3
3*B*b^2*x^2 + 33*A*b*c*x^2 - 21*A*b^2)/(b^3*x^(11/2))
```

3.194.9 Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 563, normalized size of antiderivative = 2.03

$$\int \frac{A+Bx^2}{x^{9/2}\left(bx^2+cx^4\right)}\,dx = \frac{(-c)^{7/4}\,\mathrm{atan}\left(\frac{A^3\,c^{10}\,\sqrt{x}-B^3\,b^3\,c^7\,\sqrt{x}-3\,A^2\,B\,b\,c^9\,\sqrt{x}+3\,A\,B^2\,b^2\,c^8\,\sqrt{x}}{b^{1/4}\,(-c)^{27/4}\,\left(c\,\left(c\,\left(A^3\,c-3\,A^2\,B\,b\right)+3\,A\,B^2\,b^2\right)-B^3\,b^3\right)}\right)\,\left(A\,c-B\,b\right)}{b^{15/4}}$$
$$-\frac{\frac{2\,A}{11\,b}-\frac{2\,x^2\,\left(A\,c-B\,b\right)}{7\,b^2}+\frac{2\,c\,x^4\,\left(A\,c-B\,b\right)}{3\,b^3}}{x^{11/2}}$$
$$-\frac{(-c)^{7/4}\,\mathrm{atan}\left(\frac{\frac{(-c)^{7/4}\,\left(A\,c-B\,b\right)\,\left(\sqrt{x}\,\left(16\,A^2\,b^9\,c^9-32\,A\,B\,b^{10}\,c^8+16\,B^2\,b^{11}\,c^7\right)-\frac{(-c)^{7/4}\,\left(A\,c-B\,b\right)\,\left(32\,A\,b^{13}\,c^6-32\,B\,b^{14}\,c^5\right)}{2\,b^{15/4}}\right)\,1{}\mathrm{i}}{2\,b^{15/4}}+\frac{(-c)^{7/4}\,(A\,c-B\,b \cdots}{\cdots}}{\frac{(-c)^{7/4}\,\left(A\,c-B\,b\right)\,\left(\sqrt{x}\,\left(16\,A^2\,b^9\,c^9-32\,A\,B\,b^{10}\,c^8+16\,B^2\,b^{11}\,c^7\right)-\frac{(-c)^{7/4}\,\left(A\,c-B\,b\right)\,\left(32\,A\,b^{13}\,c^6-32\,B\,b^{14}\,c^5\right)}{2\,b^{15/4}}\right)}{2\,b^{15/4}}-\frac{(-c)^{7/4}\,(A\,c-B\,b \cdots}{\cdots}}\cdots\right)\cdots}{b^{15/4}}$$

input
```
int((A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)),x)
```

output
```
((-c)^(7/4)*atan((A^3*c^10*x^(1/2) - B^3*b^3*c^7*x^(1/2) - 3*A^2*B*b*c^9*x
^(1/2) + 3*A*B^2*b^2*c^8*x^(1/2))/(b^(1/4)*(-c)^(27/4)*(c*(c*(A^3*c - 3*A^
2*B*b) + 3*A*B^2*b^2) - B^3*b^3)))*(A*c - B*b))/b^(15/4) - ((-c)^(7/4)*ata
n((((-c)^(7/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^9*c^9 + 16*B^2*b^11*c^7 - 32
*A*B*b^10*c^8) - ((-c)^(7/4)*(A*c - B*b)*(32*A*b^13*c^6 - 32*B*b^14*c^5))/
(2*b^(15/4)))*1i)/(2*b^(15/4)) + ((-c)^(7/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*
b^9*c^9 + 16*B^2*b^11*c^7 - 32*A*B*b^10*c^8) + ((-c)^(7/4)*(A*c - B*b)*(32
*A*b^13*c^6 - 32*B*b^14*c^5))/(2*b^(15/4)))*1i)/(2*b^(15/4)))/(((-c)^(7/4)
*(A*c - B*b)*(x^(1/2)*(16*A^2*b^9*c^9 + 16*B^2*b^11*c^7 - 32*A*B*b^10*c^8)
 - ((-c)^(7/4)*(A*c - B*b)*(32*A*b^13*c^6 - 32*B*b^14*c^5))/(2*b^(15/4))))
/(2*b^(15/4)) - ((-c)^(7/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^9*c^9 + 16*B^2*
b^11*c^7 - 32*A*B*b^10*c^8) + ((-c)^(7/4)*(A*c - B*b)*(32*A*b^13*c^6 - 32*
B*b^14*c^5))/(2*b^(15/4))))/(2*b^(15/4))))*(A*c - B*b)*1i)/b^(15/4) - ((2*
A)/(11*b) - (2*x^2*(A*c - B*b))/(7*b^2) + (2*c*x^4*(A*c - B*b))/(3*b^3))/x
^(11/2)
```

3.194. $\int \frac{A+Bx^2}{x^{9/2}(bx^2+cx^4)}\,dx$

3.195 $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.195.1 Optimal result

Integrand size = 26, antiderivative size = 332

$$\begin{aligned}\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx &= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4}-\frac{(13bB-9Ac)x^{5/2}}{10c^3}\\&+\frac{(13bB-9Ac)x^{9/2}}{18bc^2}-\frac{(bB-Ac)x^{13/2}}{2bc(b+cx^2)}+\frac{b^{5/4}(13bB-9Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{17/4}}\\&-\frac{b^{5/4}(13bB-9Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{17/4}}\\&+\frac{b^{5/4}(13bB-9Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{17/4}}\\&-\frac{b^{5/4}(13bB-9Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{17/4}}\end{aligned}$$

output
```
-1/10*(-9*A*c+13*B*b)*x^(5/2)/c^3+1/18*(-9*A*c+13*B*b)*x^(9/2)/b/c^2-1/2*(
-A*c+B*b)*x^(13/2)/b/c/(c*x^2+b)+1/8*b^(5/4)*(-9*A*c+13*B*b)*arctan(1-c^(1
/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(17/4)*2^(1/2)-1/8*b^(5/4)*(-9*A*c+13*B*b)*
arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(17/4)*2^(1/2)+1/16*b^(5/4)*(-
9*A*c+13*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(17/
4)*2^(1/2)-1/16*b^(5/4)*(-9*A*c+13*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/
4)*2^(1/2)*x^(1/2))/c^(17/4)*2^(1/2)+1/2*b*(-9*A*c+13*B*b)*x^(1/2)/c^4
```

3.195.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.62

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\frac{4\sqrt[4]{c}\sqrt{x}(585b^3B-9b^2c(45A-52Bx^2)+4c^3x^4(9A+5Bx^2)-4bc^2x^2(81A+13Bx^2))}{b+cx^2}+45\sqrt{2}b^{5/4}(13bB-9}{360c^{17/}}$$

input
```
Integrate[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
((4*c^(1/4)*Sqrt[x]*(585*b^3*B - 9*b^2*c*(45*A - 52*B*x^2) + 4*c^3*x^4*(9*
A + 5*B*x^2) - 4*b*c^2*x^2*(81*A + 13*B*x^2)))/(b + c*x^2) + 45*Sqrt[2]*b^
(5/4)*(13*b*B - 9*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x])] - 45*Sqrt[2]*b^(5/4)*(13*b*B - 9*A*c)*ArcTanh[(Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(360*c^(17/4))
```

3.195.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.96, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {9, 362, 262, 262, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^{11/2}(A+Bx^2)}{(b+cx^2)^2}dx$$

$$\Big\downarrow \ 362$$

$$\frac{(13bB-9Ac)\int \frac{x^{11/2}}{cx^2+b}dx}{4bc}-\frac{x^{13/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$$\Big\downarrow \ 262$$

$$\frac{(13bB-9Ac)\left(\frac{2x^{9/2}}{9c}-\frac{b\int \frac{x^{7/2}}{cx^2+b}dx}{c}\right)}{4bc}-\frac{x^{13/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$$\Big\downarrow \text{262}$$

$$\frac{(13bB-9Ac)\left(\frac{2x^{9/2}}{9c}-\frac{b\left(\frac{2x^{5/2}}{5c}-\frac{b\int\frac{x^{3/2}}{cx^2+b}dx}{c}\right)}{c}\right)}{4bc}-\frac{x^{13/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$$\Big\downarrow \text{262}$$

$$\frac{(13bB-9Ac)\left(\frac{2x^{9/2}}{9c}-\frac{b\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{b\int\frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{c}\right)}{c}\right)}{c}\right)}{4bc}-\frac{x^{13/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$$\Big\downarrow \text{266}$$

$$\frac{(13bB-9Ac)\left(\frac{2x^{9/2}}{9c}-\frac{b\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\int\frac{1}{cx^2+b}d\sqrt{x}}{c}\right)}{c}\right)}{c}\right)}{4bc}-\frac{x^{13/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$$\Big\downarrow \text{755}$$

3.195. $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

$$\frac{(13bB-9Ac)\left(\frac{2x^{9/2}}{9c}-\frac{b\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{c}\right)}{4bc}-\frac{x^{13/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$$\Big\downarrow \; 1476$$

3.195. $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

$$\frac{(13bB-9Ac)\left(\frac{2x^{9/2}}{9c}-\frac{b\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{c}\right)}{4bc}-\frac{x^{13/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

3.195. $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

↓ 1082

3.195. $\int \frac{x^{19/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$\frac{(13bB-9Ac)\left(\frac{2x^{9/2}}{9c}-\frac{b\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{c}\right)}{4bc}-\frac{x^{13/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

3.195. $\int\frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.195. $\int \frac{x^{19/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$\downarrow 217$$

$$\frac{(13bB-9Ac)\left(\frac{2x^{9/2}}{9c}-\frac{b\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}\,d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{c}\right)}{4bc}-\frac{x^{13/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

3.195. $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.195. $\downarrow$ 1479

3.195. $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

$$(13bB-9Ac)\left(\frac{2x^{9/2}}{9c}-\frac{b\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}\cdots}{\sqrt[4]{b}}\cdots\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\cdots}\cdots\right)}{c}\cdots\right)}{c}\cdots\right)}{c}\cdots\right)$$

3.195. $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

$$\downarrow 25$$

3.195. $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

$$(13bB-9Ac)\left(\frac{2x^{9/2}}{9c}-\frac{b\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}\cdots\right.}{c}\right.}{c}\right.}{c}\right.$$

3.195. $\int\frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

$$\Big\downarrow 27$$

3.195. $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

$$(13bB-9Ac)\left(\frac{2x^{9/2}}{9c}-\frac{b\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt{2}\sqrt[4]{b}\cdots}}{2\sqrt{b}}\cdots\right)}{c}\cdots\right)}{c}\cdots\right)}{c}\cdots\right)$$

3.195. $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

$\downarrow$ 1103

3.195. $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\, dx$

$$\frac{(13bB-9Ac)\left(\frac{2x^{9/2}}{9c}-\frac{b\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\right.}{}}{2\sqrt{b}}}{c}\right)}{c}\right)}{c}\right)}{4bc}$$

$$\frac{x^{13/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

3.195. $\int \frac{x^{19/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

input
```
Int[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*((b*B - A*c)*x^(13/2))/(b*c*(b + c*x^2)) + ((13*b*B - 9*A*c)*((2*x^(9
/2))/(9*c) - (b*((2*x^(5/2))/(5*c) - (b*((2*Sqrt[x])/c - (2*b*((-(ArcTan[1
 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[
1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[
b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sq
rt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + S
qrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/c))/c))/c))/(4*b*c)
```

3.195.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.195. $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.195.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{2\left(-5B\,c^{2}x^{4}-9A\,c^{2}x^{2}+18Bbc\,x^{2}+90Abc-135B\,b^{2}\right)\sqrt{x}}{45c^{4}}+\frac{b^{2}\left(\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{c\,x^{2}+b}+\frac{(9Ac-13Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{}\right.}{}$
derivativedivides	$-\frac{2\left(-\frac{B\,x^{\frac{9}{2}}c^{2}}{9}-\frac{A\,c^{2}x^{\frac{5}{2}}}{5}+\frac{2Bbc\,x^{\frac{5}{2}}}{5}+2Abc\sqrt{x}-3B\,b^{2}\sqrt{x}\right)}{c^{4}}+\frac{2b^{2}\left(\frac{\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{c\,x^{2}+b}+\frac{(9Ac-13Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{}\right.}{}$
default	$-\frac{2\left(-\frac{B\,x^{\frac{9}{2}}c^{2}}{9}-\frac{A\,c^{2}x^{\frac{5}{2}}}{5}+\frac{2Bbc\,x^{\frac{5}{2}}}{5}+2Abc\sqrt{x}-3B\,b^{2}\sqrt{x}\right)}{c^{4}}+\frac{2b^{2}\left(\frac{\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{c\,x^{2}+b}+\frac{(9Ac-13Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{}\right.}{}$

input
```
int(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/45*(-5*B*c^2*x^4-9*A*c^2*x^2+18*B*b*c*x^2+90*A*b*c-135*B*b^2)*x^(1/2)/c
^4+b^2/c^4*(2*(-1/4*A*c+1/4*B*b)*x^(1/2)/(c*x^2+b)+1/16*(9*A*c-13*B*b)*(1/
c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(
x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(
1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

3.195.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 748, normalized size of antiderivative = 2.25

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{45\left(c^5x^2+bc^4\right)\left(-\frac{28561\,B^4b^9-79092\,AB^3b^8c+82134\,A^2B^2b^7c^2-37908\,A^3Bb^6c^3+6561\,A^4b^5c^4}{c^{17}}\right)^{\frac{1}{4}}\log\left(\right.}{}$$

input
```
integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/360*(45*(c^5*x^2 + b*c^4)*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A
^2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4)*log(c
^4*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^2*b^7*c^2 - 37908*A^
3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4) - (13*B*b^2 - 9*A*b*c)*sqrt(x)
) - 45*(-I*c^5*x^2 - I*b*c^4)*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134
*A^2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4)*log
(I*c^4*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^2*b^7*c^2 - 3790
8*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4) - (13*B*b^2 - 9*A*b*c)*sqr
t(x)) - 45*(I*c^5*x^2 + I*b*c^4)*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82
134*A^2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4)*
log(-I*c^4*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^2*b^7*c^2 -
37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4) - (13*B*b^2 - 9*A*b*c)
*sqrt(x)) - 45*(c^5*x^2 + b*c^4)*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82
134*A^2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4)*
log(-c^4*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^2*b^7*c^2 - 37
908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4) - (13*B*b^2 - 9*A*b*c)*s
qrt(x)) + 4*(20*B*c^3*x^6 - 4*(13*B*b*c^2 - 9*A*c^3)*x^4 + 585*B*b^3 - 405
*A*b^2*c + 36*(13*B*b^2*c - 9*A*b*c^2)*x^2)*sqrt(x))/(c^5*x^2 + b*c^4)
```

3.195.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(19/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

3.195.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.90

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb^3-Ab^2c)\sqrt{x}}{2\,(c^5x^2+bc^4)} - \frac{\frac{2\sqrt{2}(13\,Bb-9\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}(13\,Bb-9\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}(13\,Bb-9\,Ac)\log\left(\sqrt{2}b\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{16\,c^4} + \frac{2\left(5\,Bc^2x^{\frac{9}{2}}-9\,(2\,Bbc-Ac^2)x^{\frac{5}{2}}+45\,(3\,Bb^2-2\,Abc)\sqrt{x}\right)}{45\,c^4}$$

input
```
integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b^3 - A*b^2*c)*sqrt(x)/(c^5*x^2 + b*c^4) - 1/16*(2*sqrt(2)*(13*B*b
- 9*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/
sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(13*B*b
 - 9*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x)
)/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(13*B*b
 - 9*A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3
/4)*c^(1/4)) - sqrt(2)*(13*B*b - 9*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(
x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))*b^2/c^4 + 2/45*(5*B*c^2*x^(9/
2) - 9*(2*B*b*c - A*c^2)*x^(5/2) + 45*(3*B*b^2 - 2*A*b*c)*sqrt(x))/c^4
```

3.195.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.01

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx =$$

$$-\frac{\sqrt{2}\left(13\,(bc^3)^{\frac{1}{4}}\,Bb^2-9\,(bc^3)^{\frac{1}{4}}\,Abc\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^5}$$
$$-\frac{\sqrt{2}\left(13\,(bc^3)^{\frac{1}{4}}\,Bb^2-9\,(bc^3)^{\frac{1}{4}}\,Abc\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^5}$$
$$-\frac{\sqrt{2}\left(13\,(bc^3)^{\frac{1}{4}}\,Bb^2-9\,(bc^3)^{\frac{1}{4}}\,Abc\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^5}$$
$$+\frac{\sqrt{2}\left(13\,(bc^3)^{\frac{1}{4}}\,Bb^2-9\,(bc^3)^{\frac{1}{4}}\,Abc\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^5}$$
$$+\frac{Bb^3\sqrt{x}-Ab^2c\sqrt{x}}{2\,(cx^2+b)c^4}$$
$$+\frac{2\left(5\,Bc^{16}x^{\frac{9}{2}}-18\,Bbc^{15}x^{\frac{5}{2}}+9\,Ac^{16}x^{\frac{5}{2}}+135\,Bb^2c^{14}\sqrt{x}-90\,Abc^{15}\sqrt{x}\right)}{45\,c^{18}}$$

input
```
integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*(13*(b*c^3)^(1/4)*B*b^2 - 9*(b*c^3)^(1/4)*A*b*c)*arctan(1/2*s
qrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^5 - 1/8*sqrt(2)*(1
3*(b*c^3)^(1/4)*B*b^2 - 9*(b*c^3)^(1/4)*A*b*c)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^5 - 1/16*sqrt(2)*(13*(b*c^3)^(1/
4)*B*b^2 - 9*(b*c^3)^(1/4)*A*b*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sq
rt(b/c))/c^5 + 1/16*sqrt(2)*(13*(b*c^3)^(1/4)*B*b^2 - 9*(b*c^3)^(1/4)*A*b*
c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 1/2*(B*b^3*sqrt
(x) - A*b^2*c*sqrt(x))/((c*x^2 + b)*c^4) + 2/45*(5*B*c^16*x^(9/2) - 18*B*b
*c^15*x^(5/2) + 9*A*c^16*x^(5/2) + 135*B*b^2*c^14*sqrt(x) - 90*A*b*c^15*sq
rt(x))/c^18
```

3.195.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 857, normalized size of antiderivative = 2.58

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = x^{5/2}\left(\frac{2A}{5c^2}-\frac{4Bb}{5c^3}\right)$$

$$-\sqrt{x}\left(\frac{2b\left(\frac{2A}{c^2}-\frac{4Bb}{c^3}\right)}{c}+\frac{2Bb^2}{c^4}\right)+\frac{2Bx^{9/2}}{9c^2}+\frac{\sqrt{x}\left(\frac{Bb^3}{2}-\frac{Ab^2c}{2}\right)}{c^5x^2+bc^4}$$

$$+\frac{(-b)^{5/4}\operatorname{atan}\left(\frac{\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81A^2b^4c^2-234ABb^5c+169B^2b^6\right)}{c^5}+\frac{(-b)^{5/4}(9Ac-13Bb)\left(13Bb^4-9Ab^3c\right)}{c^{21/4}}\right)(9Ac-13Bb)\,1i}{8c^{17/4}}+\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81A^2\right.}{}\right.}{}}{\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81A^2b^4c^2-234ABb^5c+169B^2b^6\right)}{c^5}+\frac{(-b)^{5/4}(9Ac-13Bb)\left(13Bb^4-9Ab^3c\right)}{c^{21/4}}\right)(9Ac-13Bb)}{8c^{17/4}}-\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81A^2\right.}{}\right.}{}}\right)}{4c^{17/4}}$$

$$-\frac{(-b)^{5/4}\operatorname{atan}\left(\frac{\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81A^2b^4c^2-234ABb^5c+169B^2b^6\right)}{c^5}-\frac{(-b)^{5/4}(9Ac-13Bb)\left(13Bb^4-9Ab^3c\right)1i}{c^{21/4}}\right)(9Ac-13Bb)}{8c^{17/4}}+\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81\right.}{}\right.}{}}{\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81A^2b^4c^2-234ABb^5c+169B^2b^6\right)}{c^5}-\frac{(-b)^{5/4}(9Ac-13Bb)\left(13Bb^4-9Ab^3c\right)1i}{c^{21/4}}\right)(9Ac-13Bb)\,1i}{8c^{17/4}}-\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81\right.}{}\right.}{}}\right)}{4c^{17/4}}$$

input
```
int((x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output

```
x^(5/2)*((2*A)/(5*c^2) - (4*B*b)/(5*c^3)) - x^(1/2)*((2*b*((2*A)/c^2 - (4*
B*b)/c^3))/c + (2*B*b^2)/c^4) + (2*B*x^(9/2))/(9*c^2) + (x^(1/2)*((B*b^3)/
2 - (A*b^2*c)/2))/(b*c^4 + c^5*x^2) + ((-b)^(5/4)*atan((((-b)^(5/4)*((x^(1
/2)*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 + ((-b)^(5/4)*(9*A
*c - 13*B*b)*(13*B*b^4 - 9*A*b^3*c))/c^(21/4))*(9*A*c - 13*B*b)*1i)/(8*c^(
17/4)) + ((-b)^(5/4)*((x^(1/2)*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5
*c))/c^5 - ((-b)^(5/4)*(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*b^3*c))/c^(21/4))*
(9*A*c - 13*B*b)*1i)/(8*c^(17/4)))/(((-b)^(5/4)*((x^(1/2)*(169*B^2*b^6 + 8
1*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 + ((-b)^(5/4)*(9*A*c - 13*B*b)*(13*B*b
^4 - 9*A*b^3*c))/c^(21/4))*(9*A*c - 13*B*b))/(8*c^(17/4)) - ((-b)^(5/4)*((
x^(1/2)*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 - ((-b)^(5/4)*
(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*b^3*c))/c^(21/4))*(9*A*c - 13*B*b))/(8*c^
(17/4))))*(9*A*c - 13*B*b)*1i)/(4*c^(17/4)) - ((-b)^(5/4)*atan((((-b)^(5/4
)*((x^(1/2)*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 - ((-b)^(5
/4)*(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*b^3*c)*1i)/c^(21/4))*(9*A*c - 13*B*b)
)/(8*c^(17/4)) + ((-b)^(5/4)*((x^(1/2)*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234
*A*B*b^5*c))/c^5 + ((-b)^(5/4)*(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*b^3*c)*1i)
/c^(21/4))*(9*A*c - 13*B*b))/(8*c^(17/4)))/(((-b)^(5/4)*((x^(1/2)*(169*B^2
*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 - ((-b)^(5/4)*(9*A*c - 13*B*b)
*(13*B*b^4 - 9*A*b^3*c)*1i)/c^(21/4))*(9*A*c - 13*B*b)*1i)/(8*c^(17/4))...
```

3.195. $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.196 $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.196.1 Optimal result

Integrand size = 26, antiderivative size = 310

$$\begin{aligned}\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx &= -\frac{(11bB-7Ac)x^{3/2}}{6c^3}+\frac{(11bB-7Ac)x^{7/2}}{14bc^2}\\ &-\frac{(bB-Ac)x^{11/2}}{2bc(b+cx^2)}-\frac{b^{3/4}(11bB-7Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{15/4}}\\ &+\frac{b^{3/4}(11bB-7Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{15/4}}\\ &+\frac{b^{3/4}(11bB-7Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{15/4}}\\ &-\frac{b^{3/4}(11bB-7Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{15/4}}\end{aligned}$$

output
```
-1/6*(-7*A*c+11*B*b)*x^(3/2)/c^3+1/14*(-7*A*c+11*B*b)*x^(7/2)/b/c^2-1/2*(-
A*c+B*b)*x^(11/2)/b/c/(c*x^2+b)-1/8*b^(3/4)*(-7*A*c+11*B*b)*arctan(1-c^(1/
4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(15/4)*2^(1/2)+1/8*b^(3/4)*(-7*A*c+11*B*b)*a
rctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(15/4)*2^(1/2)+1/16*b^(3/4)*(-7
*A*c+11*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(15/4
)*2^(1/2)-1/16*b^(3/4)*(-7*A*c+11*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4
)*2^(1/2)*x^(1/2))/c^(15/4)*2^(1/2)
```

3.196.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.59

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\frac{4c^{3/4}x^{3/2}(-77b^2B+bc(49A-44Bx^2)+4c^2x^2(7A+3Bx^2))}{b+cx^2} - 21\sqrt{2}b^{3/4}(11bB-7Ac)\arctan\left(\frac{\sqrt{\ }}{\sqrt{2}\sqrt[4]{\ }}\right.}{168c^{15/4}}$$

input
```
Integrate[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
((4*c^(3/4)*x^(3/2)*(-77*b^2*B + b*c*(49*A - 44*B*x^2) + 4*c^2*x^2*(7*A +
3*B*x^2)))/(b + c*x^2) - 21*Sqrt[2]*b^(3/4)*(11*b*B - 7*A*c)*ArcTan[(Sqrt[
b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 21*Sqrt[2]*b^(3/4)*(1
1*b*B - 7*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c
]*x)])/(168*c^(15/4))
```

3.196.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {9, 362, 262, 262, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^{9/2}(A+Bx^2)}{(b+cx^2)^2}dx$$

$$\Big\downarrow \; 362$$

$$\frac{(11bB-7Ac)\int \frac{x^{9/2}}{cx^2+b}dx}{4bc} - \frac{x^{11/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$$\Big\downarrow \; 262$$

$$\frac{(11bB-7Ac)\left(\frac{2x^{7/2}}{7c} - \frac{b\int \frac{x^{5/2}}{cx^2+b}dx}{c}\right)}{4bc} - \frac{x^{11/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$$\Big\downarrow \text{262}$$

$$\frac{(11bB-7Ac)\left(\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{b\int\frac{\sqrt{x}}{cx^2+b}dx}{c}\right)}{c}\right)}{4bc}-\frac{x^{11/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$$\Big\downarrow \text{266}$$

$$\frac{(11bB-7Ac)\left(\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\int\frac{x}{cx^2+b}d\sqrt{x}}{c}\right)}{c}\right)}{4bc}-\frac{x^{11/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$$\Big\downarrow \text{826}$$

$$\frac{(11bB-7Ac)\left(\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{c}\right)}{4bc}-\frac{x^{11/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$$\Big\downarrow \text{1476}$$

3.196. $\int \frac{x^{17/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$\frac{(11bB-7Ac)\left(\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{c}\right)}{4bc}-\frac{x^{11/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$\downarrow$ 1082

3.196. $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

$$\frac{(11bB-7Ac)\left(\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{c}\right)}{4bc}-\frac{x^{11/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$$\Big\downarrow \ 217$$

3.196. $\int \frac{x^{17/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$\frac{(11bB-7Ac)\left(\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{c}\right)}{4bc}-\frac{x^{11/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$\downarrow$ 1479

3.196. $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

$$\frac{(11bB-7Ac)\left(\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\cdots\right.}{\sqrt[4]{c}\left(x+\cdots\right.}}{2\sqrt{\cdots}}}{2\sqrt{c}}\cdots\right)}{c}\right)}{c}\right)}{4bc}$$

$$\frac{x^{11/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\downarrow$ 25

3.196. $\int \frac{x^{17/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$\frac{(11bB-7Ac)\left(\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\ldots\right.}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{\ldots}}{\sqrt[4]{\ldots}}\right.}}{2\sqrt{2}\sqrt[4]{\ldots}}}{2\sqrt{c}}\ldots\right.}{c}\ldots\right)}{c}\ldots\right)}{4bc}\ldots$$

$$\frac{x^{11/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\downarrow$ 27

3.196. $\int \frac{x^{17/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$\frac{(11bB-7Ac)\left(\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{\cdots}}{2\sqrt[4]{b}\sqrt{c}}\cdots}{2\sqrt{c}}\cdots\right)}{c}\cdots\right)}{c}\cdots\right)}{4bc}$$

$$\frac{x^{11/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$\downarrow$ 1103

3.196. $\int\frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

$$\frac{(11bB-7Ac)\left(\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{\ldots}\right.}{2\sqrt{2}\sqrt[4]{b}\ldots}}{2\sqrt{c}}\ldots\right)}{c}\ldots\right)}{c}\ldots\right)}{4bc}\ldots$$

$$\frac{x^{11/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

input
```
Int[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*((b*B - A*c)*x^(11/2))/(b*c*(b + c*x^2)) + ((11*b*B - 7*A*c)*((2*x^(7
/2))/(7*c) - (b*((2*x^(3/2))/(3*c) - (2*b*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*
Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)
*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt
[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4
)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]
*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/c))/c))/(4*b*c)
```

3.196. $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.196.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

3.196. $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.196.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.55

method	result
risch	$\frac{2x^{\frac{3}{2}}(3Bc\,x^2+7Ac-14Bb)}{21c^3}-\frac{b\left(\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{7Ac}{4}-\frac{11Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\right.}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{c^3}$
derivativedivides	$\frac{\frac{2Bc\,x^{\frac{7}{2}}}{7}+\frac{2(Ac-2Bb)x^{\frac{3}{2}}}{3}}{c^3}-\frac{2b\left(\frac{\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{7Ac}{4}-\frac{11Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arc}\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{c^3}$
default	$\frac{\frac{2Bc\,x^{\frac{7}{2}}}{7}+\frac{2(Ac-2Bb)x^{\frac{3}{2}}}{3}}{c^3}-\frac{2b\left(\frac{\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{7Ac}{4}-\frac{11Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arc}\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{c^3}$

input
```
int(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2/21*x^(3/2)*(3*B*c*x^2+7*A*c-14*B*b)/c^3-b/c^3*(2*(-1/4*A*c+1/4*B*b)*x^(3
/2)/(c*x^2+b)+1/4*(7/4*A*c-11/4*B*b)/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b
)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1
/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(
1/c*b)^(1/4)*x^(1/2)-1)))
```

3.196.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 848, normalized size of antiderivative = 2.74

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx =$$

$$-\frac{21\,(c^4x^2+bc^3)\left(-\frac{14641\,B^4b^7-37268\,AB^3b^6c+35574\,A^2B^2b^5c^2-15092\,A^3Bb^4c^3+2401\,A^4b^3c^4}{c^{15}}\right)^{\frac{1}{4}}\log\left(c^{11}\left(-\frac{14641\,B^4b^7-37268\,A\right.\right.}{}$$

input
```
integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
-1/168*(21*(c^4*x^2 + b*c^3)*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*
A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(1/4)*log(
c^11*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*
A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(3/4) - (1331*B^3*b^5 - 2541*A*B^2
*b^4*c + 1617*A^2*B*b^3*c^2 - 343*A^3*b^2*c^3)*sqrt(x)) + 21*(-I*c^4*x^2 -
 I*b*c^3)*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 1
5092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(1/4)*log(I*c^11*(-(14641*B^4
*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2
401*A^4*b^3*c^4)/c^15)^(3/4) - (1331*B^3*b^5 - 2541*A*B^2*b^4*c + 1617*A^2
*B*b^3*c^2 - 343*A^3*b^2*c^3)*sqrt(x)) + 21*(I*c^4*x^2 + I*b*c^3)*(-(14641
*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3
 + 2401*A^4*b^3*c^4)/c^15)^(1/4)*log(-I*c^11*(-(14641*B^4*b^7 - 37268*A*B^
3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/
c^15)^(3/4) - (1331*B^3*b^5 - 2541*A*B^2*b^4*c + 1617*A^2*B*b^3*c^2 - 343*
A^3*b^2*c^3)*sqrt(x)) - 21*(c^4*x^2 + b*c^3)*(-(14641*B^4*b^7 - 37268*A*B^
3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/
c^15)^(1/4)*log(-c^11*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2
*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(3/4) - (1331*B^3
*b^5 - 2541*A*B^2*b^4*c + 1617*A^2*B*b^3*c^2 - 343*A^3*b^2*c^3)*sqrt(x)) -
 4*(12*B*c^2*x^5 - 4*(11*B*b*c - 7*A*c^2)*x^3 - 7*(11*B*b^2 - 7*A*b*c)*...
```

3.196.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(17/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

3.196.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.80

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(Bb^2-Abc)x^{\frac{3}{2}}}{2\,(c^4x^2+bc^3)} + \frac{(11\,Bb^2-7\,Abc)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right.}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{16\,c^3} + \frac{2\left(3\,Bcx^{\frac{7}{2}}-7\,(2\,Bb-Ac)x^{\frac{3}{2}}\right)}{21\,c^3}$$

input
```
integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*b^2 - A*b*c)*x^(3/2)/(c^4*x^2 + b*c^3) + 1/16*(11*B*b^2 - 7*A*b*c)
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x
))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)
*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c
^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqr
t(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/c^3
 + 2/21*(3*B*c*x^(7/2) - 7*(2*B*b - A*c)*x^(3/2))/c^3
```

3.196.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.96

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{Bb^2x^{\frac{3}{2}}-Abcx^{\frac{3}{2}}}{2\,(cx^2+b)c^3} + \frac{\sqrt{2}\left(11\,(bc^3)^{\frac{3}{4}}\,Bb-7\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^6} + \frac{\sqrt{2}\left(11\,(bc^3)^{\frac{3}{4}}\,Bb-7\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^6} - \frac{\sqrt{2}\left(11\,(bc^3)^{\frac{3}{4}}\,Bb-7\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^6} + \frac{\sqrt{2}\left(11\,(bc^3)^{\frac{3}{4}}\,Bb-7\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^6} + \frac{2\left(3\,Bc^{12}x^{\frac{7}{2}}-14\,Bbc^{11}x^{\frac{3}{2}}+7\,Ac^{12}x^{\frac{3}{2}}\right)}{21\,c^{14}}$$

input
```
integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-1/2*(B*b^2*x^(3/2) - A*b*c*x^(3/2))/((c*x^2 + b)*c^3) + 1/8*sqrt(2)*(11*(
b*c^3)^(3/4)*B*b - 7*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^
(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^6 + 1/8*sqrt(2)*(11*(b*c^3)^(3/4)*B*b -
7*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))
/(b/c)^(1/4))/c^6 - 1/16*sqrt(2)*(11*(b*c^3)^(3/4)*B*b - 7*(b*c^3)^(3/4)*A
*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^6 + 1/16*sqrt(2)*(1
1*(b*c^3)^(3/4)*B*b - 7*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4
) + x + sqrt(b/c))/c^6 + 2/21*(3*B*c^12*x^(7/2) - 14*B*b*c^11*x^(3/2) + 7*
A*c^12*x^(3/2))/c^14
```

3.196.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.41

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = x^{3/2}\left(\frac{2\,A}{3\,c^2}-\frac{4\,B\,b}{3\,c^3}\right)+\frac{2\,B\,x^{7/2}}{7\,c^2}-\frac{x^{3/2}\left(\frac{B\,b^2}{2}-\frac{A\,b\,c}{2}\right)}{c^4\,x^2+b\,c^3} + \frac{(-b)^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(7\,A\,c-11\,B\,b)}{4\,c^{15/4}}+\frac{(-b)^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right)\,(7\,A\,c-11\,B\,b)\,1\mathrm{i}}{4\,c^{15/4}}$$

input
```
int((x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output
```
x^(3/2)*((2*A)/(3*c^2) - (4*B*b)/(3*c^3)) + (2*B*x^(7/2))/(7*c^2) - (x^(3/
2)*((B*b^2)/2 - (A*b*c)/2))/(b*c^3 + c^4*x^2) + ((-b)^(3/4)*atan((c^(1/4)*
x^(1/2))/(-b)^(1/4))*(7*A*c - 11*B*b))/(4*c^(15/4)) + ((-b)^(3/4)*atan((c^
(1/4)*x^(1/2)*1i)/(-b)^(1/4))*(7*A*c - 11*B*b)*1i)/(4*c^(15/4))
```

3.197 $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.197.1 Optimal result

Integrand size = 26, antiderivative size = 310

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(9bB-5Ac)\sqrt{x}}{2c^3}+\frac{(9bB-5Ac)x^{5/2}}{10bc^2}-\frac{(bB-Ac)x^{9/2}}{2bc\,(b+cx^2)} -\frac{\sqrt[4]{b}(9bB-5Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}}+\frac{\sqrt[4]{b}(9bB-5Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}} -\frac{\sqrt[4]{b}(9bB-5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}} +\frac{\sqrt[4]{b}(9bB-5Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}$$

output
```
1/10*(-5*A*c+9*B*b)*x^(5/2)/b/c^2-1/2*(-A*c+B*b)*x^(9/2)/b/c/(c*x^2+b)-1/8
*b^(1/4)*(-5*A*c+9*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(13/4)
*2^(1/2)+1/8*b^(1/4)*(-5*A*c+9*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/
4))/c^(13/4)*2^(1/2)-1/16*b^(1/4)*(-5*A*c+9*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1
/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)*2^(1/2)+1/16*b^(1/4)*(-5*A*c+9*B*b)*
ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)*2^(1/2)-1/2
*(-5*A*c+9*B*b)*x^(1/2)/c^3
```

3.197.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.59

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\frac{4\sqrt[4]{c}\sqrt{x}(-45b^2B+bc(25A-36Bx^2)+4c^2x^2(5A+Bx^2))}{b+cx^2} - 5\sqrt{2}\sqrt[4]{b}(9bB-5Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}\right.}{40c^{13/4}}$$

input
```
Integrate[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
((4*c^(1/4)*Sqrt[x]*(-45*b^2*B + b*c*(25*A - 36*B*x^2) + 4*c^2*x^2*(5*A +
B*x^2)))/(b + c*x^2) - 5*Sqrt[2]*b^(1/4)*(9*b*B - 5*A*c)*ArcTan[(Sqrt[b] -
 Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 5*Sqrt[2]*b^(1/4)*(9*b*B
- 5*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])
/(40*c^(13/4))
```

3.197.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {9, 362, 262, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^{7/2}(A+Bx^2)}{(b+cx^2)^2}dx$$

$$\Big\downarrow \text{ 362}$$

$$\frac{(9bB-5Ac)\int \frac{x^{7/2}}{cx^2+b}dx}{4bc} - \frac{x^{9/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{(9bB-5Ac)\left(\frac{2x^{5/2}}{5c} - \frac{b\int \frac{x^{3/2}}{cx^2+b}dx}{c}\right)}{4bc} - \frac{x^{9/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$\downarrow$ 262

$$\frac{(9bB-5Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{b\int\frac{1}{\sqrt{x}(cx^2+b)}dx}{c}\right)}{c}\right)}{4bc}-\frac{x^{9/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$\downarrow$ 266

$$\frac{(9bB-5Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\int\frac{1}{cx^2+b}d\sqrt{x}}{c}\right)}{c}\right)}{4bc}-\frac{x^{9/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$\downarrow$ 755

$$\frac{(9bB-5Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{4bc}-\frac{x^{9/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$\downarrow$ 1476

3.197. $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

$$\frac{(9bB-5Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{4bc}-\frac{x^{9/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\Big\downarrow$ 1082

3.197. $\int\frac{x^{15/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$\frac{(9bB-5Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{4bc}-\frac{x^{9/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\Big\downarrow$ 217

3.197. $\int \frac{x^{15/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$\frac{(9bB-5Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{4bc}-\frac{x^{9/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$\Big\downarrow$ 1479

3.197. $\int\frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

$$\frac{(9bB-5Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\text{arcta}\ldots}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{4bc}$$

$$\frac{x^{9/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\downarrow$ 25

3.197. $\int \frac{x^{15/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$\frac{(9bB-5Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\cdots\right)}{\sqrt{2}\cdots}}{2\sqrt{b}}\cdots\right)}{c}\right)}{c}\cdots\right)}{4bc}\cdots$$

$$\frac{x^{9/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\downarrow$ 27

3.197. $\int \frac{x^{15/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$\frac{(9bB-5Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{4bc}$$

$$\frac{x^{9/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\downarrow$ 1103

3.197. $\int \frac{x^{15/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$\frac{(9bB-5Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\cdots\right.}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right.}{c}\right.}{c}\right.}{4bc}$$

$$\frac{x^{9/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

input
```
Int[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*((b*B - A*c)*x^(9/2))/(b*c*(b + c*x^2)) + ((9*b*B - 5*A*c)*((2*x^(5/2
))/(5*c) - (b*((2*Sqrt[x])/c - (2*b*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x
])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[
x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] -
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + L
og[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/
4)*c^(1/4)))/(2*Sqrt[b])))/c))/c))/(4*b*c)
```

3.197. $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.197.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

3.197. $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.197.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.55

method	result
risch	$\frac{2(Bc\,x^2+5Ac-10Bb)\sqrt{x}}{5c^3}-\frac{b\left(\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(5Ac-9Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)\right.}{16b}}{c^3}$
derivativedivides	$\frac{\frac{2Bc\,x^{\frac{5}{2}}}{5}+2Ac\sqrt{x}-4bB\sqrt{x}}{c^3}-\frac{2b\left(\frac{\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(5Ac-9Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)\right.}{32b}}{c^3}$
default	$\frac{\frac{2Bc\,x^{\frac{5}{2}}}{5}+2Ac\sqrt{x}-4bB\sqrt{x}}{c^3}-\frac{2b\left(\frac{\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(5Ac-9Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)\right.}{32b}}{c^3}$

input
```
int(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2/5*(B*c*x^2+5*A*c-10*B*b)*x^(1/2)/c^3-b/c^3*(2*(-1/4*A*c+1/4*B*b)*x^(1/2)
/(c*x^2+b)+1/16*(5*A*c-9*B*b)*(1/c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)
*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(
1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^
(1/4)*x^(1/2)-1)))
```

3.197.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 696, normalized size of antiderivative = 2.25

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx =$$

$$-\frac{5\,(c^4x^2+bc^3)\left(-\frac{6561\,B^4b^5-14580\,AB^3b^4c+12150\,A^2B^2b^3c^2-4500\,A^3Bb^2c^3+625\,A^4bc^4}{c^{13}}\right)^{\frac{1}{4}}\log\left(c^3\left(-\frac{6561\,B^4b^5-14580\,AB^3b^4c+1}{}\right.\right.}{}$$

input
```
integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
-1/40*(5*(c^4*x^2 + b*c^3)*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2
*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(1/4)*log(c^3*(-(
6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*
c^3 + 625*A^4*b*c^4)/c^13)^(1/4) - (9*B*b - 5*A*c)*sqrt(x)) + 5*(I*c^4*x^2
 + I*b*c^3)*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 -
4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(1/4)*log(I*c^3*(-(6561*B^4*b^5
- 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4
*b*c^4)/c^13)^(1/4) - (9*B*b - 5*A*c)*sqrt(x)) + 5*(-I*c^4*x^2 - I*b*c^3)*
(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500*A^3*B*b
^2*c^3 + 625*A^4*b*c^4)/c^13)^(1/4)*log(-I*c^3*(-(6561*B^4*b^5 - 14580*A*B
^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^1
3)^(1/4) - (9*B*b - 5*A*c)*sqrt(x)) - 5*(c^4*x^2 + b*c^3)*(-(6561*B^4*b^5
- 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4
*b*c^4)/c^13)^(1/4)*log(-c^3*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A
^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(1/4) - (9*B*b
- 5*A*c)*sqrt(x)) - 4*(4*B*c^2*x^4 - 45*B*b^2 + 25*A*b*c - 4*(9*B*b*c - 5*
A*c^2)*x^2)*sqrt(x))/(c^4*x^2 + b*c^3)
```

3.197.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(15/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

3.197.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.87

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(Bb^2-Abc)\sqrt{x}}{2\,(c^4x^2+bc^3)} + \frac{\frac{2\,\sqrt{2}(9\,Bb-5\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\,\sqrt{2}(9\,Bb-5\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}(9\,Bb-5\,Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{16\,c^3} + \frac{2\left(Bcx^{\frac{5}{2}}-5\,(2\,Bb-Ac)\sqrt{x}\right)}{5\,c^3}$$

input
```
integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*b^2 - A*b*c)*sqrt(x)/(c^4*x^2 + b*c^3) + 1/16*(2*sqrt(2)*(9*B*b -
5*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sq
rt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(9*B*b -
5*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/s
qrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(9*B*b - 5
*A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*
c^(1/4)) - sqrt(2)*(9*B*b - 5*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) +
sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))*b/c^3 + 2/5*(B*c*x^(5/2) - 5*(2*B*
b - A*c)*sqrt(x))/c^3
```

3.197.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.96

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\sqrt{2}\left(9\,(bc^3)^{\frac{1}{4}}\,Bb-5\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^4}$$
$$+\frac{\sqrt{2}\left(9\,(bc^3)^{\frac{1}{4}}\,Bb-5\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^4}$$
$$+\frac{\sqrt{2}\left(9\,(bc^3)^{\frac{1}{4}}\,Bb-5\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^4}$$
$$-\frac{\sqrt{2}\left(9\,(bc^3)^{\frac{1}{4}}\,Bb-5\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^4}$$
$$-\frac{Bb^2\sqrt{x}-Abc\sqrt{x}}{2\,(cx^2+b)c^3}+\frac{2\left(Bc^8x^{\frac{5}{2}}-10\,Bbc^7\sqrt{x}+5\,Ac^8\sqrt{x}\right)}{5\,c^{10}}$$

input
```
integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^4 + 1/8*sqrt(2)*(9*(b*c^
3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/
4) - 2*sqrt(x))/(b/c)^(1/4))/c^4 + 1/16*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - 5*(
b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 1
/16*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(
x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 1/2*(B*b^2*sqrt(x) - A*b*c*sqrt(x))/
((c*x^2 + b)*c^3) + 2/5*(B*c^8*x^(5/2) - 10*B*b*c^7*sqrt(x) + 5*A*c^8*sqrt
(x))/c^10
```

3.197.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 823, normalized size of antiderivative = 2.65

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \sqrt{x}\left(\frac{2A}{c^2}-\frac{4Bb}{c^3}\right)+\frac{2Bx^{5/2}}{5c^2}-\frac{\sqrt{x}\left(\frac{Bb^2}{2}-\frac{Abc}{2}\right)}{c^4x^2+bc^3}$$

$$+\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25A^2b^2c^2-90ABb^3c+81B^2b^4\right)}{c^3}-\frac{(-b)^{1/4}(5Ac-9Bb)\left(72Bb^3-40Ab^2c\right)}{8c^{13/4}}\right)(5Ac-9Bb)\,1\mathrm{i}}{8c^{13/4}}+\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25A^2b^2\cdots\right.\right.}{\cdots}}{\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25A^2b^2c^2-90ABb^3c+81B^2b^4\right)}{c^3}-\frac{(-b)^{1/4}(5Ac-9Bb)\left(72Bb^3-40Ab^2c\right)}{8c^{13/4}}\right)(5Ac-9Bb)}{8c^{13/4}}-\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25A^2b^2\cdots\right.\right.}{\cdots}}\right)\cdots}{4c^{13/4}}$$

$$+\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25A^2b^2c^2-90ABb^3c+81B^2b^4\right)}{c^3}-\frac{(-b)^{1/4}(5Ac-9Bb)\left(72Bb^3-40Ab^2c\right)1\mathrm{i}}{8c^{13/4}}\right)(5Ac-9Bb)}{8c^{13/4}}+\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25A^2\cdots\right.\right.}{\cdots}}{\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25A^2b^2c^2-90ABb^3c+81B^2b^4\right)}{c^3}-\frac{(-b)^{1/4}(5Ac-9Bb)\left(72Bb^3-40Ab^2c\right)1\mathrm{i}}{8c^{13/4}}\right)(5Ac-9Bb)\,1\mathrm{i}}{8c^{13/4}}-\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25A^2\cdots\right.\right.}{\cdots}}\right)\cdots}{4c^{13/4}}$$

input
```
int((x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output
```
x^(1/2)*((2*A)/c^2 - (4*B*b)/c^3) + (2*B*x^(5/2))/(5*c^2) - (x^(1/2)*((B*b
^2)/2 - (A*b*c)/2))/(b*c^3 + c^4*x^2) + ((-b)^(1/4)*atan((((-b)^(1/4)*((x^
(1/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c))/c^3 - ((-b)^(1/4)*(5*A
*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c))/(8*c^(13/4)))*(5*A*c - 9*B*b)*1i)/(8*
c^(13/4)) + ((-b)^(1/4)*((x^(1/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^
3*c))/c^3 + ((-b)^(1/4)*(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c))/(8*c^(13/
4)))*(5*A*c - 9*B*b)*1i)/(8*c^(13/4)))/(((-b)^(1/4)*((x^(1/2)*(81*B^2*b^4
+ 25*A^2*b^2*c^2 - 90*A*B*b^3*c))/c^3 - ((-b)^(1/4)*(5*A*c - 9*B*b)*(72*B*
b^3 - 40*A*b^2*c))/(8*c^(13/4)))*(5*A*c - 9*B*b))/(8*c^(13/4)) - ((-b)^(1/
4)*((x^(1/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c))/c^3 + ((-b)^(1/
4)*(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c))/(8*c^(13/4)))*(5*A*c - 9*B*b))
/(8*c^(13/4))))*(5*A*c - 9*B*b)*1i)/(4*c^(13/4)) + ((-b)^(1/4)*atan((((-b)
^(1/4)*((x^(1/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c))/c^3 - ((-b)
^(1/4)*(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c)*1i)/(8*c^(13/4)))*(5*A*c -
9*B*b))/(8*c^(13/4)) + ((-b)^(1/4)*((x^(1/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2
- 90*A*B*b^3*c))/c^3 + ((-b)^(1/4)*(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c)
*1i)/(8*c^(13/4)))*(5*A*c - 9*B*b))/(8*c^(13/4)))/(((-b)^(1/4)*((x^(1/2)*(
81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c))/c^3 - ((-b)^(1/4)*(5*A*c - 9*
B*b)*(72*B*b^3 - 40*A*b^2*c)*1i)/(8*c^(13/4)))*(5*A*c - 9*B*b)*1i)/(8*c^(1
3/4)) - ((-b)^(1/4)*((x^(1/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3...
```

3.198 $\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.198.1 Optimal result

Integrand size = 26, antiderivative size = 289

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(7bB-3Ac)x^{3/2}}{6bc^2}-\frac{(bB-Ac)x^{7/2}}{2bc\,(b+cx^2)}+\frac{(7bB-3Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{(7bB-3Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{(7bB-3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}+\frac{(7bB-3Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}$$

output
```
1/6*(-3*A*c+7*B*b)*x^(3/2)/b/c^2-1/2*(-A*c+B*b)*x^(7/2)/b/c/(c*x^2+b)+1/8*
(-3*A*c+7*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c^(11/4)*
2^(1/2)-1/8*(-3*A*c+7*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/
4)/c^(11/4)*2^(1/2)-1/16*(-3*A*c+7*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/
4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(11/4)*2^(1/2)+1/16*(-3*A*c+7*B*b)*ln(b^(1/2
)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(11/4)*2^(1/2)
```

3.198.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.56

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\frac{4c^{3/4}x^{3/2}(7bB-3Ac+4Bcx^2)}{b+cx^2}+\frac{3\sqrt{2}(7bB-3Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt[4]{b}}+\frac{3\sqrt{2}(7bB-3Ac)\text{arctanh}\left(\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt[4]{b}}}{24c^{11/4}}$$

input
```
Integrate[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
((4*c^(3/4)*x^(3/2)*(7*b*B - 3*A*c + 4*B*c*x^2))/(b + c*x^2) + (3*Sqrt[2]*
(7*b*B - 3*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x])])/b^(1/4) + (3*Sqrt[2]*(7*b*B - 3*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/b^(1/4))/(24*c^(11/4))
```

3.198.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {9, 362, 262, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^{5/2}(A+Bx^2)}{(b+cx^2)^2}dx$$

$$\Big\downarrow \; 362$$

$$\frac{(7bB-3Ac)\int \frac{x^{5/2}}{cx^2+b}dx}{4bc}-\frac{x^{7/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$$\Big\downarrow \; 262$$

$$\frac{(7bB-3Ac)\left(\frac{2x^{3/2}}{3c}-\frac{b\int \frac{\sqrt{x}}{cx^2+b}dx}{c}\right)}{4bc}-\frac{x^{7/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$\downarrow$ 266

$$\frac{(7bB-3Ac)\left(\frac{2x^{3/2}}{3c}-\frac{2b\int\frac{x}{cx^2+b}d\sqrt{x}}{c}\right)}{4bc}-\frac{x^{7/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$\downarrow$ 826

$$\frac{(7bB-3Ac)\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{4bc}-\frac{x^{7/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$\downarrow$ 1476

$$\frac{(7bB-3Ac)\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{4bc}-\frac{x^{7/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$\downarrow$ 1082

$$\frac{(7bB-3Ac)\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{4bc}-\frac{x^{7/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$\downarrow$ 217

3.198. $\int\frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

$$\frac{(7bB-3Ac)\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{4bc}-\frac{x^{7/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\downarrow$ 1479

$$\frac{(7bB-3Ac)\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{\cdots}\right.}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\cdots\right.}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}\right)}{4bc}$$

$$\frac{x^{7/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\downarrow$ 25

3.198. $\int\frac{x^{13/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$\frac{(7bB-3Ac)\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}\right)}{4bc}$$

$$\frac{x^{7/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{(7bB-3Ac)\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{c}\right)}{4bc}-$$

$$\frac{x^{7/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\Big\downarrow$ 1103

3.198. $\int \frac{x^{13/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$\frac{(7bB-3Ac)\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}\right)}{4bc}$$

$$\frac{x^{7/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

input
```
Int[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*((b*B - A*c)*x^(7/2))/(b*c*(b + c*x^2)) + ((7*b*B - 3*A*c)*((2*x^(3/2
))/(3*c) - (2*b*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]
*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2
]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]
*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt
[c])))/c))/(4*b*c)
```

3.198.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.198. $\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.198. $\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.198.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.53

method	result
derivativedivides	$\frac{2Bx^{\frac{3}{2}}}{3c^2}+\frac{\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)x^{\frac{3}{2}}}{cx^2+b}+\frac{\left(-\frac{7Bb}{4}+\frac{3Ac}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{c^2}$
default	$\frac{2Bx^{\frac{3}{2}}}{3c^2}+\frac{\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)x^{\frac{3}{2}}}{cx^2+b}+\frac{\left(-\frac{7Bb}{4}+\frac{3Ac}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{c^2}$
risch	$\frac{2Bx^{\frac{3}{2}}}{3c^2}+\frac{\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)x^{\frac{3}{2}}}{cx^2+b}+\frac{\left(-\frac{7Bb}{4}+\frac{3Ac}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{c^2}$

input
```
int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*B/c^2*x^(3/2)+2/c^2*((-1/4*A*c+1/4*B*b)*x^(3/2)/(c*x^2+b)+1/8*(-7/4*B*
b+3/4*A*c)/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1
/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(
1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

3.198.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 793, normalized size of antiderivative = 2.74

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{3\left(c^3x^2+bc^2\right)\left(-\frac{2401\,B^4b^4-4116\,AB^3b^3c+2646\,A^2B^2b^2c^2-756\,A^3Bbc^3+81\,A^4c^4}{bc^{11}}\right)^{\frac{1}{4}}\log\left(bc^8\left(-\frac{240}{}\right.\right.}{}$$

input
```
integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/24*(3*(c^3*x^2 + b*c^2)*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^
2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(1/4)*log(b*c^8*(-(240
1*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81
*A^4*c^4)/(b*c^11))^(3/4) - (343*B^3*b^3 - 441*A*B^2*b^2*c + 189*A^2*B*b*c
^2 - 27*A^3*c^3)*sqrt(x)) - 3*(I*c^3*x^2 + I*b*c^2)*(-(2401*B^4*b^4 - 4116
*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^1
1))^(1/4)*log(I*b*c^8*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b^
2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(3/4) - (343*B^3*b^3 - 441
*A*B^2*b^2*c + 189*A^2*B*b*c^2 - 27*A^3*c^3)*sqrt(x)) - 3*(-I*c^3*x^2 - I*
b*c^2)*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3
*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(1/4)*log(-I*b*c^8*(-(2401*B^4*b^4 - 4116
*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^1
1))^(3/4) - (343*B^3*b^3 - 441*A*B^2*b^2*c + 189*A^2*B*b*c^2 - 27*A^3*c^3)
*sqrt(x)) - 3*(c^3*x^2 + b*c^2)*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*
A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(1/4)*log(-b*c^8
*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c
^3 + 81*A^4*c^4)/(b*c^11))^(3/4) - (343*B^3*b^3 - 441*A*B^2*b^2*c + 189*A^
2*B*b*c^2 - 27*A^3*c^3)*sqrt(x)) + 4*(4*B*c*x^3 + (7*B*b - 3*A*c)*x)*sqrt(
x))/(c^3*x^2 + b*c^2)
```

3.198.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

3.198.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.77

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb-Ac)x^{\frac{3}{2}}}{2\,(c^3x^2+bc^2)} + \frac{2\,Bx^{\frac{3}{2}}}{3\,c^2} - \frac{(7\,Bb-3\,Ac)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{16\,c^2}$$

input
```
integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b - A*c)*x^(3/2)/(c^3*x^2 + b*c^2) + 2/3*B*x^(3/2)/c^2 - 1/16*(7*B*
b - 3*A*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt
(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sq
rt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)
*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2
)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(
3/4)))/c^2
```

3.198.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.98

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,c^2} + \frac{Bbx^{\frac{3}{2}} - Acx^{\frac{3}{2}}}{2\,(cx^2+b)c^2} - \frac{\sqrt{2}\left(7\,(bc^3)^{\frac{3}{4}}\,Bb - 3\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^5} - \frac{\sqrt{2}\left(7\,(bc^3)^{\frac{3}{4}}\,Bb - 3\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^5} + \frac{\sqrt{2}\left(7\,(bc^3)^{\frac{3}{4}}\,Bb - 3\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{16\,bc^5} - \frac{\sqrt{2}\left(7\,(bc^3)^{\frac{3}{4}}\,Bb - 3\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{16\,bc^5}$$

input
```
integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
2/3*B*x^(3/2)/c^2 + 1/2*(B*b*x^(3/2) - A*c*x^(3/2))/((c*x^2 + b)*c^2) - 1/
8*sqrt(2)*(7*(b*c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(
sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^5) - 1/8*sqrt(2)*(7*(b*
c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(
1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^5) + 1/16*sqrt(2)*(7*(b*c^3)^(3/4)*B*b
 - 3*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(
b*c^5) - 1/16*sqrt(2)*(7*(b*c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*log(-sqr
t(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^5)
```

3.198.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.37

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{2\,B\,x^{3/2}}{3\,c^2} - \frac{x^{3/2}\left(\frac{A\,c}{2} - \frac{B\,b}{2}\right)}{c^3\,x^2 + b\,c^2} + \frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(3\,A\,c - 7\,B\,b)}{4\,(-b)^{1/4}\,c^{11/4}} + \frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right)\,(3\,A\,c - 7\,B\,b)\,1\mathrm{i}}{4\,(-b)^{1/4}\,c^{11/4}}$$

input
```
int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output
```
(2*B*x^(3/2))/(3*c^2) - (x^(3/2)*((A*c)/2 - (B*b)/2))/(b*c^2 + c^3*x^2) +
(atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(3*A*c - 7*B*b))/(4*(-b)^(1/4)*c^(11/4
)) + (atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*(3*A*c - 7*B*b)*1i)/(4*(-b)^(1
/4)*c^(11/4))
```

3.198. $\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.199 $\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.199.1 Optimal result

Integrand size = 26, antiderivative size = 289

$$\begin{aligned}\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx &= \frac{(5bB-Ac)\sqrt{x}}{2bc^2}-\frac{(bB-Ac)x^{5/2}}{2bc\,(b+cx^2)} \\ &\quad+\frac{(5bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{3/4}c^{9/4}}-\frac{(5bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{3/4}c^{9/4}} \\ &\quad+\frac{(5bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{9/4}} \\ &\quad-\frac{(5bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{9/4}}\end{aligned}$$

output
```
-1/2*(-A*c+B*b)*x^(5/2)/b/c/(c*x^2+b)+1/8*(-A*c+5*B*b)*arctan(1-c^(1/4)*2^
(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(9/4)*2^(1/2)-1/8*(-A*c+5*B*b)*arctan(1+c
^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(9/4)*2^(1/2)+1/16*(-A*c+5*B*b)*
ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(9/4)*2^(1
/2)-1/16*(-A*c+5*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2)
)/b^(3/4)/c^(9/4)*2^(1/2)+1/2*(-A*c+5*B*b)*x^(1/2)/b/c^2
```

3.199.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.56

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\frac{4\sqrt[4]{c}\sqrt{x}(5bB-Ac+4Bcx^2)}{b+cx^2} + \frac{\sqrt{2}(5bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{b^{3/4}} - \frac{\sqrt{2}(5bB-Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{\ldots}}{\sqrt{b}+\sqrt{\ldots}}\right.}{b^{3/4}}}{8c^{9/4}}$$

input
```
Integrate[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
((4*c^(1/4)*Sqrt[x]*(5*b*B - A*c + 4*B*c*x^2))/(b + c*x^2) + (Sqrt[2]*(5*b
*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])
/b^(3/4) - (Sqrt[2]*(5*b*B - A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
)/(Sqrt[b] + Sqrt[c]*x)])/b^(3/4))/(8*c^(9/4))
```

3.199.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {9, 362, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^{3/2}(A+Bx^2)}{(b+cx^2)^2}dx$$

$$\Big\downarrow \; 362$$

$$\frac{(5bB-Ac)\int \frac{x^{3/2}}{cx^2+b}dx}{4bc} - \frac{x^{5/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$$\Big\downarrow \; 262$$

$$\frac{(5bB-Ac)\left(\frac{2\sqrt{x}}{c} - \frac{b\int \frac{1}{\sqrt{x}(cx^2+b)}dx}{c}\right)}{4bc} - \frac{x^{5/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$\Big\downarrow$ 266

$$\frac{(5bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2b\int\frac{1}{cx^2+b}d\sqrt{x}}{c}\right)}{4bc}-\frac{x^{5/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\Big\downarrow$ 755

$$\frac{(5bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{c}\right)}{4bc}-\frac{x^{5/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\Big\downarrow$ 1476

$$\frac{(5bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{c}\right)}{4bc}-\frac{x^{5/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\Big\downarrow$ 1082

$$\frac{(5bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4bc}-\frac{x^{5/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\Big\downarrow$ 217

3.199. $\int\frac{x^{11/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$\frac{(5bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4bc}-\frac{x^{5/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\big\downarrow$ 1479

$$\frac{(5bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4bc}-\frac{x^{5/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\big\downarrow$ 25

3.199. $\int \frac{x^{11/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$\frac{(5bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\cdots\right)}{c}\cdots\right)}{4bc}$$

$$\frac{x^{5/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{(5bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4bc}-$$

$$\frac{x^{5/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\Big\downarrow$ 1103

3.199. $\int \frac{x^{11/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$\frac{(5bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4bc}$$

$$\frac{x^{5/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

input
```
Int[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*((b*B - A*c)*x^(5/2))/(b*c*(b + c*x^2)) + ((5*b*B - A*c)*((2*Sqrt[x])
/c - (2*b*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/
4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1
/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sq
rt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))
/c))/(4*b*c)
```

3.199.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.199. $\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2} \, dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.199.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.53

method	result
derivativedivides	$\frac{2B\sqrt{x}}{c^2}+\frac{\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(Ac-5Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16b}}{c^2}$
default	$\frac{2B\sqrt{x}}{c^2}+\frac{\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(Ac-5Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16b}}{c^2}$
risch	$\frac{2B\sqrt{x}}{c^2}+\frac{\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(Ac-5Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16b}}{c^2}$

input
```
int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2*B/c^2*x^(1/2)+2/c^2*((-1/4*A*c+1/4*B*b)*x^(1/2)/(c*x^2+b)+1/32*(A*c-5*B*
b)*(1/c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1
/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/
c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

3.199.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 669, normalized size of antiderivative = 2.31

$$\int \frac{x^{11/2}(A+Bx^2)}{\left(bx^2+cx^4\right)^2}\,dx = \frac{(c^3x^2+bc^2)\left(-\frac{625\,B^4b^4-500\,AB^3b^3c+150\,A^2B^2b^2c^2-20\,A^3Bbc^3+A^4c^4}{b^3c^9}\right)^{\frac{1}{4}}\log\left(bc^2\left(-\frac{625\,B^4b^4-50}{}\right.\right.}{}$$

input
```
integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/8*((c^3*x^2 + b*c^2)*(-(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*
c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(1/4)*log(b*c^2*(-(625*B^4*b^4
- 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c
^9))^(1/4) - (5*B*b - A*c)*sqrt(x)) - (-I*c^3*x^2 - I*b*c^2)*(-(625*B^4*b^
4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3
*c^9))^(1/4)*log(I*b*c^2*(-(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^
2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(1/4) - (5*B*b - A*c)*sqrt(x)
) - (I*c^3*x^2 + I*b*c^2)*(-(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b
^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(1/4)*log(-I*b*c^2*(-(625*B^
4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/
(b^3*c^9))^(1/4) - (5*B*b - A*c)*sqrt(x)) - (c^3*x^2 + b*c^2)*(-(625*B^4*b
^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^
3*c^9))^(1/4)*log(-b*c^2*(-(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^
2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(1/4) - (5*B*b - A*c)*sqrt(x)
) + 4*(4*B*c*x^2 + 5*B*b - A*c)*sqrt(x))/(c^3*x^2 + b*c^2)
```

3.199.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}(A+Bx^2)}{\left(bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

3.199.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.87

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb-Ac)\sqrt{x}}{2\,(c^3x^2+bc^2)} + \frac{2\,B\sqrt{x}}{c^2} - \frac{\frac{2\,\sqrt{2}(5\,Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\,\sqrt{2}(5\,Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}(5\,Bb-Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{\quad}\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{16\,c^2}$$

input
```
integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b - A*c)*sqrt(x)/(c^3*x^2 + b*c^2) + 2*B*sqrt(x)/c^2 - 1/16*(2*sqrt
(2)*(5*B*b - A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*
sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2
)*(5*B*b - A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*s
qrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(
5*B*b - A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b
^(3/4)*c^(1/4)) - sqrt(2)*(5*B*b - A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(
x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/c^2
```

3.199.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.98

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{2\,B\sqrt{x}}{c^2} - \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^3} - \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^3} - \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^3} + \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^3} + \frac{Bb\sqrt{x}-Ac\sqrt{x}}{2\,(cx^2+b)c^2}$$

input
```
integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
2*B*sqrt(x)/c^2 - 1/8*sqrt(2)*(5*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) -
1/8*sqrt(2)*(5*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) - 1/16*sqrt(2)*(5*(
b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x
+ sqrt(b/c))/(b*c^3) + 1/16*sqrt(2)*(5*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A
*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3) + 1/2*(B*b*s
qrt(x) - A*c*sqrt(x))/((c*x^2 + b)*c^2)
```

3.199.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 744, normalized size of antiderivative = 2.57

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{2\,B\,\sqrt{x}}{c^2} - \frac{\sqrt{x}\,\left(\frac{A\,c}{2}-\frac{B\,b}{2}\right)}{c^3\,x^2+b\,c^2}$$

$$+\frac{\mathrm{atan}\left(\frac{\frac{(A\,c-5\,B\,b)\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}-\frac{(A\,c-5\,B\,b)\,\left(8\,A\,b\,c^2-40\,B\,b^2\,c\right)}{8\,(-b)^{3/4}\,c^{9/4}}\right)\,1\mathrm{i}}{8\,(-b)^{3/4}\,c^{9/4}}+\frac{(A\,c-5\,B\,b)\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}+\frac{(A\,c-5\,B\,b)\,(\cdots}{8\,(-\cdots}\right.}{8\,(-b)^{3/4}\,c^{9/4}}}{\frac{(A\,c-5\,B\,b)\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}-\frac{(A\,c-5\,B\,b)\,\left(8\,A\,b\,c^2-40\,B\,b^2\,c\right)}{8\,(-b)^{3/4}\,c^{9/4}}\right)}{8\,(-b)^{3/4}\,c^{9/4}}-\frac{(A\,c-5\,B\,b)\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}+\frac{(A\,c-5\,B\,b)\,(\cdots}{8\,(-\cdots}\right.}{8\,(-b)^{3/4}\,c^{9/4}}}\right.}{4\,(-b)^{3/4}\,c^{9/4}}$$

$$+\frac{\mathrm{atan}\left(\frac{\frac{(A\,c-5\,B\,b)\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}-\frac{(A\,c-5\,B\,b)\,\left(8\,A\,b\,c^2-40\,B\,b^2\,c\right)\,1\mathrm{i}}{8\,(-b)^{3/4}\,c^{9/4}}\right)}{8\,(-b)^{3/4}\,c^{9/4}}+\frac{(A\,c-5\,B\,b)\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}+\frac{(A\,c-5\,B\,b\cdots}{8\cdots}\right.}{8\,(-b)^{3/4}\,c^{9/4}}}{\frac{(A\,c-5\,B\,b)\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}-\frac{(A\,c-5\,B\,b)\,\left(8\,A\,b\,c^2-40\,B\,b^2\,c\right)\,1\mathrm{i}}{8\,(-b)^{3/4}\,c^{9/4}}\right)\,1\mathrm{i}}{8\,(-b)^{3/4}\,c^{9/4}}-\frac{(A\,c-5\,B\,b)\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}+\frac{(A\,c-5\,B\,b\cdots}{8\cdots}\right.}{8\,(-b)^{3/4}\,c^{9/4}}}\right.}{4\,(-b)^{3/4}\,c^{9/4}}$$

input
```
int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output
```
(2*B*x^(1/2))/c^2 - (x^(1/2)*((A*c)/2 - (B*b)/2))/(b*c^2 + c^3*x^2) + (ata
n((((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2 - 10*A*B*b*c))/c - ((A*c
 - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c))/(8*(-b)^(3/4)*c^(9/4)))*1i)/(8*(-b)^(3
/4)*c^(9/4)) + ((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2 - 10*A*B*b*c
))/c + ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c))/(8*(-b)^(3/4)*c^(9/4)))*1i
)/(8*(-b)^(3/4)*c^(9/4)))/(((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2
 - 10*A*B*b*c))/c - ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c))/(8*(-b)^(3/4)*
c^(9/4))))/(8*(-b)^(3/4)*c^(9/4)) - ((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25
*B^2*b^2 - 10*A*B*b*c))/c + ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c))/(8*(-
b)^(3/4)*c^(9/4))))/(8*(-b)^(3/4)*c^(9/4))))*(A*c - 5*B*b)*1i)/(4*(-b)^(3/
4)*c^(9/4)) + (atan((((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2 - 10*A
*B*b*c))/c - ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c)*1i)/(8*(-b)^(3/4)*c^(
9/4))))/(8*(-b)^(3/4)*c^(9/4)) + ((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^
2*b^2 - 10*A*B*b*c))/c + ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c)*1i)/(8*(-
b)^(3/4)*c^(9/4))))/(8*(-b)^(3/4)*c^(9/4)))/(((A*c - 5*B*b)*((x^(1/2)*(A^2
*c^2 + 25*B^2*b^2 - 10*A*B*b*c))/c - ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*
c)*1i)/(8*(-b)^(3/4)*c^(9/4)))*1i)/(8*(-b)^(3/4)*c^(9/4)) - ((A*c - 5*B*b)
*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2 - 10*A*B*b*c))/c + ((A*c - 5*B*b)*(8*A*b*
c^2 - 40*B*b^2*c)*1i)/(8*(-b)^(3/4)*c^(9/4)))*1i)/(8*(-b)^(3/4)*c^(9/4))))
*(A*c - 5*B*b))/(4*(-b)^(3/4)*c^(9/4))
```

3.200 $\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$

3.200.1 Optimal result

Integrand size = 26, antiderivative size = 261

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx = -\frac{(bB-Ac)x^{3/2}}{2bc(b+cx^2)} - \frac{(3bB+Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{5/4}c^{7/4}} + \frac{(3bB+Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{5/4}c^{7/4}} + \frac{(3bB+Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{7/4}} - \frac{(3bB+Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{7/4}}$$

output
```
-1/2*(-A*c+B*b)*x^(3/2)/b/c/(c*x^2+b)-1/8*(A*c+3*B*b)*arctan(1-c^(1/4)*2^(
1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^(7/4)*2^(1/2)+1/8*(A*c+3*B*b)*arctan(1+c^(
1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^(7/4)*2^(1/2)+1/16*(A*c+3*B*b)*ln(
b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/c^(7/4)*2^(1/2)
-1/16*(A*c+3*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^
(5/4)/c^(7/4)*2^(1/2)
```

3.200.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.58

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\frac{4\sqrt[4]{b}c^{3/4}(-bB+Ac)x^{3/2}}{b+cx^2} - \sqrt{2}(3bB+Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) - \sqrt{2}(3bB+Ac)\text{arctan}}{8b^{5/4}c^{7/4}}$$

input
```
Integrate[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
((4*b^(1/4)*c^(3/4)*(-(b*B) + A*c)*x^(3/2))/(b + c*x^2) - Sqrt[2]*(3*b*B +
 A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - Sq
rt[2]*(3*b*B + A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + S
qrt[c]*x)])/(8*b^(5/4)*c^(7/4))
```

3.200.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {9, 362, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{\sqrt{x}(A+Bx^2)}{(b+cx^2)^2}dx$$

$$\Big\downarrow \text{ 362}$$

$$\frac{(Ac+3bB)\int \frac{\sqrt{x}}{cx^2+b}dx}{4bc} - \frac{x^{3/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{(Ac+3bB)\int \frac{x}{cx^2+b}d\sqrt{x}}{2bc} - \frac{x^{3/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$$\Big\downarrow \text{ 826}$$

$$\frac{(Ac+3bB)\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{2bc}-\frac{x^{3/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$\Big\downarrow$ 1476

$$\frac{(Ac+3bB)\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{2bc}-\frac{x^{3/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$\Big\downarrow$ 1082

$$\frac{(Ac+3bB)\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{2bc}-\frac{x^{3/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$\Big\downarrow$ 217

$$\frac{(Ac+3bB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{2bc}-\frac{x^{3/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$\Big\downarrow$ 1479

$$\frac{(Ac+3bB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{2bc}-\frac{x^{3/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

$\Big\downarrow$ 25

3.200. $\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

$$\frac{(Ac+3bB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{2bc}-\frac{x^{3/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{(Ac+3bB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{2bc}-\frac{x^{3/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{(Ac+3bB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{2bc}-\frac{x^{3/2}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

input
```
Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

3.200. $\int \frac{x^{9/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

output
```
-1/2*((b*B - A*c)*x^(3/2))/(b*c*(b + c*x^2)) + ((3*b*B + A*c)*((-(ArcTan[1
 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[
1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[
c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sq
rt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + S
qrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/(2*b*c)
```

3.200.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

3.200. $\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.200.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.56

method	result	size
derivativedivides	$\frac{(Ac-Bb)x^{\frac{3}{2}}}{2bc(c\,x^{2}+b)}+\frac{(Ac+3Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16b\,c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	146
default	$\frac{(Ac-Bb)x^{\frac{3}{2}}}{2bc(c\,x^{2}+b)}+\frac{(Ac+3Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16b\,c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	146

input
```
int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(A*c-B*b)/b/c*x^(3/2)/(c*x^2+b)+1/16*(A*c+3*B*b)/b/c^2/(1/c*b)^(1/4)*2
^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4
)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1
)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

3.200.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 776, normalized size of antiderivative = 2.97

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx =$$

$$-\frac{4\,(Bb-Ac)x^{\frac{3}{2}} - (bc^2x^2+b^2c)\left(-\frac{81\,B^4b^4+108\,AB^3b^3c+54\,A^2B^2b^2c^2+12\,A^3Bbc^3+A^4c^4}{b^5c^7}\right)^{\frac{1}{4}}\log\left(b^4c^5\left(-\frac{81\,B^4b^4+108\,AB^3b^3}{}\right.\right.}{}$$

input
```
integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*(B*b - A*c)*x^(3/2) - (b*c^2*x^2 + b^2*c)*(-(81*B^4*b^4 + 108*A*B^
3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(1/4)*
log(b^4*c^5*(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*
B*b*c^3 + A^4*c^4)/(b^5*c^7))^(3/4) + (27*B^3*b^3 + 27*A*B^2*b^2*c + 9*A^2
*B*b*c^2 + A^3*c^3)*sqrt(x)) + (I*b*c^2*x^2 + I*b^2*c)*(-(81*B^4*b^4 + 108
*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(
1/4)*log(I*b^4*c^5*(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 +
12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(3/4) + (27*B^3*b^3 + 27*A*B^2*b^2*c
+ 9*A^2*B*b*c^2 + A^3*c^3)*sqrt(x)) + (-I*b*c^2*x^2 - I*b^2*c)*(-(81*B^4*b
^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5
*c^7))^(1/4)*log(-I*b^4*c^5*(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b
^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(3/4) + (27*B^3*b^3 + 27*A*B
^2*b^2*c + 9*A^2*B*b*c^2 + A^3*c^3)*sqrt(x)) + (b*c^2*x^2 + b^2*c)*(-(81*B
^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/
(b^5*c^7))^(1/4)*log(-b^4*c^5*(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2
*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(3/4) + (27*B^3*b^3 + 27*A
*B^2*b^2*c + 9*A^2*B*b*c^2 + A^3*c^3)*sqrt(x)))/(b*c^2*x^2 + b^2*c)
```

3.200.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

3.200.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.83

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(Bb-Ac)x^{\frac{3}{2}}}{2\,(bc^2x^2+b^2c)} + \frac{(3\,Bb+Ac)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \cdots}{16\,bc}$$

input
```
integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*b - A*c)*x^(3/2)/(b*c^2*x^2 + b^2*c) + 1/16*(3*B*b + A*c)*(2*sqrt(
2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(s
qrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*s
qrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c))
)/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sq
rt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1
/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/(b*c)
```

3.200.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.05

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{Bbx^{\frac{3}{2}}-Acx^{\frac{3}{2}}}{2\,(cx^2+b)bc} + \frac{\sqrt{2}\Big(3\,(bc^3)^{\frac{3}{4}}\,Bb+(bc^3)^{\frac{3}{4}}\,Ac\Big)\arctan\left(\frac{\sqrt{2}\Big(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\Big)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^4} + \frac{\sqrt{2}\Big(3\,(bc^3)^{\frac{3}{4}}\,Bb+(bc^3)^{\frac{3}{4}}\,Ac\Big)\arctan\left(-\frac{\sqrt{2}\Big(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\Big)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^4} - \frac{\sqrt{2}\Big(3\,(bc^3)^{\frac{3}{4}}\,Bb+(bc^3)^{\frac{3}{4}}\,Ac\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^2c^4} + \frac{\sqrt{2}\Big(3\,(bc^3)^{\frac{3}{4}}\,Bb+(bc^3)^{\frac{3}{4}}\,Ac\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^2c^4}$$

input
```
integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-1/2*(B*b*x^(3/2) - A*c*x^(3/2))/((c*x^2 + b)*b*c) + 1/8*sqrt(2)*(3*(b*c^3
)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) +
 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^4) + 1/8*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + (b
*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/
c)^(1/4))/(b^2*c^4) - 1/16*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*
c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^4) + 1/16*sqrt(
2)*(3*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1
/4) + x + sqrt(b/c))/(b^2*c^4)
```

3.200.9 Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.35

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\mathrm{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)\,(A\,c+3\,B\,b)}{4\,(-b)^{5/4}\,c^{7/4}} - \frac{\mathrm{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)\,(A\,c+3\,B\,b)}{4\,(-b)^{5/4}\,c^{7/4}} + \frac{x^{3/2}\,(A\,c-B\,b)}{2\,b\,c\,(c\,x^2+b)}$$

input
```
int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output
```
(atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c + 3*B*b))/(4*(-b)^(5/4)*c^(7/4))
 - (atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c + 3*B*b))/(4*(-b)^(5/4)*c^(7/4
)) + (x^(3/2)*(A*c - B*b))/(2*b*c*(b + c*x^2))
```

3.200. $\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.201 $\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.201.1 Optimal result

Integrand size = 26, antiderivative size = 261

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(bB-Ac)\sqrt{x}}{2bc\,(b+cx^2)} - \frac{(bB+3Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}c^{5/4}} + \frac{(bB+3Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}c^{5/4}} - \frac{(bB+3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}c^{5/4}} + \frac{(bB+3Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}c^{5/4}}$$

output
```
-1/8*(3*A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c^(5/4)
*2^(1/2)+1/8*(3*A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)
/c^(5/4)*2^(1/2)-1/16*(3*A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(
1/2)*x^(1/2))/b^(7/4)/c^(5/4)*2^(1/2)+1/16*(3*A*c+B*b)*ln(b^(1/2)+x*c^(1/2
)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(5/4)*2^(1/2)-1/2*(-A*c+B*b)*
x^(1/2)/b/c/(c*x^2+b)
```

3.201.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.58

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\frac{4b^{3/4}\sqrt[4]{c}(-bB+Ac)\sqrt{x}}{b+cx^2} - \sqrt{2}(bB+3Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) + \sqrt{2}(bB+3Ac)\text{arctanh}}{8b^{7/4}c^{5/4}}$$

input
```
Integrate[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
((4*b^(3/4)*c^(1/4)*(-(b*B) + A*c)*Sqrt[x])/(b + c*x^2) - Sqrt[2]*(b*B + 3
*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + Sq
rt[2]*(b*B + 3*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + S
qrt[c]*x)])/(8*b^(7/4)*c^(5/4))
```

3.201.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {9, 362, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{A+Bx^2}{\sqrt{x}\,(b+cx^2)^2}dx$$

$$\Big\downarrow \; 362$$

$$\frac{(3Ac+bB)\int \frac{1}{\sqrt{x}(cx^2+b)}dx}{4bc} - \frac{\sqrt{x}(bB-Ac)}{2bc\,(b+cx^2)}$$

$$\Big\downarrow \; 266$$

$$\frac{(3Ac+bB)\int \frac{1}{cx^2+b}d\sqrt{x}}{2bc} - \frac{\sqrt{x}(bB-Ac)}{2bc\,(b+cx^2)}$$

$$\Big\downarrow \; 755$$

$$\frac{(3Ac+bB)\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{2bc}-\frac{\sqrt{x}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{(3Ac+bB)\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{2bc}-\frac{\sqrt{x}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{(3Ac+bB)\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2bc}-\frac{\sqrt{x}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{(3Ac+bB)\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2bc}-\frac{\sqrt{x}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{(3Ac+bB)\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2bc}-\frac{\sqrt{x}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$$\Big\downarrow \text{ 25}$$

3.201. $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$\frac{(3Ac+bB)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2bc}-\frac{\sqrt{x}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{(3Ac+bB)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2bc}-\frac{\sqrt{x}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

$\Big\downarrow$ 1103

$$\frac{(3Ac+bB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2bc}-\frac{\sqrt{x}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

input
```
Int[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

3.201. $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

output
```
-1/2*((b*B - A*c)*Sqrt[x])/(b*c*(b + c*x^2)) + ((b*B + 3*A*c)*((-(ArcTan[1
 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[
1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[
b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sq
rt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + S
qrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/(2*b*c)
```

3.201.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

3.201. $\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2} \, dx$

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.201.4 Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.56

method	result
derivativedivides	$\frac{(Ac-Bb)\sqrt{x}}{2bc(c\,x^2+b)}+\frac{(3Ac+Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16b^2c}$
default	$\frac{(Ac-Bb)\sqrt{x}}{2bc(c\,x^2+b)}+\frac{(3Ac+Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16b^2c}$

input
```
int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(A*c-B*b)/b/c*x^(1/2)/(c*x^2+b)+1/16*(3*A*c+B*b)/b^2/c*(1/c*b)^(1/4)*2
^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4
)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1
)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

3.201.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 658, normalized size of antiderivative = 2.52

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(bc^2x^2+b^2c)\left(-\frac{B^4b^4+12\,AB^3b^3c+54\,A^2B^2b^2c^2+108\,A^3Bbc^3+81\,A^4c^4}{b^7c^5}\right)^{\frac{1}{4}}\log\left(b^2c\left(-\frac{B^4b^4+12\,AB^3b\cdots}{}\right.\right.}{}$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/8*((b*c^2*x^2 + b^2*c)*(-(B^4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2
+ 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4)*log(b^2*c*(-(B^4*b^4 + 12
*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5
))^(1/4) + (B*b + 3*A*c)*sqrt(x)) - (-I*b*c^2*x^2 - I*b^2*c)*(-(B^4*b^4 +
12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c
^5))^(1/4)*log(I*b^2*c*(-(B^4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 +
108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4) + (B*b + 3*A*c)*sqrt(x)) -
(I*b*c^2*x^2 + I*b^2*c)*(-(B^4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 +
 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4)*log(-I*b^2*c*(-(B^4*b^4 +
12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c
^5))^(1/4) + (B*b + 3*A*c)*sqrt(x)) - (b*c^2*x^2 + b^2*c)*(-(B^4*b^4 + 12*
A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5)
)^(1/4)*log(-b^2*c*(-(B^4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*
A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4) + (B*b + 3*A*c)*sqrt(x)) - 4*(B
*b - A*c)*sqrt(x))/(b*c^2*x^2 + b^2*c)
```

3.201.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

3.201.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.92

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(Bb-Ac)\sqrt{x}}{2\,(bc^2x^2+b^2c)} + \frac{\frac{2\sqrt{2}(Bb+3\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}(Bb+3\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}(Bb+3\,Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\cdots\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{16\,bc}$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*b - A*c)*sqrt(x)/(b*c^2*x^2 + b^2*c) + 1/16*(2*sqrt(2)*(B*b + 3*A*
c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(s
qrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(B*b + 3*A*c)
*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sq
rt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(B*b + 3*A*c)*lo
g(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4))
 - sqrt(2)*(B*b + 3*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x
+ sqrt(b))/(b^(3/4)*c^(1/4)))/(b*c)
```

3.201.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.05

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+3\,(bc^3)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^2} + \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+3\,(bc^3)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^2} + \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+3\,(bc^3)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^2c^2} - \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+3\,(bc^3)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^2c^2} - \frac{Bb\sqrt{x}-Ac\sqrt{x}}{2\,(cx^2+b)bc}$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/8*sqrt(2)*((b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(
sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) + 1/8*sqrt(2)*((b*
c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(
1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) + 1/16*sqrt(2)*((b*c^3)^(1/4)*B*b
 + 3*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(
b^2*c^2) - 1/16*sqrt(2)*((b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*log(-sqr
t(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^2) - 1/2*(B*b*sqrt(x) - A
*c*sqrt(x))/((c*x^2 + b)*b*c)
```

3.201.9 Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 750, normalized size of antiderivative = 2.87

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\frac{(3\,A\,c+B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^3+6\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{b^2}-\frac{(3\,A\,c+B\,b)\left(24\,A\,c^3+8\,B\,b\,c^2\right)1\mathrm{i}}{8\,(-b)^{7/4}\,c^{5/4}}\right)}{8\,(-b)^{7/4}\,c^{5/4}}+\frac{(3\,A\,c+B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^3+\cdots\right.}{}\right.}{}}{\frac{(3\,A\,c+B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^3+6\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{b^2}-\frac{(3\,A\,c+B\,b)\left(24\,A\,c^3+8\,B\,b\,c^2\right)1\mathrm{i}}{8\,(-b)^{7/4}\,c^{5/4}}\right)1\mathrm{i}}{8\,(-b)^{7/4}\,c^{5/4}}-\frac{(3\,A\,c+B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^3+\cdots\right.}{}\right.}{}}\right)\cdots}{4\,(-b)^{7/4}\,c^{5/4}}$$
$$+\frac{\sqrt{x}\,(A\,c-B\,b)}{2\,b\,c\,\left(c\,x^2+b\right)}$$
$$+\frac{\operatorname{atan}\left(\frac{\frac{(3\,A\,c+B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^3+6\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{b^2}-\frac{(3\,A\,c+B\,b)\left(24\,A\,c^3+8\,B\,b\,c^2\right)}{8\,(-b)^{7/4}\,c^{5/4}}\right)1\mathrm{i}}{8\,(-b)^{7/4}\,c^{5/4}}+\frac{(3\,A\,c+B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^3+6\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{b^2}+\frac{(3\,A\,c+B\,b)\left(\cdots\right.}{8\,(-b\cdots}\right.}{8\,(-b)^{7/4}\,c^{5/4}}}{\frac{(3\,A\,c+B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^3+6\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{b^2}-\frac{(3\,A\,c+B\,b)\left(24\,A\,c^3+8\,B\,b\,c^2\right)}{8\,(-b)^{7/4}\,c^{5/4}}\right)}{8\,(-b)^{7/4}\,c^{5/4}}-\frac{(3\,A\,c+B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^3+6\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{b^2}+\frac{(3\,A\,c+B\,b)\left(\cdots\right.}{8\,(-b\cdots}\right.}{8\,(-b)^{7/4}\,c^{5/4}}}\right)\cdots}{4\,(-b)^{7/4}\,c^{5/4}}$$

input
```
int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output
```
(atan((((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3 + B^2*b^2*c + 6*A*B*b*c^2))/b^2
 - ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2))/(8*(-b)^(7/4)*c^(5/4)))*1i)/(8*(
-b)^(7/4)*c^(5/4)) + ((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3 + B^2*b^2*c + 6*A
*B*b*c^2))/b^2 + ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2))/(8*(-b)^(7/4)*c^(5
/4)))*1i)/(8*(-b)^(7/4)*c^(5/4)))/(((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3 + B
^2*b^2*c + 6*A*B*b*c^2))/b^2 - ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2))/(8*(
-b)^(7/4)*c^(5/4))))/(8*(-b)^(7/4)*c^(5/4)) - ((3*A*c + B*b)*((x^(1/2)*(9*
A^2*c^3 + B^2*b^2*c + 6*A*B*b*c^2))/b^2 + ((3*A*c + B*b)*(24*A*c^3 + 8*B*b
*c^2))/(8*(-b)^(7/4)*c^(5/4))))/(8*(-b)^(7/4)*c^(5/4))))*(3*A*c + B*b)*1i)
/(4*(-b)^(7/4)*c^(5/4)) + (atan((((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3 + B^2
*b^2*c + 6*A*B*b*c^2))/b^2 - ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2)*1i)/(8*
(-b)^(7/4)*c^(5/4))))/(8*(-b)^(7/4)*c^(5/4)) + ((3*A*c + B*b)*((x^(1/2)*(9
*A^2*c^3 + B^2*b^2*c + 6*A*B*b*c^2))/b^2 + ((3*A*c + B*b)*(24*A*c^3 + 8*B*
b*c^2)*1i)/(8*(-b)^(7/4)*c^(5/4))))/(8*(-b)^(7/4)*c^(5/4)))/(((3*A*c + B*b
)*((x^(1/2)*(9*A^2*c^3 + B^2*b^2*c + 6*A*B*b*c^2))/b^2 - ((3*A*c + B*b)*(2
4*A*c^3 + 8*B*b*c^2)*1i)/(8*(-b)^(7/4)*c^(5/4)))*1i)/(8*(-b)^(7/4)*c^(5/4)
) - ((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3 + B^2*b^2*c + 6*A*B*b*c^2))/b^2 +
((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2)*1i)/(8*(-b)^(7/4)*c^(5/4)))*1i)/(8*(
-b)^(7/4)*c^(5/4))))*(3*A*c + B*b))/(4*(-b)^(7/4)*c^(5/4)) + (x^(1/2)*(A*c
 - B*b))/(2*b*c*(b + c*x^2))
```

3.202 $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$

3.202.1 Optimal result

Integrand size = 26, antiderivative size = 284

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx = \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}\,(b+cx^2)} - \frac{(bB-5Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{9/4}c^{3/4}} + \frac{(bB-5Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{9/4}c^{3/4}} + \frac{(bB-5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}c^{3/4}} - \frac{(bB-5Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}c^{3/4}}$$

output
```
-1/8*(-5*A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(9/4)/c^(3/4
)*2^(1/2)+1/8*(-5*A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(9/
4)/c^(3/4)*2^(1/2)+1/16*(-5*A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*
2^(1/2)*x^(1/2))/b^(9/4)/c^(3/4)*2^(1/2)-1/16*(-5*A*c+B*b)*ln(b^(1/2)+x*c^
(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)/c^(3/4)*2^(1/2)+1/2*(-5*A*c
+B*b)/b^2/c/x^(1/2)+1/2*(A*c-B*b)/b/c/(c*x^2+b)/x^(1/2)
```

3.202.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.57

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\frac{4\sqrt[4]{b}(-4Ab+bBx^2-5Acx^2)}{\sqrt{x}(b+cx^2)}+\frac{\sqrt{2}(-bB+5Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{c^{3/4}}+\frac{\sqrt{2}(-bB+5Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt{b}\cdots}\right)}{c^{3/4}}}{8b^{9/4}}$$

input
```
Integrate[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
((4*b^(1/4)*(-4*A*b + b*B*x^2 - 5*A*c*x^2))/(Sqrt[x]*(b + c*x^2)) + (Sqrt[
2]*(-(b*B) + 5*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*
Sqrt[x])])/c^(3/4) + (Sqrt[2]*(-(b*B) + 5*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/c^(3/4))/(8*b^(9/4))
```

3.202.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {9, 362, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{A+Bx^2}{x^{3/2}(b+cx^2)^2}dx$$

$$\Big\downarrow \; 362$$

$$-\frac{(bB-5Ac)\int \frac{1}{x^{3/2}(cx^2+b)}dx}{4bc}-\frac{bB-Ac}{2bc\sqrt{x}(b+cx^2)}$$

$$\Big\downarrow \; 264$$

$$-\frac{(bB-5Ac)\left(-\frac{c\int \frac{\sqrt{x}}{cx^2+b}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{4bc}-\frac{bB-Ac}{2bc\sqrt{x}(b+cx^2)}$$

$$\Big\downarrow \ 266$$

$$-\frac{(bB-5Ac)\left(-\frac{2c\int \frac{x}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{4bc}-\frac{bB-Ac}{2bc\sqrt{x}\,(b+cx^2)}$$

$$\Big\downarrow \ 826$$

$$-\frac{(bB-5Ac)\left(-\frac{2c\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4bc}-\frac{bB-Ac}{2bc\sqrt{x}\,(b+cx^2)}$$

$$\Big\downarrow \ 1476$$

$$-\frac{(bB-5Ac)\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4bc}-\frac{bB-Ac}{2bc\sqrt{x}\,(b+cx^2)}$$

$$\Big\downarrow \ 1082$$

$$-\frac{(bB-5Ac)\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4bc}-\frac{bB-Ac}{2bc\sqrt{x}\,(b+cx^2)}$$

$$\Big\downarrow \ 217$$

3.202. $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

$$-\frac{(bB-5Ac)\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4bc}-\frac{bB-Ac}{2bc\sqrt{x}\left(b+cx^2\right)}$$

$$\Big\downarrow \; 1479$$

$$-\frac{(bB-5Ac)\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{\cdots}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}\cdots}{2\sqrt{c}}\cdots\right)}{b}\cdots\right)}{4bc}$$

$$-\frac{bB-Ac}{2bc\sqrt{x}\left(b+cx^2\right)}$$

$$\Big\downarrow \; 25$$

3.202. $\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$-\frac{(bB-5Ac)\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}\right.}{4bc}$$

$$\frac{bB-Ac}{2bc\sqrt{x}\left(b+cx^2\right)}$$

$\big\downarrow$ 27

$$-\frac{(bB-5Ac)\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4bc}-$$

$$\frac{bB-Ac}{2bc\sqrt{x}\left(b+cx^2\right)}$$

$\big\downarrow$ 1103

3.202. $\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$-\frac{(bB-5Ac)\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\cdots\right)}{4bc}$$

$$\frac{bB-Ac}{2bc\sqrt{x}\,(b+cx^2)}$$

input
```
Int[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(b*B - A*c)/(b*c*Sqrt[x]*(b + c*x^2)) - ((b*B - 5*A*c)*(-2/(b*Sqrt[x]
) - (2*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4
)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/
4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqr
t[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/
b))/(4*b*c)
```

3.202.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.202. $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.202.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.54

method	result
derivativedivides	$-\frac{2A}{b^2\sqrt{x}}-\frac{2\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{5Ac}{4}-\frac{Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^2}$
default	$-\frac{2A}{b^2\sqrt{x}}-\frac{2\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{5Ac}{4}-\frac{Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^2}$
risch	$-\frac{2A}{b^2\sqrt{x}}-\frac{\frac{2\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{5Ac}{4}-\frac{Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b^2}$

input
```
int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*A/b^2/x^(1/2)-2/b^2*((1/4*A*c-1/4*B*b)*x^(3/2)/(c*x^2+b)+1/8*(5/4*A*c-1
/4*B*b)/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*
b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2
)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

3.202.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 789, normalized size of antiderivative = 2.78

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx =$$

$$-\frac{(b^2cx^3+b^3x)\left(-\frac{B^4b^4-20\,AB^3b^3c+150\,A^2B^2b^2c^2-500\,A^3Bbc^3+625\,A^4c^4}{b^9c^3}\right)^{\frac{1}{4}}\log\left(b^7c^2\left(-\frac{B^4b^4-20\,AB^3b^3c+150\,A^2B^2b^2c^2-500\,A}{b^9c^3}\right.\right.}{}$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
-1/8*((b^2*c*x^3 + b^3*x)*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^
2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4)*log(b^7*c^2*(-(B^4*b^4
 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(
b^9*c^3))^(3/4) - (B^3*b^3 - 15*A*B^2*b^2*c + 75*A^2*B*b*c^2 - 125*A^3*c^3
)*sqrt(x)) + (-I*b^2*c*x^3 - I*b^3*x)*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^
2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4)*log(I*b^7*
c^2*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 +
625*A^4*c^4)/(b^9*c^3))^(3/4) - (B^3*b^3 - 15*A*B^2*b^2*c + 75*A^2*B*b*c^2
 - 125*A^3*c^3)*sqrt(x)) + (I*b^2*c*x^3 + I*b^3*x)*(-(B^4*b^4 - 20*A*B^3*b
^3*c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/
4)*log(-I*b^7*c^2*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 500*
A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(3/4) - (B^3*b^3 - 15*A*B^2*b^2*c +
75*A^2*B*b*c^2 - 125*A^3*c^3)*sqrt(x)) - (b^2*c*x^3 + b^3*x)*(-(B^4*b^4 -
20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9
*c^3))^(1/4)*log(-b^7*c^2*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^
2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(3/4) - (B^3*b^3 - 15*A*B^2*
b^2*c + 75*A^2*B*b*c^2 - 125*A^3*c^3)*sqrt(x)) - 4*((B*b - 5*A*c)*x^2 - 4*
A*b)*sqrt(x))/(b^2*c*x^3 + b^3*x)
```

3.202.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

3.202.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.78

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb-5\,Ac)x^2-4\,Ab}{2\left(b^2cx^{\frac{5}{2}}+b^3\sqrt{x}\right)} + \frac{(Bb-5\,Ac)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac14}c^{\frac14}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac14}c^{\frac14}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac14}c^{\frac14}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac14}c^{\frac34}} + \cdots\right.}{16\,b^2}$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/2*((B*b - 5*A*c)*x^2 - 4*A*b)/(b^2*c*x^(5/2) + b^3*sqrt(x)) + 1/16*(B*b
- 5*A*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c
)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt
(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b
^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*
log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/
4)))/b^2
```

3.202.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.98

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bbx^2-5\,Acx^2-4\,Ab}{2\left(cx^{\frac{5}{2}}+b\sqrt{x}\right)b^2} + \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}\,Bb-5\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c^3} + \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}\,Bb-5\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c^3} - \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}\,Bb-5\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^3c^3} + \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}\,Bb-5\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^3c^3}$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/2*(B*b*x^2 - 5*A*c*x^2 - 4*A*b)/((c*x^(5/2) + b*sqrt(x))*b^2) + 1/8*sqrt
(2)*((b*c^3)^(3/4)*B*b - 5*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*
(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^3) + 1/8*sqrt(2)*((b*c^3)^(3/
4)*B*b - 5*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2
*sqrt(x))/(b/c)^(1/4))/(b^3*c^3) - 1/16*sqrt(2)*((b*c^3)^(3/4)*B*b - 5*(b*
c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3)
 + 1/16*sqrt(2)*((b*c^3)^(3/4)*B*b - 5*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqr
t(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3)
```

3.202.9 Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.37

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\operatorname{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(5\,A\,c-B\,b)}{4\,(-b)^{9/4}\,c^{3/4}} - \frac{\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(5\,A\,c-B\,b)}{4\,(-b)^{9/4}\,c^{3/4}} - \frac{\frac{2\,A}{b}+\frac{x^2\,(5\,A\,c-B\,b)}{2\,b^2}}{b\,\sqrt{x}+c\,x^{5/2}}$$

input

```
int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output

```
(atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(5*A*c - B*b))/(4*(-b)^(9/4)*c^(3/4))
 - (atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(5*A*c - B*b))/(4*(-b)^(9/4)*c^(3/4
)) - ((2*A)/b + (x^2*(5*A*c - B*b))/(2*b^2))/(b*x^(1/2) + c*x^(5/2))
```

3.203 $\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.203.1 Optimal result

Integrand size = 26, antiderivative size = 289

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{3bB-7Ac}{6b^2cx^{3/2}} - \frac{bB-Ac}{2bcx^{3/2}(b+cx^2)} - \frac{(3bB-7Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{11/4}\sqrt[4]{c}} + \frac{(3bB-7Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{11/4}\sqrt[4]{c}} - \frac{(3bB-7Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}} + \frac{(3bB-7Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}}$$

output
```
1/6*(-7*A*c+3*B*b)/b^2/c/x^(3/2)+1/2*(A*c-B*b)/b/c/x^(3/2)/(c*x^2+b)-1/8*(
-7*A*c+3*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)/c^(1/4)*2
^(1/2)+1/8*(-7*A*c+3*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/
4)/c^(1/4)*2^(1/2)-1/16*(-7*A*c+3*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4
)*2^(1/2)*x^(1/2))/b^(11/4)/c^(1/4)*2^(1/2)+1/16*(-7*A*c+3*B*b)*ln(b^(1/2)
+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)/c^(1/4)*2^(1/2)
```

3.203.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.57

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\frac{4b^{3/4}(-4Ab+3bBx^2-7Acx^2)}{x^{3/2}(b+cx^2)} + \frac{3\sqrt{2}(-3bB+7Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt[4]{c}} + \frac{3\sqrt{2}(3bB-7Ac)\operatorname{arctanh}\left(\frac{\sqrt{2}\cdots}{}\right)}{\sqrt[4]{c}}}{24b^{11/4}}$$

input
```
Integrate[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
((4*b^(3/4)*(-4*A*b + 3*b*B*x^2 - 7*A*c*x^2))/(x^(3/2)*(b + c*x^2)) + (3*S
qrt[2]*(-3*b*B + 7*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x])])/c^(1/4) + (3*Sqrt[2]*(3*b*B - 7*A*c)*ArcTanh[(Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/c^(1/4))/(24*b^(11/4))
```

3.203.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {9, 362, 264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{A+Bx^2}{x^{5/2}(b+cx^2)^2}dx$$

$$\Big\downarrow \; 362$$

$$-\frac{(3bB-7Ac)\int \frac{1}{x^{5/2}(cx^2+b)}dx}{4bc} - \frac{bB-Ac}{2bcx^{3/2}(b+cx^2)}$$

$$\Big\downarrow \; 264$$

$$-\frac{(3bB-7Ac)\left(-\frac{c\int \frac{1}{\sqrt{x}(cx^2+b)}dx}{b} - \frac{2}{3bx^{3/2}}\right)}{4bc} - \frac{bB-Ac}{2bcx^{3/2}(b+cx^2)}$$

$$\Big\downarrow \text{266}$$

$$-\frac{(3bB-7Ac)\left(-\frac{2c\int\frac{1}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{3bx^{3/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \text{755}$$

$$-\frac{(3bB-7Ac)\left(-\frac{2c\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{(3bB-7Ac)\left(-\frac{2c\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{(3bB-7Ac)\left(-\frac{2c\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \text{217}$$

3.203. $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$-\frac{(3bB-7Ac)\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}$$

$\downarrow$ 1479

$$-\frac{(3bB-7Ac)\left(-\frac{2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}\cdots\right)}{4bc}-\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}$$

$\downarrow$ 25

3.203. $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$-\frac{(3bB-7Ac)\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}\right.}{4bc}-\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \; 27$$

$$-\frac{(3bB-7Ac)\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \; 1103$$

3.203. $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$-\frac{(3bB-7Ac)\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\cdots\right)}{4bc}$$

$$\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}$$

input
```
Int[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(b*B - A*c)/(b*c*x^(3/2)*(b + c*x^2)) - ((3*b*B - 7*A*c)*(-2/(3*b*x^(
3/2)) - (2*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^
(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b
^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)
*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]
)))/b))/(4*b*c)
```

3.203.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.203. $\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.203.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.53

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(7Ac-3Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32b}\right)}{b^2}-\cdots$
default	$-\frac{2\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(7Ac-3Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32b}\right)}{b^2}-\cdots$
risch	$-\frac{2A}{3b^2x^{\frac{3}{2}}}-\frac{\frac{2\left(\frac{Ac}{4}-\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(7Ac-3Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\cdots}{16b}}{b^2}$

input
```
int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/b^2*((1/4*A*c-1/4*B*b)*x^(1/2)/(c*x^2+b)+1/32*(7*A*c-3*B*b)*(1/c*b)^(1/
4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b
)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(
1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))-2/3*A/b^2/x^(3/2)
```

3.203.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 691, normalized size of antiderivative = 2.39

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx =$$

$$-\frac{3\,(b^2cx^4+b^3x^2)\left(-\frac{81\,B^4b^4-756\,AB^3b^3c+2646\,A^2B^2b^2c^2-4116\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c}\right)^{\frac{1}{4}}\log\left(b^3\left(-\frac{81\,B^4b^4-756\,AB^3b^3c+2646\,A^2B}{b^{11}}\right.\right.}{}$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
-1/24*(3*(b^2*c*x^4 + b^3*x^2)*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*
B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(1/4)*log(b^3*(-(
81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2
401*A^4*c^4)/(b^11*c))^(1/4) - (3*B*b - 7*A*c)*sqrt(x)) + 3*(I*b^2*c*x^4 +
 I*b^3*x^2)*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*
A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(1/4)*log(I*b^3*(-(81*B^4*b^4 - 756*
A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^1
1*c))^(1/4) - (3*B*b - 7*A*c)*sqrt(x)) + 3*(-I*b^2*c*x^4 - I*b^3*x^2)*(-(8
1*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 24
01*A^4*c^4)/(b^11*c))^(1/4)*log(-I*b^3*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2
646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(1/4) - (
3*B*b - 7*A*c)*sqrt(x)) - 3*(b^2*c*x^4 + b^3*x^2)*(-(81*B^4*b^4 - 756*A*B^
3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c)
)^(1/4)*log(-b^3*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 -
4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(1/4) - (3*B*b - 7*A*c)*sqrt(x)
) - 4*((3*B*b - 7*A*c)*x^2 - 4*A*b)*sqrt(x))/(b^2*c*x^4 + b^3*x^2)
```

3.203.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

3.203.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.87

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(3\,Bb-7\,Ac)x^2-4\,Ab}{6\left(b^2cx^{\frac{7}{2}}+b^3x^{\frac{3}{2}}\right)} + \frac{\frac{2\,\sqrt{2}(3\,Bb-7\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\,\sqrt{2}(3\,Bb-7\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}(3\,Bb-7\,Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{16\,b^2}$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/6*((3*B*b - 7*A*c)*x^2 - 4*A*b)/(b^2*c*x^(7/2) + b^3*x^(3/2)) + 1/16*(2*
sqrt(2)*(3*B*b - 7*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sq
rt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*
sqrt(2)*(3*B*b - 7*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*s
qrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + s
qrt(2)*(3*B*b - 7*A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + s
qrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(3*B*b - 7*A*c)*log(-sqrt(2)*b^(1/4)*c
^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^2
```

3.203.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.98

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\sqrt{2}\left(3\,(bc^3)^{\frac{1}{4}}\,Bb-7\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c} + \frac{\sqrt{2}\left(3\,(bc^3)^{\frac{1}{4}}\,Bb-7\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c} + \frac{\sqrt{2}\left(3\,(bc^3)^{\frac{1}{4}}\,Bb-7\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^3c} - \frac{\sqrt{2}\left(3\,(bc^3)^{\frac{1}{4}}\,Bb-7\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^3c} + \frac{Bb\sqrt{x}-Ac\sqrt{x}}{2\,(cx^2+b)b^2} - \frac{2\,A}{3\,b^2x^{\frac{3}{2}}}$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(3*(b*c^3)^(1/4)*B*b - 7*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) + 1/8*sqrt(2)*(3*(
b*c^3)^(1/4)*B*b - 7*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)
^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) + 1/16*sqrt(2)*(3*(b*c^3)^(1/4)*B
*b - 7*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))
/(b^3*c) - 1/16*sqrt(2)*(3*(b*c^3)^(1/4)*B*b - 7*(b*c^3)^(1/4)*A*c)*log(-s
qrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) + 1/2*(B*b*sqrt(x) - A
*c*sqrt(x))/((c*x^2 + b)*b^2) - 2/3*A/(b^2*x^(3/2))
```

3.203.9 Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 859, normalized size of antiderivative = 2.97

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{\frac{2A}{3b}+\frac{x^2\,(7\,A\,c-3\,B\,b)}{6\,b^2}}{b\,x^{3/2}+c\,x^{7/2}}$$

$$-\frac{\operatorname{atan}\left(\frac{\frac{(7\,A\,c-3\,B\,b)\left(\sqrt{x}\left(1568\,A^2\,b^6\,c^5-1344\,A\,B\,b^7\,c^4+288\,B^2\,b^8\,c^3\right)-\frac{(7\,A\,c-3\,B\,b)\left(1792\,A\,b^9\,c^4-768\,B\,b^{10}\,c^3\right)}{8\,(-b)^{11/4}\,c^{1/4}}\right)1\mathrm{i}}{8\,(-b)^{11/4}\,c^{1/4}}+\frac{(7\,A\,c-3\,B\,b)\left(\sqrt{x}\left(1568\,A^2\,b^6\,c^5-\right.\right.}{}}{\frac{(7\,A\,c-3\,B\,b)\left(\sqrt{x}\left(1568\,A^2\,b^6\,c^5-1344\,A\,B\,b^7\,c^4+288\,B^2\,b^8\,c^3\right)-\frac{(7\,A\,c-3\,B\,b)\left(1792\,A\,b^9\,c^4-768\,B\,b^{10}\,c^3\right)}{8\,(-b)^{11/4}\,c^{1/4}}\right)}{8\,(-b)^{11/4}\,c^{1/4}}-\frac{(7\,A\,c-3\,B\,b)\left(\sqrt{x}\left(1568\,A^2\,b^6\,c^5-\right.\right.}{}}\right)}{4\,(-b)^{11/4}\,c^{1/4}}$$

$$-\frac{\operatorname{atan}\left(\frac{\frac{(7\,A\,c-3\,B\,b)\left(\sqrt{x}\left(1568\,A^2\,b^6\,c^5-1344\,A\,B\,b^7\,c^4+288\,B^2\,b^8\,c^3\right)-\frac{(7\,A\,c-3\,B\,b)\left(1792\,A\,b^9\,c^4-768\,B\,b^{10}\,c^3\right)1\mathrm{i}}{8\,(-b)^{11/4}\,c^{1/4}}\right)}{8\,(-b)^{11/4}\,c^{1/4}}+\frac{(7\,A\,c-3\,B\,b)\left(\sqrt{x}\left(1568\,A^2\,b^6\,c^5\right.\right.}{}}{\frac{(7\,A\,c-3\,B\,b)\left(\sqrt{x}\left(1568\,A^2\,b^6\,c^5-1344\,A\,B\,b^7\,c^4+288\,B^2\,b^8\,c^3\right)-\frac{(7\,A\,c-3\,B\,b)\left(1792\,A\,b^9\,c^4-768\,B\,b^{10}\,c^3\right)1\mathrm{i}}{8\,(-b)^{11/4}\,c^{1/4}}\right)1\mathrm{i}}{8\,(-b)^{11/4}\,c^{1/4}}-\frac{(7\,A\,c-3\,B\,b)\left(\sqrt{x}\left(1568\,A^2\,b^6\,c^5\right.\right.}{}}\right)}{4\,(-b)^{11/4}\,c^{1/4}}$$

input
```
int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output
```
- ((2*A)/(3*b) + (x^2*(7*A*c - 3*B*b))/(6*b^2))/(b*x^(3/2) + c*x^(7/2)) -
(atan((((7*A*c - 3*B*b)*(x^(1/2)*(1568*A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 134
4*A*B*b^7*c^4) - ((7*A*c - 3*B*b)*(1792*A*b^9*c^4 - 768*B*b^10*c^3))/(8*(-
b)^(11/4)*c^(1/4)))*1i)/(8*(-b)^(11/4)*c^(1/4)) + ((7*A*c - 3*B*b)*(x^(1/2
)*(1568*A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7*c^4) + ((7*A*c - 3*B*
b)*(1792*A*b^9*c^4 - 768*B*b^10*c^3))/(8*(-b)^(11/4)*c^(1/4)))*1i)/(8*(-b)
^(11/4)*c^(1/4)))/(((7*A*c - 3*B*b)*(x^(1/2)*(1568*A^2*b^6*c^5 + 288*B^2*b
^8*c^3 - 1344*A*B*b^7*c^4) - ((7*A*c - 3*B*b)*(1792*A*b^9*c^4 - 768*B*b^10
*c^3))/(8*(-b)^(11/4)*c^(1/4))))/(8*(-b)^(11/4)*c^(1/4)) - ((7*A*c - 3*B*b
)*(x^(1/2)*(1568*A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7*c^4) + ((7*A
*c - 3*B*b)*(1792*A*b^9*c^4 - 768*B*b^10*c^3))/(8*(-b)^(11/4)*c^(1/4))))/(
8*(-b)^(11/4)*c^(1/4))))*(7*A*c - 3*B*b)*1i)/(4*(-b)^(11/4)*c^(1/4)) - (at
an((((7*A*c - 3*B*b)*(x^(1/2)*(1568*A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*A
*B*b^7*c^4) - ((7*A*c - 3*B*b)*(1792*A*b^9*c^4 - 768*B*b^10*c^3)*1i)/(8*(-
b)^(11/4)*c^(1/4))))/(8*(-b)^(11/4)*c^(1/4)) + ((7*A*c - 3*B*b)*(x^(1/2)*(
1568*A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7*c^4) + ((7*A*c - 3*B*b)*
(1792*A*b^9*c^4 - 768*B*b^10*c^3)*1i)/(8*(-b)^(11/4)*c^(1/4))))/(8*(-b)^(1
1/4)*c^(1/4)))/(((7*A*c - 3*B*b)*(x^(1/2)*(1568*A^2*b^6*c^5 + 288*B^2*b^8*
c^3 - 1344*A*B*b^7*c^4) - ((7*A*c - 3*B*b)*(1792*A*b^9*c^4 - 768*B*b^10*c^
3)*1i)/(8*(-b)^(11/4)*c^(1/4)))*1i)/(8*(-b)^(11/4)*c^(1/4)) - ((7*A*c -...
```

3.204 $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.204.1 Optimal result

Integrand size = 26, antiderivative size = 310

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}(b+cx^2)} + \frac{\sqrt[4]{c}(5bB-9Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}} - \frac{\sqrt[4]{c}(5bB-9Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}} - \frac{\sqrt[4]{c}(5bB-9Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}} + \frac{\sqrt[4]{c}(5bB-9Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}}$$

output
```
1/10*(-9*A*c+5*B*b)/b^2/c/x^(5/2)+1/2*(A*c-B*b)/b/c/x^(5/2)/(c*x^2+b)+1/8*
c^(1/4)*(-9*A*c+5*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(13/4)*
2^(1/2)-1/8*c^(1/4)*(-9*A*c+5*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4
))/b^(13/4)*2^(1/2)-1/16*c^(1/4)*(-9*A*c+5*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/
4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)+1/16*c^(1/4)*(-9*A*c+5*B*b)*l
n(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)+1/2*
(9*A*c-5*B*b)/b^3/x^(1/2)
```

3.204.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$
$$= \frac{-\frac{4\sqrt[4]{b}(5bBx^2(4b+5cx^2)+A(4b^2-36bcx^2-45c^2x^4))}{x^{5/2}(b+cx^2)}+5\sqrt{2}\sqrt[4]{c}(5bB-9Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+5\sqrt{2}\sqrt[4]{c}(5bB-9}{40b^{13/4}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
((-4*b^(1/4)*(5*b*B*x^2*(4*b + 5*c*x^2) + A*(4*b^2 - 36*b*c*x^2 - 45*c^2*x
^4)))/(x^(5/2)*(b + c*x^2)) + 5*Sqrt[2]*c^(1/4)*(5*b*B - 9*A*c)*ArcTan[(Sq
rt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 5*Sqrt[2]*c^(1/4)*
(5*b*B - 9*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[
c]*x)])/(40*b^(13/4))
```

3.204.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {9, 362, 264, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{A+Bx^2}{x^{7/2}(b+cx^2)^2}dx$$
$$\Big\downarrow \text{ 362}$$
$$-\frac{(5bB-9Ac)\int \frac{1}{x^{7/2}(cx^2+b)}dx}{4bc}-\frac{bB-Ac}{2bcx^{5/2}(b+cx^2)}$$
$$\Big\downarrow \text{ 264}$$

$$-\frac{(5bB-9Ac)\left(-\frac{c\int \frac{1}{x^{3/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{5bx^{5/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)}$$

$\Big\downarrow$ 264

$$-\frac{(5bB-9Ac)\left(-\frac{c\left(-\frac{c\int \frac{\sqrt{x}}{cx^2+b}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)}$$

$\Big\downarrow$ 266

$$-\frac{(5bB-9Ac)\left(-\frac{c\left(-\frac{2c\int \frac{x}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)}$$

$\Big\downarrow$ 826

$$-\frac{(5bB-9Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)}$$

$\Big\downarrow$ 1476

3.204. $\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$-\frac{(5bB-9Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)}$$

$\Big\downarrow$ 1082

3.204. $\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$-\frac{(5bB-9Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)}$$

$\Big\downarrow$ 217

3.204. $\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$-\frac{(5bB-9Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)}$$

$\Big\downarrow$ 1479

3.204. $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

$$-\frac{(5bB-9Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\cdots\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\cdots\right)}\cdots}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{4bc}\cdots$$

$$\frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \; 25$$

3.204. $\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$-\frac{(5bB-9Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}}{b}}{b}}{4bc}$$

$$\frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)}$$

$\downarrow$ 27

3.204. $\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$-\frac{(5bB-9Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}\right)}{4bc}$$

$$\frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)}$$

$\downarrow$ 1103

3.204. $\int\frac{\sqrt{x}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

$$-\frac{(5bB-9Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}\right)}{b}\right)}{4bc}$$

$$\frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)}$$

input
```
Int[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(b*B - A*c)/(b*c*x^(5/2)*(b + c*x^2)) - ((5*b*B - 9*A*c)*(-2/(5*b*x^(
5/2)) - (c*(-2/(b*Sqrt[x]) - (2*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])
/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[b] - Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log
[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)
*c^(1/4)))/(2*Sqrt[c])))/b))/b))/(4*b*c)
```

3.204. $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2} \, dx$

3.204.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

3.204. $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.204.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.55

method	result
derivativedivides	$-\frac{2A}{5b^2x^{\frac{5}{2}}}-\frac{2(-2Ac+Bb)}{b^3\sqrt{x}}+\frac{2c\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{9Ac}{4}-\frac{5Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{a}\cdots\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b^3}$
default	$-\frac{2A}{5b^2x^{\frac{5}{2}}}-\frac{2(-2Ac+Bb)}{b^3\sqrt{x}}+\frac{2c\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{9Ac}{4}-\frac{5Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{a}\cdots\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b^3}$
risch	$-\frac{2\left(-10Ac\,x^2+5bB\,x^2+Ab\right)}{5b^3x^{\frac{5}{2}}}+\frac{c\left(\frac{2\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{9Ac}{4}-\frac{5Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+\cdots\right.}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b^3}$

input
```
int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/5*A/b^2/x^(5/2)-2*(-2*A*c+B*b)/b^3/x^(1/2)+2/b^3*c*((1/4*A*c-1/4*B*b)*x
^(3/2)/(c*x^2+b)+1/8*(9/4*A*c-5/4*B*b)/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c
*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+
(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)
/(1/c*b)^(1/4)*x^(1/2)-1)))
```

3.204.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 838, normalized size of antiderivative = 2.70

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$

$$= \frac{5\left(b^3cx^5+b^4x^3\right)\left(-\frac{625\,B^4b^4c-4500\,AB^3b^3c^2+12150\,A^2B^2b^2c^3-14580\,A^3Bbc^4+6561\,A^4c^5}{b^{13}}\right)^{\frac{1}{4}}\log\left(b^{10}\left(-\frac{625\,B^4b^4c-4500\,AB^3b^3c\cdots}{}\right.\right.}{}$$

input
```
integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/40*(5*(b^3*c*x^5 + b^4*x^3)*(-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 1215
0*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)^(1/4)*log(b^10
*(-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3
*B*b*c^4 + 6561*A^4*c^5)/b^13)^(3/4) - (125*B^3*b^3*c - 675*A*B^2*b^2*c^2
+ 1215*A^2*B*b*c^3 - 729*A^3*c^4)*sqrt(x)) - 5*(I*b^3*c*x^5 + I*b^4*x^3)*(
-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B
*b*c^4 + 6561*A^4*c^5)/b^13)^(1/4)*log(I*b^10*(-(625*B^4*b^4*c - 4500*A*B^
3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13
)^(3/4) - (125*B^3*b^3*c - 675*A*B^2*b^2*c^2 + 1215*A^2*B*b*c^3 - 729*A^3*
c^4)*sqrt(x)) - 5*(-I*b^3*c*x^5 - I*b^4*x^3)*(-(625*B^4*b^4*c - 4500*A*B^3
*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)
^(1/4)*log(-I*b^10*(-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b
^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)^(3/4) - (125*B^3*b^3*c -
675*A*B^2*b^2*c^2 + 1215*A^2*B*b*c^3 - 729*A^3*c^4)*sqrt(x)) - 5*(b^3*c*x^
5 + b^4*x^3)*(-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3
 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)^(1/4)*log(-b^10*(-(625*B^4*b^4*
c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*
A^4*c^5)/b^13)^(3/4) - (125*B^3*b^3*c - 675*A*B^2*b^2*c^2 + 1215*A^2*B*b*c
^3 - 729*A^3*c^4)*sqrt(x)) - 4*(5*(5*B*b*c - 9*A*c^2)*x^4 + 4*A*b^2 + 4*(5
*B*b^2 - 9*A*b*c)*x^2)*sqrt(x))/(b^3*c*x^5 + b^4*x^3)
```

3.204.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

3.204.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{5\,(5\,Bbc-9\,Ac^2)x^4+4\,Ab^2+4\,(5\,Bb^2-9\,Abc)x^2}{10\left(b^3cx^{\frac{9}{2}}+b^4x^{\frac{5}{2}}\right)} - \frac{(5\,Bbc-9\,Ac^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\right.}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{16\,b^3}$$

input
```
integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/10*(5*(5*B*b*c - 9*A*c^2)*x^4 + 4*A*b^2 + 4*(5*B*b^2 - 9*A*b*c)*x^2)/(b
^3*c*x^(9/2) + b^4*x^(5/2)) - 1/16*(5*B*b*c - 9*A*c^2)*(2*sqrt(2)*arctan(1
/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt
(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqr
t(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqr
t(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqr
t(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)
*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^3
```

3.204.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{Bbcx^{\frac{3}{2}} - Ac^2x^{\frac{3}{2}}}{2\,(cx^2+b)b^3} - \frac{\sqrt{2}\Big(5\,(bc^3)^{\frac{3}{4}}\,Bb - 9\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4c^2} - \frac{\sqrt{2}\Big(5\,(bc^3)^{\frac{3}{4}}\,Bb - 9\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4c^2} + \frac{\sqrt{2}\Big(5\,(bc^3)^{\frac{3}{4}}\,Bb - 9\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4c^2} - \frac{\sqrt{2}\Big(5\,(bc^3)^{\frac{3}{4}}\,Bb - 9\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4c^2} - \frac{2\,(5\,Bbx^2 - 10\,Acx^2 + Ab)}{5\,b^3x^{\frac{5}{2}}}$$

input
```
integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-1/2*(B*b*c*x^(3/2) - A*c^2*x^(3/2))/((c*x^2 + b)*b^3) - 1/8*sqrt(2)*(5*(b
*c^3)^(3/4)*B*b - 9*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(
1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c^2) - 1/8*sqrt(2)*(5*(b*c^3)^(3/4)*B*
b - 9*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt
(x))/(b/c)^(1/4))/(b^4*c^2) + 1/16*sqrt(2)*(5*(b*c^3)^(3/4)*B*b - 9*(b*c^3
)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c^2) -
1/16*sqrt(2)*(5*(b*c^3)^(3/4)*B*b - 9*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt
(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c^2) - 2/5*(5*B*b*x^2 - 10*A*c*x^2 +
 A*b)/(b^3*x^(5/2))
```

3.204.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\frac{2\,x^2\,(9\,A\,c-5\,B\,b)}{5\,b^2}-\frac{2\,A}{5\,b}+\frac{c\,x^4\,(9\,A\,c-5\,B\,b)}{2\,b^3}}{b\,x^{5/2}+c\,x^{9/2}} + \frac{{(-c)}^{1/4}\,\mathrm{atan}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(9\,A\,c-5\,B\,b)}{4\,b^{13/4}} - \frac{{(-c)}^{1/4}\,\mathrm{atanh}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(9\,A\,c-5\,B\,b)}{4\,b^{13/4}}$$

input
```
int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output
```
((2*x^2*(9*A*c - 5*B*b))/(5*b^2) - (2*A)/(5*b) + (c*x^4*(9*A*c - 5*B*b))/(
2*b^3))/(b*x^(5/2) + c*x^(9/2)) + ((-c)^(1/4)*atan(((-c)^(1/4)*x^(1/2))/b^
(1/4))*(9*A*c - 5*B*b))/(4*b^(13/4)) - ((-c)^(1/4)*atanh(((-c)^(1/4)*x^(1/
2))/b^(1/4))*(9*A*c - 5*B*b))/(4*b^(13/4))
```

3.205 $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^2} dx$

3.205.1 Optimal result

Integrand size = 26, antiderivative size = 310

$$\begin{aligned}\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx = {} & \frac{7bB-11Ac}{14b^2cx^{7/2}}-\frac{7bB-11Ac}{6b^3x^{3/2}}-\frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} \\ & +\frac{c^{3/4}(7bB-11Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{15/4}} \\ & -\frac{c^{3/4}(7bB-11Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{15/4}} \\ & +\frac{c^{3/4}(7bB-11Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}} \\ & -\frac{c^{3/4}(7bB-11Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}}\end{aligned}$$

output
```
1/14*(-11*A*c+7*B*b)/b^2/c/x^(7/2)+1/6*(11*A*c-7*B*b)/b^3/x^(3/2)+1/2*(A*c
-B*b)/b/c/x^(7/2)/(c*x^2+b)+1/8*c^(3/4)*(-11*A*c+7*B*b)*arctan(1-c^(1/4)*2
^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)*2^(1/2)-1/8*c^(3/4)*(-11*A*c+7*B*b)*arcta
n(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)*2^(1/2)+1/16*c^(3/4)*(-11*A*
c+7*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(15/4)*2^
(1/2)-1/16*c^(3/4)*(-11*A*c+7*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^
(1/2)*x^(1/2))/b^(15/4)*2^(1/2)
```

3.205.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx$$
$$=\frac{-\frac{4b^{3/4}(7bBx^2(4b+7cx^2)+A(12b^2-44bcx^2-77c^2x^4))}{x^{7/2}(b+cx^2)}+21\sqrt{2}c^{3/4}(7bB-11Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+21\sqrt{2}c^{3/4}(-7}{168b^{15/4}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

output
```
((-4*b^(3/4)*(7*b*B*x^2*(4*b + 7*c*x^2) + A*(12*b^2 - 44*b*c*x^2 - 77*c^2*
x^4)))/(x^(7/2)*(b + c*x^2)) + 21*Sqrt[2]*c^(3/4)*(7*b*B - 11*A*c)*ArcTan[
(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 21*Sqrt[2]*c^(3
/4)*(-7*b*B + 11*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] +
 Sqrt[c]*x)])/(168*b^(15/4))
```

3.205.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {9, 362, 264, 264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx$$
$$\Big\downarrow \ 9$$
$$\int \frac{A+Bx^2}{x^{9/2}\left(b+cx^2\right)^2}dx$$
$$\Big\downarrow \ 362$$
$$-\frac{(7bB-11Ac)\int \frac{1}{x^{9/2}(cx^2+b)}dx}{4bc}-\frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)}$$
$$\Big\downarrow \ 264$$

$$-\frac{(7bB-11Ac)\left(-\frac{c\int \frac{1}{x^{5/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{7bx^{7/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)}$$

$\downarrow$ 264

$$-\frac{(7bB-11Ac)\left(-\frac{c\left(-\frac{c\int \frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)}$$

$\downarrow$ 266

$$-\frac{(7bB-11Ac)\left(-\frac{c\left(-\frac{2c\int \frac{1}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)}$$

$\downarrow$ 755

$$-\frac{(7bB-11Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)}$$

$\downarrow$ 1476

3.205. $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^2}\,dx$

$$-\frac{(7bB-11Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \text{ 1082}$$

3.205. $\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx$

$$-\frac{(7bB-11Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \; 217$$

3.205. $\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx$

$$-\frac{(7bB-11Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)}$$

$\downarrow$ 1479

3.205. $\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx$

$$-\frac{(7bB-11Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\sqrt{2}\cdots\right)}{\sqrt{2}\sqrt[4]{b}\cdots}}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{4bc}$$

$$\frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)}$$

$\downarrow$ 25

3.205. $\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx$

$$-\frac{(7bB-11Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}\cdots}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{4bc}$$

$$\frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)}$$

$$\downarrow 27$$

3.205. $\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx$

$$-\frac{(7bB-11Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}} \cdots\right)}{b} \cdots\right)}{4bc}$$

$$\frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \ 1103$$

3.205. $\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx$

$$-\frac{(7bB-11Ac)\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}}{b}\right)}{4bc}$$

$$\frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)}$$

input
```
Int[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

output
```
-1/2*(b*B - A*c)/(b*c*x^(7/2)*(b + c*x^2)) - ((7*b*B - 11*A*c)*(-2/(7*b*x^
(7/2)) - (c*(-2/(3*b*x^(3/2)) - (2*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[
x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt
[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] -
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) +
Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1
/4)*c^(1/4)))/(2*Sqrt[b])))/b))/b))/(4*b*c)
```

3.205. $\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx$

3.205.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.205.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.55

method	result
derivativedivides	$\frac{2c\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(11Ac-7Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32b}\right)}{b^3}-\frac{}{7}$
default	$\frac{2c\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(11Ac-7Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32b}\right)}{b^3}-\frac{}{7}$
risch	$-\frac{2(-14Ac\,x^2+7bB\,x^2+3Ab)}{21b^3x^{\frac{7}{2}}}+\frac{c\left(\frac{2\left(\frac{Ac}{4}-\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(11Ac-7Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{16b}\right.}{b^3}$

input
```
int((B*x^2+A)/(c*x^4+b*x^2)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^3*c*((1/4*A*c-1/4*B*b)*x^(1/2)/(c*x^2+b)+1/32*(11*A*c-7*B*b)*(1/c*b)^(
1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c
*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x
^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))-2/7*A/b^2/x^(7/2)-2/
3*(-2*A*c+B*b)/b^3/x^(3/2)
```

3.205.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.39

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^2}\,dx$$

$$= \frac{21\,(b^3cx^6+b^4x^4)\left(-\frac{2401\,B^4b^4c^3-15092\,AB^3b^3c^4+35574\,A^2B^2b^2c^5-37268\,A^3Bbc^6+14641\,A^4c^7}{b^{15}}\right)^{\frac{1}{4}}\log\left(b^4\left(-\frac{2401\,B^4b^4c^3-15092}{}\right.\right.}{}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="fricas")
```

output
```
1/168*(21*(b^3*c*x^6 + b^4*x^4)*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4
+ 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4)*l
og(b^4*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 -
 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4) - (7*B*b*c - 11*A*c^2)*sqr
t(x)) - 21*(-I*b^3*c*x^6 - I*b^4*x^4)*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^
3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(
1/4)*log(I*b^4*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b
^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4) - (7*B*b*c - 11*A*
c^2)*sqrt(x)) - 21*(I*b^3*c*x^6 + I*b^4*x^4)*(-(2401*B^4*b^4*c^3 - 15092*A
*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/
b^15)^(1/4)*log(-I*b^4*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A
^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4) - (7*B*b*c
 - 11*A*c^2)*sqrt(x)) - 21*(b^3*c*x^6 + b^4*x^4)*(-(2401*B^4*b^4*c^3 - 150
92*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c
^7)/b^15)^(1/4)*log(-b^4*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574
*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4) - (7*B*b
*c - 11*A*c^2)*sqrt(x)) - 4*(7*(7*B*b*c - 11*A*c^2)*x^4 + 12*A*b^2 + 4*(7*
B*b^2 - 11*A*b*c)*x^2)*sqrt(x))/(b^3*c*x^6 + b^4*x^4)
```

3.205.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)/(c*x**4+b*x**2)**2/x**(1/2),x)
```

output
```
Timed out
```

3.205.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^2}\,dx = -\frac{7\,(7\,Bbc-11\,Ac^2)x^4+12\,Ab^2+4\,(7\,Bb^2-11\,Abc)x^2}{42\left(b^3cx^{\frac{11}{2}}+b^4x^{\frac{7}{2}}\right)} - \frac{\frac{2\sqrt{2}(7\,Bbc-11\,Ac^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(7\,Bbc-11\,Ac^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\sqrt{2}(7\,Bbc-11\,Ac^2)\,\mathrm{l}}{16\,b^3}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="maxima")
```

output
```
-1/42*(7*(7*B*b*c - 11*A*c^2)*x^4 + 12*A*b^2 + 4*(7*B*b^2 - 11*A*b*c)*x^2)
/(b^3*c*x^(11/2) + b^4*x^(7/2)) - 1/16*(2*sqrt(2)*(7*B*b*c - 11*A*c^2)*arc
tan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)
*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(7*B*b*c - 11*A*c^2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(s
qrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(7*B*b*c - 11*A
*c^2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*
c^(1/4)) - sqrt(2)*(7*B*b*c - 11*A*c^2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(
x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^3
```

3.205.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx = -\frac{\sqrt{2}\left(7\left(bc^3\right)^{\frac{1}{4}}Bb-11\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4} - \frac{\sqrt{2}\left(7\left(bc^3\right)^{\frac{1}{4}}Bb-11\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4} - \frac{\sqrt{2}\left(7\left(bc^3\right)^{\frac{1}{4}}Bb-11\left(bc^3\right)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4} + \frac{\sqrt{2}\left(7\left(bc^3\right)^{\frac{1}{4}}Bb-11\left(bc^3\right)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4} - \frac{Bbc\sqrt{x}-Ac^2\sqrt{x}}{2\left(cx^2+b\right)b^3} - \frac{2\left(7\,Bbx^2-14\,Acx^2+3\,Ab\right)}{21\,b^3x^{\frac{7}{2}}}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(
2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^4 - 1/8*sqrt(2)*(7*(b*
c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^
(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 - 1/16*sqrt(2)*(7*(b*c^3)^(1/4)*B*b -
11*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4
 + 1/16*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)*
sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 1/2*(B*b*c*sqrt(x) - A*c^2*sqrt
(x))/((c*x^2 + b)*b^3) - 2/21*(7*B*b*x^2 - 14*A*c*x^2 + 3*A*b)/(b^3*x^(7/2
))
```

3.205.9 Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.92

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^2}\,dx = \frac{\frac{2\,x^2\,(11\,A\,c-7\,B\,b)}{21\,b^2}-\frac{2\,A}{7\,b}+\frac{c\,x^4\,(11\,A\,c-7\,B\,b)}{6\,b^3}}{b\,x^{7/2}+c\,x^{11/2}}$$
$$+\frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{\frac{(-c)^{3/4}\,(11\,A\,c-7\,B\,b)\,\left(\sqrt{x}\,\left(3872\,A^2\,b^9\,c^7-4928\,A\,B\,b^{10}\,c^6+1568\,B^2\,b^{11}\,c^5\right)-\frac{(-c)^{3/4}\,(11\,A\,c-7\,B\,b)\,\left(2816\,A\,b^{13}\,c^5-1792\,B\,b^{14}\,c^4\cdots\right.}{8\,b^{15/4}}\cdots\right.}{8\,b^{15/4}}}{\frac{(-c)^{3/4}\,(11\,A\,c-7\,B\,b)\,\left(\sqrt{x}\,\left(3872\,A^2\,b^9\,c^7-4928\,A\,B\,b^{10}\,c^6+1568\,B^2\,b^{11}\,c^5\right)-\frac{(-c)^{3/4}\,(11\,A\,c-7\,B\,b)\,\left(2816\,A\,b^{13}\,c^5-1792\,B\,b^{14}\,c^4\right)\cdots}{8\,b^{15/4}}\cdots\right.}{8\,b^{15/4}}}\cdots\right.}{\cdots}$$
$$-\frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{A^3\,c^8\,\sqrt{x}\,1331\mathrm{i}-B^3\,b^3\,c^5\,\sqrt{x}\,343\mathrm{i}-A^2\,B\,b\,c^7\,\sqrt{x}\,2541\mathrm{i}+A\,B^2\,b^2\,c^6\,\sqrt{x}\,1617\mathrm{i}}{b^{1/4}\,(-c)^{19/4}\,\left(c\,\left(c\,\left(1331\,A^3\,c-2541\,A^2\,B\,b\right)+1617\,A\,B^2\,b^2\right)-343\,B^3\,b^3\right)}\right)\,(11\,A\,c-7\,B\,b)\,1\mathrm{i}}{4\,b^{15/4}}$$

input
```
int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^2),x)
```

output
```
((2*x^2*(11*A*c - 7*B*b))/(21*b^2) - (2*A)/(7*b) + (c*x^4*(11*A*c - 7*B*b)
)/(6*b^3))/(b*x^(7/2) + c*x^(11/2)) + ((-c)^(3/4)*atan((((-c)^(3/4)*(11*A*
c - 7*B*b)*(x^(1/2)*(3872*A^2*b^9*c^7 + 1568*B^2*b^11*c^5 - 4928*A*B*b^10*
c^6) - ((-c)^(3/4)*(11*A*c - 7*B*b)*(2816*A*b^13*c^5 - 1792*B*b^14*c^4)*1i
)/(8*b^(15/4))))/(8*b^(15/4)) + ((-c)^(3/4)*(11*A*c - 7*B*b)*(x^(1/2)*(387
2*A^2*b^9*c^7 + 1568*B^2*b^11*c^5 - 4928*A*B*b^10*c^6) + ((-c)^(3/4)*(11*A
*c - 7*B*b)*(2816*A*b^13*c^5 - 1792*B*b^14*c^4)*1i)/(8*b^(15/4))))/(8*b^(1
5/4)))/(((-c)^(3/4)*(11*A*c - 7*B*b)*(x^(1/2)*(3872*A^2*b^9*c^7 + 1568*B^2
*b^11*c^5 - 4928*A*B*b^10*c^6) - ((-c)^(3/4)*(11*A*c - 7*B*b)*(2816*A*b^13
*c^5 - 1792*B*b^14*c^4)*1i)/(8*b^(15/4)))*1i)/(8*b^(15/4)) - ((-c)^(3/4)*(
11*A*c - 7*B*b)*(x^(1/2)*(3872*A^2*b^9*c^7 + 1568*B^2*b^11*c^5 - 4928*A*B*
b^10*c^6) + ((-c)^(3/4)*(11*A*c - 7*B*b)*(2816*A*b^13*c^5 - 1792*B*b^14*c^
4)*1i)/(8*b^(15/4)))*1i)/(8*b^(15/4))))*(11*A*c - 7*B*b))/(4*b^(15/4)) - (
(-c)^(3/4)*atan((A^3*c^8*x^(1/2)*1331i - B^3*b^3*c^5*x^(1/2)*343i - A^2*B*
b*c^7*x^(1/2)*2541i + A*B^2*b^2*c^6*x^(1/2)*1617i)/(b^(1/4)*(-c)^(19/4)*(c
*(c*(1331*A^3*c - 2541*A^2*B*b) + 1617*A*B^2*b^2) - 343*B^3*b^3)))*(11*A*c
 - 7*B*b)*1i)/(4*b^(15/4))
```

3.205. $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^2}\,dx$

3.206 $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^2}\,dx$

3.206.1 Optimal result

Integrand size = 26, antiderivative size = 332

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx = \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} - \frac{c^{5/4}(9bB-13Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{17/4}} + \frac{c^{5/4}(9bB-13Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{17/4}} + \frac{c^{5/4}(9bB-13Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}} - \frac{c^{5/4}(9bB-13Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}$$

output
```
1/18*(-13*A*c+9*B*b)/b^2/c/x^(9/2)+1/10*(13*A*c-9*B*b)/b^3/x^(5/2)+1/2*(A*
c-B*b)/b/c/x^(9/2)/(c*x^2+b)-1/8*c^(5/4)*(-13*A*c+9*B*b)*arctan(1-c^(1/4)*
2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)+1/8*c^(5/4)*(-13*A*c+9*B*b)*arct
an(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)+1/16*c^(5/4)*(-13*A
*c+9*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(17/4)*2
^(1/2)-1/16*c^(5/4)*(-13*A*c+9*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2
^(1/2)*x^(1/2))/b^(17/4)*2^(1/2)+1/2*c*(-13*A*c+9*B*b)/b^4/x^(1/2)
```

3.206.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx = \frac{-\frac{4\sqrt[4]{b}\left(9bBx^2\left(4b^2-36bcx^2-45c^2x^4\right)+A\left(20b^3-52b^2cx^2+468bc^2x^4+585c^3x^6\right)\right)}{x^{9/2}\left(b+cx^2\right)}+45\sqrt{2}c^{5/4}(-9bB+}{360b^{1}}$$

input
```
Integrate[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```

output
```
((-4*b^(1/4)*(9*b*B*x^2*(4*b^2 - 36*b*c*x^2 - 45*c^2*x^4) + A*(20*b^3 - 52
*b^2*c*x^2 + 468*b*c^2*x^4 + 585*c^3*x^6)))/(x^(9/2)*(b + c*x^2)) + 45*Sqr
t[2]*c^(5/4)*(-9*b*B + 13*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x])] + 45*Sqrt[2]*c^(5/4)*(-9*b*B + 13*A*c)*ArcTanh[(Sqrt[
2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(360*b^(17/4))
```

3.206.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.96, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {9, 362, 264, 264, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{A+Bx^2}{x^{11/2}\left(b+cx^2\right)^2}dx$$

$$\Big\downarrow \text{ 362}$$

$$-\frac{(9bB-13Ac)\int \frac{1}{x^{11/2}\left(cx^2+b\right)}dx}{4bc}-\frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{(9bB-13Ac)\left(-\frac{c\int \frac{1}{x^{7/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{9bx^{9/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \text{264}$$

$$-\frac{(9bB-13Ac)\left(-\frac{c\left(-\frac{c\int \frac{1}{x^{3/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \text{264}$$

$$-\frac{(9bB-13Ac)\left(-\frac{c\left(-\frac{c\left(-\frac{c\int \frac{\sqrt{x}}{cx^2+b}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \text{266}$$

$$-\frac{(9bB-13Ac)\left(-\frac{c\left(-\frac{c\left(-\frac{2c\int \frac{x}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \text{826}$$

3.206. $\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx$

$$-\frac{(9bB-13Ac)\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)}$$

$\downarrow$ 1476

3.206. $\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx$

$$-\frac{(9bB-13Ac)\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)}-$$

3.206. $\int\frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx$

$\downarrow$ 1082

3.206. $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^2}\,dx$

$$-\frac{(9bB-13Ac)\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{-x-1}\,d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}\,d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}\,d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)}$$

3.206. $\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx$

↓ 217

3.206. $\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx$

$$-\frac{(9bB-13Ac)\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}\,d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4bc}-\frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)}$$

3.206. $\int\frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx$

↓ 1479

3.206. $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^2}\,dx$

$$(9bB-13Ac)\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}\cdots\right.}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\cdots}{\sqrt[4]{c}}\cdots\right.}\cdots}{2\sqrt{2}\sqrt[4]{b}\cdots}}{2\sqrt{c}}\cdots\right.}{b}\cdots\right.}{b}\cdots\right.}{b}\cdots\right.$$

3.206. $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^2}\,dx$

$$\downarrow 25$$

3.206. $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^2}\,dx$

$$(9bB-13Ac)\left(-\frac{c\left(c-\frac{c\left(c-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{\cdots}\right.}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\cdots\right.}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\cdots\right.}{b}\cdots\right)}{b}\cdots\right)}{b}\cdots\right.$$

3.206. $\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx$

3.206. $\downarrow$ 27

3.206. $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^2}\,dx$

$$(9bB-13Ac)\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{\cdots}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right.$$

3.206. $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^2}\,dx$

$$\downarrow 1103$$

3.206. $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^2}\,dx$

$$-\frac{(9bB-13Ac)\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\cdots\right.}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}\cdots}{2\sqrt{c}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{4bc}\cdots$$

$$\frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)}$$

3.206. $\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx$

input
```
Int[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```

output
```
-1/2*(b*B - A*c)/(b*c*x^(9/2)*(b + c*x^2)) - ((9*b*B - 13*A*c)*(-2/(9*b*x^
(9/2)) - (c*(-2/(5*b*x^(5/2)) - (c*(-2/(b*Sqrt[x]) - (2*c*((-(ArcTan[1 - (
Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 +
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])
- (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2
]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/b))/b))/b))/(4*b*c)
```

3.206.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.206. $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^2}\,dx$

3.206.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.57

method	result
derivativedivides	$-\frac{2A}{9b^{2}x^{\frac{9}{2}}}-\frac{2(-2Ac+Bb)}{5b^{3}x^{\frac{5}{2}}}-\frac{2c(3Ac-2Bb)}{b^{4}\sqrt{x}}-\frac{2c^{2}\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^{2}+b}+\frac{\left(\frac{13Ac}{4}-\frac{9Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\,\mathrm{a}\cdots\right)}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}\cdots}\cdots\right)}{b^{4}}\cdots$
default	$-\frac{2A}{9b^{2}x^{\frac{9}{2}}}-\frac{2(-2Ac+Bb)}{5b^{3}x^{\frac{5}{2}}}-\frac{2c(3Ac-2Bb)}{b^{4}\sqrt{x}}-\frac{2c^{2}\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^{2}+b}+\frac{\left(\frac{13Ac}{4}-\frac{9Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\,\mathrm{a}\cdots\right)}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}\cdots}\cdots\right)}{b^{4}}\cdots$
risch	$-\frac{2\left(135A\,c^{2}x^{4}-90x^{4}Bbc-18Abc\,x^{2}+9b^{2}B\,x^{2}+5b^{2}A\right)}{45b^{4}x^{\frac{9}{2}}}-\frac{c^{2}\left(\frac{2\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^{2}+b}+\frac{\left(\frac{13Ac}{4}-\frac{9Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\cdots}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\cdots}\right)\cdots\right)}{\cdots}\cdots\right)}{b^{4}}\cdots$

input
```
int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/9*A/b^2/x^(9/2)-2/5*(-2*A*c+B*b)/b^3/x^(5/2)-2*c*(3*A*c-2*B*b)/b^4/x^(1
/2)-2/b^4*c^2*((1/4*A*c-1/4*B*b)*x^(3/2)/(c*x^2+b)+1/8*(13/4*A*c-9/4*B*b)/
c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)
)/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b
)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

3.206.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 886, normalized size of antiderivative = 2.67

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx =$$

$$-\frac{45\left(b^4cx^7+b^5x^5\right)\left(-\frac{6561\,B^4b^4c^5-37908\,AB^3b^3c^6+82134\,A^2B^2b^2c^7-79092\,A^3Bbc^8+28561\,A^4c^9}{b^{17}}\right)^{\frac{1}{4}}\log\left(b^{13}\left(-\frac{6561\,B^4b^4c^5-3790\cdots}{\cdots}\right)\cdots\right)\cdots}{\cdots}$$

input
```
integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
-1/360*(45*(b^4*c*x^7 + b^5*x^5)*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6
 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(1/4)*
log(b^13*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7
 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(3/4) - (729*B^3*b^3*c^4 - 315
9*A*B^2*b^2*c^5 + 4563*A^2*B*b*c^6 - 2197*A^3*c^7)*sqrt(x)) + 45*(-I*b^4*c
*x^7 - I*b^5*x^5)*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^
2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(1/4)*log(I*b^13*(-(6
561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*
B*b*c^8 + 28561*A^4*c^9)/b^17)^(3/4) - (729*B^3*b^3*c^4 - 3159*A*B^2*b^2*c
^5 + 4563*A^2*B*b*c^6 - 2197*A^3*c^7)*sqrt(x)) + 45*(I*b^4*c*x^7 + I*b^5*x
^5)*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79
092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(1/4)*log(-I*b^13*(-(6561*B^4*b^4*c
^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 285
61*A^4*c^9)/b^17)^(3/4) - (729*B^3*b^3*c^4 - 3159*A*B^2*b^2*c^5 + 4563*A^2
*B*b*c^6 - 2197*A^3*c^7)*sqrt(x)) - 45*(b^4*c*x^7 + b^5*x^5)*(-(6561*B^4*b
^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 +
 28561*A^4*c^9)/b^17)^(1/4)*log(-b^13*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^
3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(
3/4) - (729*B^3*b^3*c^4 - 3159*A*B^2*b^2*c^5 + 4563*A^2*B*b*c^6 - 2197*A^3
*c^7)*sqrt(x)) - 4*(45*(9*B*b*c^2 - 13*A*c^3)*x^6 + 36*(9*B*b^2*c - 13*...
```

3.206.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)/x**(3/2)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

3.206.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx = \frac{45\,(9\,Bbc^2-13\,Ac^3)x^6+36\,(9\,Bb^2c-13\,Abc^2)x^4-20\,Ab^3-4\,(9\,Bb^3-13\,Ab^2c)x^2}{90\left(b^4cx^{\frac{13}{2}}+b^5x^{\frac{9}{2}}\right)} + \frac{(9\,Bbc^2-13\,Ac^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac14}c^{\frac14}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac14}c^{\frac14}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac14}c^{\frac14}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac14}c^{\frac34}} \cdots\right)}{16\,b^4}$$

input
```
integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/90*(45*(9*B*b*c^2 - 13*A*c^3)*x^6 + 36*(9*B*b^2*c - 13*A*b*c^2)*x^4 - 20
*A*b^3 - 4*(9*B*b^3 - 13*A*b^2*c)*x^2)/(b^4*c*x^(13/2) + b^5*x^(9/2)) + 1/
16*(9*B*b*c^2 - 13*A*c^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c
^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*
sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt
(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt
(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^
(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b
))/(b^(1/4)*c^(3/4)))/b^4
```

3.206.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx = \frac{\sqrt{2}\left(9\left(bc^3\right)^{\frac{3}{4}}Bb-13\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^5c}$$
$$+\frac{\sqrt{2}\left(9\left(bc^3\right)^{\frac{3}{4}}Bb-13\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^5c}$$
$$-\frac{\sqrt{2}\left(9\left(bc^3\right)^{\frac{3}{4}}Bb-13\left(bc^3\right)^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^5c}$$
$$+\frac{\sqrt{2}\left(9\left(bc^3\right)^{\frac{3}{4}}Bb-13\left(bc^3\right)^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^5c}$$
$$+\frac{Bbc^2x^{\frac{3}{2}}-Ac^3x^{\frac{3}{2}}}{2\left(cx^2+b\right)b^4}+\frac{2\left(90\,Bbcx^4-135\,Ac^2x^4-9\,Bb^2x^2+18\,Abcx^2-5\,Ab^2\right)}{45\,b^4x^{\frac{9}{2}}}$$

input
```
integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(9*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^5*c) + 1/8*sqrt(2)*(9*
(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/
c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^5*c) - 1/16*sqrt(2)*(9*(b*c^3)^(3/4)
*B*b - 13*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/
c))/(b^5*c) + 1/16*sqrt(2)*(9*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*lo
g(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^5*c) + 1/2*(B*b*c^2*x^(
3/2) - A*c^3*x^(3/2))/((c*x^2 + b)*b^4) + 2/45*(90*B*b*c*x^4 - 135*A*c^2*x
^4 - 9*B*b^2*x^2 + 18*A*b*c*x^2 - 5*A*b^2)/(b^4*x^(9/2))
```

3.206.9 Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.43

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx = \frac{(-c)^{5/4}\operatorname{atan}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)(13\,A\,c-9\,B\,b)}{4\,b^{17/4}} - \frac{\frac{2\,A}{9\,b}-\frac{2\,x^2\,(13\,A\,c-9\,B\,b)}{45\,b^2}+\frac{c^2\,x^6\,(13\,A\,c-9\,B\,b)}{2\,b^4}+\frac{2\,c\,x^4\,(13\,A\,c-9\,B\,b)}{5\,b^3}}{b\,x^{9/2}+c\,x^{13/2}} - \frac{(-c)^{5/4}\operatorname{atanh}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)(13\,A\,c-9\,B\,b)}{4\,b^{17/4}}$$

input

```
int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^2),x)
```

output

```
((-c)^(5/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4))*(13*A*c - 9*B*b))/(4*b^(17/
4)) - ((2*A)/(9*b) - (2*x^2*(13*A*c - 9*B*b))/(45*b^2) + (c^2*x^6*(13*A*c
- 9*B*b))/(2*b^4) + (2*c*x^4*(13*A*c - 9*B*b))/(5*b^3))/(b*x^(9/2) + c*x^(
13/2)) - ((-c)^(5/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4))*(13*A*c - 9*B*b))
/(4*b^(17/4))
```

3.207 $\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.207.1 Optimal result

Integrand size = 26, antiderivative size = 343

$$\begin{aligned}\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= -\frac{9(13bB-5Ac)\sqrt{x}}{16c^4}+\frac{9(13bB-5Ac)x^{5/2}}{80bc^3} \\ &\quad -\frac{(bB-Ac)x^{13/2}}{4bc\,(b+cx^2)^2}-\frac{(13bB-5Ac)x^{9/2}}{16bc^2\,(b+cx^2)}-\frac{9\sqrt[4]{b}(13bB-5Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}c^{17/4}} \\ &\quad +\frac{9\sqrt[4]{b}(13bB-5Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}c^{17/4}} \\ &\quad -\frac{9\sqrt[4]{b}(13bB-5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}c^{17/4}} \\ &\quad +\frac{9\sqrt[4]{b}(13bB-5Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}c^{17/4}}\end{aligned}$$

output
```
9/80*(-5*A*c+13*B*b)*x^(5/2)/b/c^3-1/4*(-A*c+B*b)*x^(13/2)/b/c/(c*x^2+b)^2
-1/16*(-5*A*c+13*B*b)*x^(9/2)/b/c^2/(c*x^2+b)-9/64*b^(1/4)*(-5*A*c+13*B*b)
*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(17/4)*2^(1/2)+9/64*b^(1/4)*(
-5*A*c+13*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(17/4)*2^(1/2)-
9/128*b^(1/4)*(-5*A*c+13*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)
*x^(1/2))/c^(17/4)*2^(1/2)+9/128*b^(1/4)*(-5*A*c+13*B*b)*ln(b^(1/2)+x*c^(1
/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(17/4)*2^(1/2)-9/16*(-5*A*c+13*B*b)
*x^(1/2)/c^4
```

3.207.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.59

$$\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{4\sqrt[4]{c}\sqrt{x}(-585b^3B+bc^2x^2(405A-416Bx^2)+9b^2c(25A-117Bx^2)+32c^3x^4(5A+Bx^2))}{(b+cx^2)^2} - 45\sqrt{2}\sqrt[4]{b}(13bB \cdots}{320c^{17}\cdots}$$

input
```
Integrate[(x^(23/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
((4*c^(1/4)*Sqrt[x]*(-585*b^3*B + b*c^2*x^2*(405*A - 416*B*x^2) + 9*b^2*c*
(25*A - 117*B*x^2) + 32*c^3*x^4*(5*A + B*x^2)))/(b + c*x^2)^2 - 45*Sqrt[2]
*b^(1/4)*(13*b*B - 5*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x])] + 45*Sqrt[2]*b^(1/4)*(13*b*B - 5*A*c)*ArcTanh[(Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(320*c^(17/4))
```

3.207.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.96, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {9, 362, 252, 262, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^{11/2}(A+Bx^2)}{(b+cx^2)^3}dx$$

$$\Big\downarrow \; 362$$

$$\frac{(13bB-5Ac)\int \frac{x^{11/2}}{(cx^2+b)^2}dx}{8bc} - \frac{x^{13/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \; 252$$

$$\frac{(13bB-5Ac)\left(\frac{9\int \frac{x^{7/2}}{cx^2+b}dx}{4c} - \frac{x^{9/2}}{2c(b+cx^2)}\right)}{8bc} - \frac{x^{13/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{(13bB-5Ac)\left(\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\int \frac{x^{3/2}}{cx^2+b}dx}{c}\right)}{4c}-\frac{x^{9/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{13/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{(13bB-5Ac)\left(\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{b\int \frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{c}\right)}{c}\right)}{4c}-\frac{x^{9/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{13/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{(13bB-5Ac)\left(\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\int \frac{1}{cx^2+b}d\sqrt{x}}{c}\right)}{c}\right)}{4c}-\frac{x^{9/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{13/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{755}$$

3.207. $\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\frac{(13bB-5Ac)\left(\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{4c}-\frac{x^{9/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{13/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$\downarrow$ 1476

3.207. $\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\frac{(13bB-5Ac)\left(\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}\,d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}\,d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}\,d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{4c}-\frac{x^{9/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{13/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

3.207. $\int\frac{x^{23/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$\downarrow$ 1082

3.207. $\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\frac{(13bB-5Ac)\left(\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{4c}-\frac{x^{9/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{13/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

3.207. $\int\frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\downarrow 217$$

3.207. $\int \frac{x^{23/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$\frac{(13bB-5Ac)\left(\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{4c}-\frac{x^{9/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{13/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

3.207. $\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.207. $\displaystyle\int \frac{x^{23/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$\downarrow$ 1479

$$(13bB-5Ac)\left(\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\cdots}{2\sqrt{b}}\cdots}{c}\right)}{c}\cdots}{4c}\cdots$$

3.207. $\int \frac{x^{23/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$\downarrow 25$$

3.207. $\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$(13bB-5Ac)\left(\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(\cdots\right)}{\sqrt{\cdots}}}{2\sqrt{b}}\cdots\right)}{c}\cdots\right)}{c}\cdots\right)}{4c}\cdots\right.$$

3.207. $\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\Big\downarrow 27$$

3.207. $\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$(13bB-5Ac)\left(\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right.}{c}\right.}{c}\right.}{4c}\right.$$

3.207. $\int\frac{x^{23/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$\downarrow$ 1103

3.207. $\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\frac{(13bB-5Ac)\left(\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\cdots\right.}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{\cdots}}}{2\sqrt{b}}\cdots\right.}{c}\cdots\right.}{c}\cdots\right.}{4c}\cdots\right.}{8bc\cdots}$$

$$\frac{x^{13/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

3.207. $\int \frac{x^{23/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

input
```
Int[(x^(23/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*((b*B - A*c)*x^(13/2))/(b*c*(b + c*x^2)^2) + ((13*b*B - 5*A*c)*(-1/2*
x^(9/2)/(c*(b + c*x^2)) + (9*((2*x^(5/2))/(5*c) - (b*((2*Sqrt[x])/c - (2*b
*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4
))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/
4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + S
qrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/c))/c))/
(4*c)))/(8*b*c)
```

3.207.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.207. $\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.207.4 Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.55

method	result
risch	$\frac{2(Bc\,x^2+5Ac-15Bb)\sqrt{x}}{5c^4}-\frac{b\left(\frac{2\left(-\frac{17}{32}A\,c^2+\frac{25}{32}Bbc\right)x^{\frac{5}{2}}-\frac{b(13Ac-21Bb)\sqrt{x}}{16}}{(c\,x^2+b)^2}+\frac{9(5Ac-13Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+}\right.\right.}{}\right.}{c^4}$
derivativedivides	$\frac{\frac{2Bc\,x^{\frac{5}{2}}}{5}+2Ac\sqrt{x}-6bB\sqrt{x}}{c^4}-\frac{2b\left(\frac{\left(-\frac{17}{32}A\,c^2+\frac{25}{32}Bbc\right)x^{\frac{5}{2}}-\frac{b(13Ac-21Bb)\sqrt{x}}{32}}{(c\,x^2+b)^2}+\frac{9(5Ac-13Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+}\right.\right.}{}\right.}{c^4}$
default	$\frac{\frac{2Bc\,x^{\frac{5}{2}}}{5}+2Ac\sqrt{x}-6bB\sqrt{x}}{c^4}-\frac{2b\left(\frac{\left(-\frac{17}{32}A\,c^2+\frac{25}{32}Bbc\right)x^{\frac{5}{2}}-\frac{b(13Ac-21Bb)\sqrt{x}}{32}}{(c\,x^2+b)^2}+\frac{9(5Ac-13Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+}\right.\right.}{}\right.}{c^4}$

input
```
int(x^(23/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2/5*(B*c*x^2+5*A*c-15*B*b)*x^(1/2)/c^4-b/c^4*(2*((-17/32*A*c^2+25/32*B*b*c
)*x^(5/2)-1/32*b*(13*A*c-21*B*b)*x^(1/2))/(c*x^2+b)^2+9/128*(5*A*c-13*B*b)
*(1/c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2
))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*
b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

3.207.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 776, normalized size of antiderivative = 2.26

$$\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx =$$

$$-\frac{45\,(c^6x^4+2\,bc^5x^2+b^2c^4)\left(-\frac{28561\,B^4b^5-43940\,AB^3b^4c+25350\,A^2B^2b^3c^2-6500\,A^3Bb^2c^3+625\,A^4bc^4}{c^{17}}\right)^{\frac{1}{4}}\log\left(9\,c^4\left(-\frac{28561\,B^4b^5\cdots}{}\right.\right.}{}$$

input
```
integrate(x^(23/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
-1/320*(45*(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)*(-(28561*B^4*b^5 - 43940*A*B^
3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17
)^(1/4)*log(9*c^4*(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c + 25350*A^2*B^2*b^3
*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)^(1/4) - 9*(13*B*b - 5*A*c
)*sqrt(x)) + 45*(I*c^6*x^4 + 2*I*b*c^5*x^2 + I*b^2*c^4)*(-(28561*B^4*b^5 -
 43940*A*B^3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*
b*c^4)/c^17)^(1/4)*log(9*I*c^4*(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c + 2535
0*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)^(1/4) - 9*(1
3*B*b - 5*A*c)*sqrt(x)) + 45*(-I*c^6*x^4 - 2*I*b*c^5*x^2 - I*b^2*c^4)*(-(2
8561*B^4*b^5 - 43940*A*B^3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*
c^3 + 625*A^4*b*c^4)/c^17)^(1/4)*log(-9*I*c^4*(-(28561*B^4*b^5 - 43940*A*B
^3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^1
7)^(1/4) - 9*(13*B*b - 5*A*c)*sqrt(x)) - 45*(c^6*x^4 + 2*b*c^5*x^2 + b^2*c
^4)*(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^
3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)^(1/4)*log(-9*c^4*(-(28561*B^4*b^5 - 439
40*A*B^3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^
4)/c^17)^(1/4) - 9*(13*B*b - 5*A*c)*sqrt(x)) - 4*(32*B*c^3*x^6 - 32*(13*B*
b*c^2 - 5*A*c^3)*x^4 - 585*B*b^3 + 225*A*b^2*c - 81*(13*B*b^2*c - 5*A*b*c^
2)*x^2)*sqrt(x))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)
```

3.207.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(23/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

3.207.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.89

$$\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{(25\,Bb^2c-17\,Abc^2)x^{\frac{5}{2}}+(21\,Bb^3-13\,Ab^2c)\sqrt{x}}{16\,(c^6x^4+2\,bc^5x^2+b^2c^4)} + \frac{9\left(\frac{2\sqrt{2}(13\,Bb-5\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}(13\,Bb-5\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}(13\,Bb-5\,Ac)\log\left(\sqrt{2}\right.}{b^{\frac{3}{4}}c}\right.}{128\,c^4} + \frac{2\left(Bcx^{\frac{5}{2}}-5\,(3\,Bb-Ac)\sqrt{x}\right)}{5\,c^4}$$

input
```
integrate(x^(23/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/16*((25*B*b^2*c - 17*A*b*c^2)*x^(5/2) + (21*B*b^3 - 13*A*b^2*c)*sqrt(x)
)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4) + 9/128*(2*sqrt(2)*(13*B*b - 5*A*c)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b
)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(13*B*b - 5*A*c)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt
(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(13*B*b - 5*A*c)*l
og(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)
) - sqrt(2)*(13*B*b - 5*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c
)*x + sqrt(b))/(b^(3/4)*c^(1/4)))*b/c^4 + 2/5*(B*c*x^(5/2) - 5*(3*B*b - A*
c)*sqrt(x))/c^4
```

3.207.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.94

$$\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{9\sqrt{2}\left(13\,(bc^3)^{\frac{1}{4}}\,Bb-5\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,c^5}$$
$$+\frac{9\sqrt{2}\left(13\,(bc^3)^{\frac{1}{4}}\,Bb-5\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,c^5}$$
$$+\frac{9\sqrt{2}\left(13\,(bc^3)^{\frac{1}{4}}\,Bb-5\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,c^5}$$
$$-\frac{9\sqrt{2}\left(13\,(bc^3)^{\frac{1}{4}}\,Bb-5\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,c^5}$$
$$-\frac{25\,Bb^2cx^{\frac{5}{2}}-17\,Abc^2x^{\frac{5}{2}}+21\,Bb^3\sqrt{x}-13\,Ab^2c\sqrt{x}}{16\,(cx^2+b)^2c^4}$$
$$+\frac{2\left(Bc^{12}x^{\frac{5}{2}}-15\,Bbc^{11}\sqrt{x}+5\,Ac^{12}\sqrt{x}\right)}{5\,c^{15}}$$

input
```
integrate(x^(23/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
9/64*sqrt(2)*(13*(b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(
2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^5 + 9/64*sqrt(2)*(13*(
b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)
^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^5 + 9/128*sqrt(2)*(13*(b*c^3)^(1/4)*B*b
 - 5*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c
^5 - 9/128*sqrt(2)*(13*(b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*log(-sqrt(
2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 - 1/16*(25*B*b^2*c*x^(5/2) - 1
7*A*b*c^2*x^(5/2) + 21*B*b^3*sqrt(x) - 13*A*b^2*c*sqrt(x))/((c*x^2 + b)^2*
c^4) + 2/5*(B*c^12*x^(5/2) - 15*B*b*c^11*sqrt(x) + 5*A*c^12*sqrt(x))/c^15
```

3.207.9 Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 865, normalized size of antiderivative = 2.52

$$\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((x^(23/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
(x^(5/2)*((17*A*b*c^2)/16 - (25*B*b^2*c)/16) - x^(1/2)*((21*B*b^3)/16 - (1
3*A*b^2*c)/16))/(b^2*c^4 + c^6*x^4 + 2*b*c^5*x^2) + x^(1/2)*((2*A)/c^3 - (
6*B*b)/c^4) + (2*B*x^(5/2))/(5*c^3) + ((-b)^(1/4)*atan((((-b)^(1/4)*((81*x
^(1/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) - (81*(-b)
^(1/4)*(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*b^2*c))/(64*c^(21/4)))*(5*A*c - 13
*B*b)*9i)/(64*c^(17/4)) + ((-b)^(1/4)*((81*x^(1/2)*(169*B^2*b^4 + 25*A^2*b
^2*c^2 - 130*A*B*b^3*c))/(64*c^5) + (81*(-b)^(1/4)*(5*A*c - 13*B*b)*(13*B*
b^3 - 5*A*b^2*c))/(64*c^(21/4)))*(5*A*c - 13*B*b)*9i)/(64*c^(17/4)))/((9*(
-b)^(1/4)*((81*x^(1/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2 - 130*A*B*b^3*c))/(64
*c^5) - (81*(-b)^(1/4)*(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*b^2*c))/(64*c^(21/
4)))*(5*A*c - 13*B*b))/(64*c^(17/4)) - (9*(-b)^(1/4)*((81*x^(1/2)*(169*B^2
*b^4 + 25*A^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) + (81*(-b)^(1/4)*(5*A*c -
 13*B*b)*(13*B*b^3 - 5*A*b^2*c))/(64*c^(21/4)))*(5*A*c - 13*B*b))/(64*c^(1
7/4))))*(5*A*c - 13*B*b)*9i)/(32*c^(17/4)) + (9*(-b)^(1/4)*atan(((9*(-b)^(
1/4)*((81*x^(1/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5)
 - ((-b)^(1/4)*(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*b^2*c)*81i)/(64*c^(21/4)))
*(5*A*c - 13*B*b))/(64*c^(17/4)) + (9*(-b)^(1/4)*((81*x^(1/2)*(169*B^2*b^4
 + 25*A^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) + ((-b)^(1/4)*(5*A*c - 13*B*b
)*(13*B*b^3 - 5*A*b^2*c)*81i)/(64*c^(21/4)))*(5*A*c - 13*B*b))/(64*c^(17/4
)))/(((-b)^(1/4)*((81*x^(1/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2 - 130*A*B*b...
```

3.208 $\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$

3.208.1 Optimal result

Integrand size = 26, antiderivative size = 322

$$\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx = \frac{7(11bB-3Ac)x^{3/2}}{48bc^3} - \frac{(bB-Ac)x^{11/2}}{4bc(b+cx^2)^2} - \frac{(11bB-3Ac)x^{7/2}}{16bc^2(b+cx^2)} + \frac{7(11bB-3Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{15/4}} - \frac{7(11bB-3Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{15/4}} - \frac{7(11bB-3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{15/4}} + \frac{7(11bB-3Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{15/4}}$$

output
```
7/48*(-3*A*c+11*B*b)*x^(3/2)/b/c^3-1/4*(-A*c+B*b)*x^(11/2)/b/c/(c*x^2+b)^2
-1/16*(-3*A*c+11*B*b)*x^(7/2)/b/c^2/(c*x^2+b)+7/64*(-3*A*c+11*B*b)*arctan(
1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c^(15/4)*2^(1/2)-7/64*(-3*A*c+1
1*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c^(15/4)*2^(1/2)-
7/128*(-3*A*c+11*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2)
)/b^(1/4)/c^(15/4)*2^(1/2)+7/128*(-3*A*c+11*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1
/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(15/4)*2^(1/2)
```

3.208.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.57

$$\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{4c^{3/4}x^{3/2}(77b^2B-21Abc+121bBcx^2-33Ac^2x^2+32Bc^2x^4)}{(b+cx^2)^2}+\frac{21\sqrt{2}(11bB-3Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt[4]{b}}+}{192c^{15/4}}$$

input
```
Integrate[(x^(21/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
((4*c^(3/4)*x^(3/2)*(77*b^2*B - 21*A*b*c + 121*b*B*c*x^2 - 33*A*c^2*x^2 +
32*B*c^2*x^4))/(b + c*x^2)^2 + (21*Sqrt[2]*(11*b*B - 3*A*c)*ArcTan[(Sqrt[b
] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])]])/b^(1/4) + (21*Sqrt[2]*(
11*b*B - 3*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[
c]*x)])/b^(1/4))/(192*c^(15/4))
```

3.208.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {9, 362, 252, 262, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{x^{9/2}(A+Bx^2)}{(b+cx^2)^3}\,dx$$

$$\Big\downarrow \text{362}$$

$$\frac{(11bB-3Ac)\int \frac{x^{9/2}}{(cx^2+b)^2}dx}{8bc}-\frac{x^{11/2}(bB-Ac)}{4bc(b+cx^2)^2}$$

$$\Big\downarrow \text{252}$$

$$\frac{(11bB-3Ac)\left(\frac{7\int \frac{x^{5/2}}{cx^2+b}dx}{4c}-\frac{x^{7/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{11/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$\big\downarrow$ 262

$$\frac{(11bB-3Ac)\left(\frac{7\left(\frac{2x^{3/2}}{3c}-\frac{b\int \frac{\sqrt{x}}{cx^2+b}dx}{c}\right)}{4c}-\frac{x^{7/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{11/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$\big\downarrow$ 266

$$\frac{(11bB-3Ac)\left(\frac{7\left(\frac{2x^{3/2}}{3c}-\frac{2b\int \frac{x}{cx^2+b}d\sqrt{x}}{c}\right)}{4c}-\frac{x^{7/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{11/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$\big\downarrow$ 826

$$\frac{(11bB-3Ac)\left(\frac{7\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{4c}-\frac{x^{7/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{11/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$\big\downarrow$ 1476

3.208. $\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\frac{(11bB-3Ac)\left(\frac{7\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{4c}-\frac{x^{7/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{11/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{1082}$$

3.208. $\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\frac{(11bB-3Ac)\left(\frac{7\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{4c}-\frac{x^{7/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{11/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$\downarrow$ 217

3.208. $\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\frac{(11bB-3Ac)\left(\frac{7\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{4c}-\frac{x^{7/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{11/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 1479$$

3.208. $\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\frac{(11bB-3Ac)\left(\frac{7\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{\cdots}\right.}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}\cdots\right.}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\cdots}}{2\sqrt{c}}\cdots\right)}{c}\cdots\right)}{4c}\cdots\right)}{8bc}$$

$$\frac{x^{11/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$\downarrow$ 25

3.208. $\int \frac{x^{21/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$\frac{(11bB-3Ac)\left(\frac{7\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}\right)}{4c}\right)}{8bc}$$

$$\frac{x^{11/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$\downarrow$ 27

3.208. $\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\frac{(11bB-3Ac)\left(\frac{7\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{c}\right)}{4c}-\cdots\right)}{8bc}$$

$$\frac{x^{11/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$\big\downarrow$ 1103

3.208. $\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\frac{(11bB-3Ac)\left(\frac{7\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}\right)}{4c}\right)}{8bc}$$

$$\frac{x^{11/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

input
```
Int[(x^(21/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*((b*B - A*c)*x^(11/2))/(b*c*(b + c*x^2)^2) + ((11*b*B - 3*A*c)*(-1/2*
x^(7/2)/(c*(b + c*x^2)) + (7*((2*x^(3/2))/(3*c) - (2*b*((-(ArcTan[1 - (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sq
rt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (
-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b
^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/c))/(4*c)))/(8*b*c)
```

3.208. $\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.208.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

3.208. $\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

rule 362

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 826

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.208. $\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.208.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.54

method	result
derivativedivides	$\frac{2B\,x^{\frac{3}{2}}}{3c^{3}}+\frac{\frac{2\left(-\frac{c(11Ac-19Bb)x^{\frac{7}{2}}}{32}+\left(-\frac{7}{32}Abc+\frac{15}{32}B\,b^{2}\right)x^{\frac{3}{2}}\right)}{\left(c\,x^{2}+b\right)^{2}}+\frac{\left(\frac{21Ac}{32}-\frac{77Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{}}{\left(\frac{b}{c}\right)^{}}\right.\right.}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{c^{3}}$
default	$\frac{2B\,x^{\frac{3}{2}}}{3c^{3}}+\frac{\frac{2\left(-\frac{c(11Ac-19Bb)x^{\frac{7}{2}}}{32}+\left(-\frac{7}{32}Abc+\frac{15}{32}B\,b^{2}\right)x^{\frac{3}{2}}\right)}{\left(c\,x^{2}+b\right)^{2}}+\frac{\left(\frac{21Ac}{32}-\frac{77Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{}}{\left(\frac{b}{c}\right)^{}}\right.\right.}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{c^{3}}$
risch	$\frac{2B\,x^{\frac{3}{2}}}{3c^{3}}+\frac{\frac{-\frac{c(11Ac-19Bb)x^{\frac{7}{2}}}{16}+2\left(-\frac{7}{32}Abc+\frac{15}{32}B\,b^{2}\right)x^{\frac{3}{2}}}{\left(c\,x^{2}+b\right)^{2}}+\frac{\left(\frac{21Ac}{32}-\frac{77Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+\right.\right.}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{c^{3}}$

input
```
int(x^(21/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*B/c^3*x^(3/2)+2/c^3*((-1/32*c*(11*A*c-19*B*b)*x^(7/2)+(-7/32*A*b*c+15/
32*B*b^2)*x^(3/2))/(c*x^2+b)^2+1/8*(21/32*A*c-77/32*B*b)/c/(1/c*b)^(1/4)*2
^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4
)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1
)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

3.208.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 872, normalized size of antiderivative = 2.71

$$\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{21\left(c^5x^4+2\,bc^4x^2+b^2c^3\right)\left(-\frac{14641\,B^4b^4-15972\,AB^3b^3c+6534\,A^2B^2b^2c^2-1188\,A^3Bbc^3+81\,A^4c^4}{bc^{15}}\right)^{\frac{1}{4}}}{}$$

input
```
integrate(x^(21/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/192*(21*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-(14641*B^4*b^4 - 15972*A*B^3
*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(
1/4)*log(343*b*c^11*(-(14641*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^
2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(3/4) - 343*(1331*B^3*b^3
 - 1089*A*B^2*b^2*c + 297*A^2*B*b*c^2 - 27*A^3*c^3)*sqrt(x)) - 21*(I*c^5*x
^4 + 2*I*b*c^4*x^2 + I*b^2*c^3)*(-(14641*B^4*b^4 - 15972*A*B^3*b^3*c + 653
4*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(1/4)*log(343
*I*b*c^11*(-(14641*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 11
88*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(3/4) - 343*(1331*B^3*b^3 - 1089*A*
B^2*b^2*c + 297*A^2*B*b*c^2 - 27*A^3*c^3)*sqrt(x)) - 21*(-I*c^5*x^4 - 2*I*
b*c^4*x^2 - I*b^2*c^3)*(-(14641*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2
*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(1/4)*log(-343*I*b*c^1
1*(-(14641*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B
*b*c^3 + 81*A^4*c^4)/(b*c^15))^(3/4) - 343*(1331*B^3*b^3 - 1089*A*B^2*b^2*
c + 297*A^2*B*b*c^2 - 27*A^3*c^3)*sqrt(x)) - 21*(c^5*x^4 + 2*b*c^4*x^2 + b
^2*c^3)*(-(14641*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188
*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(1/4)*log(-343*b*c^11*(-(14641*B^4*b^
4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c
^4)/(b*c^15))^(3/4) - 343*(1331*B^3*b^3 - 1089*A*B^2*b^2*c + 297*A^2*B*b*c
^2 - 27*A^3*c^3)*sqrt(x)) + 4*(32*B*c^2*x^5 + 11*(11*B*b*c - 3*A*c^2)*x...
```

3.208.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(21/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

3.208.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.80

$$\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(19\,Bbc-11\,Ac^2)x^{\frac{7}{2}}+(15\,Bb^2-7\,Abc)x^{\frac{3}{2}}}{16\,(c^5x^4+2\,bc^4x^2+b^2c^3)}+\frac{2\,Bx^{\frac{3}{2}}}{3\,c^3} - \frac{7\,(11\,Bb-3\,Ac)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right.}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{128\,c^3}$$

input
```
integrate(x^(21/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/16*((19*B*b*c - 11*A*c^2)*x^(7/2) + (15*B*b^2 - 7*A*b*c)*x^(3/2))/(c^5*x
^4 + 2*b*c^4*x^2 + b^2*c^3) + 2/3*B*x^(3/2)/c^3 - 7/128*(11*B*b - 3*A*c)*(
2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))
/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*s
qrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(
1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(
2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/c^3
```

3.208.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.94

$$\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,c^3} + \frac{19\,Bbcx^{\frac{7}{2}} - 11\,Ac^2x^{\frac{7}{2}} + 15\,Bb^2x^{\frac{3}{2}} - 7\,Abcx^{\frac{3}{2}}}{16\,(cx^2+b)^2c^3}$$
$$- \frac{7\sqrt{2}\Big(11\,(bc^3)^{\frac{3}{4}}\,Bb - 3\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^6}$$
$$- \frac{7\sqrt{2}\Big(11\,(bc^3)^{\frac{3}{4}}\,Bb - 3\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^6}$$
$$+ \frac{7\sqrt{2}\Big(11\,(bc^3)^{\frac{3}{4}}\,Bb - 3\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{128\,bc^6}$$
$$- \frac{7\sqrt{2}\Big(11\,(bc^3)^{\frac{3}{4}}\,Bb - 3\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{128\,bc^6}$$

input
```
integrate(x^(21/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
2/3*B*x^(3/2)/c^3 + 1/16*(19*B*b*c*x^(7/2) - 11*A*c^2*x^(7/2) + 15*B*b^2*x
^(3/2) - 7*A*b*c*x^(3/2))/((c*x^2 + b)^2*c^3) - 7/64*sqrt(2)*(11*(b*c^3)^(
3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) +
2*sqrt(x))/(b/c)^(1/4))/(b*c^6) - 7/64*sqrt(2)*(11*(b*c^3)^(3/4)*B*b - 3*(
b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b
/c)^(1/4))/(b*c^6) + 7/128*sqrt(2)*(11*(b*c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)
*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^6) - 7/128*sqr
t(2)*(11*(b*c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/
c)^(1/4) + x + sqrt(b/c))/(b*c^6)
```

3.208.9 Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.43

$$\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{x^{3/2}\left(\frac{15\,B\,b^2}{16}-\frac{7\,A\,b\,c}{16}\right)-x^{7/2}\left(\frac{11\,A\,c^2}{16}-\frac{19\,B\,b\,c}{16}\right)}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4}+\frac{2\,B\,x^{3/2}}{3\,c^3} + \frac{7\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(3\,A\,c-11\,B\,b)}{32\,(-b)^{1/4}\,c^{15/4}}+\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right)\,(3\,A\,c-11\,B\,b)\,7\mathrm{i}}{32\,(-b)^{1/4}\,c^{15/4}}$$

input
```
int((x^(21/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
(x^(3/2)*((15*B*b^2)/16 - (7*A*b*c)/16) - x^(7/2)*((11*A*c^2)/16 - (19*B*b
*c)/16))/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + (2*B*x^(3/2))/(3*c^3) + (7*at
an((c^(1/4)*x^(1/2))/(-b)^(1/4))*(3*A*c - 11*B*b))/(32*(-b)^(1/4)*c^(15/4)
) + (atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*(3*A*c - 11*B*b)*7i)/(32*(-b)^(
1/4)*c^(15/4))
```

3.209 $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.209.1 Optimal result

Integrand size = 26, antiderivative size = 322

$$\begin{aligned}\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= \frac{5(9bB-Ac)\sqrt{x}}{16bc^3}-\frac{(bB-Ac)x^{9/2}}{4bc\,(b+cx^2)^2}-\frac{(9bB-Ac)x^{5/2}}{16bc^2\,(b+cx^2)}\\&+\frac{5(9bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{3/4}c^{13/4}}-\frac{5(9bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{3/4}c^{13/4}}\\&+\frac{5(9bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{13/4}}\\&-\frac{5(9bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{13/4}}\end{aligned}$$

output
```
-1/4*(-A*c+B*b)*x^(9/2)/b/c/(c*x^2+b)^2-1/16*(-A*c+9*B*b)*x^(5/2)/b/c^2/(c
*x^2+b)+5/64*(-A*c+9*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4
)/c^(13/4)*2^(1/2)-5/64*(-A*c+9*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1
/4))/b^(3/4)/c^(13/4)*2^(1/2)+5/128*(-A*c+9*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1
/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(13/4)*2^(1/2)-5/128*(-A*c+9*B*b)*l
n(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(13/4)*2^(1
/2)+5/16*(-A*c+9*B*b)*x^(1/2)/b/c^3
```

3.209.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.57

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{4\sqrt[4]{c}\sqrt{x}(45b^2B-5Abc+81bBcx^2-9Ac^2x^2+32Bc^2x^4)}{(b+cx^2)^2}+\frac{5\sqrt{2}(9bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{b^{3/4}}-\frac{5\sqrt{2}(9b\cdots}{}}{64c^{13/4}}$$

input
```
Integrate[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
((4*c^(1/4)*Sqrt[x]*(45*b^2*B - 5*A*b*c + 81*b*B*c*x^2 - 9*A*c^2*x^2 + 32*
B*c^2*x^4))/(b + c*x^2)^2 + (5*Sqrt[2]*(9*b*B - A*c)*ArcTan[(Sqrt[b] - Sqr
t[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/b^(3/4) - (5*Sqrt[2]*(9*b*B -
A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/b^(
3/4))/(64*c^(13/4))
```

3.209.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {9, 362, 252, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^{7/2}(A+Bx^2)}{(b+cx^2)^3}dx$$

$$\Big\downarrow \ 362$$

$$\frac{(9bB-Ac)\int \frac{x^{7/2}}{(cx^2+b)^2}dx}{8bc}-\frac{x^{9/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \ 252$$

$$\frac{(9bB-Ac)\left(\frac{5\int\frac{x^{3/2}}{cx^2+b}dx}{4c}-\frac{x^{5/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{9/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \; 262$$

$$\frac{(9bB-Ac)\left(\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{b\int\frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{c}\right)}{4c}-\frac{x^{5/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{9/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \; 266$$

$$\frac{(9bB-Ac)\left(\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\int\frac{1}{cx^2+b}d\sqrt{x}}{c}\right)}{4c}-\frac{x^{5/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{9/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \; 755$$

$$\frac{(9bB-Ac)\left(\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{c}\right)}{4c}-\frac{x^{5/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{9/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \; 1476$$

3.209. $\int\frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\frac{(9bB-Ac)\left(\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{c}\right)}{4c}-\frac{x^{5/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{9/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 1082$$

3.209. $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\frac{(9bB-Ac)\left(\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4c}-\frac{x^{5/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{9/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$\downarrow$ 217

3.209. $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\frac{(9bB-Ac)\left(\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4c}-\frac{x^{5/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{9/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{1479}$$

3.209. $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\frac{(9bB-Ac)\left(\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{\cdots}}\cdots}{2\sqrt{b}}\cdots\right)}{c}\cdots\right)}{4c}\cdots\right)}{8bc}\cdots$$

$$\frac{x^{9/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$\big\downarrow$ 25

3.209. $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\frac{(9bB-Ac)\left(\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4c}\right)}{8bc}$$

$$\frac{x^{9/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 27$$

3.209. $\int \frac{x^{19/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$\frac{(9bB-Ac)\left(\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4c}-\frac{x^5}{2c(b\cdots}\right)}{8bc}$$

$$\frac{x^{9/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$\downarrow$ 1103

3.209. $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$(9bB-Ac)\frac{\left(\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4c}\right.}{8bc}$$

$$\frac{x^{9/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

input
```
Int[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*((b*B - A*c)*x^(9/2))/(b*c*(b + c*x^2)^2) + ((9*b*B - A*c)*(-1/2*x^(5
/2)/(c*(b + c*x^2)) + (5*((2*Sqrt[x])/c - (2*b*((-(ArcTan[1 - (Sqrt[2]*c^(
1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^
(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log
[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c
^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sq
rt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/c))/(4*c)))/(8*b*c)
```

3.209. $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.209.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

3.209. $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.209. $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.209.4 Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.53

method	result
derivativedivides	$\frac{2B\sqrt{x}}{c^3}+\frac{\frac{2\left(\left(-\frac{9}{32}A\,c^2+\frac{17}{32}Bbc\right)x^{\frac{5}{2}}-\frac{b(5Ac-13Bb)\sqrt{x}}{32}\right)}{\left(c\,x^2+b\right)^2}+\frac{5(Ac-9Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{128b}}{c^3}$
default	$\frac{2B\sqrt{x}}{c^3}+\frac{\frac{2\left(\left(-\frac{9}{32}A\,c^2+\frac{17}{32}Bbc\right)x^{\frac{5}{2}}-\frac{b(5Ac-13Bb)\sqrt{x}}{32}\right)}{\left(c\,x^2+b\right)^2}+\frac{5(Ac-9Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{128b}}{c^3}$
risch	$\frac{2B\sqrt{x}}{c^3}+\frac{\frac{2\left(-\frac{9}{32}A\,c^2+\frac{17}{32}Bbc\right)x^{\frac{5}{2}}-\frac{b(5Ac-13Bb)\sqrt{x}}{16}}{\left(c\,x^2+b\right)^2}+\frac{5(Ac-9Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right.\right.}{128b}}{c^3}$

input
```
int(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*B/c^3*x^(1/2)+2/c^3*(((-9/32*A*c^2+17/32*B*b*c)*x^(5/2)-1/32*b*(5*A*c-13
*B*b)*x^(1/2))/(c*x^2+b)^2+5/256*(A*c-9*B*b)*(1/c*b)^(1/4)/b*2^(1/2)*(ln((
x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^
(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2
^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

3.209.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 748, normalized size of antiderivative = 2.32

$$\int \frac{x^{19/2}(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx = \frac{5\left(c^5x^4+2\,bc^4x^2+b^2c^3\right)\left(-\frac{6561\,B^4b^4-2916\,AB^3b^3c+486\,A^2B^2b^2c^2-36\,A^3Bbc^3+A^4c^4}{b^3c^{13}}\right)^{\frac{1}{4}}\log\left(5\,b\right.}{}$$

input
```
integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/64*(5*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-(6561*B^4*b^4 - 2916*A*B^3*b^3
*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/4)*log
(5*b*c^3*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3
*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/4) - 5*(9*B*b - A*c)*sqrt(x)) - 5*(-I*c
^5*x^4 - 2*I*b*c^4*x^2 - I*b^2*c^3)*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 4
86*A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/4)*log(5*I*b
*c^3*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*B*b
*c^3 + A^4*c^4)/(b^3*c^13))^(1/4) - 5*(9*B*b - A*c)*sqrt(x)) - 5*(I*c^5*x^
4 + 2*I*b*c^4*x^2 + I*b^2*c^3)*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^
2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/4)*log(-5*I*b*c^3
*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3
 + A^4*c^4)/(b^3*c^13))^(1/4) - 5*(9*B*b - A*c)*sqrt(x)) - 5*(c^5*x^4 + 2*
b*c^4*x^2 + b^2*c^3)*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*
c^2 - 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/4)*log(-5*b*c^3*(-(6561*B^4
*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)/
(b^3*c^13))^(1/4) - 5*(9*B*b - A*c)*sqrt(x)) + 4*(32*B*c^2*x^4 + 45*B*b^2
- 5*A*b*c + 9*(9*B*b*c - A*c^2)*x^2)*sqrt(x))/(c^5*x^4 + 2*b*c^4*x^2 + b^2
*c^3)
```

3.209.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(19/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

3.209.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.88

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(17\,Bbc-9\,Ac^2)x^{\frac{5}{2}}+(13\,Bb^2-5\,Abc)\sqrt{x}}{16\,(c^5x^4+2\,bc^4x^2+b^2c^3)}+\frac{2\,B\sqrt{x}}{c^3} - \frac{5\left(\frac{2\sqrt{2}(9\,Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(9\,Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(9\,Bb-Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{\right.}}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right.}{128\,c^3}$$

input
```
integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/16*((17*B*b*c - 9*A*c^2)*x^(5/2) + (13*B*b^2 - 5*A*b*c)*sqrt(x))/(c^5*x^
4 + 2*b*c^4*x^2 + b^2*c^3) + 2*B*sqrt(x)/c^3 - 5/128*(2*sqrt(2)*(9*B*b - A
*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(
sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(9*B*b - A*c
)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(s
qrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(9*B*b - A*c)*l
og(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)
) - sqrt(2)*(9*B*b - A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x
 + sqrt(b))/(b^(3/4)*c^(1/4)))/c^3
```

3.209.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.94

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{2B\sqrt{x}}{c^3} - \frac{5\sqrt{2}\left(9\,(bc^3)^{\frac14}Bb-(bc^3)^{\frac14}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac14}}\right)}{64\,bc^4} - \frac{5\sqrt{2}\left(9\,(bc^3)^{\frac14}Bb-(bc^3)^{\frac14}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac14}}\right)}{64\,bc^4} - \frac{5\sqrt{2}\left(9\,(bc^3)^{\frac14}Bb-(bc^3)^{\frac14}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac14}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^4} + \frac{5\sqrt{2}\left(9\,(bc^3)^{\frac14}Bb-(bc^3)^{\frac14}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac14}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^4} + \frac{17\,Bbcx^{\frac52}-9\,Ac^2x^{\frac52}+13\,Bb^2\sqrt{x}-5\,Abc\sqrt{x}}{16\,(cx^2+b)^2c^3}$$

input
```
integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
2*B*sqrt(x)/c^3 - 5/64*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*a
rctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) -
 5/64*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) - 5/128*sqrt(2)*(
9*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) +
 x + sqrt(b/c))/(b*c^4) + 5/128*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/
4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^4) + 1/16*(
17*B*b*c*x^(5/2) - 9*A*c^2*x^(5/2) + 13*B*b^2*sqrt(x) - 5*A*b*c*sqrt(x))/(
(c*x^2 + b)^2*c^3)
```

3.209.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 760, normalized size of antiderivative = 2.36

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\sqrt{x}\left(\frac{13\,B\,b^2}{16}-\frac{5\,A\,b\,c}{16}\right)-x^{5/2}\left(\frac{9\,A\,c^2}{16}-\frac{17\,B\,b\,c}{16}\right)}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4}+\frac{2\,B\,\sqrt{x}}{c^3}$$

$$-\frac{\operatorname{atan}\left(\frac{\frac{(A\,c-9\,B\,b)\left(\frac{25\,\sqrt{x}\,(A^2\,c^2-18\,A\,B\,b\,c+81\,B^2\,b^2)}{64\,c^3}-\frac{5\,(45\,B\,b^2-5\,A\,b\,c)\,(A\,c-9\,B\,b)}{64\,(-b)^{3/4}\,c^{13/4}}\right)5\mathrm{i}}{64\,(-b)^{3/4}\,c^{13/4}}+\frac{(A\,c-9\,B\,b)\left(\frac{25\,\sqrt{x}\,(A^2\,c^2-18\,A\,B\,b\,c+81\,B^2\,b^2)}{64\,c^3}+\frac{5\,(45\,B\ldots}{\ldots}\right.}{64\,(-b)^{3/4}\,c^{13/4}}}{\frac{5\,(A\,c-9\,B\,b)\left(\frac{25\,\sqrt{x}\,(A^2\,c^2-18\,A\,B\,b\,c+81\,B^2\,b^2)}{64\,c^3}-\frac{5\,(45\,B\,b^2-5\,A\,b\,c)\,(A\,c-9\,B\,b)}{64\,(-b)^{3/4}\,c^{13/4}}\right)}{64\,(-b)^{3/4}\,c^{13/4}}-\frac{5\,(A\,c-9\,B\,b)\left(\frac{25\,\sqrt{x}\,(A^2\,c^2-18\,A\,B\,b\,c+81\,B^2\,b^2)}{64\,c^3}+\frac{5\,(45\ldots}{\ldots}\right.}{64\,(-b)^{3/4}\,c^{13/4}}}\right.}{32\,(-b)^{3/4}\,c^{13/4}}$$

$$-\frac{5\operatorname{atan}\left(\frac{\frac{5\,(A\,c-9\,B\,b)\left(\frac{25\,\sqrt{x}\,(A^2\,c^2-18\,A\,B\,b\,c+81\,B^2\,b^2)}{64\,c^3}-\frac{(45\,B\,b^2-5\,A\,b\,c)\,(A\,c-9\,B\,b)\,5\mathrm{i}}{64\,(-b)^{3/4}\,c^{13/4}}\right)}{64\,(-b)^{3/4}\,c^{13/4}}+\frac{5\,(A\,c-9\,B\,b)\left(\frac{25\,\sqrt{x}\,(A^2\,c^2-18\,A\,B\,b\,c+81\,B^2\,b^2)}{64\,c^3}+\frac{(45\ldots}{\ldots}\right.}{64\,(-b)^{3/4}\,c^{13/4}}}{\frac{(A\,c-9\,B\,b)\left(\frac{25\,\sqrt{x}\,(A^2\,c^2-18\,A\,B\,b\,c+81\,B^2\,b^2)}{64\,c^3}-\frac{(45\,B\,b^2-5\,A\,b\,c)\,(A\,c-9\,B\,b)\,5\mathrm{i}}{64\,(-b)^{3/4}\,c^{13/4}}\right)5\mathrm{i}}{64\,(-b)^{3/4}\,c^{13/4}}-\frac{(A\,c-9\,B\,b)\left(\frac{25\,\sqrt{x}\,(A^2\,c^2-18\,A\,B\,b\,c+81\,B^2\,b^2)}{64\,c^3}+\frac{(45\ldots}{\ldots}\right.}{64\,(-b)^{3/4}\,c^{13/4}}}\right.}{32\,(-b)^{3/4}\,c^{13/4}}$$

input
```
int((x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
(x^(1/2)*((13*B*b^2)/16 - (5*A*b*c)/16) - x^(5/2)*((9*A*c^2)/16 - (17*B*b*
c)/16))/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + (2*B*x^(1/2))/c^3 - (atan((((A
*c - 9*B*b)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) - (
5*(45*B*b^2 - 5*A*b*c)*(A*c - 9*B*b))/(64*(-b)^(3/4)*c^(13/4)))*5i)/(64*(-
b)^(3/4)*c^(13/4)) + ((A*c - 9*B*b)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 - 1
8*A*B*b*c))/(64*c^3) + (5*(45*B*b^2 - 5*A*b*c)*(A*c - 9*B*b))/(64*(-b)^(3/
4)*c^(13/4)))*5i)/(64*(-b)^(3/4)*c^(13/4)))/((5*(A*c - 9*B*b)*((25*x^(1/2)
*(A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) - (5*(45*B*b^2 - 5*A*b*c)*(
A*c - 9*B*b))/(64*(-b)^(3/4)*c^(13/4))))/(64*(-b)^(3/4)*c^(13/4)) - (5*(A*
c - 9*B*b)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) + (5
*(45*B*b^2 - 5*A*b*c)*(A*c - 9*B*b))/(64*(-b)^(3/4)*c^(13/4))))/(64*(-b)^(
3/4)*c^(13/4))))*(A*c - 9*B*b)*5i)/(32*(-b)^(3/4)*c^(13/4)) - (5*atan(((5*
(A*c - 9*B*b)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) -
 ((45*B*b^2 - 5*A*b*c)*(A*c - 9*B*b)*5i)/(64*(-b)^(3/4)*c^(13/4))))/(64*(-
b)^(3/4)*c^(13/4)) + (5*(A*c - 9*B*b)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 -
 18*A*B*b*c))/(64*c^3) + ((45*B*b^2 - 5*A*b*c)*(A*c - 9*B*b)*5i)/(64*(-b)^
(3/4)*c^(13/4))))/(64*(-b)^(3/4)*c^(13/4)))/(((A*c - 9*B*b)*((25*x^(1/2)*(
A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) - ((45*B*b^2 - 5*A*b*c)*(A*c
- 9*B*b)*5i)/(64*(-b)^(3/4)*c^(13/4)))*5i)/(64*(-b)^(3/4)*c^(13/4)) - ((A*
c - 9*B*b)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) +...
```

3.210 $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.210.1 Optimal result

Integrand size = 26, antiderivative size = 293

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{(bB-Ac)x^{7/2}}{4bc\,(b+cx^2)^2} - \frac{(7bB+Ac)x^{3/2}}{16bc^2\,(b+cx^2)} - \frac{3(7bB+Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{5/4}c^{11/4}} + \frac{3(7bB+Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{5/4}c^{11/4}} + \frac{3(7bB+Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{11/4}} - \frac{3(7bB+Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{11/4}}$$

output
```
-1/4*(-A*c+B*b)*x^(7/2)/b/c/(c*x^2+b)^2-1/16*(A*c+7*B*b)*x^(3/2)/b/c^2/(c*
x^2+b)-3/64*(A*c+7*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/
c^(11/4)*2^(1/2)+3/64*(A*c+7*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4)
)/b^(5/4)/c^(11/4)*2^(1/2)+3/128*(A*c+7*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*
c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/c^(11/4)*2^(1/2)-3/128*(A*c+7*B*b)*ln(b^(
1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/c^(11/4)*2^(1/2)
```

3.210.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.58

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{-\frac{4\sqrt[4]{b}c^{3/4}x^{3/2}(7b^2B-3Ac^2x^2+bc(A+11Bx^2))}{(b+cx^2)^2}-3\sqrt{2}(7bB+Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)-3}{64b^{5/4}c^{11/4}}$$

input
```
Integrate[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
((-4*b^(1/4)*c^(3/4)*x^(3/2)*(7*b^2*B - 3*A*c^2*x^2 + b*c*(A + 11*B*x^2)))
/(b + c*x^2)^2 - 3*Sqrt[2]*(7*b*B + A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqr
t[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 3*Sqrt[2]*(7*b*B + A*c)*ArcTanh[(Sqrt[2]*
b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(64*b^(5/4)*c^(11/4))
```

3.210.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {9, 362, 252, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^{5/2}(A+Bx^2)}{(b+cx^2)^3}dx$$

$$\Big\downarrow \; 362$$

$$\frac{(Ac+7bB)\int \frac{x^{5/2}}{(cx^2+b)^2}dx}{8bc}-\frac{x^{7/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \; 252$$

$$\frac{(Ac+7bB)\left(\frac{3\int \frac{\sqrt{x}}{cx^2+b}dx}{4c}-\frac{x^{3/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{7/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{(Ac+7bB)\left(\frac{3\int\frac{x}{cx^2+b}d\sqrt{x}}{2c}-\frac{x^{3/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{7/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{(Ac+7bB)\left(\frac{3\left(\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{7/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{(Ac+7bB)\left(\frac{3\left(\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{7/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(Ac+7bB)\left(\frac{3\left(\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{7/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

3.210. $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$\downarrow$ 217

$$\frac{(Ac+7bB)\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{7/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$\downarrow$ 1479

$$\frac{(Ac+7bB)\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{2c}\right)}{8bc}$$

$$\frac{x^{7/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$\downarrow$ 25

3.210. $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\frac{(Ac+7bB)\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{2c}-\frac{}{2c}\right)}{8bc}$$

$$\frac{x^{7/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$\downarrow$ 27

$$\frac{(Ac+7bB)\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c(b+cx^2)}\right)}{8bc}-$$

$$\frac{x^{7/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$\downarrow$ 1103

3.210. $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\frac{(Ac+7bB)\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{2c}-\frac{x^3}{2c(b+}\right.}{8bc}$$

$$\frac{x^{7/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

input
```
Int[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*((b*B - A*c)*x^(7/2))/(b*c*(b + c*x^2)^2) + ((7*b*B + A*c)*(-1/2*x^(3
/2)/(c*(b + c*x^2)) + (3*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)
]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b]
+ Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4))
)/(2*Sqrt[c])))/(2*c)))/(8*b*c)
```

3.210.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.210. $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.210.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.57

method	result
derivativedivides	$\frac{\frac{(3Ac-11Bb)x^{\frac{7}{2}}}{16bc}-\frac{(Ac+7Bb)x^{\frac{3}{2}}}{16c^2}}{(c\,x^2+b)^2}+\frac{3(Ac+7Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{128c^3b\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$\frac{\frac{(3Ac-11Bb)x^{\frac{7}{2}}}{16bc}-\frac{(Ac+7Bb)x^{\frac{3}{2}}}{16c^2}}{(c\,x^2+b)^2}+\frac{3(Ac+7Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{128c^3b\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

input
```
int(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/32*(3*A*c-11*B*b)/b/c*x^(7/2)-1/32*(A*c+7*B*b)/c^2*x^(3/2))/(c*x^2+b)
^2+3/128*(A*c+7*B*b)/c^3/b/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1
/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))
)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)
*x^(1/2)-1))
```

3.210.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 871, normalized size of antiderivative = 2.97

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3\left(bc^4x^4+2\,b^2c^3x^2+b^3c^2\right)\left(-\frac{2401\,B^4b^4+1372\,AB^3b^3c+294\,A^2B^2b^2c^2+28\,A^3Bbc^3+A^4c^4}{b^5c^{11}}\right)^{\frac{1}{4}}\log\left(\right.}{}$$

input
```
integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/64*(3*(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(2401*B^4*b^4 + 1372*A*B^3
*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(1/4)
*log(27*b^4*c^8*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 +
 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(3/4) + 27*(343*B^3*b^3 + 147*A*B^2
*b^2*c + 21*A^2*B*b*c^2 + A^3*c^3)*sqrt(x)) - 3*(I*b*c^4*x^4 + 2*I*b^2*c^3
*x^2 + I*b^3*c^2)*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2
 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(1/4)*log(27*I*b^4*c^8*(-(2401*B^
4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)
/(b^5*c^11))^(3/4) + 27*(343*B^3*b^3 + 147*A*B^2*b^2*c + 21*A^2*B*b*c^2 +
A^3*c^3)*sqrt(x)) - 3*(-I*b*c^4*x^4 - 2*I*b^2*c^3*x^2 - I*b^3*c^2)*(-(2401
*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c
^4)/(b^5*c^11))^(1/4)*log(-27*I*b^4*c^8*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c
 + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(3/4) + 27*
(343*B^3*b^3 + 147*A*B^2*b^2*c + 21*A^2*B*b*c^2 + A^3*c^3)*sqrt(x)) - 3*(b
*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 2
94*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(1/4)*log(-27*b
^4*c^8*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B
*b*c^3 + A^4*c^4)/(b^5*c^11))^(3/4) + 27*(343*B^3*b^3 + 147*A*B^2*b^2*c +
21*A^2*B*b*c^2 + A^3*c^3)*sqrt(x)) - 4*((11*B*b*c - 3*A*c^2)*x^3 + (7*B*b^
2 + A*b*c)*x)*sqrt(x))/(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)
```

3.210.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(17/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

3.210.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.86

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{(11\,Bbc-3\,Ac^2)x^{\frac{7}{2}}+(7\,Bb^2+Abc)x^{\frac{3}{2}}}{16\,(bc^4x^4+2\,b^2c^3x^2+b^3c^2)} + \frac{3\,(7\,Bb+Ac)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\right.}{128\,bc^2}$$

input
```
integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/16*((11*B*b*c - 3*A*c^2)*x^(7/2) + (7*B*b^2 + A*b*c)*x^(3/2))/(b*c^4*x^
4 + 2*b^2*c^3*x^2 + b^3*c^2) + 3/128*(7*B*b + A*c)*(2*sqrt(2)*arctan(1/2*s
qrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c))
)/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)
*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)
*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)
*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqr
t(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/(b*c^2)
```

3.210.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{11\,Bbcx^{\frac{7}{2}} - 3\,Ac^2x^{\frac{7}{2}} + 7\,Bb^2x^{\frac{3}{2}} + Abcx^{\frac{3}{2}}}{16\,(cx^2+b)^2bc^2} + \frac{3\,\sqrt{2}\left(7\,(bc^3)^{\frac{3}{4}}\,Bb + (bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^5} + \frac{3\,\sqrt{2}\left(7\,(bc^3)^{\frac{3}{4}}\,Bb + (bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^5} - \frac{3\,\sqrt{2}\left(7\,(bc^3)^{\frac{3}{4}}\,Bb + (bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{128\,b^2c^5} + \frac{3\,\sqrt{2}\left(7\,(bc^3)^{\frac{3}{4}}\,Bb + (bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{128\,b^2c^5}$$

input
```
integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-1/16*(11*B*b*c*x^(7/2) - 3*A*c^2*x^(7/2) + 7*B*b^2*x^(3/2) + A*b*c*x^(3/2
))/((c*x^2 + b)^2*b*c^2) + 3/64*sqrt(2)*(7*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/
4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/
(b^2*c^5) + 3/64*sqrt(2)*(7*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*arctan(
-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^5) - 3/
128*sqrt(2)*(7*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*
(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^5) + 3/128*sqrt(2)*(7*(b*c^3)^(3/4)*B*
b + (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(
b^2*c^5)
```

3.210.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.42

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(A\,c+7\,B\,b)}{32\,(-b)^{5/4}\,c^{11/4}} - \frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(A\,c+7\,B\,b)}{32\,(-b)^{5/4}\,c^{11/4}} - \frac{\frac{x^{3/2}\,(A\,c+7\,B\,b)}{16\,c^2} - \frac{x^{7/2}\,(3\,A\,c-11\,B\,b)}{16\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

input
```
int((x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
(3*atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c + 7*B*b))/(32*(-b)^(5/4)*c^(11
/4)) - (3*atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c + 7*B*b))/(32*(-b)^(5/4)
*c^(11/4)) - ((x^(3/2)*(A*c + 7*B*b))/(16*c^2) - (x^(7/2)*(3*A*c - 11*B*b)
)/(16*b*c))/(b^2 + c^2*x^4 + 2*b*c*x^2)
```

3.211 $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.211.1 Optimal result

Integrand size = 26, antiderivative size = 298

$$\begin{aligned}\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= -\frac{(bB-Ac)x^{5/2}}{4bc\,(b+cx^2)^2}-\frac{(5bB+3Ac)\sqrt{x}}{16bc^2\,(b+cx^2)}\\ &-\frac{(5bB+3Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{7/4}c^{9/4}}+\frac{(5bB+3Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{7/4}c^{9/4}}\\ &-\frac{(5bB+3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{9/4}}\\ &+\frac{(5bB+3Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{9/4}}\end{aligned}$$

output
```
-1/4*(-A*c+B*b)*x^(5/2)/b/c/(c*x^2+b)^2-1/64*(3*A*c+5*B*b)*arctan(1-c^(1/4
)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c^(9/4)*2^(1/2)+1/64*(3*A*c+5*B*b)*arct
an(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c^(9/4)*2^(1/2)-1/128*(3*A*c
+5*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(9
/4)*2^(1/2)+1/128*(3*A*c+5*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/
2)*x^(1/2))/b^(7/4)/c^(9/4)*2^(1/2)-1/16*(3*A*c+5*B*b)*x^(1/2)/b/c^2/(c*x^
2+b)
```

3.211.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.58

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{-\frac{4b^{3/4}\sqrt[4]{c}\sqrt{x}(5b^2B-Ac^2x^2+3bc(A+3Bx^2))}{(b+cx^2)^2}-\sqrt{2}(5bB+3Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+\sqrt{2}}{64b^{7/4}c^{9/4}}$$

input
```
Integrate[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
((-4*b^(3/4)*c^(1/4)*Sqrt[x]*(5*b^2*B - A*c^2*x^2 + 3*b*c*(A + 3*B*x^2)))/
(b + c*x^2)^2 - Sqrt[2]*(5*b*B + 3*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + Sqrt[2]*(5*b*B + 3*A*c)*ArcTanh[(Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(64*b^(7/4)*c^(9/4))
```

3.211.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {9, 362, 252, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^{3/2}(A+Bx^2)}{(b+cx^2)^3}dx$$

$$\Big\downarrow \ 362$$

$$\frac{(3Ac+5bB)\int \frac{x^{3/2}}{(cx^2+b)^2}dx}{8bc}-\frac{x^{5/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \ 252$$

$$\frac{(3Ac+5bB)\left(\frac{\int \frac{1}{\sqrt{x}(cx^2+b)}dx}{4c}-\frac{\sqrt{x}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{5/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{(3Ac+5bB)\left(\frac{\int \frac{1}{cx^2+b}d\sqrt{x}}{2c}-\frac{\sqrt{x}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{5/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{755}$$

$$\frac{(3Ac+5bB)\left(\frac{\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{5/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{(3Ac+5bB)\left(\frac{\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{5/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(3Ac+5bB)\left(\frac{\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{5/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{(3Ac+5bB)\left(\frac{\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c(b+cx^2)}\right)}{8bc}-\frac{x^{5/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

3.211. $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\Big\downarrow \text{1479}$$

$$\frac{(3Ac+5bB)\left(\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}}{2c}-\ldots\right)}{8bc}\ldots$$

$$\frac{x^{5/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(3Ac+5bB)\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}}{2c}-\frac{\sqrt{\ldots}}{2c(b+\ldots}\right)}{8bc}\ldots$$

$$\frac{x^{5/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(3Ac+5bB)\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c(b+cx^2)}\right)}{8bc}-$$

$$\frac{x^{5/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

3.211. $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\downarrow \text{1103}$$

$$\frac{(3Ac+5bB)\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c(b+cx^2}\right.}{8bc}$$

$$\frac{x^{5/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

input
```
Int[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*((b*B - A*c)*x^(5/2))/(b*c*(b + c*x^2)^2) + ((5*b*B + 3*A*c)*(-1/2*Sq
rt[x]/(c*(b + c*x^2)) + ((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/
(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] +
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))
/(2*Sqrt[b]))/(2*c)))/(8*b*c)
```

3.211.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.211. $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.211.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.56

method	result
derivativedivides	$\frac{\frac{(Ac-9Bb)x^{\frac{5}{2}}}{16bc}-\frac{(3Ac+5Bb)\sqrt{x}}{16c^2}}{(cx^2+b)^2}+\frac{(3Ac+5Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{128c^2b^2}$
default	$\frac{\frac{(Ac-9Bb)x^{\frac{5}{2}}}{16bc}-\frac{(3Ac+5Bb)\sqrt{x}}{16c^2}}{(cx^2+b)^2}+\frac{(3Ac+5Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{128c^2b^2}$

input
```
int(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/32*(A*c-9*B*b)/b/c*x^(5/2)-1/32*(3*A*c+5*B*b)/c^2*x^(1/2))/(c*x^2+b)^
2+1/128*(3*A*c+5*B*b)/c^2/b^2*(1/c*b)^(1/4)*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x
^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/
2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1
/4)*x^(1/2)-1))
```

3.211.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 763, normalized size of antiderivative = 2.56

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(bc^4x^4+2\,b^2c^3x^2+b^3c^2)\left(-\frac{625\,B^4b^4+1500\,AB^3b^3c+1350\,A^2B^2b^2c^2+540\,A^3Bbc^3+81\,A^4c^4}{b^7c^9}\right)^{\frac{1}{4}}\log\left(\right.}{}$$

input
```
integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/64*((b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(625*B^4*b^4 + 1500*A*B^3*b^
3*c + 1350*A^2*B^2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4
)*log(b^2*c^2*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 5
40*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4) + (5*B*b + 3*A*c)*sqrt(x)) -
 (-I*b*c^4*x^4 - 2*I*b^2*c^3*x^2 - I*b^3*c^2)*(-(625*B^4*b^4 + 1500*A*B^3*
b^3*c + 1350*A^2*B^2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1
/4)*log(I*b^2*c^2*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2
 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4) + (5*B*b + 3*A*c)*sqrt(x
)) - (I*b*c^4*x^4 + 2*I*b^2*c^3*x^2 + I*b^3*c^2)*(-(625*B^4*b^4 + 1500*A*B
^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))
^(1/4)*log(-I*b^2*c^2*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2*b^2
*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4) + (5*B*b + 3*A*c)*sq
rt(x)) - (b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(625*B^4*b^4 + 1500*A*B^3
*b^3*c + 1350*A^2*B^2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(
1/4)*log(-b^2*c^2*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2
 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4) + (5*B*b + 3*A*c)*sqrt(x
)) - 4*(5*B*b^2 + 3*A*b*c + (9*B*b*c - A*c^2)*x^2)*sqrt(x))/(b*c^4*x^4 + 2
*b^2*c^3*x^2 + b^3*c^2)
```

3.211.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(15/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

3.211.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.94

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{(9\,Bbc-Ac^2)x^{\frac{5}{2}}+(5\,Bb^2+3\,Abc)\sqrt{x}}{16\,(bc^4x^4+2\,b^2c^3x^2+b^3c^2)} + \frac{\frac{2\sqrt{2}(5\,Bb+3\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}(5\,Bb+3\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}(5\,Bb+3\,Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{128\,bc^2}$$

input
```
integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/16*((9*B*b*c - A*c^2)*x^(5/2) + (5*B*b^2 + 3*A*b*c)*sqrt(x))/(b*c^4*x^4
 + 2*b^2*c^3*x^2 + b^3*c^2) + 1/128*(2*sqrt(2)*(5*B*b + 3*A*c)*arctan(1/2*
sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)
))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(5*B*b + 3*A*c)*arctan(-1/2
*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c
)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(5*B*b + 3*A*c)*log(sqrt(2)*
b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)
*(5*B*b + 3*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b
))/(b^(3/4)*c^(1/4)))/(b*c^2)
```

3.211.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00

$$\int \frac{x^{15/2}(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx = \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb+3\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^3} + \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb+3\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^3} + \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb+3\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^3} - \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb+3\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^3} - \frac{9\,Bbcx^{\frac{5}{2}}-Ac^2x^{\frac{5}{2}}+5\,Bb^2\sqrt{x}+3\,Abc\sqrt{x}}{16\,(cx^2+b)^2bc^2}$$

input
```
integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
1/64*sqrt(2)*(5*(b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) + 1/64*sqrt(2)*
(5*(b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) + 1/128*sqrt(2)*(5*(b*c^3)^
(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqr
t(b/c))/(b^2*c^3) - 1/128*sqrt(2)*(5*(b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A
*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^3) - 1/16*(9*
B*b*c*x^(5/2) - A*c^2*x^(5/2) + 5*B*b^2*sqrt(x) + 3*A*b*c*sqrt(x))/((c*x^2
 + b)^2*b*c^2)
```

3.211.9 Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 799, normalized size of antiderivative = 2.68

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{\frac{\sqrt{x}\,(3\,A\,c+5\,B\,b)}{16\,c^2}-\frac{x^{5/2}\,(A\,c-9\,B\,b)}{16\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$
$$+\frac{\operatorname{atan}\left(\frac{\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2\right)}{64\,b^2\,c}-\frac{\left(3\,A\,c^2+5\,B\,b\,c\right)(3\,A\,c+5\,B\,b)}{64\,(-b)^{7/4}\,c^{9/4}}\right)1\mathrm{i}}{64\,(-b)^{7/4}\,c^{9/4}}+\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2\right)}{64\,b^2\,c}+\frac{\left(3\,A\,c^2+\cdots\right.}{64\cdots}\right.}{64\,(-b)^{7/4}\,c^{9/4}}}{\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2\right)}{64\,b^2\,c}-\frac{\left(3\,A\,c^2+5\,B\,b\,c\right)(3\,A\,c+5\,B\,b)}{64\,(-b)^{7/4}\,c^{9/4}}\right)}{64\,(-b)^{7/4}\,c^{9/4}}-\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2\right)}{64\,b^2\,c}+\frac{\left(3\,A\,c^2+\cdots\right.}{64\cdots}\right.}{64\,(-b)^{7/4}\,c^{9/4}}}\cdots\right)\cdots}{32\,(-b)^{7/4}\,c^{9/4}}$$
$$+\frac{\operatorname{atan}\left(\frac{\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2\right)}{64\,b^2\,c}-\frac{\left(3\,A\,c^2+5\,B\,b\,c\right)(3\,A\,c+5\,B\,b)\,1\mathrm{i}}{64\,(-b)^{7/4}\,c^{9/4}}\right)}{64\,(-b)^{7/4}\,c^{9/4}}+\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2\right)}{64\,b^2\,c}+\frac{\left(3\,A\,c^2\cdots\right.}{\cdots}\right.}{64\,(-b)^{7/4}\,c^{9/4}}}{\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2\right)}{64\,b^2\,c}-\frac{\left(3\,A\,c^2+5\,B\,b\,c\right)(3\,A\,c+5\,B\,b)\,1\mathrm{i}}{64\,(-b)^{7/4}\,c^{9/4}}\right)1\mathrm{i}}{64\,(-b)^{7/4}\,c^{9/4}}-\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2\right)}{64\,b^2\,c}+\frac{\left(3\,A\,c^2\cdots\right.}{\cdots}\right.}{64\,(-b)^{7/4}\,c^{9/4}}}\cdots\right)\cdots}{32\,(-b)^{7/4}\,c^{9/4}}$$

input
```
int((x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
(atan((((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*A*B*b*c))/(
64*b^2*c) - ((3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b))/(64*(-b)^(7/4)*c^(9/4)))
*1i)/(64*(-b)^(7/4)*c^(9/4)) + ((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*
B^2*b^2 + 30*A*B*b*c))/(64*b^2*c) + ((3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b))/
(64*(-b)^(7/4)*c^(9/4)))*1i)/(64*(-b)^(7/4)*c^(9/4)))/(((3*A*c + 5*B*b)*((
x^(1/2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*A*B*b*c))/(64*b^2*c) - ((3*A*c^2 + 5*
B*b*c)*(3*A*c + 5*B*b))/(64*(-b)^(7/4)*c^(9/4))))/(64*(-b)^(7/4)*c^(9/4))
- ((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*A*B*b*c))/(64*b^
2*c) + ((3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b))/(64*(-b)^(7/4)*c^(9/4))))/(64
*(-b)^(7/4)*c^(9/4))))*(3*A*c + 5*B*b)*1i)/(32*(-b)^(7/4)*c^(9/4)) - ((x^(
1/2)*(3*A*c + 5*B*b))/(16*c^2) - (x^(5/2)*(A*c - 9*B*b))/(16*b*c))/(b^2 +
c^2*x^4 + 2*b*c*x^2) + (atan((((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B
^2*b^2 + 30*A*B*b*c))/(64*b^2*c) - ((3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b)*1i
)/(64*(-b)^(7/4)*c^(9/4))))/(64*(-b)^(7/4)*c^(9/4)) + ((3*A*c + 5*B*b)*((x
^(1/2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*A*B*b*c))/(64*b^2*c) + ((3*A*c^2 + 5*B
*b*c)*(3*A*c + 5*B*b)*1i)/(64*(-b)^(7/4)*c^(9/4))))/(64*(-b)^(7/4)*c^(9/4)
))/(((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*A*B*b*c))/(64*
b^2*c) - ((3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b)*1i)/(64*(-b)^(7/4)*c^(9/4)))
*1i)/(64*(-b)^(7/4)*c^(9/4)) - ((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*
B^2*b^2 + 30*A*B*b*c))/(64*b^2*c) + ((3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b...
```

3.212 $\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.212.1 Optimal result

Integrand size = 26, antiderivative size = 298

$$\begin{aligned}\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= -\frac{(bB-Ac)x^{3/2}}{4bc\,(b+cx^2)^2}+\frac{(3bB+5Ac)x^{3/2}}{16b^2c\,(b+cx^2)} \\ &- \frac{(3bB+5Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{9/4}c^{7/4}}+\frac{(3bB+5Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{9/4}c^{7/4}} \\ &+ \frac{(3bB+5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{7/4}} \\ &- \frac{(3bB+5Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{7/4}}\end{aligned}$$

output
```
-1/4*(-A*c+B*b)*x^(3/2)/b/c/(c*x^2+b)^2+1/16*(5*A*c+3*B*b)*x^(3/2)/b^2/c/(
c*x^2+b)-1/64*(5*A*c+3*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(9
/4)/c^(7/4)*2^(1/2)+1/64*(5*A*c+3*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^
(1/4))/b^(9/4)/c^(7/4)*2^(1/2)+1/128*(5*A*c+3*B*b)*ln(b^(1/2)+x*c^(1/2)-b^
(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)/c^(7/4)*2^(1/2)-1/128*(5*A*c+3*B*b)
*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)/c^(7/4)*2^(
1/2)
```

3.212.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.59

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{4\sqrt[4]{b}c^{3/4}x^{3/2}(-b^2B+9Abc+3bBcx^2+5Ac^2x^2)}{(b+cx^2)^2} - \sqrt{2}(3bB+5Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) - \sqrt{2}}{64b^{9/4}c^{7/4}}$$

input
```
Integrate[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
((4*b^(1/4)*c^(3/4)*x^(3/2)*(-(b^2*B) + 9*A*b*c + 3*b*B*c*x^2 + 5*A*c^2*x^
2))/(b + c*x^2)^2 - Sqrt[2]*(3*b*B + 5*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - Sqrt[2]*(3*b*B + 5*A*c)*ArcTanh[(Sqrt[
2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(64*b^(9/4)*c^(7/4))
```

3.212.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {9, 362, 253, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{\sqrt{x}(A+Bx^2)}{(b+cx^2)^3}dx$$

$$\Big\downarrow \; 362$$

$$\frac{(5Ac+3bB)\int \frac{\sqrt{x}}{(cx^2+b)^2}dx}{8bc} - \frac{x^{3/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \; 253$$

$$\frac{(5Ac+3bB)\left(\frac{\int \frac{\sqrt{x}}{cx^2+b}dx}{4b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8bc} - \frac{x^{3/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \; 266$$

$$\frac{(5Ac+3bB)\left(\frac{\int \frac{x}{cx^2+b}d\sqrt{x}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8bc}-\frac{x^{3/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \; 826$$

$$\frac{(5Ac+3bB)\left(\frac{\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8bc}-\frac{x^{3/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \; 1476$$

$$\frac{(5Ac+3bB)\left(\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8bc}-\frac{x^{3/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{(5Ac+3bB)\left(\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8bc}-\frac{x^{3/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{(5Ac+3bB)\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8bc}-\frac{x^{3/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

3.212. $\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\Big\downarrow \; 1479$$

$$\frac{(5Ac+3bB)\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}}{2b}+\cdots\right.}{8bc}$$

$$\frac{x^{3/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{(5Ac+3bB)\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}}{2b}+\frac{x^{3}\cdots}{2b(b+\cdots}\right.}{8bc}$$

$$\frac{x^{3/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(5Ac+3bB)\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8bc}-$$

$$\frac{x^{3/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

3.212. $\int \frac{x^{13/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$\downarrow 1103$$

$$\frac{(5Ac+3bB)\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2}\right.}{8bc}$$

$$\frac{x^{3/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

input
```
Int[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*((b*B - A*c)*x^(3/2))/(b*c*(b + c*x^2)^2) + ((3*b*B + 5*A*c)*(x^(3/2)
/(2*b*(b + c*x^2)) + ((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sq
rt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(S
qrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)
*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2
*Sqrt[c]))/(2*b)))/(8*b*c)
```

3.212.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.212. $\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3} \, dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.212.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.56

method	result
derivativedivides	$\frac{\frac{(5Ac+3Bb)x^{\frac{7}{2}}}{16b^2}+\frac{(9Ac-Bb)x^{\frac{3}{2}}}{16bc}}{(cx^2+b)^2}+\frac{(5Ac+3Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{128b^2c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$\frac{\frac{(5Ac+3Bb)x^{\frac{7}{2}}}{16b^2}+\frac{(9Ac-Bb)x^{\frac{3}{2}}}{16bc}}{(cx^2+b)^2}+\frac{(5Ac+3Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{128b^2c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

input
```
int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/32*(5*A*c+3*B*b)/b^2*x^(7/2)+1/32*(9*A*c-B*b)/b/c*x^(3/2))/(c*x^2+b)^
2+1/128*(5*A*c+3*B*b)/b^2/c^2/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x
^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/
2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1
/4)*x^(1/2)-1))
```

3.212.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 878, normalized size of antiderivative = 2.95

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(b^2c^3x^4+2\,b^3c^2x^2+b^4c)\left(-\frac{81\,B^4b^4+540\,AB^3b^3c+1350\,A^2B^2b^2c^2+1500\,A^3Bbc^3+625\,A^4c^4}{b^9c^7}\right)^{\frac{1}{4}}\log\left(\right.}{}$$

input
```
integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/64*((b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)*(-(81*B^4*b^4 + 540*A*B^3*b^3*
c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(1/4
)*log(b^7*c^5*(-(81*B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 150
0*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(3/4) + (27*B^3*b^3 + 135*A*B^2*b^
2*c + 225*A^2*B*b*c^2 + 125*A^3*c^3)*sqrt(x)) - (I*b^2*c^3*x^4 + 2*I*b^3*c
^2*x^2 + I*b^4*c)*(-(81*B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 +
 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(1/4)*log(I*b^7*c^5*(-(81*B^4*
b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*
c^4)/(b^9*c^7))^(3/4) + (27*B^3*b^3 + 135*A*B^2*b^2*c + 225*A^2*B*b*c^2 +
125*A^3*c^3)*sqrt(x)) - (-I*b^2*c^3*x^4 - 2*I*b^3*c^2*x^2 - I*b^4*c)*(-(81
*B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625
*A^4*c^4)/(b^9*c^7))^(1/4)*log(-I*b^7*c^5*(-(81*B^4*b^4 + 540*A*B^3*b^3*c
+ 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(3/4)
+ (27*B^3*b^3 + 135*A*B^2*b^2*c + 225*A^2*B*b*c^2 + 125*A^3*c^3)*sqrt(x))
- (b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)*(-(81*B^4*b^4 + 540*A*B^3*b^3*c +
1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(1/4)*lo
g(-b^7*c^5*(-(81*B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A
^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(3/4) + (27*B^3*b^3 + 135*A*B^2*b^2*c
 + 225*A^2*B*b*c^2 + 125*A^3*c^3)*sqrt(x)) + 4*((3*B*b*c + 5*A*c^2)*x^3 -
(B*b^2 - 9*A*b*c)*x)*sqrt(x))/(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)
```

3.212.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

3.212.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.85

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(3\,Bbc+5\,Ac^2)x^{\frac{7}{2}}-(Bb^2-9\,Abc)x^{\frac{3}{2}}}{16\,(b^2c^3x^4+2\,b^3c^2x^2+b^4c)} + \frac{(3\,Bb+5\,Ac)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+ \right.}{128\,b^2c}$$

input
```
integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/16*((3*B*b*c + 5*A*c^2)*x^(7/2) - (B*b^2 - 9*A*b*c)*x^(3/2))/(b^2*c^3*x^
4 + 2*b^3*c^2*x^2 + b^4*c) + 1/128*(3*B*b + 5*A*c)*(2*sqrt(2)*arctan(1/2*s
qrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c))
)/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)
*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)
*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)
*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqr
t(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/(b^2*c)
```

3.212.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3\,Bbcx^{\frac{7}{2}}+5\,Ac^2x^{\frac{7}{2}}-Bb^2x^{\frac{3}{2}}+9\,Abcx^{\frac{3}{2}}}{16\,(cx^2+b)^2b^2c}$$
$$+\frac{\sqrt{2}\left(3\,(bc^3)^{\frac{3}{4}}\,Bb+5\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c^4}$$
$$+\frac{\sqrt{2}\left(3\,(bc^3)^{\frac{3}{4}}\,Bb+5\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c^4}$$
$$-\frac{\sqrt{2}\left(3\,(bc^3)^{\frac{3}{4}}\,Bb+5\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^3c^4}$$
$$+\frac{\sqrt{2}\left(3\,(bc^3)^{\frac{3}{4}}\,Bb+5\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^3c^4}$$

input
```
integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
1/16*(3*B*b*c*x^(7/2) + 5*A*c^2*x^(7/2) - B*b^2*x^(3/2) + 9*A*b*c*x^(3/2))
/((c*x^2 + b)^2*b^2*c) + 1/64*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + 5*(b*c^3)^(3/
4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/
(b^3*c^4) + 1/64*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + 5*(b*c^3)^(3/4)*A*c)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^4) -
1/128*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + 5*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt
(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^4) + 1/128*sqrt(2)*(3*(b*c^3)^(3/4
)*B*b + 5*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b
/c))/(b^3*c^4)
```

3.212.9 Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.42

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{x^{7/2}\,(5\,A\,c+3\,B\,b)}{16\,b^2}+\frac{x^{3/2}\,(9\,A\,c-B\,b)}{16\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4} + \frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(5\,A\,c+3\,B\,b)}{32\,(-b)^{9/4}\,c^{7/4}} - \frac{\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(5\,A\,c+3\,B\,b)}{32\,(-b)^{9/4}\,c^{7/4}}$$

input
```
int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
((x^(7/2)*(5*A*c + 3*B*b))/(16*b^2) + (x^(3/2)*(9*A*c - B*b))/(16*b*c))/(b
^2 + c^2*x^4 + 2*b*c*x^2) + (atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(5*A*c + 3
*B*b))/(32*(-b)^(9/4)*c^(7/4)) - (atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(5*A
*c + 3*B*b))/(32*(-b)^(9/4)*c^(7/4))
```

3.213 $\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.213.1 Optimal result

Integrand size = 26, antiderivative size = 293

$$\begin{aligned}\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= -\frac{(bB-Ac)\sqrt{x}}{4bc(b+cx^2)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c(b+cx^2)} \\ &- \frac{3(bB+7Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{11/4}c^{5/4}}+\frac{3(bB+7Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{11/4}c^{5/4}} \\ &- \frac{3(bB+7Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}c^{5/4}} \\ &+ \frac{3(bB+7Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}c^{5/4}}\end{aligned}$$

output
```
-3/64*(7*A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)/c^(5/
4)*2^(1/2)+3/64*(7*A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1
1/4)/c^(5/4)*2^(1/2)-3/128*(7*A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4
)*2^(1/2)*x^(1/2))/b^(11/4)/c^(5/4)*2^(1/2)+3/128*(7*A*c+B*b)*ln(b^(1/2)+x
*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)/c^(5/4)*2^(1/2)-1/4*(-A
*c+B*b)*x^(1/2)/b/c/(c*x^2+b)^2+1/16*(7*A*c+B*b)*x^(1/2)/b^2/c/(c*x^2+b)
```

3.213.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.59

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{4b^{3/4}\sqrt[4]{c}\sqrt{x}(-3b^2B+11Abc+bBcx^2+7Ac^2x^2)}{(b+cx^2)^2} - 3\sqrt{2}(bB+7Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) + 3\sqrt{}}{64b^{11/4}c^{5/4}}$$

input
```
Integrate[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
((4*b^(3/4)*c^(1/4)*Sqrt[x]*(-3*b^2*B + 11*A*b*c + b*B*c*x^2 + 7*A*c^2*x^2
))/(b + c*x^2)^2 - 3*Sqrt[2]*(b*B + 7*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 3*Sqrt[2]*(b*B + 7*A*c)*ArcTanh[(Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(64*b^(11/4)*c^(5/4))
```

3.213.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {9, 362, 253, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{A+Bx^2}{\sqrt{x}\,(b+cx^2)^3}dx$$

$$\Big\downarrow \ 362$$

$$\frac{(7Ac+bB)\int \frac{1}{\sqrt{x}(cx^2+b)^2}dx}{8bc} - \frac{\sqrt{x}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \ 253$$

$$\frac{(7Ac+bB)\left(\frac{3\int \frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{4b} + \frac{\sqrt{x}}{2b(b+cx^2)}\right)}{8bc} - \frac{\sqrt{x}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

$$\Big\downarrow \; 266$$

$$\frac{(7Ac+bB)\left(\frac{3\int \frac{1}{cx^2+b}d\sqrt{x}}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}\right)}{8bc}-\frac{\sqrt{x}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 755$$

$$\frac{(7Ac+bB)\left(\frac{3\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}\right)}{8bc}-\frac{\sqrt{x}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 1476$$

$$\frac{(7Ac+bB)\left(\frac{3\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}\right)}{8bc}-\frac{\sqrt{x}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{(7Ac+bB)\left(\frac{3\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}\right)}{8bc}-\frac{\sqrt{x}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

3.213. $\int \frac{x^{11/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$\downarrow$ 217

$$\frac{(7Ac+bB)\left(\frac{3\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}\right)}{8bc}-\frac{\sqrt{x}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$\downarrow$ 1479

$$\frac{(7Ac+bB)\left(\frac{3\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}\right)}{8bc}$$

$$\frac{\sqrt{x}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$\downarrow$ 25

3.213. $\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\frac{(7Ac+bB)\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}\right)}{8bc}+\frac{\sqrt{x}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(7Ac+bB)\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}\right)}{8bc}+\frac{\sqrt{x}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 1103$$

3.213. $\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\frac{(7Ac+bB)\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{\;}}{2b(b+}\right.}{8bc}$$

$$\frac{\sqrt{x}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

input
```
Int[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*((b*B - A*c)*Sqrt[x])/(b*c*(b + c*x^2)^2) + ((b*B + 7*A*c)*(Sqrt[x]/(
2*b*(b + c*x^2)) + (3*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(S
qrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(
Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(
2*Sqrt[b])))/(2*b)))/(8*b*c)
```

3.213.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.213. $\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.213. $\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3} \, dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.213.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.57

method	result
derivativedivides	$\frac{\frac{(7Ac+Bb)x^{\frac{5}{2}}}{16b^2}+\frac{(11Ac-3Bb)\sqrt{x}}{16bc}}{(c\,x^2+b)^2}+\frac{3(7Ac+Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.}{128b^3c}$
default	$\frac{\frac{(7Ac+Bb)x^{\frac{5}{2}}}{16b^2}+\frac{(11Ac-3Bb)\sqrt{x}}{16bc}}{(c\,x^2+b)^2}+\frac{3(7Ac+Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.}{128b^3c}$

input
```
int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/32*(7*A*c+B*b)/b^2*x^(5/2)+1/32*(11*A*c-3*B*b)/b/c*x^(1/2))/(c*x^2+b)
^2+3/128*(7*A*c+B*b)/b^3/c*(1/c*b)^(1/4)*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1
/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))
)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)
*x^(1/2)-1))
```

3.213.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 749, normalized size of antiderivative = 2.56

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3\left(b^2c^3x^4+2\,b^3c^2x^2+b^4c\right)\left(-\frac{B^4b^4+28\,AB^3b^3c+294\,A^2B^2b^2c^2+1372\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c^5}\right)^{\frac{1}{4}}\log\left(\right.}{}$$

input
```
integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/64*(3*(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)*(-(B^4*b^4 + 28*A*B^3*b^3*c
+ 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4)
*log(3*b^3*c*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*
B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4) + 3*(B*b + 7*A*c)*sqrt(x)) - 3*(
-I*b^2*c^3*x^4 - 2*I*b^3*c^2*x^2 - I*b^4*c)*(-(B^4*b^4 + 28*A*B^3*b^3*c +
294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4)*l
og(3*I*b^3*c*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*
B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4) + 3*(B*b + 7*A*c)*sqrt(x)) - 3*(
I*b^2*c^3*x^4 + 2*I*b^3*c^2*x^2 + I*b^4*c)*(-(B^4*b^4 + 28*A*B^3*b^3*c + 2
94*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4)*lo
g(-3*I*b^3*c*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*
B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4) + 3*(B*b + 7*A*c)*sqrt(x)) - 3*(
b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2
*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4)*log(-3*b
^3*c*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3
+ 2401*A^4*c^4)/(b^11*c^5))^(1/4) + 3*(B*b + 7*A*c)*sqrt(x)) - 4*(3*B*b^2
- 11*A*b*c - (B*b*c + 7*A*c^2)*x^2)*sqrt(x))/(b^2*c^3*x^4 + 2*b^3*c^2*x^2
+ b^4*c)
```

3.213.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

3.213.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.94

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(Bbc+7\,Ac^2)x^{\frac{5}{2}}-(3\,Bb^2-11\,Abc)\sqrt{x}}{16\,(b^2c^3x^4+2\,b^3c^2x^2+b^4c)} + \frac{3\left(\frac{2\sqrt{2}(Bb+7\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(Bb+7\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(Bb+7\,Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right.}{128\,b^2c}$$

input
```
integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/16*((B*b*c + 7*A*c^2)*x^(5/2) - (3*B*b^2 - 11*A*b*c)*sqrt(x))/(b^2*c^3*x
^4 + 2*b^3*c^2*x^2 + b^4*c) + 3/128*(2*sqrt(2)*(B*b + 7*A*c)*arctan(1/2*sq
rt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))
/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(B*b + 7*A*c)*arctan(-1/2*sqr
t(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/
(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(B*b + 7*A*c)*log(sqrt(2)*b^(1/4
)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(B*b
+ 7*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3
/4)*c^(1/4)))/(b^2*c)
```

3.213.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+7\,(bc^3)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c^2}$$
$$+\frac{3\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+7\,(bc^3)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c^2}$$
$$+\frac{3\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+7\,(bc^3)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^3c^2}$$
$$-\frac{3\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+7\,(bc^3)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^3c^2}$$
$$+\frac{Bbcx^{\frac{5}{2}}+7\,Ac^2x^{\frac{5}{2}}-3\,Bb^2\sqrt{x}+11\,Abc\sqrt{x}}{16\,(cx^2+b)^2b^2c}$$

input
```
integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
3/64*sqrt(2)*((b*c^3)^(1/4)*B*b + 7*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 3/64*sqrt(2)*((
b*c^3)^(1/4)*B*b + 7*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)
^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 3/128*sqrt(2)*((b*c^3)^(1/4)*
B*b + 7*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c)
)/(b^3*c^2) - 3/128*sqrt(2)*((b*c^3)^(1/4)*B*b + 7*(b*c^3)^(1/4)*A*c)*log(
-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) + 1/16*(B*b*c*x^(5
/2) + 7*A*c^2*x^(5/2) - 3*B*b^2*sqrt(x) + 11*A*b*c*sqrt(x))/((c*x^2 + b)^2
*b^2*c)
```

3.213.9 Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 780, normalized size of antiderivative = 2.66

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{x^{5/2}\,(7\,A\,c+B\,b)}{16\,b^2}+\frac{\sqrt{x}\,(11\,A\,c-3\,B\,b)}{16\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4} - \frac{\operatorname{atan}\left(\frac{\frac{(7\,A\,c+B\,b)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,c^3+14\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{64\,b^4}-\frac{9\,(7\,A\,c+B\,b)\,\left(7\,A\,c^3+B\,b\,c^2\right)}{64\,(-b)^{15/4}\,c^{5/4}}\right)3\mathrm{i}}{64\,(-b)^{11/4}\,c^{5/4}}+\frac{(7\,A\,c+B\,b)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,c^3+14\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{64\,b^4}+\frac{9\,(7\,A \cdots}{\cdots}\right)}{64\,(-b)^{11/4}\,c^{5/4}}}{\frac{3\,(7\,A\,c+B\,b)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,c^3+14\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{64\,b^4}-\frac{9\,(7\,A\,c+B\,b)\,\left(7\,A\,c^3+B\,b\,c^2\right)}{64\,(-b)^{15/4}\,c^{5/4}}\right)}{64\,(-b)^{11/4}\,c^{5/4}}-\frac{3\,(7\,A\,c+B\,b)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,c^3+14\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{64\,b^4}+\frac{9\,(7 \cdots}{\cdots}\right)}{64\,(-b)^{11/4}\,c^{5/4}}}\right) \cdots}{32\,(-b)^{11/4}\,c^{5/4}} - \frac{3\operatorname{atan}\left(\frac{\frac{3\,(7\,A\,c+B\,b)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,c^3+14\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{64\,b^4}-\frac{(7\,A\,c+B\,b)\,\left(7\,A\,c^3+B\,b\,c^2\right)\,9\mathrm{i}}{64\,(-b)^{15/4}\,c^{5/4}}\right)}{64\,(-b)^{11/4}\,c^{5/4}}+\frac{3\,(7\,A\,c+B\,b)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,c^3+14\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{64\,b^4}+\frac{(7 \cdots}{\cdots}\right)}{64\,(-b)^{11/4}\,c^{5/4}}}{\frac{(7\,A\,c+B\,b)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,c^3+14\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{64\,b^4}-\frac{(7\,A\,c+B\,b)\,\left(7\,A\,c^3+B\,b\,c^2\right)\,9\mathrm{i}}{64\,(-b)^{15/4}\,c^{5/4}}\right)3\mathrm{i}}{64\,(-b)^{11/4}\,c^{5/4}}-\frac{(7\,A\,c+B\,b)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,c^3+14\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{64\,b^4}+\frac{(7 \cdots}{\cdots}\right)}{64\,(-b)^{11/4}\,c^{5/4}}}\right) \cdots}{32\,(-b)^{11/4}\,c^{5/4}}$$

input
```
int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
((x^(5/2)*(7*A*c + B*b))/(16*b^2) + (x^(1/2)*(11*A*c - 3*B*b))/(16*b*c))/(
b^2 + c^2*x^4 + 2*b*c*x^2) - (atan((((7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3
 + B^2*b^2*c + 14*A*B*b*c^2))/(64*b^4) - (9*(7*A*c + B*b)*(7*A*c^3 + B*b*c
^2))/(64*(-b)^(15/4)*c^(5/4)))*3i)/(64*(-b)^(11/4)*c^(5/4)) + ((7*A*c + B*
b)*((9*x^(1/2)*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2))/(64*b^4) + (9*(7*A
*c + B*b)*(7*A*c^3 + B*b*c^2))/(64*(-b)^(15/4)*c^(5/4)))*3i)/(64*(-b)^(11/
4)*c^(5/4)))/((3*(7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 + B^2*b^2*c + 14*A*
B*b*c^2))/(64*b^4) - (9*(7*A*c + B*b)*(7*A*c^3 + B*b*c^2))/(64*(-b)^(15/4)
*c^(5/4))))/(64*(-b)^(11/4)*c^(5/4)) - (3*(7*A*c + B*b)*((9*x^(1/2)*(49*A^
2*c^3 + B^2*b^2*c + 14*A*B*b*c^2))/(64*b^4) + (9*(7*A*c + B*b)*(7*A*c^3 +
B*b*c^2))/(64*(-b)^(15/4)*c^(5/4))))/(64*(-b)^(11/4)*c^(5/4))))*(7*A*c + B
*b)*3i)/(32*(-b)^(11/4)*c^(5/4)) - (3*atan(((3*(7*A*c + B*b)*((9*x^(1/2)*(
49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2))/(64*b^4) - ((7*A*c + B*b)*(7*A*c^3
 + B*b*c^2)*9i)/(64*(-b)^(15/4)*c^(5/4))))/(64*(-b)^(11/4)*c^(5/4)) + (3*(
7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2))/(64*b^4)
 + ((7*A*c + B*b)*(7*A*c^3 + B*b*c^2)*9i)/(64*(-b)^(15/4)*c^(5/4))))/(64*(
-b)^(11/4)*c^(5/4)))/(((7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 + B^2*b^2*c +
 14*A*B*b*c^2))/(64*b^4) - ((7*A*c + B*b)*(7*A*c^3 + B*b*c^2)*9i)/(64*(-b)
^(15/4)*c^(5/4)))*3i)/(64*(-b)^(11/4)*c^(5/4)) - ((7*A*c + B*b)*((9*x^(1/2
)*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2))/(64*b^4) + ((7*A*c + B*b)*(7...
```

3.214 $\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.214.1 Optimal result

Integrand size = 26, antiderivative size = 316

$$\begin{aligned}\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-Ac}{4bc\sqrt{x}\,(b+cx^2)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\,(b+cx^2)} \\ &\quad - \frac{5(bB-9Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{13/4}c^{3/4}} + \frac{5(bB-9Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{13/4}c^{3/4}} \\ &\quad + \frac{5(bB-9Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}c^{3/4}} \\ &\quad - \frac{5(bB-9Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}c^{3/4}}\end{aligned}$$

output
```
-5/64*(-9*A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(13/4)/c^(3
/4)*2^(1/2)+5/64*(-9*A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^
(13/4)/c^(3/4)*2^(1/2)+5/128*(-9*A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(
1/4)*2^(1/2)*x^(1/2))/b^(13/4)/c^(3/4)*2^(1/2)-5/128*(-9*A*c+B*b)*ln(b^(1/
2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)/c^(3/4)*2^(1/2)+5/1
6*(-9*A*c+B*b)/b^3/c/x^(1/2)+1/4*(A*c-B*b)/b/c/(c*x^2+b)^2/x^(1/2)+1/16*(9
*A*c-B*b)/b^2/c/(c*x^2+b)/x^(1/2)
```

3.214.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.59

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{-\frac{4\sqrt[4]{b}(-bBx^2(9b+5cx^2)+A(32b^2+81bcx^2+45c^2x^4))}{\sqrt{x}(b+cx^2)^2}+\frac{5\sqrt{2}(-bB+9Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{c^{3/4}}+\frac{5\sqrt{2}(-}{}}{64b^{13/4}}$$

input
```
Integrate[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
((-4*b^(1/4)*(-(b*B*x^2*(9*b + 5*c*x^2)) + A*(32*b^2 + 81*b*c*x^2 + 45*c^2
*x^4)))/(Sqrt[x]*(b + c*x^2)^2) + (5*Sqrt[2]*(-(b*B) + 9*A*c)*ArcTan[(Sqrt
[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])]])/c^(3/4) + (5*Sqrt[2]*
(-(b*B) + 9*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt
[c]*x)]])/c^(3/4))/(64*b^(13/4))
```

3.214.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {9, 362, 253, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{A+Bx^2}{x^{3/2}(b+cx^2)^3}dx$$

$$\Big\downarrow \; 362$$

$$-\frac{(bB-9Ac)\int \frac{1}{x^{3/2}(cx^2+b)^2}dx}{8bc}-\frac{bB-Ac}{4bc\sqrt{x}(b+cx^2)^2}$$

$$\Big\downarrow \; 253$$

$$-\frac{(bB-9Ac)\left(\frac{5\int \frac{1}{x^{3/2}\left(cx^2+b\right)}dx}{4b}+\frac{1}{2b\sqrt{x}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 264

$$-\frac{(bB-9Ac)\left(\frac{5\left(-\frac{c\int \frac{\sqrt{x}}{cx^2+b}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 266

$$-\frac{(bB-9Ac)\left(\frac{5\left(-\frac{2c\int \frac{x}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 826

$$-\frac{(bB-9Ac)\left(\frac{5\left(-\frac{2c\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 1476

3.214. $\int \frac{x^{9/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}dx$

$$-\frac{(bB-9Ac)\left(\frac{5\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bc\sqrt{x}\,(b+cx^2)^2}$$

$\downarrow$ 1082

3.214. $\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$-\frac{(bB-9Ac)\left(\frac{5\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 217$$

3.214. $\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$-\frac{(bB-9Ac)\left(\frac{5\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bc\sqrt{x}\,(b+cx^2)^2}$$

$$\Big\downarrow \text{1479}$$

3.214. $\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$-\frac{(bB-9Ac)\left(\frac{5\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}\right)}{4b}\right)}{8bc}$$

$$\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 25$$

3.214. $\int \frac{x^{9/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$-\frac{(bB-9Ac)\left(\frac{5\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\cdots\right)}{b}\cdots\right)}{4b}\cdots\right)}{8bc}\cdots$$

$$\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2}$$

$$\downarrow 27$$

3.214. $\int \frac{x^{9/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$-\frac{(bB-9Ac)\left(\frac{5\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\cdots\right)}{8bc}$$

$$\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2}$$

$\downarrow$ 1103

3.214. $\int\frac{x^{9/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$-\frac{(bB-9Ac)\left(\frac{5\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\cdots\right)}{4b}\cdots\right)}{8bc}$$

$$\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2}$$

input
```
Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*(b*B - A*c)/(b*c*Sqrt[x]*(b + c*x^2)^2) - ((b*B - 9*A*c)*(1/(2*b*Sqrt
[x]*(b + c*x^2)) + (5*(-2/(b*Sqrt[x]) - (2*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[S
qrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(
1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt
[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/b))/(4*b)))/(8*b*c)
```

3.214. $\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.214.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

3.214. $\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3} \, dx$

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.214. $\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.214.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.55

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{13}{32}Ac^2-\frac{5}{32}Bbc\right)x^{\frac{7}{2}}+\frac{b(17Ac-9Bb)x^{\frac{3}{2}}}{32}}{\left(cx^2+b\right)^2}+\frac{\left(\frac{45Ac}{32}-\frac{5Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b^3}$
default	$-\frac{2\left(\frac{\left(\frac{13}{32}Ac^2-\frac{5}{32}Bbc\right)x^{\frac{7}{2}}+\frac{b(17Ac-9Bb)x^{\frac{3}{2}}}{32}}{\left(cx^2+b\right)^2}+\frac{\left(\frac{45Ac}{32}-\frac{5Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b^3}$
risch	$-\frac{2A}{b^3\sqrt{x}}-\frac{\frac{2\left(\frac{13}{32}Ac^2-\frac{5}{32}Bbc\right)x^{\frac{7}{2}}+\frac{b(17Ac-9Bb)x^{\frac{3}{2}}}{16}}{\left(cx^2+b\right)^2}+\frac{\left(\frac{45Ac}{32}-\frac{5Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+\right.}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b^3}$

input
```
int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/b^3*(((13/32*A*c^2-5/32*B*b*c)*x^(7/2)+1/32*b*(17*A*c-9*B*b)*x^(3/2))/(
c*x^2+b)^2+1/8*(45/32*A*c-5/32*B*b)/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)
^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/
c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1
/c*b)^(1/4)*x^(1/2)-1)))-2*A/b^3/x^(1/2)
```

3.214.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 870, normalized size of antiderivative = 2.75

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx =$$

$$-\frac{5\left(b^3c^2x^5+2\,b^4cx^3+b^5x\right)\left(-\frac{B^4b^4-36\,AB^3b^3c+486\,A^2B^2b^2c^2-2916\,A^3Bbc^3+6561\,A^4c^4}{b^{13}c^3}\right)^{\frac{1}{4}}\log\left(125\,b^{10}c^2\left(-\frac{B^4b^4-36\,AB^3b\right.\right.}{}$$

input
```
integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
-1/64*(5*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-(B^4*b^4 - 36*A*B^3*b^3*c +
 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(1/4)*
log(125*b^10*c^2*(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*
A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(3/4) - 125*(B^3*b^3 - 27*A*B^2*b^
2*c + 243*A^2*B*b*c^2 - 729*A^3*c^3)*sqrt(x)) + 5*(-I*b^3*c^2*x^5 - 2*I*b^
4*c*x^3 - I*b^5*x)*(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 291
6*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(1/4)*log(125*I*b^10*c^2*(-(B^4*
b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c
^4)/(b^13*c^3))^(3/4) - 125*(B^3*b^3 - 27*A*B^2*b^2*c + 243*A^2*B*b*c^2 -
729*A^3*c^3)*sqrt(x)) + 5*(I*b^3*c^2*x^5 + 2*I*b^4*c*x^3 + I*b^5*x)*(-(B^4
*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*
c^4)/(b^13*c^3))^(1/4)*log(-125*I*b^10*c^2*(-(B^4*b^4 - 36*A*B^3*b^3*c + 4
86*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(3/4) -
125*(B^3*b^3 - 27*A*B^2*b^2*c + 243*A^2*B*b*c^2 - 729*A^3*c^3)*sqrt(x)) -
5*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^
2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(1/4)*log(-12
5*b^10*c^2*(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*
b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(3/4) - 125*(B^3*b^3 - 27*A*B^2*b^2*c +
243*A^2*B*b*c^2 - 729*A^3*c^3)*sqrt(x)) - 4*(5*(B*b*c - 9*A*c^2)*x^4 - 32*
A*b^2 + 9*(B*b^2 - 9*A*b*c)*x^2)*sqrt(x))/(b^3*c^2*x^5 + 2*b^4*c*x^3 + ...
```

3.214.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

3.214.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.81

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{5\,(Bbc-9\,Ac^2)x^4-32\,Ab^2+9\,(Bb^2-9\,Abc)x^2}{16\left(b^3c^2x^{\frac{9}{2}}+2\,b^4cx^{\frac{5}{2}}+b^5\sqrt{x}\right)} + \frac{5\,(Bb-9\,Ac)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\cdots\right)}{128\,b^3}$$

input
```
integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/16*(5*(B*b*c - 9*A*c^2)*x^4 - 32*A*b^2 + 9*(B*b^2 - 9*A*b*c)*x^2)/(b^3*c
^2*x^(9/2) + 2*b^4*c*x^(5/2) + b^5*sqrt(x)) + 5/128*(B*b - 9*A*c)*(2*sqrt(
2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(s
qrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*s
qrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c))
)/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sq
rt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1
/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^3
```

3.214.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.95

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{2\,A}{b^3\sqrt{x}} + \frac{5\,Bbcx^{\frac{7}{2}} - 13\,Ac^2x^{\frac{7}{2}} + 9\,Bb^2x^{\frac{3}{2}} - 17\,Abcx^{\frac{3}{2}}}{16\,(cx^2+b)^2b^3}$$
$$+ \frac{5\,\sqrt{2}\left((bc^3)^{\frac{3}{4}}\,Bb - 9\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4c^3}$$
$$+ \frac{5\,\sqrt{2}\left((bc^3)^{\frac{3}{4}}\,Bb - 9\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4c^3}$$
$$- \frac{5\,\sqrt{2}\left((bc^3)^{\frac{3}{4}}\,Bb - 9\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{128\,b^4c^3}$$
$$+ \frac{5\,\sqrt{2}\left((bc^3)^{\frac{3}{4}}\,Bb - 9\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{128\,b^4c^3}$$

input
```
integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-2*A/(b^3*sqrt(x)) + 1/16*(5*B*b*c*x^(7/2) - 13*A*c^2*x^(7/2) + 9*B*b^2*x^
(3/2) - 17*A*b*c*x^(3/2))/((c*x^2 + b)^2*b^3) + 5/64*sqrt(2)*((b*c^3)^(3/4
)*B*b - 9*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*s
qrt(x))/(b/c)^(1/4))/(b^4*c^3) + 5/64*sqrt(2)*((b*c^3)^(3/4)*B*b - 9*(b*c^
3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^
(1/4))/(b^4*c^3) - 5/128*sqrt(2)*((b*c^3)^(3/4)*B*b - 9*(b*c^3)^(3/4)*A*c)
*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c^3) + 5/128*sqrt(2
)*((b*c^3)^(3/4)*B*b - 9*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/
4) + x + sqrt(b/c))/(b^4*c^3)
```

3.214.9 Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.42

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{5\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(9\,A\,c-B\,b)}{32\,(-b)^{13/4}\,c^{3/4}} - \frac{\frac{2\,A}{b}+\frac{9\,x^2\,(9\,A\,c-B\,b)}{16\,b^2}+\frac{5\,c\,x^4\,(9\,A\,c-B\,b)}{16\,b^3}}{b^2\,\sqrt{x}+c^2\,x^{9/2}+2\,b\,c\,x^{5/2}} - \frac{5\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(9\,A\,c-B\,b)}{32\,(-b)^{13/4}\,c^{3/4}}$$

input
```
int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
(5*atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(9*A*c - B*b))/(32*(-b)^(13/4)*c^(3/
4)) - ((2*A)/b + (9*x^2*(9*A*c - B*b))/(16*b^2) + (5*c*x^4*(9*A*c - B*b))/
(16*b^3))/(b^2*x^(1/2) + c^2*x^(9/2) + 2*b*c*x^(5/2)) - (5*atanh((c^(1/4)*
x^(1/2))/(-b)^(1/4))*(9*A*c - B*b))/(32*(-b)^(13/4)*c^(3/4))
```

3.215 $\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3} \, dx$

3.215.1 Optimal result

Integrand size = 26, antiderivative size = 322

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3} \, dx = \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}(b+cx^2)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}(b+cx^2)} - \frac{7(3bB-11Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{15/4}\sqrt[4]{c}} + \frac{7(3bB-11Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{15/4}\sqrt[4]{c}} - \frac{7(3bB-11Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{15/4}\sqrt[4]{c}} + \frac{7(3bB-11Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{15/4}\sqrt[4]{c}}$$

output
```
7/48*(-11*A*c+3*B*b)/b^3/c/x^(3/2)+1/4*(A*c-B*b)/b/c/x^(3/2)/(c*x^2+b)^2+1
/16*(11*A*c-3*B*b)/b^2/c/x^(3/2)/(c*x^2+b)-7/64*(-11*A*c+3*B*b)*arctan(1-c
^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)/c^(1/4)*2^(1/2)+7/64*(-11*A*c+3*B
*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)/c^(1/4)*2^(1/2)-7/1
28*(-11*A*c+3*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b
^(15/4)/c^(1/4)*2^(1/2)+7/128*(-11*A*c+3*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)
*c^(1/4)*2^(1/2)*x^(1/2))/b^(15/4)/c^(1/4)*2^(1/2)
```

3.215.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.58

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{-\frac{4b^{3/4}(-3bBx^2(11b+7cx^2)+A(32b^2+121bcx^2+77c^2x^4))}{x^{3/2}(b+cx^2)^2}+\frac{21\sqrt{2}(-3bB+11Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{cx}}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt[4]{c}}+}{192b^{15/4}}$$

input
```
Integrate[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
((-4*b^(3/4)*(-3*b*B*x^2*(11*b + 7*c*x^2) + A*(32*b^2 + 121*b*c*x^2 + 77*c
^2*x^4)))/(x^(3/2)*(b + c*x^2)^2) + (21*Sqrt[2]*(-3*b*B + 11*A*c)*ArcTan[(
Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/c^(1/4) + (21*Sqr
t[2]*(3*b*B - 11*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] +
 Sqrt[c]*x)])/c^(1/4))/(192*b^(15/4))
```

3.215.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {9, 362, 253, 264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{A+Bx^2}{x^{5/2}(b+cx^2)^3}dx$$

$$\Big\downarrow \text{ 362}$$

$$-\frac{(3bB-11Ac)\int \frac{1}{x^{5/2}(cx^2+b)^2}dx}{8bc}-\frac{bB-Ac}{4bcx^{3/2}(b+cx^2)^2}$$

$$\Big\downarrow \text{ 253}$$

$$-\frac{(3bB-11Ac)\left(\frac{7\int \frac{1}{x^{5/2}\left(cx^2+b\right)}dx}{4b}+\frac{1}{2bx^{3/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2}$$

$\downarrow$ 264

$$-\frac{(3bB-11Ac)\left(\frac{7\left(-\frac{c\int \frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\frac{1}{2bx^{3/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2}$$

$\downarrow$ 266

$$-\frac{(3bB-11Ac)\left(\frac{7\left(-\frac{2c\int \frac{1}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\frac{1}{2bx^{3/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2}$$

$\downarrow$ 755

$$-\frac{(3bB-11Ac)\left(\frac{7\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\frac{1}{2bx^{3/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2}$$

$\downarrow$ 1476

3.215. $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$-\frac{(3bB-11Ac)\left(\frac{7\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\frac{1}{2bx^{3/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2}$$

$\big\downarrow$ 1082

3.215. $\displaystyle\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$-\frac{(3bB-11Ac)\left(\frac{7\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\frac{1}{2bx^{3/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2}$$

$\big\downarrow$ 217

3.215. $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$-\frac{(3bB-11Ac)\left(\frac{7\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\frac{1}{2bx^{3/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 1479$$

3.215. $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$-\frac{(3bB-11Ac)\left(\frac{7\left(-\frac{2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{\cdots}}}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{4b}\cdots\right)}{8bc}\cdots$$

$$\frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2}$$

$\big\downarrow$ 25

3.215. $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$-\frac{(3bB-11Ac)\left(\frac{7\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\cdots\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}\cdots}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)\cdots}{4b}\cdots\right)\cdots}{8bc}\cdots$$

$$\frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 27$$

3.215. $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$-\frac{(3bB-11Ac)\left(\frac{7\left(-\frac{2c\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}\right)}{8bc}$$

$$\frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2}$$

$\downarrow$ 1103

3.215. $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$-\frac{(3bB-11Ac)\left(\frac{7\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}\right.}{4b}\right.}{8bc}$$

$$\frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2}$$

input
```
Int[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*(b*B - A*c)/(b*c*x^(3/2)*(b + c*x^2)^2) - ((3*b*B - 11*A*c)*(1/(2*b*x
^(3/2)*(b + c*x^2)) + (7*(-2/(3*b*x^(3/2)) - (2*c*((-(ArcTan[1 - (Sqrt[2]*
c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]
*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*
Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4
)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2
*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/b))/(4*b)))/(8*b*c)
```

3.215. $\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.215.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

3.215. $\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.215. $\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.215.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.54

method	result
derivativedivides	$-\frac{2A}{3b^3x^{\frac{3}{2}}}-\frac{2\left(\frac{\left(\frac{15}{32}Ac^2-\frac{7}{32}Bbc\right)x^{\frac{5}{2}}+\frac{b(19Ac-11Bb)\sqrt{x}}{32}}{\left(cx^2+b\right)^2}+\frac{7(11Ac-3Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\cdots}{\left(\frac{b}{c}\right)\cdots}\right)\cdots\right)}{256b}\right)}{b^3}$
default	$-\frac{2A}{3b^3x^{\frac{3}{2}}}-\frac{2\left(\frac{\left(\frac{15}{32}Ac^2-\frac{7}{32}Bbc\right)x^{\frac{5}{2}}+\frac{b(19Ac-11Bb)\sqrt{x}}{32}}{\left(cx^2+b\right)^2}+\frac{7(11Ac-3Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\cdots}{\left(\frac{b}{c}\right)\cdots}\right)\cdots\right)}{256b}\right)}{b^3}$
risch	$-\frac{2A}{3b^3x^{\frac{3}{2}}}-\frac{\frac{2\left(\frac{15}{32}Ac^2-\frac{7}{32}Bbc\right)x^{\frac{5}{2}}+\frac{b(19Ac-11Bb)\sqrt{x}}{16}}{\left(cx^2+b\right)^2}+\frac{7(11Ac-3Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cdots}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}\cdots}\right)\cdots\right)}{128b}}{b^3}$

input
```
int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3*A/b^3/x^(3/2)-2/b^3*(((15/32*A*c^2-7/32*B*b*c)*x^(5/2)+1/32*b*(19*A*c
-11*B*b)*x^(1/2))/(c*x^2+b)^2+7/256*(11*A*c-3*B*b)*(1/c*b)^(1/4)/b*2^(1/2)
*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1
/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*ar
ctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

3.215.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.39

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx =$$

$$-\frac{21\left(b^3c^2x^6+2b^4cx^4+b^5x^2\right)\left(-\frac{81B^4b^4-1188AB^3b^3c+6534A^2B^2b^2c^2-15972A^3Bbc^3+14641A^4c^4}{b^{15}c}\right)^{\frac{1}{4}}\log\left(7b^4\left(-\frac{81B^4b^4-1\cdots}{\cdots}\right.\right.\cdots}{\cdots}$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
-1/192*(21*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-(81*B^4*b^4 - 1188*A*B^
3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*
c))^(1/4)*log(7*b^4*(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^
2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c))^(1/4) - 7*(3*B*b - 11*A*c
)*sqrt(x)) + 21*(I*b^3*c^2*x^6 + 2*I*b^4*c*x^4 + I*b^5*x^2)*(-(81*B^4*b^4
- 1188*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*
c^4)/(b^15*c))^(1/4)*log(7*I*b^4*(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A
^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c))^(1/4) - 7*(3
*B*b - 11*A*c)*sqrt(x)) + 21*(-I*b^3*c^2*x^6 - 2*I*b^4*c*x^4 - I*b^5*x^2)*
(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^
3 + 14641*A^4*c^4)/(b^15*c))^(1/4)*log(-7*I*b^4*(-(81*B^4*b^4 - 1188*A*B^3
*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c
))^(1/4) - 7*(3*B*b - 11*A*c)*sqrt(x)) - 21*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b
^5*x^2)*(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^
3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c))^(1/4)*log(-7*b^4*(-(81*B^4*b^4 - 1188
*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(
b^15*c))^(1/4) - 7*(3*B*b - 11*A*c)*sqrt(x)) - 4*(7*(3*B*b*c - 11*A*c^2)*x
^4 - 32*A*b^2 + 11*(3*B*b^2 - 11*A*b*c)*x^2)*sqrt(x))/(b^3*c^2*x^6 + 2*b^4
*c*x^4 + b^5*x^2)
```

3.215.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

3.215.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.89

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{7\,(3\,Bbc-11\,Ac^2)x^4-32\,Ab^2+11\,(3\,Bb^2-11\,Abc)x^2}{48\left(b^3c^2x^{\frac{11}{2}}+2\,b^4cx^{\frac{7}{2}}+b^5x^{\frac{3}{2}}\right)}$$
$$+\frac{7\left(\frac{2\,\sqrt{2}(3\,Bb-11\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\,\sqrt{2}(3\,Bb-11\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(3\,Bb-11\,Ac)\log\left(\sqrt{2}\right.}{b^{\frac{3}{4}}c}\right.}{128\,b^3}$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/48*(7*(3*B*b*c - 11*A*c^2)*x^4 - 32*A*b^2 + 11*(3*B*b^2 - 11*A*b*c)*x^2)
/(b^3*c^2*x^(11/2) + 2*b^4*c*x^(7/2) + b^5*x^(3/2)) + 7/128*(2*sqrt(2)*(3*
B*b - 11*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt
(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(3
*B*b - 11*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sq
rt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(3
*B*b - 11*A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/
(b^(3/4)*c^(1/4)) - sqrt(2)*(3*B*b - 11*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*
sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^3
```

3.215.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.94

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{7\sqrt{2}\left(3\,(bc^3)^{\frac{1}{4}}\,Bb-11\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4c} + \frac{7\sqrt{2}\left(3\,(bc^3)^{\frac{1}{4}}\,Bb-11\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4c} + \frac{7\sqrt{2}\left(3\,(bc^3)^{\frac{1}{4}}\,Bb-11\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^4c} - \frac{7\sqrt{2}\left(3\,(bc^3)^{\frac{1}{4}}\,Bb-11\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^4c} - \frac{2\,A}{3\,b^3x^{\frac{3}{2}}} + \frac{7\,Bbcx^{\frac{5}{2}}-15\,Ac^2x^{\frac{5}{2}}+11\,Bb^2\sqrt{x}-19\,Abc\sqrt{x}}{16\,(cx^2+b)^2b^3}$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
7/64*sqrt(2)*(3*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(
2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) + 7/64*sqrt(2)*(
3*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) + 7/128*sqrt(2)*(3*(b*c^3)^(1
/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt
(b/c))/(b^4*c) - 7/128*sqrt(2)*(3*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c
)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c) - 2/3*A/(b^3*x
^(3/2)) + 1/16*(7*B*b*c*x^(5/2) - 15*A*c^2*x^(5/2) + 11*B*b^2*sqrt(x) - 19
*A*b*c*sqrt(x))/((c*x^2 + b)^2*b^3)
```

3.215.9 Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 888, normalized size of antiderivative = 2.76

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
- ((2*A)/(3*b) + (11*x^2*(11*A*c - 3*B*b))/(48*b^2) + (7*c*x^4*(11*A*c - 3
*B*b))/(48*b^3))/(b^2*x^(3/2) + c^2*x^(11/2) + 2*b*c*x^(7/2)) - (atan((((1
1*A*c - 3*B*b)*(x^(1/2)*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 529
85856*A*B*b^10*c^4) - (7*(11*A*c - 3*B*b)*(80740352*A*b^13*c^4 - 22020096*
B*b^14*c^3))/(64*(-b)^(15/4)*c^(1/4)))*7i)/(64*(-b)^(15/4)*c^(1/4)) + ((11
*A*c - 3*B*b)*(x^(1/2)*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 5298
5856*A*B*b^10*c^4) + (7*(11*A*c - 3*B*b)*(80740352*A*b^13*c^4 - 22020096*B
*b^14*c^3))/(64*(-b)^(15/4)*c^(1/4)))*7i)/(64*(-b)^(15/4)*c^(1/4)))/((7*(1
1*A*c - 3*B*b)*(x^(1/2)*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 529
85856*A*B*b^10*c^4) - (7*(11*A*c - 3*B*b)*(80740352*A*b^13*c^4 - 22020096*
B*b^14*c^3))/(64*(-b)^(15/4)*c^(1/4))))/(64*(-b)^(15/4)*c^(1/4)) - (7*(11*
A*c - 3*B*b)*(x^(1/2)*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 52985
856*A*B*b^10*c^4) + (7*(11*A*c - 3*B*b)*(80740352*A*b^13*c^4 - 22020096*B*
b^14*c^3))/(64*(-b)^(15/4)*c^(1/4))))/(64*(-b)^(15/4)*c^(1/4))))*(11*A*c -
 3*B*b)*7i)/(32*(-b)^(15/4)*c^(1/4)) - (7*atan(((7*(11*A*c - 3*B*b)*(x^(1/
2)*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 52985856*A*B*b^10*c^4) -
 ((11*A*c - 3*B*b)*(80740352*A*b^13*c^4 - 22020096*B*b^14*c^3)*7i)/(64*(-b
)^(15/4)*c^(1/4))))/(64*(-b)^(15/4)*c^(1/4)) + (7*(11*A*c - 3*B*b)*(x^(1/2
)*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 52985856*A*B*b^10*c^4) +
((11*A*c - 3*B*b)*(80740352*A*b^13*c^4 - 22020096*B*b^14*c^3)*7i)/(64*(...
```

3.216 $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.216.1 Optimal result

Integrand size = 26, antiderivative size = 343

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB-13Ac)}{16b^4\sqrt{x}} - \frac{bB-Ac}{4bcx^{5/2}(b+cx^2)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}(b+cx^2)} + \frac{9\sqrt[4]{c}(5bB-13Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{17/4}} - \frac{9\sqrt[4]{c}(5bB-13Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{17/4}} - \frac{9\sqrt[4]{c}(5bB-13Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}} + \frac{9\sqrt[4]{c}(5bB-13Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}}$$

output
```
9/80*(-13*A*c+5*B*b)/b^3/c/x^(5/2)+1/4*(A*c-B*b)/b/c/x^(5/2)/(c*x^2+b)^2+1
/16*(13*A*c-5*B*b)/b^2/c/x^(5/2)/(c*x^2+b)+9/64*c^(1/4)*(-13*A*c+5*B*b)*ar
ctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)-9/64*c^(1/4)*(-13
*A*c+5*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)-9/1
28*c^(1/4)*(-13*A*c+5*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^
(1/2))/b^(17/4)*2^(1/2)+9/128*c^(1/4)*(-13*A*c+5*B*b)*ln(b^(1/2)+x*c^(1/2)
+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(17/4)*2^(1/2)-9/16*(-13*A*c+5*B*b)/b^
4/x^(1/2)
```

3.216.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.61

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{-\frac{4\sqrt[4]{b}(5bBx^2(32b^2+81bcx^2+45c^2x^4)+A(32b^3-416b^2cx^2-1053bc^2x^4-585c^3x^6))}{x^{5/2}(b+cx^2)^2}+45\sqrt{2}\sqrt[4]{c}(5bB-13}{320b^{17/4}}$$

input
```
Integrate[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
((-4*b^(1/4)*(5*b*B*x^2*(32*b^2 + 81*b*c*x^2 + 45*c^2*x^4) + A*(32*b^3 - 4
16*b^2*c*x^2 - 1053*b*c^2*x^4 - 585*c^3*x^6)))/(x^(5/2)*(b + c*x^2)^2) + 4
5*Sqrt[2]*c^(1/4)*(5*b*B - 13*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x])] + 45*Sqrt[2]*c^(1/4)*(5*b*B - 13*A*c)*ArcTanh[(Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(320*b^(17/4))
```

3.216.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.96, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {9, 362, 253, 264, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{A+Bx^2}{x^{7/2}(b+cx^2)^3}dx$$

$$\Big\downarrow \ 362$$

$$-\frac{(5bB-13Ac)\int \frac{1}{x^{7/2}(cx^2+b)^2}dx}{8bc}-\frac{bB-Ac}{4bcx^{5/2}(b+cx^2)^2}$$

$$\Big\downarrow \ 253$$

$$-\frac{(5bB-13Ac)\left(\frac{9\int \frac{1}{x^{7/2}(cx^2+b)}dx}{4b}+\frac{1}{2bx^{5/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{5/2}(b+cx^2)^2}$$

$$\Big\downarrow \text{264}$$

$$-\frac{(5bB-13Ac)\left(\frac{9\left(-\frac{c\int\frac{1}{x^{3/2}(cx^2+b)}dx}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{264}$$

$$-\frac{(5bB-13Ac)\left(\frac{9\left(-\frac{c\left(-\frac{c\int\frac{\sqrt{x}}{cx^2+b}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{266}$$

$$-\frac{(5bB-13Ac)\left(\frac{9\left(-\frac{c\left(-\frac{2c\int\frac{x}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{826}$$

3.216. $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$-\frac{(5bB-13Ac)\left(\frac{9\left(-\frac{c\left(-\frac{2c\left(\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 1476$$

3.216. $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$-\frac{(5bB-13Ac)\left(\frac{9\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}(b+cx^2)}\right)}{8bc}$$

$$\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2}$$

3.216. $\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$\downarrow$ 1082

3.216. $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$-\frac{(5bB-13Ac)\left(\frac{9\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}(b+cx^{2}}\right.}{8bc}$$

$$\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2}$$

3.216. $\int\frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

↓ 217

3.216. $\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$-\frac{(5bB-13Ac)\left(\frac{9\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2}$$

3.216. $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\downarrow 1479$$

3.216. $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$(5bB-13Ac)\left(\frac{9\left(c-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\cdots\right.}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}\cdots\right.}\cdots}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\cdots}}{2\sqrt{c}}\cdots\right)}{b}\cdots\right)}{b}\cdots}{4b}\cdots\right.$$

3.216. $\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$\downarrow 25$$

3.216. $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$(5bB-13Ac)\left(\frac{\left(9\left(c-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\ldots\right)}\ldots}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\ldots\right)}{b}\ldots\right)}{b}\ldots\right)}{4b}\ldots\right)$$

3.216. $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$\Big\downarrow \text{27}$$

3.216. $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$(5bB-13Ac)\left(\frac{9\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}\right)}{4b}\right.$$

3.216. $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$\downarrow$ 1103

3.216. $\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$-\frac{(5bB-13Ac)\left(\frac{9\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\cdots\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{4b}\cdots\right)}{8bc}$$

$$\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2}$$

3.216. $\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

input
```
Int[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*(b*B - A*c)/(b*c*x^(5/2)*(b + c*x^2)^2) - ((5*b*B - 13*A*c)*(1/(2*b*x
^(5/2)*(b + c*x^2)) + (9*(-2/(5*b*x^(5/2)) - (c*(-2/(b*Sqrt[x]) - (2*c*((-
(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))
+ ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))
/(2*Sqrt[c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sq
rt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/b))/b))/(4*b
)))/(8*b*c)
```

3.216.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.216. $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.216.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.55

method	result
derivativedivides	$\frac{2c\left(\frac{\left(\frac{21}{32}A\,c^2-\frac{13}{32}Bbc\right)x^{\frac{7}{2}}+\frac{b(25Ac-17Bb)x^{\frac{3}{2}}}{32}}{\left(c\,x^2+b\right)^2}+\frac{\left(\frac{117Ac}{32}-\frac{45Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arcta\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b^4}$
default	$\frac{2c\left(\frac{\left(\frac{21}{32}A\,c^2-\frac{13}{32}Bbc\right)x^{\frac{7}{2}}+\frac{b(25Ac-17Bb)x^{\frac{3}{2}}}{32}}{\left(c\,x^2+b\right)^2}+\frac{\left(\frac{117Ac}{32}-\frac{45Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arcta\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b^4}$
risch	$-\frac{2\left(-15Ac\,x^2+5bB\,x^2+Ab\right)}{5b^4x^{\frac{5}{2}}}+\frac{c\left(\frac{2\left(\frac{21}{32}A\,c^2-\frac{13}{32}Bbc\right)x^{\frac{7}{2}}+\frac{b(25Ac-17Bb)x^{\frac{3}{2}}}{16}}{\left(c\,x^2+b\right)^2}+\frac{\left(\frac{117Ac}{32}-\frac{45Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+}\right.\right.}{}\right.}{b^4}$

input
```
int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2/b^4*c*(((21/32*A*c^2-13/32*B*b*c)*x^(7/2)+1/32*b*(25*A*c-17*B*b)*x^(3/2)
)/(c*x^2+b)^2+1/8*(117/32*A*c-45/32*B*b)/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1
/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2
)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/
2)/(1/c*b)^(1/4)*x^(1/2)-1)))-2/5*A/b^3/x^(5/2)-2*(-3*A*c+B*b)/b^4/x^(1/2)
```

3.216.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 918, normalized size of antiderivative = 2.68

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{45\,(b^4c^2x^7+2\,b^5cx^5+b^6x^3)\left(-\frac{625\,B^4b^4c-6500\,AB^3b^3c^2+25350\,A^2B^2b^2c^3-43940\,A^3Bbc^4+28561\,A^4c^5}{b^{17}}\right.}{}$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/320*(45*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-(625*B^4*b^4*c - 6500*A*
B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b
^17)^(1/4)*log(729*b^13*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*
B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(3/4) - 729*(125*B^
3*b^3*c - 975*A*B^2*b^2*c^2 + 2535*A^2*B*b*c^3 - 2197*A^3*c^4)*sqrt(x)) -
45*(I*b^4*c^2*x^7 + 2*I*b^5*c*x^5 + I*b^6*x^3)*(-(625*B^4*b^4*c - 6500*A*B
^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^
17)^(1/4)*log(729*I*b^13*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2
*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(3/4) - 729*(125*B
^3*b^3*c - 975*A*B^2*b^2*c^2 + 2535*A^2*B*b*c^3 - 2197*A^3*c^4)*sqrt(x)) -
 45*(-I*b^4*c^2*x^7 - 2*I*b^5*c*x^5 - I*b^6*x^3)*(-(625*B^4*b^4*c - 6500*A
*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/
b^17)^(1/4)*log(-729*I*b^13*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*
A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(3/4) - 729*(12
5*B^3*b^3*c - 975*A*B^2*b^2*c^2 + 2535*A^2*B*b*c^3 - 2197*A^3*c^4)*sqrt(x)
) - 45*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-(625*B^4*b^4*c - 6500*A*B^3
*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17
)^(1/4)*log(-729*b^13*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^
2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(3/4) - 729*(125*B^3*
b^3*c - 975*A*B^2*b^2*c^2 + 2535*A^2*B*b*c^3 - 2197*A^3*c^4)*sqrt(x)) -...
```

3.216.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

3.216.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.83

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{45\,(5\,Bbc^2-13\,Ac^3)x^6+81\,(5\,Bb^2c-13\,Abc^2)x^4+32\,Ab^3+32\,(5\,Bb^3-13\,Ab^2c)x^2}{80\left(b^4c^2x^{\frac{13}{2}}+2\,b^5cx^{\frac{9}{2}}+b^6x^{\frac{5}{2}}\right)} - \frac{9\,(5\,Bbc-13\,Ac^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\right.}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{128\,b^4}$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/80*(45*(5*B*b*c^2 - 13*A*c^3)*x^6 + 81*(5*B*b^2*c - 13*A*b*c^2)*x^4 + 3
2*A*b^3 + 32*(5*B*b^3 - 13*A*b^2*c)*x^2)/(b^4*c^2*x^(13/2) + 2*b^5*c*x^(9/
2) + b^6*x^(5/2)) - 9/128*(5*B*b*c - 13*A*c^2)*(2*sqrt(2)*arctan(1/2*sqrt(
2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(s
qrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(
1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqr
t(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x +
 sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x)
 + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^4
```

3.216.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.95

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx =$$
$$-\frac{9\sqrt{2}\left(5\,(bc^3)^{\frac{3}{4}}\,Bb-13\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5c^2}$$
$$-\frac{9\sqrt{2}\left(5\,(bc^3)^{\frac{3}{4}}\,Bb-13\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5c^2}$$
$$+\frac{9\sqrt{2}\left(5\,(bc^3)^{\frac{3}{4}}\,Bb-13\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5c^2}$$
$$-\frac{9\sqrt{2}\left(5\,(bc^3)^{\frac{3}{4}}\,Bb-13\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5c^2}$$
$$-\frac{13\,Bbc^2x^{\frac{7}{2}}-21\,Ac^3x^{\frac{7}{2}}+17\,Bb^2cx^{\frac{3}{2}}-25\,Abc^2x^{\frac{3}{2}}}{16\,(cx^2+b)^2b^4}-\frac{2\,(5\,Bbx^2-15\,Acx^2+Ab)}{5\,b^4x^{\frac{5}{2}}}$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-9/64*sqrt(2)*(5*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt
(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^5*c^2) - 9/64*sqrt(2
)*(5*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^5*c^2) + 9/128*sqrt(2)*(5*(b*c^
3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
 sqrt(b/c))/(b^5*c^2) - 9/128*sqrt(2)*(5*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3
/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^5*c^2) - 1/1
6*(13*B*b*c^2*x^(7/2) - 21*A*c^3*x^(7/2) + 17*B*b^2*c*x^(3/2) - 25*A*b*c^2
*x^(3/2))/((c*x^2 + b)^2*b^4) - 2/5*(5*B*b*x^2 - 15*A*c*x^2 + A*b)/(b^4*x^
(5/2))
```

3.216.9 Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.44

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{2\,x^2\,(13\,A\,c-5\,B\,b)}{5\,b^2}-\frac{2\,A}{5\,b}+\frac{9\,c^2\,x^6\,(13\,A\,c-5\,B\,b)}{16\,b^4}+\frac{81\,c\,x^4\,(13\,A\,c-5\,B\,b)}{80\,b^3}}{b^2\,x^{5/2}+c^2\,x^{13/2}+2\,b\,c\,x^{9/2}} + \frac{9\,(-c)^{1/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(13\,A\,c-5\,B\,b)}{32\,b^{17/4}} - \frac{9\,(-c)^{1/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(13\,A\,c-5\,B\,b)}{32\,b^{17/4}}$$

input
```
int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
((2*x^2*(13*A*c - 5*B*b))/(5*b^2) - (2*A)/(5*b) + (9*c^2*x^6*(13*A*c - 5*B
*b))/(16*b^4) + (81*c*x^4*(13*A*c - 5*B*b))/(80*b^3))/(b^2*x^(5/2) + c^2*x
^(13/2) + 2*b*c*x^(9/2)) + (9*(-c)^(1/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)
)*(13*A*c - 5*B*b))/(32*b^(17/4)) - (9*(-c)^(1/4)*atanh(((-c)^(1/4)*x^(1/2
))/b^(1/4))*(13*A*c - 5*B*b))/(32*b^(17/4))
```

3.217 $\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.217.1 Optimal result

Integrand size = 26, antiderivative size = 343

$$\begin{aligned}\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= \frac{11(7bB-15Ac)}{112b^3cx^{7/2}}-\frac{11(7bB-15Ac)}{48b^4x^{3/2}}-\frac{bB-Ac}{4bcx^{7/2}(b+cx^2)^2}\\ &-\frac{7bB-15Ac}{16b^2cx^{7/2}(b+cx^2)}+\frac{11c^{3/4}(7bB-15Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{19/4}}\\ &-\frac{11c^{3/4}(7bB-15Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{19/4}}\\ &+\frac{11c^{3/4}(7bB-15Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}\\ &-\frac{11c^{3/4}(7bB-15Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}\end{aligned}$$

output
```
11/112*(-15*A*c+7*B*b)/b^3/c/x^(7/2)-11/48*(-15*A*c+7*B*b)/b^4/x^(3/2)+1/4
*(A*c-B*b)/b/c/x^(7/2)/(c*x^2+b)^2+1/16*(15*A*c-7*B*b)/b^2/c/x^(7/2)/(c*x^
2+b)+11/64*c^(3/4)*(-15*A*c+7*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4
))/b^(19/4)*2^(1/2)-11/64*c^(3/4)*(-15*A*c+7*B*b)*arctan(1+c^(1/4)*2^(1/2)
*x^(1/2)/b^(1/4))/b^(19/4)*2^(1/2)+11/128*c^(3/4)*(-15*A*c+7*B*b)*ln(b^(1/
2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(19/4)*2^(1/2)-11/128*c^(3
/4)*(-15*A*c+7*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/
b^(19/4)*2^(1/2)
```

3.217.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.61

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{-\frac{4b^{3/4}(7bBx^2(32b^2+121bcx^2+77c^2x^4)+3A(32b^3-160b^2cx^2-605bc^2x^4-385c^3x^6))}{x^{7/2}(b+cx^2)^2}+231\sqrt{2}c^{3/4}(7bB-}{1344b}$$

input
```
Integrate[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
((-4*b^(3/4)*(7*b*B*x^2*(32*b^2 + 121*b*c*x^2 + 77*c^2*x^4) + 3*A*(32*b^3
- 160*b^2*c*x^2 - 605*b*c^2*x^4 - 385*c^3*x^6)))/(x^(7/2)*(b + c*x^2)^2) +
 231*Sqrt[2]*c^(3/4)*(7*b*B - 15*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x])] + 231*Sqrt[2]*c^(3/4)*(-7*b*B + 15*A*c)*ArcTan
h[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(1344*b^(19/4)
)
```

3.217.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {9, 362, 253, 264, 264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{A+Bx^2}{x^{9/2}(b+cx^2)^3}dx$$

$$\Big\downarrow \ 362$$

$$-\frac{(7bB-15Ac)\int \frac{1}{x^{9/2}(cx^2+b)^2}dx}{8bc}-\frac{bB-Ac}{4bcx^{7/2}(b+cx^2)^2}$$

$$\Big\downarrow \ 253$$

$$-\frac{(7bB-15Ac)\left(\frac{11\int \frac{1}{x^{9/2}\left(cx^2+b\right)}dx}{4b}+\frac{1}{2bx^{7/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}$$

$\downarrow$ 264

$$-\frac{(7bB-15Ac)\left(\frac{11\left(-\frac{c\int \frac{1}{x^{5/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}$$

$\downarrow$ 264

$$-\frac{(7bB-15Ac)\left(\frac{11\left(-\frac{c\left(-\frac{c\int \frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}$$

$\downarrow$ 266

$$-\frac{(7bB-15Ac)\left(\frac{11\left(-\frac{c\left(-\frac{2c\int \frac{1}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}$$

$\downarrow$ 755

3.217. $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$-\frac{(7bB-15Ac)\left(\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \ 1476$$

3.217. $\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$-\frac{(7bB-15Ac)\left(\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}\,d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}\,d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}\,d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}(b+c}\right.}{8bc}$$

$$\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}$$

3.217. $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

↓ 1082

3.217. $\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$-\frac{(7bB-15Ac)\left(\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}(b+}\right)}{8bc}$$

$$\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}$$

3.217. $\int\frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

↓ 217

3.217. $\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$-\frac{(7bB-15Ac)\left(\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}\,d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}(b+cx^2)}\right)}{8bc}$$

$$\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}$$

3.217. $\int\frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

↓ 1479

3.217. $\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$(7bB-15Ac)\left(\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\cdots\right)}{\sqrt{2}\cdots}}{2\sqrt{b}}\cdots\right.}{b}\cdots\right)}{b}\cdots\right)}{4b}\cdots\right.$$

3.217. $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

3.217. $\downarrow$ 25

3.217. $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$(7bB-15Ac)\left(\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\,\cdots}{\cdots}\right)}{\sqrt{2}\sqrt[4]{b}\,\cdots}}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{4b}\cdots\right.$$

3.217. $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$\downarrow 27$$

3.217. $\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$(7bB-15Ac)\left(\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{}{3bx}\cdots\right)}{b}\cdots\right)}{4b}\cdots\right.$$

3.217. $\int\frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$\downarrow$ 1103

3.217. $\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$-\frac{(7bB-15Ac)\left(\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\cdots\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}\cdots}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{4b}\cdots\right)}{8bc}\cdots$$

$$\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}$$

3.217. $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

input
```
Int[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*(b*B - A*c)/(b*c*x^(7/2)*(b + c*x^2)^2) - ((7*b*B - 15*A*c)*(1/(2*b*x
^(7/2)*(b + c*x^2)) + (11*(-2/(7*b*x^(7/2)) - (c*(-2/(3*b*x^(3/2)) - (2*c*
((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)
)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4
)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sq
rt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)
*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/b))/b))/(
4*b)))/(8*b*c)
```

3.217.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.217. $\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3} \, dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.217.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.55

method	result
derivativedivides	$-\frac{2A}{7b^3x^{\frac{7}{2}}}-\frac{2(-3Ac+Bb)}{3b^4x^{\frac{3}{2}}}+\frac{2c\left(\frac{\left(\frac{23}{32}Ac^2-\frac{15}{32}Bbc\right)x^{\frac{5}{2}}+\frac{b(27Ac-19Bb)\sqrt{x}}{32}}{(cx^2+b)^2}+\frac{11(15Ac-7Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}\cdots}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}\cdots}\right.\right.}{\cdots}\right.}{b^4}$
default	$-\frac{2A}{7b^3x^{\frac{7}{2}}}-\frac{2(-3Ac+Bb)}{3b^4x^{\frac{3}{2}}}+\frac{2c\left(\frac{\left(\frac{23}{32}Ac^2-\frac{15}{32}Bbc\right)x^{\frac{5}{2}}+\frac{b(27Ac-19Bb)\sqrt{x}}{32}}{(cx^2+b)^2}+\frac{11(15Ac-7Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}\cdots}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}\cdots}\right.\right.}{\cdots}\right.}{b^4}$
risch	$-\frac{2(-21Acx^2+7bBx^2+3Ab)}{21b^4x^{\frac{7}{2}}}+\frac{c\left(\frac{2\left(\frac{23}{32}Ac^2-\frac{15}{32}Bbc\right)x^{\frac{5}{2}}+\frac{b(27Ac-19Bb)\sqrt{x}}{16}}{(cx^2+b)^2}+\frac{11(15Ac-7Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\cdots}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\cdots}\right.\right.}{\cdots}\right.}{b^4}$

input
```
int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/7*A/b^3/x^(7/2)-2/3*(-3*A*c+B*b)/b^4/x^(3/2)+2/b^4*c*(((23/32*A*c^2-15/
32*B*b*c)*x^(5/2)+1/32*b*(27*A*c-19*B*b)*x^(1/2))/(c*x^2+b)^2+11/256*(15*A
*c-7*B*b)*(1/c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/
c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1
/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

3.217.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 822, normalized size of antiderivative = 2.40

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{231\,(b^4c^2x^8+2\,b^5cx^6+b^6x^4)\left(-\frac{2401\,B^4b^4c^3-20580\,AB^3b^3c^4+66150\,A^2B^2b^2c^5-94500\,A^3Bbc^6+5062}{b^{19}}\right.}{}$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/1344*(231*(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-(2401*B^4*b^4*c^3 - 20
580*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*
c^7)/b^19)^(1/4)*log(11*b^5*(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 66
150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19)^(1/4) - 11*
(7*B*b*c - 15*A*c^2)*sqrt(x)) - 231*(-I*b^4*c^2*x^8 - 2*I*b^5*c*x^6 - I*b^
6*x^4)*(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 -
 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19)^(1/4)*log(11*I*b^5*(-(2401*B^4*b
^4*c^3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 +
 50625*A^4*c^7)/b^19)^(1/4) - 11*(7*B*b*c - 15*A*c^2)*sqrt(x)) - 231*(I*b^
4*c^2*x^8 + 2*I*b^5*c*x^6 + I*b^6*x^4)*(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b
^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19)^
(1/4)*log(-11*I*b^5*(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*
B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19)^(1/4) - 11*(7*B*b*c
 - 15*A*c^2)*sqrt(x)) - 231*(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-(2401*
B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*
c^6 + 50625*A^4*c^7)/b^19)^(1/4)*log(-11*b^5*(-(2401*B^4*b^4*c^3 - 20580*A
*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/
b^19)^(1/4) - 11*(7*B*b*c - 15*A*c^2)*sqrt(x)) - 4*(77*(7*B*b*c^2 - 15*A*c
^3)*x^6 + 121*(7*B*b^2*c - 15*A*b*c^2)*x^4 + 96*A*b^3 + 32*(7*B*b^3 - 15*A
*b^2*c)*x^2)*sqrt(x))/(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)
```

3.217.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

3.217.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.94

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx =$$
$$-\frac{77\,(7\,Bbc^2-15\,Ac^3)x^6+121\,(7\,Bb^2c-15\,Abc^2)x^4+96\,Ab^3+32\,(7\,Bb^3-15\,Ab^2c)x^2}{336\,\left(b^4c^2x^{\frac{15}{2}}+2\,b^5cx^{\frac{11}{2}}+b^6x^{\frac{7}{2}}\right)}$$
$$-\frac{11\left(\frac{2\sqrt{2}(7\,Bbc-15\,Ac^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(7\,Bbc-15\,Ac^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(7\,Bbc-15\,Ac\cdots}{\cdots}\right.}{128\,b^4}$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/336*(77*(7*B*b*c^2 - 15*A*c^3)*x^6 + 121*(7*B*b^2*c - 15*A*b*c^2)*x^4 +
 96*A*b^3 + 32*(7*B*b^3 - 15*A*b^2*c)*x^2)/(b^4*c^2*x^(15/2) + 2*b^5*c*x^(
11/2) + b^6*x^(7/2)) - 11/128*(2*sqrt(2)*(7*B*b*c - 15*A*c^2)*arctan(1/2*s
qrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c))
)/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(7*B*b*c - 15*A*c^2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sq
rt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(7*B*b*c - 15*A*c^2)*log
(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4))
- sqrt(2)*(7*B*b*c - 15*A*c^2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt
(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^4
```

3.217.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.92

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx =$$

$$-\frac{11\sqrt{2}\left(7\,(bc^3)^{\frac{1}{4}}\,Bb-15\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5}$$
$$-\frac{11\sqrt{2}\left(7\,(bc^3)^{\frac{1}{4}}\,Bb-15\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5}$$
$$-\frac{11\sqrt{2}\left(7\,(bc^3)^{\frac{1}{4}}\,Bb-15\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5}$$
$$+\frac{11\sqrt{2}\left(7\,(bc^3)^{\frac{1}{4}}\,Bb-15\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5}$$
$$-\frac{15\,Bbc^2x^{\frac{5}{2}}-23\,Ac^3x^{\frac{5}{2}}+19\,Bb^2c\sqrt{x}-27\,Abc^2\sqrt{x}}{16\,(cx^2+b)^2b^4}$$
$$-\frac{2\,(7\,Bbx^2-21\,Acx^2+3\,Ab)}{21\,b^4x^{\frac{7}{2}}}$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-11/64*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 15*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqr
t(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^5 - 11/64*sqrt(2)*(7
*(b*c^3)^(1/4)*B*b - 15*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b
/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^5 - 11/128*sqrt(2)*(7*(b*c^3)^(1/4)*
B*b - 15*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c
))/b^5 + 11/128*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 15*(b*c^3)^(1/4)*A*c)*log(-
sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^5 - 1/16*(15*B*b*c^2*x^(5/2
) - 23*A*c^3*x^(5/2) + 19*B*b^2*c*sqrt(x) - 27*A*b*c^2*sqrt(x))/((c*x^2 +
b)^2*b^4) - 2/21*(7*B*b*x^2 - 21*A*c*x^2 + 3*A*b)/(b^4*x^(7/2))
```

3.217.9 Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 626, normalized size of antiderivative = 1.83

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{2\,x^2\,(15\,A\,c-7\,B\,b)}{21\,b^2}-\frac{2\,A}{7\,b}+\frac{11\,c^2\,x^6\,(15\,A\,c-7\,B\,b)}{48\,b^4}+\frac{121\,c\,x^4\,(15\,A\,c-7\,B\,b)}{336\,b^3}}{b^2\,x^{7/2}+c^2\,x^{15/2}+2\,b\,c\,x^{11/2}}$$

$$+\frac{11\,(-c)^{3/4}\,\mathrm{atan}\left(\frac{\frac{11\,(-c)^{3/4}\,(15\,A\,c-7\,B\,b)\,\left(\sqrt{x}\,\left(446054400\,A^2\,b^{12}\,c^7-416317440\,A\,B\,b^{13}\,c^6+97140736\,B^2\,b^{14}\,c^5\right)-\frac{(-c)^{3/4}\,(15\,A\,c-7\,B\,b)\,\left(173015040\right.}{64\,b^{19}}\right.}{64\,b^{19/4}}}{\frac{(-c)^{3/4}\,(15\,A\,c-7\,B\,b)\,\left(\sqrt{x}\,\left(446054400\,A^2\,b^{12}\,c^7-416317440\,A\,B\,b^{13}\,c^6+97140736\,B^2\,b^{14}\,c^5\right)-\frac{(-c)^{3/4}\,(15\,A\,c-7\,B\,b)\,\left(173015040\,A\,b\right.}{64\,b^{19/4}}\right.}{64\,b^{19/4}}}\right.}{}$$

$$-\frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{A^3\,c^8\,\sqrt{x}\,3375\mathrm{i}-B^3\,b^3\,c^5\,\sqrt{x}\,343\mathrm{i}-A^2\,B\,b\,c^7\,\sqrt{x}\,4725\mathrm{i}+A\,B^2\,b^2\,c^6\,\sqrt{x}\,2205\mathrm{i}}{b^{1/4}\,(-c)^{19/4}\,\left(c\,\left(c\,\left(3375\,A^3\,c-4725\,A^2\,B\,b\right)+2205\,A\,B^2\,b^2\right)-343\,B^3\,b^3\right)}\right)\,(15\,A\,c-7\,B\,b)\,11\mathrm{i}}{32\,b^{19/4}}$$

input
```
int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
((2*x^2*(15*A*c - 7*B*b))/(21*b^2) - (2*A)/(7*b) + (11*c^2*x^6*(15*A*c - 7
*B*b))/(48*b^4) + (121*c*x^4*(15*A*c - 7*B*b))/(336*b^3))/(b^2*x^(7/2) + c
^2*x^(15/2) + 2*b*c*x^(11/2)) + (11*(-c)^(3/4)*atan(((11*(-c)^(3/4)*(15*A*
c - 7*B*b)*(x^(1/2)*(446054400*A^2*b^12*c^7 + 97140736*B^2*b^14*c^5 - 4163
17440*A*B*b^13*c^6) - ((-c)^(3/4)*(15*A*c - 7*B*b)*(173015040*A*b^17*c^5 -
 80740352*B*b^18*c^4)*11i)/(64*b^(19/4))))/(64*b^(19/4)) + (11*(-c)^(3/4)*
(15*A*c - 7*B*b)*(x^(1/2)*(446054400*A^2*b^12*c^7 + 97140736*B^2*b^14*c^5
- 416317440*A*B*b^13*c^6) + ((-c)^(3/4)*(15*A*c - 7*B*b)*(173015040*A*b^17
*c^5 - 80740352*B*b^18*c^4)*11i)/(64*b^(19/4))))/(64*b^(19/4)))/(((-c)^(3/
4)*(15*A*c - 7*B*b)*(x^(1/2)*(446054400*A^2*b^12*c^7 + 97140736*B^2*b^14*c
^5 - 416317440*A*B*b^13*c^6) - ((-c)^(3/4)*(15*A*c - 7*B*b)*(173015040*A*b
^17*c^5 - 80740352*B*b^18*c^4)*11i)/(64*b^(19/4)))*11i)/(64*b^(19/4)) - ((
-c)^(3/4)*(15*A*c - 7*B*b)*(x^(1/2)*(446054400*A^2*b^12*c^7 + 97140736*B^2
*b^14*c^5 - 416317440*A*B*b^13*c^6) + ((-c)^(3/4)*(15*A*c - 7*B*b)*(173015
040*A*b^17*c^5 - 80740352*B*b^18*c^4)*11i)/(64*b^(19/4)))*11i)/(64*b^(19/4
))))*(15*A*c - 7*B*b))/(32*b^(19/4)) - ((-c)^(3/4)*atan((A^3*c^8*x^(1/2)*3
375i - B^3*b^3*c^5*x^(1/2)*343i - A^2*B*b*c^7*x^(1/2)*4725i + A*B^2*b^2*c^
6*x^(1/2)*2205i)/(b^(1/4)*(-c)^(19/4)*(c*(c*(3375*A^3*c - 4725*A^2*B*b) +
2205*A*B^2*b^2) - 343*B^3*b^3))))*(15*A*c - 7*B*b)*11i)/(32*b^(19/4))
```

3.218 $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3} dx$

3.218.1 Optimal result

Integrand size = 26, antiderivative size = 365

$$\begin{aligned}\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3} dx =& \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} \\ &- \frac{bB-Ac}{4bcx^{9/2}(b+cx^2)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}(b+cx^2)} \\ &- \frac{13c^{5/4}(9bB-17Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{21/4}} \\ &+ \frac{13c^{5/4}(9bB-17Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{21/4}} \\ &+ \frac{13c^{5/4}(9bB-17Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}} \\ &- \frac{13c^{5/4}(9bB-17Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}\end{aligned}$$

output
```
13/144*(-17*A*c+9*B*b)/b^3/c/x^(9/2)-13/80*(-17*A*c+9*B*b)/b^4/x^(5/2)+1/4
*(A*c-B*b)/b/c/x^(9/2)/(c*x^2+b)^2+1/16*(17*A*c-9*B*b)/b^2/c/x^(9/2)/(c*x^
2+b)-13/64*c^(5/4)*(-17*A*c+9*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4
))/b^(21/4)*2^(1/2)+13/64*c^(5/4)*(-17*A*c+9*B*b)*arctan(1+c^(1/4)*2^(1/2)
*x^(1/2)/b^(1/4))/b^(21/4)*2^(1/2)+13/128*c^(5/4)*(-17*A*c+9*B*b)*ln(b^(1/
2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(21/4)*2^(1/2)-13/128*c^(5
/4)*(-17*A*c+9*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/
b^(21/4)*2^(1/2)+13/16*c*(-17*A*c+9*B*b)/b^5/x^(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{-\frac{4\sqrt[4]{b}(-9bBx^2(-32b^3+416b^2cx^2+1053bc^2x^4+585c^3x^6)+A(160b^4-544b^3cx^2+7072b^2c^2x^4+17901bc^3x^6+9945c^4x^8))}{x^{9/2}(b+cx^2)^2}+585\sqrt{2}c^{5/4}(-9}{2880b^{21/4}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
((-4*b^(1/4)*(-9*b*B*x^2*(-32*b^3 + 416*b^2*c*x^2 + 1053*b*c^2*x^4 + 585*c
^3*x^6) + A*(160*b^4 - 544*b^3*c*x^2 + 7072*b^2*c^2*x^4 + 17901*b*c^3*x^6
+ 9945*c^4*x^8)))/(x^(9/2)*(b + c*x^2)^2) + 585*Sqrt[2]*c^(5/4)*(-9*b*B +
17*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] +
585*Sqrt[2]*c^(5/4)*(-9*b*B + 17*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqr
t[x])/(Sqrt[b] + Sqrt[c]*x)])/(2880*b^(21/4))
```

3.218.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.95, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {9, 362, 253, 264, 264, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{A+Bx^2}{x^{11/2}\left(b+cx^2\right)^3}dx$$

$$\Big\downarrow \ 362$$

$$-\frac{(9bB-17Ac)\int \frac{1}{x^{11/2}\left(cx^2+b\right)^2}dx}{8bc}-\frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \ 253$$

$$-\frac{(9bB-17Ac)\left(\frac{13\int \frac{1}{x^{11/2}\left(cx^2+b\right)}dx}{4b}+\frac{1}{2bx^{9/2}\left(b+cx^2\right)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \ 264$$

$$-\frac{(9bB-17Ac)\left(\frac{13\left(-\frac{c\int \frac{1}{x^{7/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{1}{2bx^{9/2}\left(b+cx^2\right)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \ 264$$

$$-\frac{(9bB-17Ac)\left(\frac{13\left(-\frac{c\left(-\frac{c\int \frac{1}{x^{3/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{1}{2bx^{9/2}\left(b+cx^2\right)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \ 264$$

3.218. $\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

$$-\frac{(9bB-17Ac)\left(\frac{13\left(-\frac{c\left(-\frac{c\left(-\frac{c\int\frac{\sqrt{x}}{cx^2+b}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{1}{2bx^{9/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 266$$

$$-\frac{(9bB-17Ac)\left(\frac{13\left(-\frac{c\left(-\frac{c\left(-\frac{2c\int\frac{x}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{1}{2bx^{9/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 826$$

3.218. $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$-\frac{(9bB-17Ac)\left(\frac{13\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{1}{2bx^{9/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2}$$

$\big\downarrow$ 1476

3.218. $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$(9bB-17Ac)\left(\frac{13\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}\right)+$$

3.218. $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$\downarrow$ 1082

3.218. $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$(9bB-17Ac)\frac{13\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}-$$

3.218. $\int\frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$\downarrow$ 217

3.218. $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$(9bB-17Ac)\frac{13\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}\,d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{\ }{2bx}$$

3.218. $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

↓ 1479

3.218. $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$13\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\cdots\right.}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\cdots}{\cdots}\right.}\cdots}{2\sqrt{2}\sqrt[4]{\cdots}}}{2\sqrt{c}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)\cdots$$

3.218. $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

3.218. $\downarrow$ 25

3.218. $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3} \, dx$

$$13\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\cdots\right.}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\cdots}}{\sqrt[4]{c}}\cdots\right.}\cdots}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{\cdots}}\cdots}{2\sqrt{c}}\cdots\right.}{b}\cdots\right.}{b}\cdots\right.}{b}\cdots\right.$$

3.218. $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$\downarrow$ 27

3.218. $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\, dx$

$$13\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{b}\right)}{b}\right)}{b}\right.$$

3.218. $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$\downarrow$ 1103

3.218. $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

$$
(9bB-17Ac)\,\frac{\left(13\,-\,\frac{\left(c\,-\,\frac{\left(c\,-\,\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\cdots\right.}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\cdots}{b}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{4b}\cdots
$$

3.218. $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

input
```
Int[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*(b*B - A*c)/(b*c*x^(9/2)*(b + c*x^2)^2) - ((9*b*B - 17*A*c)*(1/(2*b*x
^(9/2)*(b + c*x^2)) + (13*(-2/(9*b*x^(9/2)) - (c*(-2/(5*b*x^(5/2)) - (c*(-
2/(b*Sqrt[x]) - (2*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sq
rt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(S
qrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)
*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2
*Sqrt[c])))/b))/b))/b))/(4*b)))/(8*b*c)
```

3.218.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.218. $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.218.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.58

method	result
derivativedivides	$-\frac{2c^2\left(\frac{\frac{c(29Ac-21Bb)x^{\frac{7}{2}}}{32}+\left(\frac{33}{32}Abc-\frac{25}{32}Bb^2\right)x^{\frac{3}{2}}}{\left(cx^2+b\right)^2}+\frac{\left(\frac{221Ac}{32}-\frac{117Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^5}$
default	$-\frac{2c^2\left(\frac{\frac{c(29Ac-21Bb)x^{\frac{7}{2}}}{32}+\left(\frac{33}{32}Abc-\frac{25}{32}Bb^2\right)x^{\frac{3}{2}}}{\left(cx^2+b\right)^2}+\frac{\left(\frac{221Ac}{32}-\frac{117Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^5}$
risch	$-\frac{2\left(270Ac^2x^4-135x^4Bbc-27Abcx^2+9b^2Bx^2+5b^2A\right)}{45b^5x^{\frac{9}{2}}}-\frac{c^2\left(\frac{\frac{c(29Ac-21Bb)x^{\frac{7}{2}}}{16}+2\left(\frac{33}{32}Abc-\frac{25}{32}Bb^2\right)x^{\frac{3}{2}}}{\left(cx^2+b\right)^2}+\frac{\left(\frac{221Ac}{32}-\frac{117}{3}\right.}{}\right)}{}$

input
```
int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/b^5*c^2*((1/32*c*(29*A*c-21*B*b)*x^(7/2)+(33/32*A*b*c-25/32*B*b^2)*x^(3
/2))/(c*x^2+b)^2+1/8*(221/32*A*c-117/32*B*b)/c/(1/c*b)^(1/4)*2^(1/2)*(ln((
x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^
(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2
^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))-2/9*A/b^3/x^(9/2)-2/5*(-3*A*c+B*b)/b^4/x
^(5/2)-6*c*(2*A*c-B*b)/b^5/x^(1/2)
```

3.218.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 966, normalized size of antiderivative = 2.65

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx =$$

$$-\frac{585\,(b^5c^2x^9+2\,b^6cx^7+b^7x^5)\left(-\frac{6561\,B^4b^4c^5-49572\,AB^3b^3c^6+140454\,A^2B^2b^2c^7-176868\,A^3Bbc^8+83521\,A^4c^9}{b^{21}}\right)^{\frac{1}{4}}\log\left(219\right.}{}$$

input
```
integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
-1/2880*(585*(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-(6561*B^4*b^4*c^5 - 4
9572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A
^4*c^9)/b^21)^(1/4)*log(2197*b^16*(-(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^
6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^21)^(3/
4) - 2197*(729*B^3*b^3*c^4 - 4131*A*B^2*b^2*c^5 + 7803*A^2*B*b*c^6 - 4913*
A^3*c^7)*sqrt(x)) + 585*(-I*b^5*c^2*x^9 - 2*I*b^6*c*x^7 - I*b^7*x^5)*(-(65
61*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3
*B*b*c^8 + 83521*A^4*c^9)/b^21)^(1/4)*log(2197*I*b^16*(-(6561*B^4*b^4*c^5
- 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 8352
1*A^4*c^9)/b^21)^(3/4) - 2197*(729*B^3*b^3*c^4 - 4131*A*B^2*b^2*c^5 + 7803
*A^2*B*b*c^6 - 4913*A^3*c^7)*sqrt(x)) + 585*(I*b^5*c^2*x^9 + 2*I*b^6*c*x^7
 + I*b^7*x^5)*(-(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b
^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^21)^(1/4)*log(-2197*I*b^16*
(-(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 17686
8*A^3*B*b*c^8 + 83521*A^4*c^9)/b^21)^(3/4) - 2197*(729*B^3*b^3*c^4 - 4131*
A*B^2*b^2*c^5 + 7803*A^2*B*b*c^6 - 4913*A^3*c^7)*sqrt(x)) - 585*(b^5*c^2*x
^9 + 2*b^6*c*x^7 + b^7*x^5)*(-(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 14
0454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^21)^(1/4)*log
(-2197*b^16*(-(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2
*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^21)^(3/4) - 2197*(729*B^3*...
```

3.218.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

3.218.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{585\,(9\,Bbc^3-17\,Ac^4)x^8+1053\,(9\,Bb^2c^2-17\,Abc^3)x^6-160\,Ab^4+416\,(9\,Bb^3c-17\,Ab^2c^2)x^4-32\,(9\,Bb}{720\,\left(b^5c^2x^{\frac{17}{2}}+2\,b^6cx^{\frac{13}{2}}+b^7x^{\frac{9}{2}}\right)} + \frac{13\,(9\,Bbc^2-17\,Ac^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\right.}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{128\,b^5}$$

input
```
integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/720*(585*(9*B*b*c^3 - 17*A*c^4)*x^8 + 1053*(9*B*b^2*c^2 - 17*A*b*c^3)*x^
6 - 160*A*b^4 + 416*(9*B*b^3*c - 17*A*b^2*c^2)*x^4 - 32*(9*B*b^4 - 17*A*b^
3*c)*x^2)/(b^5*c^2*x^(17/2) + 2*b^6*c*x^(13/2) + b^7*x^(9/2)) + 13/128*(9*
B*b*c^2 - 17*A*c^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4)
 + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c
)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sq
rt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*lo
g(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4))
 + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^
(1/4)*c^(3/4)))/b^5
```

3.218.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{13\sqrt{2}\left(9\,(bc^3)^{\frac{3}{4}}\,Bb-17\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6c} + \frac{13\sqrt{2}\left(9\,(bc^3)^{\frac{3}{4}}\,Bb-17\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6c} - \frac{13\sqrt{2}\left(9\,(bc^3)^{\frac{3}{4}}\,Bb-17\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^6c} + \frac{13\sqrt{2}\left(9\,(bc^3)^{\frac{3}{4}}\,Bb-17\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^6c} + \frac{21\,Bbc^3x^{\frac{7}{2}}-29\,Ac^4x^{\frac{7}{2}}+25\,Bb^2c^2x^{\frac{3}{2}}-33\,Abc^3x^{\frac{3}{2}}}{16\,(cx^2+b)^2b^5} + \frac{2\,(135\,Bbcx^4-270\,Ac^2x^4-9\,Bb^2x^2+27\,Abcx^2-5\,Ab^2)}{45\,b^5x^{\frac{9}{2}}}$$

input
```
integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
13/64*sqrt(2)*(9*(b*c^3)^(3/4)*B*b - 17*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt
(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^6*c) + 13/64*sqrt(2)
*(9*(b*c^3)^(3/4)*B*b - 17*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^6*c) - 13/128*sqrt(2)*(9*(b*c^3)
^(3/4)*B*b - 17*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + s
qrt(b/c))/(b^6*c) + 13/128*sqrt(2)*(9*(b*c^3)^(3/4)*B*b - 17*(b*c^3)^(3/4)
*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^6*c) + 1/16*(21
*B*b*c^3*x^(7/2) - 29*A*c^4*x^(7/2) + 25*B*b^2*c^2*x^(3/2) - 33*A*b*c^3*x^
(3/2))/((c*x^2 + b)^2*b^5) + 2/45*(135*B*b*c*x^4 - 270*A*c^2*x^4 - 9*B*b^2
*x^2 + 27*A*b*c*x^2 - 5*A*b^2)/(b^5*x^(9/2))
```

3.218.9 Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$
$$= \frac{13\,(-c)^{5/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(17\,A\,c-9\,B\,b)}{32\,b^{21/4}} - \frac{\frac{2\,A}{9\,b}-\frac{2\,x^2\,(17\,A\,c-9\,B\,b)}{45\,b^2}+\frac{117\,c^2\,x^6\,(17\,A\,c-9\,B\,b)}{80\,b^4}+\frac{13\,c^3\,x^8\,(17\,A\,c-9\,B\,b)}{16\,b^5}+\frac{26\,c\,x^4\,(17\,A\,c-9\,B\,b)}{45\,b^3}}{b^2\,x^{9/2}+c^2\,x^{17/2}+2\,b\,c\,x^{13/2}} - \frac{13\,(-c)^{5/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(17\,A\,c-9\,B\,b)}{32\,b^{21/4}}$$

input
```
int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

output
```
(13*(-c)^(5/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4))*(17*A*c - 9*B*b))/(32*b^
(21/4)) - ((2*A)/(9*b) - (2*x^2*(17*A*c - 9*B*b))/(45*b^2) + (117*c^2*x^6*
(17*A*c - 9*B*b))/(80*b^4) + (13*c^3*x^8*(17*A*c - 9*B*b))/(16*b^5) + (26*
c*x^4*(17*A*c - 9*B*b))/(45*b^3))/(b^2*x^(9/2) + c^2*x^(17/2) + 2*b*c*x^(1
3/2)) - (13*(-c)^(5/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4))*(17*A*c - 9*B*b
))/(32*b^(21/4))
```

3.219 $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^3}\,dx$

3.219.1 Optimal result

Integrand size = 26, antiderivative size = 365

$$\begin{aligned}\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx &= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}}-\frac{15(11bB-19Ac)}{112b^4x^{7/2}}+\frac{5c(11bB-19Ac)}{16b^5x^{3/2}}\\
&-\frac{bB-Ac}{4bcx^{11/2}\,(b+cx^2)^2}-\frac{11bB-19Ac}{16b^2cx^{11/2}\,(b+cx^2)}\\
&-\frac{15c^{7/4}(11bB-19Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{23/4}}\\
&+\frac{15c^{7/4}(11bB-19Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{23/4}}\\
&-\frac{15c^{7/4}(11bB-19Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}}\\
&+\frac{15c^{7/4}(11bB-19Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}}\end{aligned}$$

output
```
15/176*(-19*A*c+11*B*b)/b^3/c/x^(11/2)-15/112*(-19*A*c+11*B*b)/b^4/x^(7/2)
+5/16*c*(-19*A*c+11*B*b)/b^5/x^(3/2)+1/4*(A*c-B*b)/b/c/x^(11/2)/(c*x^2+b)^
2+1/16*(19*A*c-11*B*b)/b^2/c/x^(11/2)/(c*x^2+b)-15/64*c^(7/4)*(-19*A*c+11*
B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(23/4)*2^(1/2)+15/64*c^(7
/4)*(-19*A*c+11*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(23/4)*2^
(1/2)-15/128*c^(7/4)*(-19*A*c+11*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)
*2^(1/2)*x^(1/2))/b^(23/4)*2^(1/2)+15/128*c^(7/4)*(-19*A*c+11*B*b)*ln(b^(1
/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(23/4)*2^(1/2)
```

3.219.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx = \frac{-\frac{4b^{3/4}(-11bBx^2(-32b^3+160b^2cx^2+605bc^2x^4+385c^3x^6)+A(224b^4-608b^3cx^2+3040b^2c^2x^4+11495bc^3x^6+7315c^4x^8))}{x^{11/2}(b+cx^2)^2}+1155\sqrt{2}c^{7/4}(-}{4928b^{23/4}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

output
```
((-4*b^(3/4)*(-11*b*B*x^2*(-32*b^3 + 160*b^2*c*x^2 + 605*b*c^2*x^4 + 385*c
^3*x^6) + A*(224*b^4 - 608*b^3*c*x^2 + 3040*b^2*c^2*x^4 + 11495*b*c^3*x^6
+ 7315*c^4*x^8)))/(x^(11/2)*(b + c*x^2)^2) + 1155*Sqrt[2]*c^(7/4)*(-11*b*B
 + 19*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])]
 + 1155*Sqrt[2]*c^(7/4)*(11*b*B - 19*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)
*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(4928*b^(23/4))
```

3.219.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {9, 362, 253, 264, 264, 264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{A+Bx^2}{x^{13/2}\left(b+cx^2\right)^3}dx$$

$$\Big\downarrow \ 362$$

$$-\frac{(11bB-19Ac)\int \frac{1}{x^{13/2}\left(cx^2+b\right)^2}dx}{8bc}-\frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \ 253$$

$$-\frac{(11bB-19Ac)\left(\frac{15\int \frac{1}{x^{13/2}\left(cx^2+b\right)}dx}{4b}+\frac{1}{2bx^{11/2}\left(b+cx^2\right)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \ 264$$

$$-\frac{(11bB-19Ac)\left(\frac{15\left(-\frac{c\int \frac{1}{x^{9/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{11bx^{11/2}}\right)}{4b}+\frac{1}{2bx^{11/2}\left(b+cx^2\right)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \ 264$$

$$-\frac{(11bB-19Ac)\left(\frac{15\left(-\frac{c\left(-\frac{c\int \frac{1}{x^{5/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2}{11bx^{11/2}}\right)}{4b}+\frac{1}{2bx^{11/2}\left(b+cx^2\right)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \ 264$$

3.219. $\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^3}\,dx$

$$-\frac{(11bB-19Ac)\left(\frac{15\left(-\frac{c\left(-\frac{c\left(-\frac{c\int\frac{1}{\sqrt{x}(cx^2+b)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2}{11bx^{11/2}}\right)}{4b}+\frac{1}{2bx^{11/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 266

$$-\frac{(11bB-19Ac)\left(\frac{15\left(-\frac{c\left(-\frac{c\left(-\frac{2c\int\frac{1}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2}{11bx^{11/2}}\right)}{4b}+\frac{1}{2bx^{11/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 755

3.219. $\int\frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^3}\,dx$

$$-\frac{(11bB-19Ac)\left(\frac{15\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2}{11bx^{11/2}}\right)}{4b}+\frac{1}{2bx^{11/2}(b+cx^2)}\right)}{8bc}-\frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2}$$

$\big\downarrow$ 1476

3.219. $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^3}\,dx$

$$(11bB-19Ac)\frac{\left(15\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2}{11bx^{11/2}}\right)\right.}{4b}$$

3.219. $\int\frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^3}\,dx$

3.219.

↓ 1082

3.219. $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^3}\,dx$

$$(11bB-19Ac)\frac{15\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2}{11bx^{11}}}{4b}$$

3.219. $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^3}\,dx$

↓ 217

3.219. $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^3}\,dx$

$$(11bB-19Ac)\frac{15\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2}{11bx^{11/2}}\right)}{4b}-$$

3.219. $\int\frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^3}\,dx$

↓ 1479

3.219. $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^3}\,dx$

$$\left(15\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\text{arct}}{}}{2\sqrt{b}}\right.}{b}\right.}{b}\right.}{b}\right.\right.$$

3.219. $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^3}\,dx$

$$\downarrow 25$$

3.219. $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^3}\,dx$

$$15\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1 \cdots\right.}{\sqrt{2}\cdots}}{2\sqrt{b}} \cdots\right.}{b}\cdots\right.}{b}\cdots\right.}{b}\cdots\right.$$

3.219. $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^3}\,dx$

$$\downarrow 27$$

3.219. $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^3}\,dx$

$$15-\frac{\left(c-\frac{\left(c-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}}{b}\right)}{b}$$

3.219. $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^3}\,dx$

$\downarrow$ 1103

3.219. $\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^3}\,dx$

$$(11bB-19Ac)\frac{\left(15-\frac{\left(c-\frac{\left(c-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\cdots\right.}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{\cdots}}\cdots}{2\sqrt{b}}\cdots\right.}{b}\cdots\right.}{b}\cdots\right.}{b}\cdots\right.}{4b}\cdots$$

3.219. $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^3}\,dx$

input
```
Int[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

output
```
-1/4*(b*B - A*c)/(b*c*x^(11/2)*(b + c*x^2)^2) - ((11*b*B - 19*A*c)*(1/(2*b
*x^(11/2)*(b + c*x^2)) + (15*(-2/(11*b*x^(11/2)) - (c*(-2/(7*b*x^(7/2)) -
(c*(-2/(3*b*x^(3/2)) - (2*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/
4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*
b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[
b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/
4)))/(2*Sqrt[b])))/b))/b))/b))/(4*b)))/(8*b*c)
```

3.219.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.219. $\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^3} \, dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.219.4 Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.58

method	result
derivativedivides	$-\frac{2c^2\left(\frac{\left(\frac{31}{32}Ac^2-\frac{23}{32}Bbc\right)x^{\frac{5}{2}}+\frac{b(35Ac-27Bb)\sqrt{x}}{32}}{\left(cx^2+b\right)^2}+\frac{15(19Ac-11Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)\right.}{256b}}{b^5}$
default	$-\frac{2c^2\left(\frac{\left(\frac{31}{32}Ac^2-\frac{23}{32}Bbc\right)x^{\frac{5}{2}}+\frac{b(35Ac-27Bb)\sqrt{x}}{32}}{\left(cx^2+b\right)^2}+\frac{15(19Ac-11Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)\right.}{256b}}{b^5}$
risch	$-\frac{2(154Ac^2x^4-77x^4Bbc-33Abcx^2+11b^2Bx^2+7b^2A)}{77b^5x^{\frac{11}{2}}}-\frac{c^2\left(\frac{2\left(\frac{31}{32}Ac^2-\frac{23}{32}Bbc\right)x^{\frac{5}{2}}+\frac{b(35Ac-27Bb)\sqrt{x}}{16}}{\left(cx^2+b\right)^2}+\frac{15(19Ac-11B}{}\right.}{}$

input
```
int((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/b^5*c^2*(((31/32*A*c^2-23/32*B*b*c)*x^(5/2)+1/32*b*(35*A*c-27*B*b)*x^(1
/2))/(c*x^2+b)^2+15/256*(19*A*c-11*B*b)*(1/c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/
c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)
+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2
)/(1/c*b)^(1/4)*x^(1/2)-1)))-2/11*A/b^3/x^(11/2)-2/7*(-3*A*c+B*b)/b^4/x^(7
/2)-2*c*(2*A*c-B*b)/b^5/x^(3/2)
```

3.219.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 854, normalized size of antiderivative = 2.34

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx =$$

$$-\frac{1155\,(b^5c^2x^{10}+2\,b^6cx^8+b^7x^6)\left(-\frac{14641\,B^4b^4c^7-101156\,AB^3b^3c^8+262086\,A^2B^2b^2c^9-301796\,A^3Bbc^{10}+130321\,A^4c^{11}}{b^{23}}\right)^{\frac{1}{4}}\log}{}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="fricas")
```

output
```
-1/4928*(1155*(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6)*(-(14641*B^4*b^4*c^7
- 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 13
0321*A^4*c^11)/b^23)^(1/4)*log(15*b^6*(-(14641*B^4*b^4*c^7 - 101156*A*B^3*
b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 130321*A^4*c^11)/
b^23)^(1/4) - 15*(11*B*b*c^2 - 19*A*c^3)*sqrt(x)) + 1155*(I*b^5*c^2*x^10 +
 2*I*b^6*c*x^8 + I*b^7*x^6)*(-(14641*B^4*b^4*c^7 - 101156*A*B^3*b^3*c^8 +
262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 130321*A^4*c^11)/b^23)^(1/4
)*log(15*I*b^6*(-(14641*B^4*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^
2*b^2*c^9 - 301796*A^3*B*b*c^10 + 130321*A^4*c^11)/b^23)^(1/4) - 15*(11*B*
b*c^2 - 19*A*c^3)*sqrt(x)) + 1155*(-I*b^5*c^2*x^10 - 2*I*b^6*c*x^8 - I*b^7
*x^6)*(-(14641*B^4*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9
 - 301796*A^3*B*b*c^10 + 130321*A^4*c^11)/b^23)^(1/4)*log(-15*I*b^6*(-(146
41*B^4*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^
3*B*b*c^10 + 130321*A^4*c^11)/b^23)^(1/4) - 15*(11*B*b*c^2 - 19*A*c^3)*sqr
t(x)) - 1155*(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6)*(-(14641*B^4*b^4*c^7 -
 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 130
321*A^4*c^11)/b^23)^(1/4)*log(-15*b^6*(-(14641*B^4*b^4*c^7 - 101156*A*B^3*
b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 130321*A^4*c^11)/
b^23)^(1/4) - 15*(11*B*b*c^2 - 19*A*c^3)*sqrt(x)) - 4*(385*(11*B*b*c^3 - 1
9*A*c^4)*x^8 + 605*(11*B*b^2*c^2 - 19*A*b*c^3)*x^6 - 224*A*b^4 + 160*(1...
```

3.219.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)/(c*x**4+b*x**2)**3/x**(1/2),x)
```

output
```
Timed out
```

3.219.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx = \frac{385\,(11\,Bbc^3-19\,Ac^4)x^8+605\,(11\,Bb^2c^2-19\,Abc^3)x^6-224\,Ab^4+160\,(11\,Bb^3c-19\,Ab^2c^2)x^4-32\,(11}{1232\left(b^5c^2x^{\frac{19}{2}}+2\,b^6cx^{\frac{15}{2}}+b^7x^{\frac{11}{2}}\right)} + \frac{15\left(\frac{2\sqrt{2}(11\,Bbc^2-19\,Ac^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(11\,Bbc^2-19\,Ac^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(11\,Bb}{}\right.}{128\,b^5}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="maxima")
```

output
```
1/1232*(385*(11*B*b*c^3 - 19*A*c^4)*x^8 + 605*(11*B*b^2*c^2 - 19*A*b*c^3)*
x^6 - 224*A*b^4 + 160*(11*B*b^3*c - 19*A*b^2*c^2)*x^4 - 32*(11*B*b^4 - 19*
A*b^3*c)*x^2)/(b^5*c^2*x^(19/2) + 2*b^6*c*x^(15/2) + b^7*x^(11/2)) + 15/12
8*(2*sqrt(2)*(11*B*b*c^2 - 19*A*c^3)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c
^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*s
qrt(c))) + 2*sqrt(2)*(11*B*b*c^2 - 19*A*c^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*
b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(
sqrt(b)*sqrt(c))) + sqrt(2)*(11*B*b*c^2 - 19*A*c^3)*log(sqrt(2)*b^(1/4)*c^
(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(11*B*b*c
^2 - 19*A*c^3)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))
/(b^(3/4)*c^(1/4)))/b^5
```

3.219.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx$$
$$= \frac{15\sqrt{2}\left(11\,(bc^3)^{\frac{1}{4}}\,Bbc-19\,(bc^3)^{\frac{1}{4}}\,Ac^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6}$$
$$+\frac{15\sqrt{2}\left(11\,(bc^3)^{\frac{1}{4}}\,Bbc-19\,(bc^3)^{\frac{1}{4}}\,Ac^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6}$$
$$+\frac{15\sqrt{2}\left(11\,(bc^3)^{\frac{1}{4}}\,Bbc-19\,(bc^3)^{\frac{1}{4}}\,Ac^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^6}$$
$$-\frac{15\sqrt{2}\left(11\,(bc^3)^{\frac{1}{4}}\,Bbc-19\,(bc^3)^{\frac{1}{4}}\,Ac^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^6}$$
$$+\frac{23\,Bbc^3x^{\frac{5}{2}}-31\,Ac^4x^{\frac{5}{2}}+27\,Bb^2c^2\sqrt{x}-35\,Abc^3\sqrt{x}}{16\,(cx^2+b)^2b^5}$$
$$+\frac{2\,(77\,Bbcx^4-154\,Ac^2x^4-11\,Bb^2x^2+33\,Abcx^2-7\,Ab^2)}{77\,b^5x^{\frac{11}{2}}}$$

input

```
integrate((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="giac")
```

output

```
15/64*sqrt(2)*(11*(b*c^3)^(1/4)*B*b*c - 19*(b*c^3)^(1/4)*A*c^2)*arctan(1/2
*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^6 + 15/64*sqrt(2
)*(11*(b*c^3)^(1/4)*B*b*c - 19*(b*c^3)^(1/4)*A*c^2)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^6 + 15/128*sqrt(2)*(11*(b*c
^3)^(1/4)*B*b*c - 19*(b*c^3)^(1/4)*A*c^2)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4)
+ x + sqrt(b/c))/b^6 - 15/128*sqrt(2)*(11*(b*c^3)^(1/4)*B*b*c - 19*(b*c^3)
^(1/4)*A*c^2)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 + 1/16
*(23*B*b*c^3*x^(5/2) - 31*A*c^4*x^(5/2) + 27*B*b^2*c^2*sqrt(x) - 35*A*b*c^
3*sqrt(x))/((c*x^2 + b)^2*b^5) + 2/77*(77*B*b*c*x^4 - 154*A*c^2*x^4 - 11*B
*b^2*x^2 + 33*A*b*c*x^2 - 7*A*b^2)/(b^5*x^(11/2))
```

3.219.9 Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.75

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx = \frac{15\,(-c)^{7/4}\,\mathrm{atan}\left(\frac{6859\,A^3\,c^{10}\,\sqrt{x}-1331\,B^3\,b^3\,c^7\,\sqrt{x}-11913\,A^2\,B\,b\,c^9\,\sqrt{x}+6897\,A\,B^2\,b^2\,c^8\,\sqrt{x}}{b^{1/4}\,(-c)^{27/4}\,(c\,(c\,(6859\,A^3\,c-11913\,A^2\,B\,b)+6897\,A\,B^2\,b^2)-1331\,B^3\,b^3)}\right)\,(19\,A\,c-11\,B\,b)}{32\,b^{23/4}} - \frac{\frac{2\,A}{11\,b}-\frac{2\,x^2\,(19\,A\,c-11\,B\,b)}{77\,b^2}+\frac{55\,c^2\,x^6\,(19\,A\,c-11\,B\,b)}{112\,b^4}+\frac{5\,c^3\,x^8\,(19\,A\,c-11\,B\,b)}{16\,b^5}+\frac{10\,c\,x^4\,(19\,A\,c-11\,B\,b)}{77\,b^3}}{b^2\,x^{11/2}+c^2\,x^{19/2}+2\,b\,c\,x^{15/2}} - \frac{(-c)^{7/4}\,\mathrm{atan}\left(\frac{\frac{(-c)^{7/4}\,(19\,A\,c-11\,B\,b)\,\left(\sqrt{x}\,\left(1330790400\,A^2\,b^{15}\,c^9-1540915200\,A\,B\,b^{16}\,c^8+446054400\,B^2\,b^{17}\,c^7\right)-\frac{15\,(-c)^{7/4}\,(19\,A\,c-11\,B\,b)\,(2988\ldots}{64\ldots}}{64\,b^{23/4}}}{\frac{15\,(-c)^{7/4}\,(19\,A\,c-11\,B\,b)\,\left(\sqrt{x}\,\left(1330790400\,A^2\,b^{15}\,c^9-1540915200\,A\,B\,b^{16}\,c^8+446054400\,B^2\,b^{17}\,c^7\right)-\frac{15\,(-c)^{7/4}\,(19\,A\,c-11\,B\,b)\,(29\ldots}{\ldots}}{64\,b^{23/4}}}\ldots$$

input
```
int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^3),x)
```

output
```
(15*(-c)^(7/4)*atan((6859*A^3*c^10*x^(1/2) - 1331*B^3*b^3*c^7*x^(1/2) - 11
913*A^2*B*b*c^9*x^(1/2) + 6897*A*B^2*b^2*c^8*x^(1/2))/(b^(1/4)*(-c)^(27/4)
*(c*(c*(6859*A^3*c - 11913*A^2*B*b) + 6897*A*B^2*b^2) - 1331*B^3*b^3)))*(1
9*A*c - 11*B*b))/(32*b^(23/4)) - ((2*A)/(11*b) - (2*x^2*(19*A*c - 11*B*b))
/(77*b^2) + (55*c^2*x^6*(19*A*c - 11*B*b))/(112*b^4) + (5*c^3*x^8*(19*A*c
- 11*B*b))/(16*b^5) + (10*c*x^4*(19*A*c - 11*B*b))/(77*b^3))/(b^2*x^(11/2)
 + c^2*x^(19/2) + 2*b*c*x^(15/2)) - ((-c)^(7/4)*atan((((-c)^(7/4)*(19*A*c
- 11*B*b)*(x^(1/2)*(1330790400*A^2*b^15*c^9 + 446054400*B^2*b^17*c^7 - 154
0915200*A*B*b^16*c^8) - (15*(-c)^(7/4)*(19*A*c - 11*B*b)*(298844160*A*b^21
*c^6 - 173015040*B*b^22*c^5))/(64*b^(23/4)))*15i)/(64*b^(23/4)) + ((-c)^(7
/4)*(19*A*c - 11*B*b)*(x^(1/2)*(1330790400*A^2*b^15*c^9 + 446054400*B^2*b^
17*c^7 - 1540915200*A*B*b^16*c^8) + (15*(-c)^(7/4)*(19*A*c - 11*B*b)*(2988
44160*A*b^21*c^6 - 173015040*B*b^22*c^5))/(64*b^(23/4)))*15i)/(64*b^(23/4)
))/((15*(-c)^(7/4)*(19*A*c - 11*B*b)*(x^(1/2)*(1330790400*A^2*b^15*c^9 + 4
46054400*B^2*b^17*c^7 - 1540915200*A*B*b^16*c^8) - (15*(-c)^(7/4)*(19*A*c
- 11*B*b)*(298844160*A*b^21*c^6 - 173015040*B*b^22*c^5))/(64*b^(23/4))))/(
64*b^(23/4)) - (15*(-c)^(7/4)*(19*A*c - 11*B*b)*(x^(1/2)*(1330790400*A^2*b
^15*c^9 + 446054400*B^2*b^17*c^7 - 1540915200*A*B*b^16*c^8) + (15*(-c)^(7/
4)*(19*A*c - 11*B*b)*(298844160*A*b^21*c^6 - 173015040*B*b^22*c^5))/(64*b^
(23/4))))/(64*b^(23/4))))*(19*A*c - 11*B*b)*15i)/(32*b^(23/4))
```

3.220 $\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

3.220.1 Optimal result

Integrand size = 28, antiderivative size = 243

$$\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx=\frac{4b^2(3bB-5Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}}-\frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2}-\frac{2(3bB-5Ac)x^{7/2}\sqrt{bx^2+cx^4}}{55c}+\frac{2Bx^{3/2}(bx^2+cx^4)^{3/2}}{15c}-\frac{2b^{11/4}(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231c^{13/4}\sqrt{bx^2+cx^4}}$$

output
```
2/15*B*x^(3/2)*(c*x^4+b*x^2)^(3/2)/c-4/385*b*(-5*A*c+3*B*b)*x^(3/2)*(c*x^4
+b*x^2)^(1/2)/c^2-2/55*(-5*A*c+3*B*b)*x^(7/2)*(c*x^4+b*x^2)^(1/2)/c+4/231*
b^2*(-5*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)-2/231*b^(11/4)*(-5*A*c+
3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/
2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(
13/4)/(c*x^4+b*x^2)^(1/2)
```

3.220.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.56

$$\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{2\sqrt{x^2(b+cx^2)}\left((b+cx^2)\sqrt{1+\frac{cx^2}{b}}(45b^2B+7c^2x^2(15A+11Bx^2)-3bc(25A+21\cdots\right.}{1155c^3\sqrt{x}\sqrt{1+\cdots}}$$

input
```
Integrate[x^(5/2)*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)*Sqrt[1 + (c*x^2)/b]*(45*b^2*B + 7*c^
2*x^2*(15*A + 11*B*x^2) - 3*b*c*(25*A + 21*B*x^2)) + 15*b^2*(-3*b*B + 5*A*
c)*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(1155*c^3*Sqrt[x]*Sqr
t[1 + (c*x^2)/b])
```

3.220.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1945, 1426, 1429, 1429, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{2Bx^{3/2}(bx^2+cx^4)^{3/2}}{15c} - \frac{(3bB-5Ac)\int x^{5/2}\sqrt{cx^4+bx^2}dx}{5c}$$

$$\Big\downarrow \text{1426}$$

$$\frac{2Bx^{3/2}(bx^2+cx^4)^{3/2}}{15c} - \frac{(3bB-5Ac)\left(\frac{2}{11}b\int \frac{x^{9/2}}{\sqrt{cx^4+bx^2}}dx + \frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}\right)}{5c}$$

$$\Big\downarrow \text{1429}$$

$$\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{3/2}}{15c}-\frac{(3bB-5Ac)\left(\frac{2}{11}b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\int\frac{x^{5/2}}{\sqrt{cx^4+bx^2}}dx}{7c}\right)+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}\right)}{5c}$$

$$\Big\downarrow \text{1429}$$

$$\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{3/2}}{15c}-\frac{(3bB-5Ac)\left(\frac{2}{11}b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int\frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{7c}\right)+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}\right)}{5c}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{3/2}}{15c}-\frac{(3bB-5Ac)\left(\frac{2}{11}b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}\right)}{5c}$$

$$\Big\downarrow \text{266}$$

$$\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{3/2}}{15c}-\frac{(3bB-5Ac)\left(\frac{2}{11}b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{2bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}\right)}{5c}$$

$$\Big\downarrow \text{761}$$

$$\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{3/2}}{15c}-\frac{(3bB-5Ac)\left(\frac{2}{11}b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\right)}{7c}\right)+\frac{2}{11}x^{7/2}\sqrt{}\right.}{5c}$$

3.220. $\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

input
```
Int[x^(5/2)*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*B*x^(3/2)*(b*x^2 + c*x^4)^(3/2))/(15*c) - ((3*b*B - 5*A*c)*((2*x^(7/2)*
Sqrt[b*x^2 + c*x^4])/11 + (2*b*((2*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(7*c) - (5
*b*((2*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - (b^(3/4)*x*(Sqrt[b] + Sqrt[c]*
x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*S
qrt[x])/b^(1/4)], 1/2])/(3*c^(5/4)*Sqrt[b*x^2 + c*x^4])))/(7*c)))/11))/(5*
c)
```

3.220.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.220.4 Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{2(-77Bc^3x^6-105Ac^3x^4-14Bbc^2x^4-30Abc^2x^2+18Bb^2cx^2+50b^2Ac-30Bb^3)\sqrt{x^2(cx^2+b)}}{1155c^3\sqrt{x}}+\frac{2b^3(5Ac-3Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\sqrt{\right.}}{\sqrt{-}}}}{}$
default	$\frac{2\sqrt{x^4c+bx^2}\left(77Bc^5x^9+105Ac^5x^7+91Bbc^4x^7+25A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b^3c+135Ab\right.}{115}$

input
```
int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/1155*(-77*B*c^3*x^6-105*A*c^3*x^4-14*B*b*c^2*x^4-30*A*b*c^2*x^2+18*B*b^
2*c*x^2+50*A*b^2*c-30*B*b^3)/c^3/x^(1/2)*(x^2*(c*x^2+b))^(1/2)+2/231*b^3*(
5*A*c-3*B*b)/c^4*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*
(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(
c*x^3+b*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2
*2^(1/2))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

3.220.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.51

$$\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx =$$
$$-\frac{2\left(10\left(3Bb^4-5Ab^3c\right)\sqrt{c}x\,\text{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)-\left(77Bc^4x^6+30Bb^3c-50Ab^2c^2+7\left(2Bbc^3\right.\right.\right.}{1155c^4x}$$

input
```
integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/1155*(10*(3*B*b^4 - 5*A*b^3*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0,
 x) - (77*B*c^4*x^6 + 30*B*b^3*c - 50*A*b^2*c^2 + 7*(2*B*b*c^3 + 15*A*c^4)
*x^4 - 6*(3*B*b^2*c^2 - 5*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^4*
x)
```

3.220.6 Sympy [F]

$$\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx=\int x^{\frac{5}{2}}\sqrt{x^2\,(b+cx^2)}\,(A+Bx^2)\;dx$$

input
```
integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**(5/2)*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

3.220.7 Maxima [F]

$$\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2}(Bx^2+A)x^{\frac{5}{2}}\,dx$$

input
```
integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*x^(5/2), x)
```

3.220.8 Giac [F]

$$\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2}(Bx^2+A)x^{\frac{5}{2}}\,dx$$

input
```
integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*x^(5/2), x)
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx=\int x^{5/2}\,\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}\,dx$$

input
```
int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2), x)
```

3.221 $\int x^{3/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

3.221.1 Optimal result

Integrand size = 28, antiderivative size = 369

$$\int x^{3/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx=\frac{4b^2(7bB-13Ac)x^{3/2}(b+cx^2)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}$$
$$-\frac{4b(7bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}$$
$$-\frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2B\sqrt{x}(bx^2+cx^4)^{3/2}}{13c}$$
$$-\frac{4b^{9/4}(7bB-13Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}$$
$$+\frac{2b^{9/4}(7bB-13Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}$$

output
```
2/13*B*(c*x^4+b*x^2)^(3/2)*x^(1/2)/c+4/195*b^2*(-13*A*c+7*B*b)*x^(3/2)*(c*
x^2+b)/c^(5/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2/117*(-13*A*c+7*B*
b)*x^(5/2)*(c*x^4+b*x^2)^(1/2)/c-4/585*b*(-13*A*c+7*B*b)*x^(1/2)*(c*x^4+b*
x^2)^(1/2)/c^2-4/195*b^(9/4)*(-13*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin
(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x
^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)+2/195*b^(9
/4)*(-13*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos
(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)
/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))
^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

3.221.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.30

$$\int x^{3/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left(-\left((b+cx^2)\sqrt{1+\frac{cx^2}{b}}(7bB-13Ac-9Bcx^2)\right)+b(7bB-13A\right.}{117c^2\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[x^(3/2)*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)*Sqrt[1 + (c*x^2)/b]*(7*b*B
 - 13*A*c - 9*B*c*x^2)) + b*(7*b*B - 13*A*c)*Hypergeometric2F1[-1/2, 3/4,
7/4, -((c*x^2)/b)]))/(117*c^2*Sqrt[1 + (c*x^2)/b])
```

3.221.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1945, 1426, 1429, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$$

$$\Big\downarrow \; 1945$$

$$\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}{13c}-\frac{(7bB-13Ac)\int x^{3/2}\sqrt{cx^4+bx^2}dx}{13c}$$

$$\Big\downarrow \; 1426$$

$$\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}{13c}-\frac{(7bB-13Ac)\left(\frac{2}{9}b\int\frac{x^{7/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}\right)}{13c}$$

$$\Big\downarrow \; 1429$$

$$\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}{13c}-\frac{(7bB-13Ac)\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3b\int\frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{5c}\right)+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}\right)}{13c}$$

$$\Big\downarrow \; 1431$$

$$\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}{13c}-\frac{(7bB-13Ac)\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3bx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5c\sqrt{bx^2+cx^4}}\right)+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}\right)}{13c}$$

$$\Big\downarrow \; 266$$

$$\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}{13c}-\frac{(7bB-13Ac)\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5c\sqrt{bx^2+cx^4}}\right)+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}\right)}{13c}$$

$$\Big\downarrow \; 834$$

$$\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}{13c}-\frac{(7bB-13Ac)\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}\right)}{13c}$$

$$\Big\downarrow \; 27$$

3.221. $\int x^{3/2}\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$

$$\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}{13c} - \frac{(7bB-13Ac)\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}} - \frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right) + \frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}\right)}{13c}$$

$\downarrow$ 761

$$\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}{13c} - \frac{(7bB-13Ac)\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}} - \frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right) + \cdots}{13c}$$

$\downarrow$ 1510

$$\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}{13c} - \frac{(7bB-13Ac)\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}} - \frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+\cdots}{\left(\sqrt{b}+\cdots\right.}}\cdots}{\cdots}\right)}{5c\sqrt{bx^2+cx^4}}\cdots\right.\right.}{13c}$$

input
```
Int[x^(3/2)*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

3.221. $\int x^{3/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

output
```
(2*B*Sqrt[x]*(b*x^2 + c*x^4)^(3/2))/(13*c) - ((7*b*B - 13*A*c)*((2*x^(5/2)
*Sqrt[b*x^2 + c*x^4])/9 + (2*b*((2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*c) - (6
*b*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)
) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^
2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b +
c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[
b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2
*c^(3/4)*Sqrt[b + c*x^2])))/(5*c*Sqrt[b*x^2 + c*x^4])))/9))/(13*c)
```

3.221.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.221.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.72

method	result
risch	$\frac{2\sqrt{x}\,(45Bc^2x^4+65Ac^2x^2+10Bbcx^2+26Abc-14Bb^2)\sqrt{x^2(cx^2+b)}}{585c^2} - \frac{2b^2(13Ac-7Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{}}{}$
default	$-\frac{2\sqrt{x^4c+bx^2}\left(-45Bx^8c^4-65Ax^6c^4-55Bx^6bc^3+78A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b^3c-39A\sqrt{\frac{cx}{}}\right.}{}$

input
```
int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/585*x^(1/2)*(45*B*c^2*x^4+65*A*c^2*x^2+10*B*b*c*x^2+26*A*b*c-14*B*b^2)/c
^2*(x^2*(c*x^2+b))^(1/2)-2/195*b^2*(13*A*c-7*B*b)/c^3*(-b*c)^(1/2)*((x+1/c
*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/
2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/2)*E
llipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1/c*(-b*
c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)
))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

3.221.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.28

$$\int x^{3/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx =$$
$$-\frac{2\left(6\left(7\,Bb^3-13\,Ab^2c\right)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\left(45\,Bc^3x^4-14\,Bb^2c\right.\right.}{585\,c^3}$$

input
```
integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/585*(6*(7*B*b^3 - 13*A*b^2*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weiers
trassPInverse(-4*b/c, 0, x)) - (45*B*c^3*x^4 - 14*B*b^2*c + 26*A*b*c^2 + 5
*(2*B*b*c^2 + 13*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/c^3
```

3.221.6 Sympy [F]

$$\int x^{3/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \int x^{\frac{3}{2}}\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)\,dx$$

input
```
integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**(3/2)*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

3.221.7 Maxima [F]

$$\int x^{3/2}\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2}(Bx^2+A)x^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*x^(3/2), x)
```

3.221.8 Giac [F]

$$\int x^{3/2}\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2}(Bx^2+A)x^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*x^(3/2), x)
```

3.221.9 Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=\int x^{3/2}\,\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}\,dx$$

input
```
int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2), x)
```

3.222 $\int \sqrt{x}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

3.222.1 Optimal result

Integrand size = 28, antiderivative size = 204

$$\int \sqrt{x}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = -\frac{4b(5bB-11Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}-\frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c}+\frac{2B(bx^2+cx^4)^{3/2}}{11c\sqrt{x}}+\frac{2b^{7/4}(5bB-11Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}$$

output
```
2/11*B*(c*x^4+b*x^2)^(3/2)/c/x^(1/2)-2/77*(-11*A*c+5*B*b)*x^(3/2)*(c*x^4+b
*x^2)^(1/2)/c-4/231*b*(-11*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/c^2/x^(1/2)+2/23
1*b^(7/4)*(-11*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/
2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^
(1/2))^2)^(1/2)/c^(9/4)/(c*x^4+b*x^2)^(1/2)
```

3.222.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.54

$$\int \sqrt{x}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{2\sqrt{x^2(b+cx^2)}\left(-\left((b+cx^2)\sqrt{1+\frac{cx^2}{b}}(5bB-11Ac-7Bcx^2)\right)+b(5bB-11Ac)\,\text{Hypergeometric2F1}\left(\right.\right.}{77c^2\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[Sqrt[x]*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)*Sqrt[1 + (c*x^2)/b]*(5*b*B - 11*A*
c - 7*B*c*x^2)) + b*(5*b*B - 11*A*c)*Hypergeometric2F1[-1/2, 1/4, 5/4, -((
c*x^2)/b)]))/(77*c^2*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

3.222.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1945, 1426, 1429, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{2B(bx^2+cx^4)^{3/2}}{11c\sqrt{x}}-\frac{(5bB-11Ac)\int\sqrt{x}\sqrt{cx^4+bx^2}dx}{11c}$$

$$\Big\downarrow \text{1426}$$

$$\frac{2B(bx^2+cx^4)^{3/2}}{11c\sqrt{x}}-\frac{(5bB-11Ac)\left(\frac{2}{7}b\int\frac{x^{5/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}\right)}{11c}$$

$$\Big\downarrow \text{1429}$$

$$\frac{2B\left(bx^2+cx^4\right)^{3/2}}{11c\sqrt{x}}-\frac{(5bB-11Ac)\left(\frac{2}{7}b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int\frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}\right)+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}\right)}{11c}$$

$$\Big\downarrow \ 1431$$

$$\frac{2B\left(bx^2+cx^4\right)^{3/2}}{11c\sqrt{x}}-\frac{(5bB-11Ac)\left(\frac{2}{7}b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}\right)+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}\right)}{11c}$$

$$\Big\downarrow \ 266$$

$$\frac{2B\left(bx^2+cx^4\right)^{3/2}}{11c\sqrt{x}}-\frac{(5bB-11Ac)\left(\frac{2}{7}b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{2bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}\right)+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}\right)}{11c}$$

$$\Big\downarrow \ 761$$

$$\frac{2B\left(bx^2+cx^4\right)^{3/2}}{11c\sqrt{x}}-\frac{(5bB-11Ac)\left(\frac{2}{7}b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\right)+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}\right)}{11c}$$

input
```
Int[Sqrt[x]*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*B*(b*x^2 + c*x^4)^(3/2))/(11*c*Sqrt[x]) - ((5*b*B - 11*A*c)*((2*x^(3/2)
*Sqrt[b*x^2 + c*x^4])/7 + (2*b*((2*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - (b
^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*E
llipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(5/4)*Sqrt[b*x^2
+ c*x^4])))/7))/(11*c)
```

3.222.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.222. $\int \sqrt{x}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

3.222.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.06

method	result
risch	$\frac{2(21B\,c^{2}x^{4}+33A\,c^{2}x^{2}+6Bbc\,x^{2}+22Abc-10B\,b^{2})\sqrt{x^{2}(c\,x^{2}+b)}}{231c^{2}\sqrt{x}}-\frac{2b^{2}(11Ac-5Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{x}{\sqrt{-}}}}{231c^{3}\sqrt{c\,x^{3}+bx}\,x^{\frac{3}{2}}}$
default	$-\frac{2\sqrt{x^{4}c+b\,x^{2}}\left(-21B\,c^{4}x^{7}+11A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b^{2}c-33A\,c^{4}x^{5}-5B\sqrt{-bc}\sqrt{\frac{cx-}{\sqrt{}}}\right.}{231x^{\frac{3}{2}}(c\,x^{2}+b}$

input
```
int((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/231*(21*B*c^2*x^4+33*A*c^2*x^2+6*B*b*c*x^2+22*A*b*c-10*B*b^2)/c^2/x^(1/2
)*(x^2*(c*x^2+b))^(1/2)-2/231*b^2*(11*A*c-5*B*b)/c^3*(-b*c)^(1/2)*((x+1/c*
(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2
))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*EllipticF(((x+1/c*(-b
*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(x^2*(c*x^2+b))^(1/2)/x^(3/2
)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

3.222.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.48

$$\int \sqrt{x}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$
$$=\frac{2\left(2\left(5\,Bb^{3}-11\,Ab^{2}c\right)\sqrt{c}x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(21\,Bc^{3}x^{4}-10\,Bb^{2}c+22\,Abc^{2}+3\left(2\,Bbc^{2}+\right.\right.\right.}{231\,c^{3}x}$$

input
```
integrate((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/231*(2*(5*B*b^3 - 11*A*b^2*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x
) + (21*B*c^3*x^4 - 10*B*b^2*c + 22*A*b*c^2 + 3*(2*B*b*c^2 + 11*A*c^3)*x^2
)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^3*x)
```

3.222.6 Sympy [F]

$$\int \sqrt{x}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \int \sqrt{x}\sqrt{x^2(b+cx^2)}(A+Bx^2)\,dx$$

input
```
integrate((B*x**2+A)*x**(1/2)*(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(sqrt(x)*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

3.222.7 Maxima [F]

$$\int \sqrt{x}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2}(Bx^2+A)\sqrt{x}\,dx$$

input
```
integrate((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x), x)
```

3.222.8 Giac [F]

$$\int \sqrt{x}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2}(Bx^2+A)\sqrt{x}\,dx$$

input
```
integrate((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x), x)
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx=\int \sqrt{x}\,\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}\,dx$$

input
```
int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2), x)
```

3.223 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$

3.223.1 Optimal result

Integrand size = 28, antiderivative size = 326

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx = -\frac{4b(bB-3Ac)x^{3/2}(b+cx^2)}{15c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2(bB-3Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{15c} + \frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}} + \frac{4b^{5/4}(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}} - \frac{2b^{5/4}(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}$$

output
```
2/9*B*(c*x^4+b*x^2)^(3/2)/c/x^(3/2)-4/15*b*(-3*A*c+B*b)*x^(3/2)*(c*x^2+b)/
c^(3/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2/15*(-3*A*c+B*b)*x^(1/2)*
(c*x^4+b*x^2)^(1/2)/c+4/15*b^(5/4)*(-3*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^
(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(
sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((
c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)-2/15*b^(
5/4)*(-3*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2
*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2
)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)
```

3.223.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.29

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left(B(b+cx^2)\sqrt{1+\frac{cx^2}{b}}+(-bB+3Ac)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^2}{b}\right)\right)}{9c\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(B*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (-(b
*B) + 3*A*c)*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^2)/b)]))/(9*c*Sqrt[1
 + (c*x^2)/b])
```

3.223.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1945, 1426, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}}-\frac{(bB-3Ac)\int \frac{\sqrt{cx^4+bx^2}}{\sqrt{x}}dx}{3c}$$

$$\Big\downarrow \text{1426}$$

$$\frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}}-\frac{(bB-3Ac)\left(\frac{2}{5}b\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)}{3c}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2B\left(bx^2+cx^4\right)^{3/2}}{9cx^{3/2}}-\frac{(bB-3Ac)\left(\frac{2bx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)}{3c}$$

$\Big\downarrow$ 266

$$\frac{2B\left(bx^2+cx^4\right)^{3/2}}{9cx^{3/2}}-\frac{(bB-3Ac)\left(\frac{4bx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)}{3c}$$

$\Big\downarrow$ 834

$$\frac{2B\left(bx^2+cx^4\right)^{3/2}}{9cx^{3/2}}-\frac{(bB-3Ac)\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)}{3c}$$

$\Big\downarrow$ 27

$$\frac{2B\left(bx^2+cx^4\right)^{3/2}}{9cx^{3/2}}-\frac{(bB-3Ac)\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)}{3c}$$

$\Big\downarrow$ 761

$$\frac{2B\left(bx^2+cx^4\right)^{3/2}}{9cx^{3/2}}-\frac{(bB-3Ac)\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)}{3c}$$

$\Big\downarrow$ 1510

3.223. $\int\frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$

$$\frac{2B\left(bx^2+cx^4\right)^{3/2}}{9cx^{3/2}}-\frac{(bB-3Ac)\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\right.}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right.}{5\sqrt{bx^2+cx^4}}}{3c}$$

input
```
Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/Sqrt[x],x]
```

output
```
(2*B*(b*x^2 + c*x^4)^(3/2))/(9*c*x^(3/2)) - ((b*B - 3*A*c)*((2*Sqrt[x]*Sqr
t[b*x^2 + c*x^4])/5 + (4*b*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2
])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2
)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqr
t[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(5*Sqrt[b*x^2 + c*x^4]
)))/(3*c)
```

3.223.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.223. $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$

3.223.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.75

method	result
risch	$\frac{2\sqrt{x}\,(5Bc\,x^2+9Ac+2Bb)\sqrt{x^2(c\,x^2+b)}}{45c}+\frac{2b(3Ac-Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-b}}{c}\right.}{\sqrt{-bc}}}\right.}{c}\right.}{15c^2\sqrt{c\,x^3+bx}\,x^{\frac{3}{2}}\,(c\,x^2}$
default	$\frac{2\sqrt{x^4c+b\,x^2}\left(5B\,c^3x^6+18A\,b^2c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-9A\,b^2c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.}{}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*x^(1/2)*(5*B*c*x^2+9*A*c+2*B*b)/c*(x^2*(c*x^2+b))^(1/2)+2/15*b*(3*A*c
-B*b)/c^2*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-
1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b
*x)^(1/2)*(-2/c*(-b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2
))^(1/2),1/2*2^(1/2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(
-b*c)^(1/2))^(1/2),1/2*2^(1/2)))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(
x*(c*x^2+b))^(1/2)
```

3.223.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.24

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$$
$$=\frac{2\left(6\left(Bb^2-3\,Abc\right)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\left(5\,Bc^2x^2+2\,Bbc+9\,Ac\right.\right.}{45\,c^2}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
2/45*(6*(B*b^2 - 3*A*b*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPI
nverse(-4*b/c, 0, x)) + (5*B*c^2*x^2 + 2*B*b*c + 9*A*c^2)*sqrt(c*x^4 + b*x
^2)*sqrt(x))/c^2
```

3.223.6 Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx=\int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{\sqrt{x}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(1/2),x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/sqrt(x), x)
```

3.223.7 Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx=\int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{\sqrt{x}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/sqrt(x), x)
```

3.223.8 Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx=\int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{\sqrt{x}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/sqrt(x), x)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{\sqrt{x}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(1/2),x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(1/2), x)
```

3.224 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx$

3.224.1 Optimal result

Integrand size = 28, antiderivative size = 165

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = -\frac{2(bB-7Ac)\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}} - \frac{2b^{3/4}(bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21c^{5/4}\sqrt{bx^2+cx^4}}$$

output
```
2/7*B*(c*x^4+b*x^2)^(3/2)/c/x^(5/2)-2/21*(-7*A*c+B*b)*(c*x^4+b*x^2)^(1/2)/
c/x^(1/2)-2/21*b^(3/4)*(-7*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan
(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^
(1/2)+x*c^(1/2))^2)^(1/2)/c^(5/4)/(c*x^4+b*x^2)^(1/2)
```

3.224.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.57

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = \frac{2\sqrt{x^2(b+cx^2)}\left(B(b+cx^2)\sqrt{1+\frac{cx^2}{b}}+(-bB+7Ac)\,\text{Hypergeometric2F1}\right.}{7c\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(3/2),x]
```

output
```
(2*Sqrt[x^2*(b + c*x^2)]*(B*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (-(b*B) + 7*
A*c)*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(7*c*Sqrt[x]*Sqrt[1
 + (c*x^2)/b])
```

3.224.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1945, 1426, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}}-\frac{(bB-7Ac)\int \frac{\sqrt{cx^4+bx^2}}{x^{3/2}}dx}{7c}$$

$$\Big\downarrow \text{1426}$$

$$\frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}}-\frac{(bB-7Ac)\left(\frac{2}{3}b\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)}{7c}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}}-\frac{(bB-7Ac)\left(\frac{2bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)}{7c}$$

$$\Big\downarrow \text{266}$$

$$\frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}}-\frac{(bB-7Ac)\left(\frac{4bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)}{7c}$$

$$\Big\downarrow \text{761}$$

$$\frac{2B\left(bx^2+cx^4\right)^{3/2}}{7cx^{5/2}}-\frac{(bB-7Ac)\left(\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)}{7c}$$

input
```
Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(3/2),x]
```

output
```
(2*B*(b*x^2 + c*x^4)^(3/2))/(7*c*x^(5/2)) - ((b*B - 7*A*c)*((2*Sqrt[b*x^2
+ c*x^4])/(3*Sqrt[x]) + (2*b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2
)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
1/2])/(3*c^(1/4)*Sqrt[b*x^2 + c*x^4])))/(7*c)
```

3.224.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.224.4 Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.17

method	result
risch	$\frac{2(3Bc\,x^2+7Ac+2Bb)\sqrt{x^2(c\,x^2+b)}}{21c\sqrt{x}}+\frac{2b(7Ac-Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x}}{21c^2\sqrt{c\,x^3+bx}\,x^{\frac{3}{2}}(c\,x^2+b)}$
default	$\frac{2\sqrt{x^4c+b\,x^2}\left(7A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc-B\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\right.}{21x^{\frac{3}{2}}(c\,x^2+b)c^2}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*(3*B*c*x^2+7*A*c+2*B*b)/c/x^(1/2)*(x^2*(c*x^2+b))^(1/2)+2/21*b*(7*A*c
-B*b)/c^2*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-
1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b
*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

3.224.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.45

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = -\frac{2\left(2\left(Bb^2-7\,Abc\right)\sqrt{c}x\,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\left(3\,Bc^2x^2+2\,Bbc+7\,Ac^2\right)\sqrt{cx^4+bx^2}\sqrt{x}\right)}{21\,c^2x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

output
```
-2/21*(2*(B*b^2 - 7*A*b*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) - (
3*B*c^2*x^2 + 2*B*b*c + 7*A*c^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^2*x)
```

3.224.6 Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = \int \frac{\sqrt{x^2(b+cx^2)}(A+Bx^2)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(3/2),x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(3/2), x)
```

3.224.7 Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(3/2), x)
```

3.224.8 Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(3/2), x)
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx=\int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(3/2),x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(3/2), x)
```

3.225 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx$

3.225.1 Optimal result

Integrand size = 28, antiderivative size = 323

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx = \frac{4(bB+5Ac)x^{3/2}(b+cx^2)}{5\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{2(bB+5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2A(bx^2+cx^4)^{3/2}}{bx^{7/2}} - \frac{4\sqrt[4]{b}(bB+5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}} + \frac{2\sqrt[4]{b}(bB+5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}$$

output
```
-2*A*(c*x^4+b*x^2)^(3/2)/b/x^(7/2)+4/5*(5*A*c+B*b)*x^(3/2)*(c*x^2+b)/c^(1/
2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+2/5*(5*A*c+B*b)*x^(1/2)*(c*x^4+
b*x^2)^(1/2)/b-4/5*b^(1/4)*(5*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(
1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/
(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2)^(1/2)+2/5*b^(1/4)*(5*A*
c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/
2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(
3/4)/(c*x^4+b*x^2)^(1/2)
```

3.225.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.30

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx = \frac{2\sqrt{x^2(b+cx^2)}\left(-3A(b+cx^2)\sqrt{1+\frac{cx^2}{b}}+(bB+5Ac)x^2\,\text{Hypergeometric2}\right.}{3bx^{3/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(5/2),x]
```

output
```
(2*Sqrt[x^2*(b + c*x^2)]*(-3*A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (b*B + 5*
A*c)*x^2*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^2)/b)]))/(3*b*x^(3/2)*Sq
rt[1 + (c*x^2)/b])
```

3.225.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1944, 1426, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(5Ac+bB)\int \frac{\sqrt{cx^4+bx^2}}{\sqrt{x}}dx}{b}-\frac{2A(bx^2+cx^4)^{3/2}}{bx^{7/2}}$$

$$\Big\downarrow \text{1426}$$

$$\frac{(5Ac+bB)\left(\frac{2}{5}b\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)}{b}-\frac{2A(bx^2+cx^4)^{3/2}}{bx^{7/2}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{(5Ac+bB)\left(\frac{2bx\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)}{b}-\frac{2A(bx^2+cx^4)^{3/2}}{bx^{7/2}}$$

$$\downarrow \text{266}$$

$$\frac{(5Ac+bB)\left(\frac{4bx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)}{b}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}}$$

$$\downarrow \text{834}$$

$$\frac{(5Ac+bB)\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)}{b}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}}$$

$$\downarrow \text{27}$$

$$\frac{(5Ac+bB)\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)}{b}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}}$$

$$\downarrow \text{761}$$

$$\frac{(5Ac+bB)\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)}{b}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}}$$

$$\downarrow \text{1510}$$

3.225. $\int\frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^{5/2}}dx$

$$\frac{(5Ac+bB)\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\right.}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right.}{5\sqrt{bx^2+cx^4}}\right.}{b}$$

$$\frac{2A\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}}$$

input
```
Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(5/2),x]
```

output
```
(-2*A*(b*x^2 + c*x^4)^(3/2))/(b*x^(7/2)) + ((b*B + 5*A*c)*((2*Sqrt[x]*Sqrt
[b*x^2 + c*x^4])/5 + (4*b*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2]
)/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1
/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*
Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(5*Sqrt[b*x^2 + c*x^4])
))/b
```

3.225.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.225. $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx$

3.225.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{2(-x^2B+5A)\sqrt{x^2(cx^2+b)}}{5x^{\frac{3}{2}}}+\frac{\left(2Ac+\frac{2Bb}{5}\right)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}\right.}{c\sqrt{cx^3+bx}\,x^{\frac{3}{2}}(cx^2+b)}$
default	$\frac{2\sqrt{x^4c+bx^2}\left(10A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc-5A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\right.\right.}{}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-B*x^2+5*A)*(x^2*(c*x^2+b))^(1/2)/x^(3/2)+(2*A*c+2/5*B*b)/c*(-b*c)^(
1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*
c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-
b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/
2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)
,1/2*2^(1/2)))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

3.225.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.22

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx =$$
$$-\frac{2\left(2\,(Bb+5\,Ac)\sqrt{c}x^2\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\sqrt{cx^4+bx^2}(Bcx^2-5\,A\right.}{5\,cx^2}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="fricas")
```

output
```
-2/5*(2*(B*b + 5*A*c)*sqrt(c)*x^2*weierstrassZeta(-4*b/c, 0, weierstrassPI
nverse(-4*b/c, 0, x)) - sqrt(c*x^4 + b*x^2)*(B*c*x^2 - 5*A*c)*sqrt(x))/(c*
x^2)
```

3.225.6 Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx=\int \frac{\sqrt{x^2(b+cx^2)}(A+Bx^2)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(5/2),x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(5/2), x)
```

3.225.7 Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx=\int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(5/2), x)
```

3.225.8 Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx=\int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(5/2), x)
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^{5/2}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(5/2),x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(5/2), x)
```

3.226 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{7/2}} dx$

3.226.1 Optimal result

Integrand size = 28, antiderivative size = 163

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx = \frac{2(bB+Ac)\sqrt{bx^2+cx^4}}{3b\sqrt{x}} - \frac{2A(bx^2+cx^4)^{3/2}}{3bx^{9/2}} + \frac{2(bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

output
```
-2/3*A*(c*x^4+b*x^2)^(3/2)/b/x^(9/2)+2/3*(A*c+B*b)*(c*x^4+b*x^2)^(1/2)/b/x
^(1/2)+2/3*(A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/co
s(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2)
)^2)^(1/2)/b^(1/4)/c^(1/4)/(c*x^4+b*x^2)^(1/2)
```

3.226.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.60

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx = \frac{2\sqrt{x^2(b+cx^2)}\left(-A(b+cx^2)\sqrt{1+\frac{cx^2}{b}}+3(bB+Ac)x^2\,\text{Hypergeometric2F}\right.}{3bx^{5/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(7/2),x]
```

output
```
(2*Sqrt[x^2*(b + c*x^2)]*(-(A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b]) + 3*(b*B +
A*c)*x^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(3*b*x^(5/2)*Sq
rt[1 + (c*x^2)/b])
```

3.226.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1944, 1426, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{7/2}}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(Ac+bB)\int \frac{\sqrt{cx^4+bx^2}}{x^{3/2}}dx}{b}-\frac{2A(bx^2+cx^4)^{3/2}}{3bx^{9/2}}$$

$$\Big\downarrow \text{1426}$$

$$\frac{(Ac+bB)\left(\frac{2}{3}b\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)}{b}-\frac{2A(bx^2+cx^4)^{3/2}}{3bx^{9/2}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{(Ac+bB)\left(\frac{2bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)}{b}-\frac{2A(bx^2+cx^4)^{3/2}}{3bx^{9/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(Ac+bB)\left(\frac{4bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)}{b}-\frac{2A(bx^2+cx^4)^{3/2}}{3bx^{9/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(Ac+bB)\left(\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)}{b}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{3bx^{9/2}}$$

input
```
Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(7/2),x]
```

output
```
(-2*A*(b*x^2 + c*x^4)^(3/2))/(3*b*x^(9/2)) + ((b*B + A*c)*((2*Sqrt[b*x^2 +
 c*x^4])/(3*Sqrt[x]) + (2*b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1
/2])/(3*c^(1/4)*Sqrt[b*x^2 + c*x^4])))/b
```

3.226.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.226.4 Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.10

method	result
risch	$-\frac{2(-x^2B+A)\sqrt{x^2(cx^2+b)}}{3x^{\frac{5}{2}}}+\frac{\left(\frac{2Ac}{3}+\frac{2Bb}{3}\right)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x^2(cx^2}}{c\sqrt{cx^3+bx}\,x^{\frac{3}{2}}(cx^2+b)}$
default	$\frac{2\sqrt{x^4c+bx^2}\left(A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)cx+B\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{x}{\sqrt{-}}}\right.}{3x^{\frac{5}{2}}(cx^2+b)c}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-B*x^2+A)/x^(5/2)*(x^2*(c*x^2+b))^(1/2)+(2/3*A*c+2/3*B*b)/c*(-b*c)^(
1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*
c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*Elliptic
F(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(x^2*(c*x^2+b))
^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

3.226.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int\frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx=\frac{2\left(2\,(Bb+Ac)\sqrt{c}x^3\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^4+bx^2}(Bcx^2-A\right.}{3\,cx^3}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="fricas")
```

output
```
2/3*(2*(B*b + A*c)*sqrt(c)*x^3*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c*
x^4 + b*x^2)*(B*c*x^2 - A*c)*sqrt(x))/(c*x^3)
```

3.226.6 Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(7/2),x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(7/2), x)
```

3.226.7 Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(7/2), x)
```

3.226.8 Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(7/2), x)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx=\int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^{7/2}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(7/2),x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(7/2), x)
```

3.227 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx$

3.227.1 Optimal result

Integrand size = 28, antiderivative size = 328

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = \frac{4\sqrt{c}(5bB+Ac)x^{3/2}(b+cx^2)}{5b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} - \frac{4\sqrt[4]{c}(5bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}} + \frac{2\sqrt[4]{c}(5bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}$$

output
```
-2/5*A*(c*x^4+b*x^2)^(3/2)/b/x^(11/2)+4/5*(A*c+5*B*b)*x^(3/2)*(c*x^2+b)*c^
(1/2)/b/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2/5*(A*c+5*B*b)*(c*x^4+b*x
^2)^(1/2)/b/x^(3/2)-4/5*c^(1/4)*(A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^
2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/(c*x^4+b*x^2)^(1/2)+2/5*c^(1/4)*
(A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)
)),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2
)/b^(3/4)/(c*x^4+b*x^2)^(1/2)
```

3.227.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.29

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = -\frac{2\sqrt{x^2(b+cx^2)}\left(A(b+cx^2)\sqrt{1+\frac{cx^2}{b}}+(5bB+Ac)x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},-\frac{cx^2}{b}\right)\right)}{5bx^{7/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(9/2),x]
```

output
```
(-2*Sqrt[x^2*(b + c*x^2)]*(A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (5*b*B + A*
c)*x^2*Hypergeometric2F1[-1/2, -1/4, 3/4, -((c*x^2)/b)]))/(5*b*x^(7/2)*Sqr
t[1 + (c*x^2)/b])
```

3.227.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1944, 1425, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(Ac+5bB)\int \frac{\sqrt{cx^4+bx^2}}{x^{5/2}}dx}{5b}-\frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}}$$

$$\Big\downarrow \text{1425}$$

$$\frac{(Ac+5bB)\left(2c\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}\right)}{5b}-\frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{(Ac+5bB)\left(\frac{2cx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}\right)}{5b}-\frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}}$$

$\Big\downarrow$ 266

$$\frac{(Ac+5bB)\left(\frac{4cx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}\right)}{5b}-\frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}}$$

$\Big\downarrow$ 834

$$\frac{(Ac+5bB)\left(\frac{4cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}\right)}{5b}-\frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}}$$

$\Big\downarrow$ 27

$$\frac{(Ac+5bB)\left(\frac{4cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}\right)}{5b}-\frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}}$$

$\Big\downarrow$ 761

$$\frac{(Ac+5bB)\left(\frac{4cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}\right)}{5b}-\frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}}$$

$\Big\downarrow$ 1510

3.227. $\int\frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}dx$

$$\frac{(Ac+5bB)\left(\frac{4cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\right|}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}}{\sqrt{bx^2+cx^4}}\right)}{5b}$$

$$\frac{2A\left(bx^2+cx^4\right)^{3/2}}{5bx^{11/2}}$$

input
```
Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(9/2),x]
```

output
```
(-2*A*(b*x^2 + c*x^4)^(3/2))/(5*b*x^(11/2)) + ((5*b*B + A*c)*((-2*Sqrt[b*x
^2 + c*x^4])/x^(3/2) + (4*c*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^
2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^
2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x
)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sq
rt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/Sqrt[b*x^2 + c*x^4]))
/(5*b)
```

3.227.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

3.227. $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx$

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.227. $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx$

3.227.4 Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{2(2Acx^2+5bBx^2+Ab)\sqrt{x^2(cx^2+b)}}{5x^{\frac{7}{2}}b}+\frac{2(Ac+5Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\right.}{c}\right.}{5b\sqrt{cx^3+bx}\,x^{\frac{3}{2}}(cx^2-}$
default	$\frac{2\sqrt{x^4c+bx^2}\left(2A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bcx^2-A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\right.\right.}{}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(2*A*c*x^2+5*B*b*x^2+A*b)/x^(7/2)/b*(x^2*(c*x^2+b))^(1/2)+2/5*(A*c+5*
B*b)/b*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c
*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)
^(1/2)*(-2/c*(-b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^
(1/2),1/2*2^(1/2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*
c)^(1/2))^(1/2),1/2*2^(1/2)))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(
c*x^2+b))^(1/2)
```

3.227.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.23

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx =$$
$$-\frac{2\left(2\left(5\,Bb+Ac\right)\sqrt{c}x^4\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}((5\,Bb+2\,A\right.}{5\,bx^4}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="fricas")
```

output
```
-2/5*(2*(5*B*b + A*c)*sqrt(c)*x^4*weierstrassZeta(-4*b/c, 0, weierstrassPI
nverse(-4*b/c, 0, x)) + sqrt(c*x^4 + b*x^2)*((5*B*b + 2*A*c)*x^2 + A*b)*sq
rt(x))/(b*x^4)
```

3.227.6 Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(9/2),x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(9/2), x)
```

3.227.7 Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(9/2), x)
```

3.227.8 Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(9/2), x)
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^{9/2}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(9/2),x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(9/2), x)
```

3.228 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx$

3.228.1 Optimal result

Integrand size = 28, antiderivative size = 167

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx = -\frac{2(7bB-Ac)\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{7bx^{13/2}} + \frac{2c^{3/4}(7bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21b^{5/4}\sqrt{bx^2+cx^4}}$$

output
```
-2/7*A*(c*x^4+b*x^2)^(3/2)/b/x^(13/2)-2/21*(-A*c+7*B*b)*(c*x^4+b*x^2)^(1/2
)/b/x^(5/2)+2/21*c^(3/4)*(-A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1
/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arct
an(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(
b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(5/4)/(c*x^4+b*x^2)^(1/2)
```

3.228.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.59

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx = -\frac{2\sqrt{x^2(b+cx^2)}\left(3A(b+cx^2)\sqrt{1+\frac{cx^2}{b}}+(7bB-Ac)x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},-\frac{cx^2}{b}\right)\right)}{21bx^{9/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(11/2),x]
```

output
```
(-2*Sqrt[x^2*(b + c*x^2)]*(3*A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (7*b*B -
A*c)*x^2*Hypergeometric2F1[-3/4, -1/2, 1/4, -((c*x^2)/b)]))/(21*b*x^(9/2)*
Sqrt[1 + (c*x^2)/b])
```

3.228.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1944, 1425, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(7bB-Ac)\int \frac{\sqrt{cx^4+bx^2}}{x^{7/2}}dx}{7b}-\frac{2A(bx^2+cx^4)^{3/2}}{7bx^{13/2}}$$

$$\Big\downarrow \text{1425}$$

$$\frac{(7bB-Ac)\left(\frac{2}{3}c\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}\right)}{7b}-\frac{2A(bx^2+cx^4)^{3/2}}{7bx^{13/2}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{(7bB-Ac)\left(\frac{2cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}\right)}{7b}-\frac{2A(bx^2+cx^4)^{3/2}}{7bx^{13/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(7bB-Ac)\left(\frac{4cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}\right)}{7b}-\frac{2A(bx^2+cx^4)^{3/2}}{7bx^{13/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(7bB-Ac)\left(\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}\right)}{7b}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{7bx^{13/2}}$$

input
```
Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(11/2),x]
```

output
```
(-2*A*(b*x^2 + c*x^4)^(3/2))/(7*b*x^(13/2)) + ((7*b*B - A*c)*((-2*Sqrt[b*x
^2 + c*x^4])/(3*x^(5/2)) + (2*c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*
x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(3*b^(1/4)*Sqrt[b*x^2 + c*x^4])))/(7*b)
```

3.228.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

3.228. $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx$

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.228.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{2(2Acx^2+7bBx^2+3Ab)\sqrt{x^2(cx^2+b)}}{21x^{\frac{9}{2}}b}-\frac{2(Ac-7Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{21b\sqrt{cx^3+bx}\,x^{\frac{3}{2}}(cx^2+b)}$
default	$-\frac{2\sqrt{x^4c+bx^2}\left(A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)cx^3-7B\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.}{21x^{\frac{9}{2}}(cx^2+b)b}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(2*A*c*x^2+7*B*b*x^2+3*A*b)/x^(9/2)/b*(x^2*(c*x^2+b))^(1/2)-2/21*(A*
c-7*B*b)/b*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x
-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+
b*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/
2))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

3.228.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.43

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx = \frac{2\left(2\left(7\,Bb-Ac\right)\sqrt{c}x^5\,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\sqrt{cx^4+bx^2}((7\,Bb+\right.}{21\,bx^5}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="fricas")
```

output
```
2/21*(2*(7*B*b - A*c)*sqrt(c)*x^5*weierstrassPInverse(-4*b/c, 0, x) - sqrt
(c*x^4 + b*x^2)*((7*B*b + 2*A*c)*x^2 + 3*A*b)*sqrt(x))/(b*x^5)
```

3.228.6 Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx=\int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(11/2),x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(11/2), x)
```

3.228.7 Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx=\int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(11/2), x)
```

3.228.8 Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx=\int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(11/2), x)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^{11/2}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(11/2),x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(11/2), x)
```

3.229 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx$

3.229.1 Optimal result

Integrand size = 28, antiderivative size = 369

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = \frac{4c^{3/2}(3bB-Ac)x^{3/2}(b+cx^2)}{15b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{9bx^{15/2}} - \frac{4c^{5/4}(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}} + \frac{2c^{5/4}(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}$$

output
```
-2/9*A*(c*x^4+b*x^2)^(3/2)/b/x^(15/2)+4/15*c^(3/2)*(-A*c+3*B*b)*x^(3/2)*(c
*x^2+b)/b^2/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2/15*(-A*c+3*B*b)*(c*x
^4+b*x^2)^(1/2)/b/x^(7/2)-4/15*c*(-A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^(3
/2)-4/15*c^(5/4)*(-A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)
^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/
4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+
x*c^(1/2))^2)^(1/2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)+2/15*c^(5/4)*(-A*c+3*B*b)*
x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^
(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/
2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/(c
*x^4+b*x^2)^(1/2)
```

3.229.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.27

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx =$$
$$-\frac{2\sqrt{x^2(b+cx^2)}\left(5A(b+cx^2)\sqrt{1+\frac{cx^2}{b}}+3(3bB-Ac)x^2\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\frac{cx^2}{b}\right)\right)}{45bx^{11/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(13/2),x]
```

output
```
(-2*Sqrt[x^2*(b + c*x^2)]*(5*A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + 3*(3*b*B
- A*c)*x^2*Hypergeometric2F1[-5/4, -1/2, -1/4, -((c*x^2)/b)]))/(45*b*x^(11
/2)*Sqrt[1 + (c*x^2)/b])
```

3.229.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1944, 1425, 1430, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx$$
$$\Big\downarrow \ 1944$$
$$\frac{(3bB-Ac)\int \frac{\sqrt{cx^4+bx^2}}{x^{9/2}}dx}{3b}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}}$$
$$\Big\downarrow \ 1425$$
$$\frac{(3bB-Ac)\left(\frac{2}{5}c\int \frac{1}{\sqrt{x}\sqrt{cx^4+bx^2}}dx-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}\right)}{3b}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}}$$
$$\Big\downarrow \ 1430$$

$$\frac{(3bB-Ac)\left(\frac{2}{5}c\left(\frac{c\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{b}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}\right)}{3b}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}}$$

$$\Big\downarrow \; 1431$$

$$\frac{(3bB-Ac)\left(\frac{2}{5}c\left(\frac{cx\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}\right)}{3b}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}}$$

$$\Big\downarrow \; 266$$

$$\frac{(3bB-Ac)\left(\frac{2}{5}c\left(\frac{2cx\sqrt{b+cx^2}\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}\right)}{3b}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}}$$

$$\Big\downarrow \; 834$$

$$\frac{(3bB-Ac)\left(\frac{2}{5}c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}\right)}{3b}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{(3bB-Ac)\left(\frac{2}{5}c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}\right)}{3b}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}}$$

$$\Big\downarrow \; 761$$

3.229. $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}dx$

$$\frac{(3bB-Ac)\left(\frac{2}{5}c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)-\frac{2\sqrt{bx}}{5x}\right.}{3b}$$

$$\frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}}$$

$\downarrow$ 1510

$$\frac{(3bB-Ac)\left(\frac{2}{5}c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}}{\sqrt[4]{}}\right.\right.}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right.}{b\sqrt{bx^2+cx^4}}\right.\right.}{3b}$$

$$\frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}}$$

input
```
Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(13/2),x]
```

output
```
(-2*A*(b*x^2 + c*x^4)^(3/2))/(9*b*x^(15/2)) + ((3*b*B - A*c)*((-2*Sqrt[b*x
^2 + c*x^4])/(5*x^(7/2)) + (2*c*((-2*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) + (2
*c*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)
) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^
2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b +
c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[
b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2
*c^(3/4)*Sqrt[b + c*x^2])))/(b*Sqrt[b*x^2 + c*x^4])))/5))/(3*b)
```

3.229. $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx$

3.229.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

3.229. $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx$

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.229.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{2\left(-6A\,c^2x^4+18x^4Bbc+2Abc\,x^2+9b^2B\,x^2+5b^2A\right)\sqrt{x^2\left(c\,x^2+b\right)}}{45x^{\frac{11}{2}}b^2}-\frac{2c(Ac-3Bb)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\ \cdots}{\cdots}$
default	$-\frac{2\sqrt{x^4c+b\,x^2}\left(6A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b\,c^2x^4-3A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\ \cdots\right)}{\cdots}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*(-6*A*c^2*x^4+18*B*b*c*x^4+2*A*b*c*x^2+9*B*b^2*x^2+5*A*b^2)/x^(11/2)
/b^2*(x^2*(c*x^2+b))^(1/2)-2/15*c*(A*c-3*B*b)/b^2*(-b*c)^(1/2)*((x+1/c*(-b
*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^
(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/2)*Ellip
ticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1/c*(-b*c)^(
1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))*(
x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

3.229.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.28

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx =$$
$$-\frac{2\left(6\left(3\,Bbc-Ac^2\right)\sqrt{c}x^6\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\left(6\left(3\,Bbc-Ac^2\right)x^4+5\right.\right.}{45\,b^2x^6}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="fricas")
```

output
```
-2/45*(6*(3*B*b*c - A*c^2)*sqrt(c)*x^6*weierstrassZeta(-4*b/c, 0, weierstr
assPInverse(-4*b/c, 0, x)) + (6*(3*B*b*c - A*c^2)*x^4 + 5*A*b^2 + (9*B*b^2
 + 2*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^2*x^6)
```

3.229.6 Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(13/2),x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(13/2), x)
```

3.229.7 Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(13/2), x)
```

3.229.8 Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(13/2), x)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^{13/2}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(13/2),x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(13/2), x)
```

3.230 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx$

3.230.1 Optimal result

Integrand size = 28, antiderivative size = 204

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}} - \frac{4c(11bB-5Ac)\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}} - \frac{2c^{7/4}(11bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}$$

output

```
-2/11*A*(c*x^4+b*x^2)^(3/2)/b/x^(17/2)-2/77*(-5*A*c+11*B*b)*(c*x^4+b*x^2)^
(1/2)/b/x^(9/2)-4/231*c*(-5*A*c+11*B*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^(5/2)-2/
231*c^(7/4)*(-5*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(
1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)
*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*
c^(1/2))^2)^(1/2)/b^(9/4)/(c*x^4+b*x^2)^(1/2)
```

3.230.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.48

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = -\frac{2\sqrt{x^2(b+cx^2)}\left(7A(b+cx^2)\sqrt{1+\frac{cx^2}{b}}+(11bB-5Ac)x^2\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{2},-\frac{3}{4},-\frac{cx^2}{b}\right)\right)}{77bx^{13/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(15/2),x]
```

output
```
(-2*Sqrt[x^2*(b + c*x^2)]*(7*A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (11*b*B -
 5*A*c)*x^2*Hypergeometric2F1[-7/4, -1/2, -3/4, -((c*x^2)/b)]))/(77*b*x^(1
3/2)*Sqrt[1 + (c*x^2)/b])
```

3.230.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1944, 1425, 1430, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(11bB-5Ac)\int \frac{\sqrt{cx^4+bx^2}}{x^{11/2}}dx}{11b}-\frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}}$$

$$\Big\downarrow \text{1425}$$

$$\frac{(11bB-5Ac)\left(\frac{2}{7}c\int \frac{1}{x^{3/2}\sqrt{cx^4+bx^2}}dx-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}\right)}{11b}-\frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{(11bB-5Ac)\left(\frac{2}{7}c\left(-\frac{c\int\frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3b}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}\right)}{11b}-\frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}}$$

$\downarrow$ 1431

$$\frac{(11bB-5Ac)\left(\frac{2}{7}c\left(-\frac{cx\sqrt{b+cx^2}\int\frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}\right)}{11b}-\frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}}$$

$\downarrow$ 266

$$\frac{(11bB-5Ac)\left(\frac{2}{7}c\left(-\frac{2cx\sqrt{b+cx^2}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}\right)}{11b}-\frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}}$$

$\downarrow$ 761

$$\frac{(11bB-5Ac)\left(\frac{2}{7}c\left(-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}\right)}{11b}-\frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}}$$

input
```
Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(15/2),x]
```

output
```
(-2*A*(b*x^2 + c*x^4)^(3/2))/(11*b*x^(17/2)) + ((11*b*B - 5*A*c)*((-2*Sqrt
[b*x^2 + c*x^4])/(7*x^(9/2)) + (2*c*((-2*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5/2)
) - (c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x
)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[
b*x^2 + c*x^4])))/7))/(11*b)
```

3.230.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.230. $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx$

3.230.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{2(-10Ac^2x^4+22x^4Bbc+6Abcx^2+33b^2Bx^2+21b^2A)\sqrt{x^2(cx^2+b)}}{231x^{\frac{13}{2}}b^2}+\frac{2c(5Ac-11Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{}}{231b^2\sqrt{cx^3+}}$
default	$\frac{2\sqrt{x^4c+bx^2}\left(5A\sqrt{-bc}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}c^2x^5-11B\sqrt{-bc}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{}\right.\right.}{231x^{\frac{13}{2}}(cx^2+b)b^2}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x,method=_RETURNVERBOSE)
```

output
```
-2/231*(-10*A*c^2*x^4+22*B*b*c*x^4+6*A*b*c*x^2+33*B*b^2*x^2+21*A*b^2)/x^(1
3/2)/b^2*(x^2*(c*x^2+b))^(1/2)+2/231*c*(5*A*c-11*B*b)/b^2*(-b*c)^(1/2)*((x
+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)
^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*EllipticF(((x+1/
c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(x^2*(c*x^2+b))^(1/2)/x
^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

3.230.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.47

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx =$$
$$-\frac{2\left(2\,(11\,Bbc-5\,Ac^2)\sqrt{c}x^7\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+(2\,(11\,Bbc-5\,Ac^2)x^4+21\,Ab^2+3\,(11\,Bb^2+\right.}{231\,b^2x^7}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="fricas")
```

output
```
-2/231*(2*(11*B*b*c - 5*A*c^2)*sqrt(c)*x^7*weierstrassPInverse(-4*b/c, 0,
x) + (2*(11*B*b*c - 5*A*c^2)*x^4 + 21*A*b^2 + 3*(11*B*b^2 + 2*A*b*c)*x^2)*
sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^2*x^7)
```

3.230.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(15/2),x)
```

output
```
Timed out
```

3.230.7 Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{15}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(15/2), x)
```

3.230.8 Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{15}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(15/2), x)
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^{15/2}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(15/2),x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(15/2), x)
```

3.231 $\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

3.231.1 Optimal result

Integrand size = 28, antiderivative size = 486

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{88b^5(3bB-5Ac)x^{3/2}(b+cx^2)}{16575c^{9/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}$$
$$-\frac{88b^4(3bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{49725c^4}+\frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+cx^4}}{69615c^3}$$
$$-\frac{8b^2(3bB-5Ac)x^{9/2}\sqrt{bx^2+cx^4}}{7735c^2}-\frac{4b(3bB-5Ac)x^{13/2}\sqrt{bx^2+cx^4}}{595c}$$
$$-\frac{2(3bB-5Ac)x^{9/2}\left(bx^2+cx^4\right)^{3/2}}{105c}+\frac{2Bx^{5/2}\left(bx^2+cx^4\right)^{5/2}}{25c}$$
$$-\frac{88b^{21/4}(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{16575c^{19/4}\sqrt{bx^2+cx^4}}$$
$$+\frac{44b^{21/4}(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{16575c^{19/4}\sqrt{bx^2+cx^4}}$$

output
```
-2/105*(-5*A*c+3*B*b)*x^(9/2)*(c*x^4+b*x^2)^(3/2)/c+2/25*B*x^(5/2)*(c*x^4+
b*x^2)^(5/2)/c+88/16575*b^5*(-5*A*c+3*B*b)*x^(3/2)*(c*x^2+b)/c^(9/2)/(b^(1
/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+88/69615*b^3*(-5*A*c+3*B*b)*x^(5/2)*(c*
x^4+b*x^2)^(1/2)/c^3-8/7735*b^2*(-5*A*c+3*B*b)*x^(9/2)*(c*x^4+b*x^2)^(1/2)
/c^2-4/595*b*(-5*A*c+3*B*b)*x^(13/2)*(c*x^4+b*x^2)^(1/2)/c-88/49725*b^4*(-
5*A*c+3*B*b)*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^4-88/16575*b^(21/4)*(-5*A*c+3*B
*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4
)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2
^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(19/
4)/(c*x^4+b*x^2)^(1/2)+44/16575*b^(21/4)*(-5*A*c+3*B*b)*x*(cos(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*El
lipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(
1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(19/4)/(c*x^4+b*x^2)^(1/2)
```

3.231.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.24 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.33

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left(-(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}(1155b^3B-221c^3x^4(25A+21Bx^2)-55b^2c(35A\cdots\right.}{116025\cdots}$$

input
```
Integrate[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(115
5*b^3*B - 221*c^3*x^4*(25*A + 21*B*x^2) - 55*b^2*c*(35*A + 39*B*x^2) + 65*
b*c^2*x^2*(55*A + 51*B*x^2))) + 385*b^4*(3*b*B - 5*A*c)*Hypergeometric2F1[
-3/2, 3/4, 7/4, -((c*x^2)/b)]))/(116025*c^4*Sqrt[1 + (c*x^2)/b])
```

3.231.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 455, normalized size of antiderivative = 0.94, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1945, 1426, 1426, 1429, 1429, 1429, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1945}$$

$$\frac{2Bx^{5/2}\left(bx^2+cx^4\right)^{5/2}}{25c}-\frac{(3bB-5Ac)\int x^{7/2}\left(cx^4+bx^2\right)^{3/2}dx}{5c}$$

$$\Big\downarrow \text{ 1426}$$

$$\frac{2Bx^{5/2}\left(bx^2+cx^4\right)^{5/2}}{25c}-\frac{(3bB-5Ac)\left(\frac{2}{7}b\int x^{11/2}\sqrt{cx^4+bx^2}dx+\frac{2}{21}x^{9/2}\left(bx^2+cx^4\right)^{3/2}\right)}{5c}$$

$$\Big\downarrow \text{ 1426}$$

$$\frac{2Bx^{5/2}\left(bx^2+cx^4\right)^{5/2}}{25c}-\frac{(3bB-5Ac)\left(\frac{2}{7}b\left(\frac{2}{17}b\int \frac{x^{15/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{17}x^{13/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{21}x^{9/2}\left(bx^2+cx^4\right)^{3/2}\right)}{5c}$$

$$\Big\downarrow \text{ 1429}$$

$$\frac{2Bx^{5/2}\left(bx^2+cx^4\right)^{5/2}}{25c}-\frac{(3bB-5Ac)\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{9/2}\sqrt{bx^2+cx^4}}{13c}-\frac{11b\int \frac{x^{11/2}}{\sqrt{cx^4+bx^2}}dx}{13c}\right)+\frac{2}{17}x^{13/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{21}x^{9/2}\left(bx^2+cx^4\right)^{3/2}\right)}{5c}$$

$$\Big\downarrow \text{ 1429}$$

$$\frac{2Bx^{5/2}\left(bx^2+cx^4\right)^{5/2}}{25c}-\frac{(3bB-5Ac)\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{9/2}\sqrt{bx^2+cx^4}}{13c}-\frac{11b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\int \frac{x^{7/2}}{\sqrt{cx^4+bx^2}}dx}{9c}\right)}{13c}\right)+\frac{2}{17}x^{13/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{21}x^{9/2}(bx\ldots}{5c}$$

$$\Big\downarrow \text{ 1429}$$

$$\frac{2Bx^{5/2}\left(bx^2+cx^4\right)^{5/2}}{25c}-\frac{(3bB-5Ac)\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{9/2}\sqrt{bx^2+cx^4}}{13c}-\frac{11b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3b\int\frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{5c}\right)}{9c}\right)}{13c}\right)+\frac{2}{17}x^{13/2}\sqrt{bx^2+c\cdots}\right.\right.}{5c}$$

$\downarrow$ 1431

$$\frac{2Bx^{5/2}\left(bx^2+cx^4\right)^{5/2}}{25c}-\frac{(3bB-5Ac)\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{9/2}\sqrt{bx^2+cx^4}}{13c}-\frac{11b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3bx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)}{13c}\right)+\frac{2}{17}x^{13/2}\sqrt{bx\cdots}\right.\right.}{5c}$$

$\downarrow$ 266

$$\frac{2Bx^{5/2}\left(bx^2+cx^4\right)^{5/2}}{25c}-\frac{(3bB-5Ac)\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{9/2}\sqrt{bx^2+cx^4}}{13c}-\frac{11b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)}{13c}\right)+\frac{2}{17}x^{13/2}\sqrt{b\cdots}\right.\right.}{5c}$$

$\downarrow$ 834

3.231. $\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

$$\frac{2Bx^{5/2}\left(bx^2+cx^4\right)^{5/2}}{25c} - \frac{(3bB-5Ac)\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{9/2}\sqrt{bx^2+cx^4}}{13c} - \frac{11b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} - \frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}} - \frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)}{13c}\right)\right)}{5c}$$

$\downarrow$ 27

$$\frac{2Bx^{5/2}\left(bx^2+cx^4\right)^{5/2}}{25c} - \frac{(3bB-5Ac)\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{9/2}\sqrt{bx^2+cx^4}}{13c} - \frac{11b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} - \frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}} - \frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)}{13c}\right)\right)}{5c}$$

$\downarrow$ 761

3.231. $\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

$$\frac{2Bx^{5/2}\left(bx^2+cx^4\right)^{5/2}}{25c}-\frac{(3bB-5Ac)\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{9/2}\sqrt{bx^2+cx^4}}{13c}-\frac{11b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}}{2c^{3/4}\sqrt{b+cx^2}}\right.}{5c\sqrt{bx^2+c}}\right.}{9c}\right.}{13c}\right.\right.\right.}{5c}$$

$\downarrow$ 1510

3.231. $\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

$$\frac{2Bx^{5/2}\left(bx^2+cx^4\right)^{5/2}}{25c}-$$

$$(3bB-5Ac)\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{9/2}\sqrt{bx^2+cx^4}}{13c}-\frac{11b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}}{2c^{3/4}\sqrt{b+cx^2}}\right.}{}\right.}{}\right.}{}\right.\right.\right.$$

3.231. $\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

input
```
Int[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*B*x^(5/2)*(b*x^2 + c*x^4)^(5/2))/(25*c) - ((3*b*B - 5*A*c)*((2*x^(9/2)*
(b*x^2 + c*x^4)^(3/2))/21 + (2*b*((2*x^(13/2)*Sqrt[b*x^2 + c*x^4])/17 + (2
*b*((2*x^(9/2)*Sqrt[b*x^2 + c*x^4])/(13*c) - (11*b*((2*x^(5/2)*Sqrt[b*x^2
+ c*x^4])/(9*c) - (7*b*((2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*c) - (6*b*x*Sqr
t[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(
1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellip
ticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))
/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqr
t[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)
*Sqrt[b + c*x^2])))/(5*c*Sqrt[b*x^2 + c*x^4])))/(9*c)))/(13*c)))/17))/7))/
(5*c)
```

3.231.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.231.4 Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.70

method	result
risch	$\frac{2\sqrt{x}\,(13923B\,c^5x^{10}+16575A\,c^5x^8+17901Bb\,c^4x^8+22425Ab\,c^4x^6+468B\,b^2c^3x^6+900A\,b^2c^3x^4-540B\,b^3c^2x^4-1100A\,b^3c^2x^2+660B}{348075c^4}$
default	$-\frac{2(x^4c+b\,x^2)^{\frac{3}{2}}\left(-13923B\,c^7x^{14}-16575A\,c^7x^{12}-31824Bb\,c^6x^{12}-39000Ab\,c^6x^{10}-18369B\,b^2c^5x^{10}-23325A\,b^2c^5x^8+72B\,b^3c^4x^8+\right.}{}$

input
```
int(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/348075/c^4*x^(1/2)*(13923*B*c^5*x^10+16575*A*c^5*x^8+17901*B*b*c^4*x^8+2
2425*A*b*c^4*x^6+468*B*b^2*c^3*x^6+900*A*b^2*c^3*x^4-540*B*b^3*c^2*x^4-110
0*A*b^3*c^2*x^2+660*B*b^4*c*x^2+1540*A*b^4*c-924*B*b^5)*(x^2*(c*x^2+b))^(1
/2)-44/16575*b^5/c^5*(5*A*c-3*B*b)*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-
b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b
*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/2)*EllipticE(((x+1/c*(-
b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1/c*(-b*c)^(1/2)*EllipticF(
((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))*(x^2*(c*x^2+b))^
(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

3.231.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.36

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx =$$
$$-\frac{2\left(924\left(3\,Bb^6-5\,Ab^5c\right)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\left(13923\,Bc^6x^{10}+66\right.}{}$$

input
```
integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-2/348075*(924*(3*B*b^6 - 5*A*b^5*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, we
ierstrassPInverse(-4*b/c, 0, x)) - (13923*B*c^6*x^10 + 663*(27*B*b*c^5 + 2
5*A*c^6)*x^8 - 924*B*b^5*c + 1540*A*b^4*c^2 + 39*(12*B*b^2*c^4 + 575*A*b*c
^5)*x^6 - 180*(3*B*b^3*c^3 - 5*A*b^2*c^4)*x^4 + 220*(3*B*b^4*c^2 - 5*A*b^3
*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/c^5
```

3.231.6 Sympy [F(-1)]

Timed out.

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

output
```
Timed out
```

3.231.7 Maxima [F]

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int\left(cx^4+bx^2\right)^{\frac{3}{2}}(Bx^2+A)x^{\frac{7}{2}}\,dx$$

input
```
integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(7/2), x)
```

3.231.8 Giac [F]

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int\left(cx^4+bx^2\right)^{\frac{3}{2}}(Bx^2+A)x^{\frac{7}{2}}\,dx$$

input
```
integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(7/2), x)
```

3.231.9 Mupad [F(-1)]

Timed out.

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int x^{7/2}\,\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}\,dx$$

input
```
int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

3.232 $\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

3.232.1 Optimal result

Integrand size = 28, antiderivative size = 321

$$\begin{aligned}\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx =& -\frac{24b^4(13bB-23Ac)\sqrt{bx^2+cx^4}}{33649c^4\sqrt{x}}+\frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+cx^4}}{168245c^3}\\&-\frac{8b^2(13bB-23Ac)x^{7/2}\sqrt{bx^2+cx^4}}{24035c^2}-\frac{4b(13bB-23Ac)x^{11/2}\sqrt{bx^2+cx^4}}{2185c}\\&-\frac{2(13bB-23Ac)x^{7/2}\left(bx^2+cx^4\right)^{3/2}}{437c}+\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{5/2}}{23c}\\&+\frac{12b^{19/4}(13bB-23Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{33649c^{17/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

output
```
-2/437*(-23*A*c+13*B*b)*x^(7/2)*(c*x^4+b*x^2)^(3/2)/c+2/23*B*x^(3/2)*(c*x^
4+b*x^2)^(5/2)/c+72/168245*b^3*(-23*A*c+13*B*b)*x^(3/2)*(c*x^4+b*x^2)^(1/2
)/c^3-8/24035*b^2*(-23*A*c+13*B*b)*x^(7/2)*(c*x^4+b*x^2)^(1/2)/c^2-4/2185*
b*(-23*A*c+13*B*b)*x^(11/2)*(c*x^4+b*x^2)^(1/2)/c-24/33649*b^4*(-23*A*c+13
*B*b)*(c*x^4+b*x^2)^(1/2)/c^4/x^(1/2)+12/33649*b^(19/4)*(-23*A*c+13*B*b)*x
*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(
1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2
))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(17/4)/(c
*x^4+b*x^2)^(1/2)
```

3.232.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.23 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.50

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}dx = \frac{2\sqrt{x^2(b+cx^2)}\left(-(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}(195b^3B-55c^3x^4(23A+19Bx^2)+11bc^2x^2(69A+65\ldots\right.}{24035c^4\sqrt{\ldots}}$$

input
```
Integrate[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(195*b^3*B -
 55*c^3*x^4*(23*A + 19*B*x^2) + 11*b*c^2*x^2*(69*A + 65*B*x^2) - 3*b^2*c*(
115*A + 143*B*x^2))) + 15*b^4*(13*b*B - 23*A*c)*Hypergeometric2F1[-3/2, 1/
4, 5/4, -((c*x^2)/b)]))/(24035*c^4*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

3.232.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1945, 1426, 1426, 1429, 1429, 1429, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1945}$$

$$\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{5/2}}{23c}-\frac{(13bB-23Ac)\int x^{5/2}\left(cx^4+bx^2\right)^{3/2}dx}{23c}$$

$$\Big\downarrow \text{ 1426}$$

$$\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{5/2}}{23c}-\frac{(13bB-23Ac)\left(\frac{6}{19}b\int x^{9/2}\sqrt{cx^4+bx^2}dx+\frac{2}{19}x^{7/2}\left(bx^2+cx^4\right)^{3/2}\right)}{23c}$$

$$\Big\downarrow \text{ 1426}$$

$$\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{5/2}}{23c}-\frac{(13bB-23Ac)\left(\frac{6}{19}b\left(\frac{2}{15}b\int\frac{x^{13/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{15}x^{11/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{19}x^{7/2}\left(bx^2+cx^4\right)^{3/2}\right)}{23c}$$

$\Big\downarrow$ 1429

$$\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{5/2}}{23c}-\frac{(13bB-23Ac)\left(\frac{6}{19}b\left(\frac{2}{15}b\left(\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\int\frac{x^{9/2}}{\sqrt{cx^4+bx^2}}dx}{11c}\right)+\frac{2}{15}x^{11/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{19}x^{7/2}\left(bx^2+cx^4\right)^{3/2}\right)}{23c}$$

$\Big\downarrow$ 1429

$$\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{5/2}}{23c}-\frac{(13bB-23Ac)\left(\frac{6}{19}b\left(\frac{2}{15}b\left(\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\int\frac{x^{5/2}}{\sqrt{cx^4+bx^2}}dx}{7c}\right)}{11c}\right)+\frac{2}{15}x^{11/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{19}x^{7/2}(\right)}{23c}$$

$\Big\downarrow$ 1429

$$\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{5/2}}{23c}-\frac{(13bB-23Ac)\left(\frac{6}{19}b\left(\frac{2}{15}b\left(\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int\frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{7c}\right)}{11c}\right)+\frac{2}{15}x^{11/2}\sqrt{bx^2+c}\right)\right)}{23c}$$

$\Big\downarrow$ 1431

3.232. $\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

$$\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{5/2}}{23c} - \frac{(13bB-23Ac)\left(\frac{6}{19}b\left(\frac{2}{15}b\left(\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{9b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c} - \frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)}{11c}\right) + \frac{2}{15}x^{11/2}\sqrt{b\cdots}\right)\cdots\right)}{23c}$$

$\downarrow$ 266

$$\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{5/2}}{23c} - \frac{(13bB-23Ac)\left(\frac{6}{19}b\left(\frac{2}{15}b\left(\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{9b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c} - \frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{2bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)}{11c}\right) + \frac{2}{15}x^{11/2}\sqrt{b\cdots}\right)\cdots\right)}{23c}$$

$\downarrow$ 761

$$\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{5/2}}{23c} - \frac{(13bB-23Ac)\left(\frac{6}{19}b\left(\frac{2}{15}b\left(\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{9b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c} - \frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\cdots\right)\cdots\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\cdots\right)}{7c}\cdots\right)}{11c}\cdots\right)\cdots\right)\cdots\right)}{23c}$$

3.232. $\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

input
```
Int[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*B*x^(3/2)*(b*x^2 + c*x^4)^(5/2))/(23*c) - ((13*b*B - 23*A*c)*((2*x^(7/2
)*(b*x^2 + c*x^4)^(3/2))/19 + (6*b*((2*x^(11/2)*Sqrt[b*x^2 + c*x^4])/15 +
(2*b*((2*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(11*c) - (9*b*((2*x^(3/2)*Sqrt[b*x^2
 + c*x^4])/(7*c) - (5*b*((2*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - (b^(3/4)*
x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(5/4)*Sqrt[b*x^2 + c*x^4
])))/(7*c)))/(11*c)))/15))/19))/(23*c)
```

3.232.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.232.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.90

method	result
risch	$\frac{2(7315Bc^5x^{10}+8855Ac^5x^8+9625Bbc^4x^8+12397Abc^4x^6+308Bb^2c^3x^6+644Ab^2c^3x^4-364Bb^3c^2x^4-828Ab^3c^2x^2+468Bb^4cx^2+}{168245c^4\sqrt{x}}$
default	$-\frac{2(x^4c+bx^2)^{\frac{3}{2}}\left(-7315Bc^7x^{13}-8855Ac^7x^{11}-16940Bbc^6x^{11}-21252Abc^6x^9-9933Bb^2c^5x^9-13041Ab^2c^5x^7+56Bb^3c^4x^7+690A\right.}{}$

input
```
int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/168245/c^4*(7315*B*c^5*x^10+8855*A*c^5*x^8+9625*B*b*c^4*x^8+12397*A*b*c^
4*x^6+308*B*b^2*c^3*x^6+644*A*b^2*c^3*x^4-364*B*b^3*c^2*x^4-828*A*b^3*c^2*
x^2+468*B*b^4*c*x^2+1380*A*b^4*c-780*B*b^5)/x^(1/2)*(x^2*(c*x^2+b))^(1/2)-
12/33649*b^5/c^5*(23*A*c-13*B*b)*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*
c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c
)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^
(1/2))^(1/2),1/2*2^(1/2))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^
2+b))^(1/2)
```

3.232.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.53

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{2\left(60\left(13\,Bb^6-23\,Ab^5c\right)\sqrt{c}x\,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(7315\,Bc^6x^{10}+385\left(25\,Bbc^5+\right.\right.\right.}{}$$

input
```
integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/168245*(60*(13*B*b^6 - 23*A*b^5*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c,
 0, x) + (7315*B*c^6*x^10 + 385*(25*B*b*c^5 + 23*A*c^6)*x^8 - 780*B*b^5*c
+ 1380*A*b^4*c^2 + 77*(4*B*b^2*c^4 + 161*A*b*c^5)*x^6 - 28*(13*B*b^3*c^3 -
 23*A*b^2*c^4)*x^4 + 36*(13*B*b^4*c^2 - 23*A*b^3*c^3)*x^2)*sqrt(c*x^4 + b*
x^2)*sqrt(x))/(c^5*x)
```

3.232.6 Sympy [F(-1)]

Timed out.

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

output
```
Timed out
```

3.232.7 Maxima [F]

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \int \left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)x^{\frac{5}{2}}\,dx$$

input
```
integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(5/2), x)
```

3.232.8 Giac [F]

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2\right)^{\frac{3}{2}}(Bx^2+A)x^{\frac{5}{2}}\,dx$$

input
```
integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(5/2), x)
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int x^{5/2}\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}\,dx$$

input
```
int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

3.233 $\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

3.233.1 Optimal result

Integrand size = 28, antiderivative size = 447

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = -\frac{8b^4(11bB-21Ac)x^{3/2}(b+cx^2)}{3315c^{7/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{8b^3(11bB-21Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{9945c^3}-\frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+cx^4}}{13923c^2}-\frac{4b(11bB-21Ac)x^{9/2}\sqrt{bx^2+cx^4}}{1547c}-\frac{2(11bB-21Ac)x^{5/2}\left(bx^2+cx^4\right)^{3/2}}{357c}+\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{5/2}}{21c}+\frac{8b^{17/4}(11bB-21Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3315c^{15/4}\sqrt{bx^2+cx^4}}-\frac{4b^{17/4}(11bB-21Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3315c^{15/4}\sqrt{bx^2+cx^4}}$$

output
```
-2/357*(-21*A*c+11*B*b)*x^(5/2)*(c*x^4+b*x^2)^(3/2)/c+2/21*B*(c*x^4+b*x^2)
^(5/2)*x^(1/2)/c-8/3315*b^4*(-21*A*c+11*B*b)*x^(3/2)*(c*x^2+b)/c^(7/2)/(b^
(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-8/13923*b^2*(-21*A*c+11*B*b)*x^(5/2)*
(c*x^4+b*x^2)^(1/2)/c^2-4/1547*b*(-21*A*c+11*B*b)*x^(9/2)*(c*x^4+b*x^2)^(1
/2)/c+8/9945*b^3*(-21*A*c+11*B*b)*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^3+8/3315*b
^(17/4)*(-21*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^
(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(
1/2))^2)^(1/2)/c^(15/4)/(c*x^4+b*x^2)^(1/2)-4/3315*b^(17/4)*(-21*A*c+11*B*
b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)
*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^
(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(15/4
)/(c*x^4+b*x^2)^(1/2)
```

3.233.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.31

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\left(\left(b+cx^2\right)^2\sqrt{1+\frac{cx^2}{b}}(77b^2B+13c^2x^2(21A+17Bx^2)-bc(147A+143Bx\right.}{4641c^3\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(77*b^
2*B + 13*c^2*x^2*(21*A + 17*B*x^2) - b*c*(147*A + 143*B*x^2)) + 7*b^3*(-11
*b*B + 21*A*c)*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)]))/(4641*c^3
*Sqrt[1 + (c*x^2)/b])
```

3.233.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 419, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1945, 1426, 1426, 1429, 1429, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1945}$$

$$\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{5/2}}{21c}-\frac{(11bB-21Ac)\int x^{3/2}\left(cx^4+bx^2\right)^{3/2}dx}{21c}$$

$$\Big\downarrow \text{ 1426}$$

$$\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{5/2}}{21c}-\frac{(11bB-21Ac)\left(\frac{6}{17}b\int x^{7/2}\sqrt{cx^4+bx^2}dx+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}\right)}{21c}$$

$$\Big\downarrow \text{ 1426}$$

$$\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{5/2}}{21c}-\frac{(11bB-21Ac)\left(\frac{6}{17}b\left(\frac{2}{13}b\int\frac{x^{11/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}\right)}{21c}$$

$$\Big\downarrow \text{ 1429}$$

$$\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{5/2}}{21c}-\frac{(11bB-21Ac)\left(\frac{6}{17}b\left(\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\int\frac{x^{7/2}}{\sqrt{cx^4+bx^2}}dx}{9c}\right)+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}\right)}{21c}$$

$$\Big\downarrow \text{ 1429}$$

$$\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{5/2}}{21c}-\frac{(11bB-21Ac)\left(\frac{6}{17}b\left(\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3b\int\frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{5c}\right)}{9c}\right)+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{17}x^{5/2}(bx}{21c}$$

$$\Big\downarrow \text{ 1431}$$

$$\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{5/2}}{21c}-\frac{(11bB-21Ac)\left(\frac{6}{17}b\left(\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3bx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{17}x^{5}}{21c}$$

$\Big\downarrow$ 266

$$\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{5/2}}{21c}-\frac{(11bB-21Ac)\left(\frac{6}{17}b\left(\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{17}x}{21c}$$

$\Big\downarrow$ 834

$$\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{5/2}}{21c}-\frac{(11bB-21Ac)\left(\frac{6}{17}b\left(\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)+\frac{2}{13}x^{9}\right)}{21c}$$

$\Big\downarrow$ 27

$$\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{5/2}}{21c}-\frac{(11bB-21Ac)\left(\frac{6}{17}b\left(\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)+\frac{2}{13}x^{9/2}\sqrt{}\right)}{21c}$$

$\Big\downarrow$ 761

3.233. $\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx$

$$\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{5/2}}{21c}-\frac{(11bB-21Ac)\Bigg(\frac{6}{17}b\Bigg(\frac{2}{13}b\Bigg(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\Bigg(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\Bigg(\frac{\sqrt[4]{b}(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}}\,\text{EllipticF}\Big(2\arctan\Big(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}}{2c^{3/4}\sqrt{b+cx^2}}}{5c\sqrt{bx^2+cx^4}}}{9c}}{21c}$$

$\downarrow$ 1510

$$\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{5/2}}{21c}-\frac{(11bB-21Ac)\Bigg(\frac{6}{17}b\Bigg(\frac{2}{13}b\Bigg(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\Bigg(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\Bigg(\frac{\sqrt[4]{b}(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}}\,\text{EllipticF}\Big(2\arctan\Big(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}}{2c^{3/4}\sqrt{b+cx^2}}}{5c}}{9c}}{21}$$

3.233. $\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

input
```
Int[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*B*Sqrt[x]*(b*x^2 + c*x^4)^(5/2))/(21*c) - ((11*b*B - 21*A*c)*((2*x^(5/2
)*(b*x^2 + c*x^4)^(3/2))/17 + (6*b*((2*x^(9/2)*Sqrt[b*x^2 + c*x^4])/13 + (
2*b*((2*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(9*c) - (7*b*((2*Sqrt[x]*Sqrt[b*x^2 +
 c*x^4])/(5*c) - (6*b*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(S
qrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sq
rt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(5*c*Sqrt[b*x^2 + c*x^4])))
/(9*c)))/13))/17))/(21*c)
```

3.233.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.233.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{2\sqrt{x}\,(-3315B\,x^8c^4-4095A\,x^6c^4-4485B\,x^6b\,c^3-5985A\,x^4b\,c^3-180B\,x^4b^2c^2-420A\,x^2b^2c^2+220B\,x^2b^3c+588A\,b^3c-308B\,b^4)\sqrt{x}}{69615c^3}$
default	$\frac{2(x^4c+b\,x^2)^{\frac{3}{2}}\left(3315B\,c^6x^{12}+4095A\,c^6x^{10}+7800Bb\,c^5x^{10}+10080Ab\,c^5x^8+4665B\,b^2c^4x^8+6405A\,b^2c^4x^6-40B\,b^3c^3x^6+1764A\,b^5c\sqrt{}\right.}{}$

input
```
int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/69615/c^3*x^(1/2)*(-3315*B*c^4*x^8-4095*A*c^4*x^6-4485*B*b*c^3*x^6-5985
*A*b*c^3*x^4-180*B*b^2*c^2*x^4-420*A*b^2*c^2*x^2+220*B*b^3*c*x^2+588*A*b^3
*c-308*B*b^4)*(x^2*(c*x^2+b))^(1/2)+4/3315*b^4/c^4*(21*A*c-11*B*b)*(-b*c)^
(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))
*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(
-b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2
),1/2*2^(1/2)))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2
)
```

3.233.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.34

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{2\left(84\left(11\,Bb^5-21\,Ab^4c\right)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+(3315$$

input
```
integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/69615*(84*(11*B*b^5 - 21*A*b^4*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, wei
erstrassPInverse(-4*b/c, 0, x)) + (3315*B*c^5*x^8 + 195*(23*B*b*c^4 + 21*A
*c^5)*x^6 + 308*B*b^4*c - 588*A*b^3*c^2 + 45*(4*B*b^2*c^3 + 133*A*b*c^4)*x
^4 - 20*(11*B*b^3*c^2 - 21*A*b^2*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/c^
4
```

3.233.6 Sympy [F(-1)]

Timed out.

$$\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

output
```
Timed out
```

3.233.7 Maxima [F]

$$\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\int\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)x^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(3/2), x)
```

3.233.8 Giac [F]

$$\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\int\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)x^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(3/2), x)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int x^{3/2}\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}\,dx$$

input
```
int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

3.234 $\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

3.234.1 Optimal result

Integrand size = 28, antiderivative size = 282

$$\begin{aligned}\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx &= \frac{8b^3(9bB-19Ac)\sqrt{bx^2+cx^4}}{4389c^3\sqrt{x}}\\ &-\frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2}-\frac{4b(9bB-19Ac)x^{7/2}\sqrt{bx^2+cx^4}}{1045c}\\ &-\frac{2(9bB-19Ac)x^{3/2}(bx^2+cx^4)^{3/2}}{285c}+\frac{2B(bx^2+cx^4)^{5/2}}{19c\sqrt{x}}\\ &-\frac{4b^{15/4}(9bB-19Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{4389c^{13/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

output
```
-2/285*(-19*A*c+9*B*b)*x^(3/2)*(c*x^4+b*x^2)^(3/2)/c+2/19*B*(c*x^4+b*x^2)^
(5/2)/c/x^(1/2)-8/7315*b^2*(-19*A*c+9*B*b)*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c^2
-4/1045*b*(-19*A*c+9*B*b)*x^(7/2)*(c*x^4+b*x^2)^(1/2)/c+8/4389*b^3*(-19*A*
c+9*B*b)*(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)-4/4389*b^(15/4)*(-19*A*c+9*B*b)*x
*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(
1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2
))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(13/4)/(c
*x^4+b*x^2)^(1/2)
```

3.234.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.49

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{2\sqrt{x^2(b+cx^2)}\left((b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}(45b^2B+11c^2x^2(19A+15Bx^2)-bc(95A+99Bx^2))+\right.}{3135c^3\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(45*b^2*B + 11
*c^2*x^2*(19*A + 15*B*x^2) - b*c*(95*A + 99*B*x^2)) + 5*b^3*(-9*b*B + 19*A
*c)*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(3135*c^3*Sqrt[x]*Sq
rt[1 + (c*x^2)/b])
```

3.234.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1945, 1426, 1426, 1429, 1429, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{19c\sqrt{x}}-\frac{(9bB-19Ac)\int\sqrt{x}\left(cx^4+bx^2\right)^{3/2}dx}{19c}$$

$$\Big\downarrow \text{1426}$$

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{19c\sqrt{x}}-\frac{(9bB-19Ac)\left(\frac{2}{5}b\int x^{5/2}\sqrt{cx^4+bx^2}dx+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}\right)}{19c}$$

$$\Big\downarrow \text{1426}$$

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{19c\sqrt{x}}-\frac{(9bB-19Ac)\left(\frac{2}{5}b\left(\frac{2}{11}b\int\frac{x^{9/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}\right)}{19c}$$

$\Big\downarrow$ 1429

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{19c\sqrt{x}}-\frac{(9bB-19Ac)\left(\frac{2}{5}b\left(\frac{2}{11}b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\int\frac{x^{5/2}}{\sqrt{cx^4+bx^2}}dx}{7c}\right)+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}\right)}{19c}$$

$\Big\downarrow$ 1429

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{19c\sqrt{x}}-\frac{(9bB-19Ac)\left(\frac{2}{5}b\left(\frac{2}{11}b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int\frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{7c}\right)+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{15}x^{3/2}\left(bx^2+c\right.\right.}{19c}$$

$\Big\downarrow$ 1431

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{19c\sqrt{x}}-\frac{(9bB-19Ac)\left(\frac{2}{5}b\left(\frac{2}{11}b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{15}x^{3/2}\left(\right.\right.}{19c}$$

$\Big\downarrow$ 266

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{19c\sqrt{x}}-\frac{(9bB-19Ac)\left(\frac{2}{5}b\left(\frac{2}{11}b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{2bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{15}x^{3/2}\left(\right.\right.}{19c}$$

$\Big\downarrow$ 761

3.234. $\int\sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

$$\frac{2B(bx^2+cx^4)^{5/2}}{19c\sqrt{x}} - \frac{(9bB-19Ac)\left(\frac{2}{5}b\left(\frac{2}{11}b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\right)}{7c}\right)+\frac{2}{11}\right.\right.}{19c}$$

input
```
Int[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*B*(b*x^2 + c*x^4)^(5/2))/(19*c*Sqrt[x]) - ((9*b*B - 19*A*c)*((2*x^(3/2)
*(b*x^2 + c*x^4)^(3/2))/15 + (2*b*((2*x^(7/2)*Sqrt[b*x^2 + c*x^4])/11 + (2
*b*((2*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(7*c) - (5*b*((2*Sqrt[b*x^2 + c*x^4])/
(3*c*Sqrt[x]) - (b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
 + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c
^(5/4)*Sqrt[b*x^2 + c*x^4])))/(7*c)))/11))/5))/(19*c)
```

3.234.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.234.4 Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{2\left(-1155B\,x^{8}c^{4}-1463A\,x^{6}c^{4}-1617B\,x^{6}b\,c^{3}-2261A\,x^{4}b\,c^{3}-84B\,x^{4}b^{2}c^{2}-228A\,x^{2}b^{2}c^{2}+108B\,x^{2}b^{3}c+380A\,b^{3}c-180B\,b^{4}\right)\sqrt{x^{2}(c\,x}}{21945c^{3}\sqrt{x}}$
default	$\frac{2\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}\left(1155B\,c^{6}x^{11}+1463A\,c^{6}x^{9}+2772Bb\,c^{5}x^{9}+3724Ab\,c^{5}x^{7}+1701B\,b^{2}c^{4}x^{7}+190A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{}\right.}{}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21945/c^3*(-1155*B*c^4*x^8-1463*A*c^4*x^6-1617*B*b*c^3*x^6-2261*A*b*c^3
*x^4-84*B*b^2*c^2*x^4-228*A*b^2*c^2*x^2+108*B*b^3*c*x^2+380*A*b^3*c-180*B*
b^4)/x^(1/2)*(x^2*(c*x^2+b))^(1/2)+4/4389*b^4/c^4*(19*A*c-9*B*b)*(-b*c)^(1
/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c
/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*EllipticF
(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(x^2*(c*x^2+b))^
(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

3.234.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.52

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx =$$
$$-\frac{2\left(20\left(9\,Bb^5-19\,Ab^4c\right)\sqrt{c}x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\left(1155\,Bc^5x^8+77\left(21\,Bbc^4+19\,Ac^5\right)x^6+18\right.}{21945\,c^4x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="fricas")
```

output
```
-2/21945*(20*(9*B*b^5 - 19*A*b^4*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c,
0, x) - (1155*B*c^5*x^8 + 77*(21*B*b*c^4 + 19*A*c^5)*x^6 + 180*B*b^4*c - 3
80*A*b^3*c^2 + 7*(12*B*b^2*c^3 + 323*A*b*c^4)*x^4 - 12*(9*B*b^3*c^2 - 19*A
*b^2*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^4*x)
```

3.234.6 Sympy [F]

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \int \sqrt{x}\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)*x**(1/2),x)
```

output
```
Integral(sqrt(x)*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

3.234.7 Maxima [F]

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int (cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)\sqrt{x}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*sqrt(x), x)
```

3.234.8 Giac [F]

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int (cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)\sqrt{x}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*sqrt(x), x)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int \sqrt{x}\,\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}\,dx$$

input
```
int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

3.235 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx$

3.235.1 Optimal result

Integrand size = 28, antiderivative size = 408

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx = \frac{8b^3(7bB-17Ac)x^{3/2}(b+cx^2)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{8b^2(7bB-17Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2} - \frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} - \frac{2(7bB-17Ac)\sqrt{x}(bx^2+cx^4)^{3/2}}{221c} + \frac{2B(bx^2+cx^4)^{5/2}}{17cx^{3/2}} - \frac{8b^{13/4}(7bB-17Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}} + \frac{4b^{13/4}(7bB-17Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}}$$

output
```
2/17*B*(c*x^4+b*x^2)^(5/2)/c/x^(3/2)-2/221*(-17*A*c+7*B*b)*(c*x^4+b*x^2)^(
3/2)*x^(1/2)/c+8/1105*b^3*(-17*A*c+7*B*b)*x^(3/2)*(c*x^2+b)/c^(5/2)/(b^(1/
2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-4/663*b*(-17*A*c+7*B*b)*x^(5/2)*(c*x^4+b
*x^2)^(1/2)/c-8/3315*b^2*(-17*A*c+7*B*b)*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^2-8
/1105*b^(13/4)*(-17*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2
)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1
/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)
+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)+4/1105*b^(13/4)*(-17*A*c
+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1
/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^
(11/4)/(c*x^4+b*x^2)^(1/2)
```

3.235.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.28

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left(-(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}(7bB-17Ac-13Bcx^2)+b^2(7\right.}{221c^2\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(7*b
*B - 17*A*c - 13*B*c*x^2)) + b^2*(7*b*B - 17*A*c)*Hypergeometric2F1[-3/2,
3/4, 7/4, -((c*x^2)/b)]))/(221*c^2*Sqrt[1 + (c*x^2)/b])
```

3.235.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1945, 1426, 1426, 1429, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{ 1945}$$

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{17cx^{3/2}}-\frac{(7bB-17Ac)\int \frac{\left(cx^4+bx^2\right)^{3/2}}{\sqrt{x}}dx}{17c}$$

$$\Big\downarrow \text{ 1426}$$

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{17cx^{3/2}}-\frac{(7bB-17Ac)\left(\frac{6}{13}b\int x^{3/2}\sqrt{cx^4+bx^2}dx+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}\right)}{17c}$$

$$\Big\downarrow \text{ 1426}$$

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{17cx^{3/2}}-\frac{(7bB-17Ac)\left(\frac{6}{13}b\left(\frac{2}{9}b\int \frac{x^{7/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}\right)}{17c}$$

$$\Big\downarrow \text{ 1429}$$

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{17cx^{3/2}}-\frac{(7bB-17Ac)\left(\frac{6}{13}b\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3b\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{5c}\right)+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}\right)}{17c}$$

$$\Big\downarrow \text{ 1431}$$

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{17cx^{3/2}}-\frac{(7bB-17Ac)\left(\frac{6}{13}b\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3bx\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5c\sqrt{bx^2+cx^4}}\right)+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}\right)}{17c}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{17cx^{3/2}}-\frac{(7bB-17Ac)\left(\frac{6}{13}b\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5c\sqrt{bx^2+cx^4}}\right)+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}\right)}{17c}$$

$$\Big\downarrow \text{ 834}$$

3.235. $\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{\sqrt{x}}dx$

$$\frac{\frac{2B\left(bx^2+cx^4\right)^{5/2}}{17cx^{3/2}}-(7bB-17Ac)\left(\frac{6}{13}b\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{13}\cdots}{17c}$$

$\downarrow$ 27

$$\frac{\frac{2B\left(bx^2+cx^4\right)^{5/2}}{17cx^{3/2}}-(7bB-17Ac)\left(\frac{6}{13}b\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{13}\sqrt{x}\cdots}{17c}$$

$\downarrow$ 761

$$\frac{\frac{2B\left(bx^2+cx^4\right)^{5/2}}{17cx^{3/2}}-(7bB-17Ac)\left(\frac{6}{13}b\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\cdots\right)}{5c\sqrt{bx^2+cx^4}}\cdots}{17c}$$

$\downarrow$ 1510

3.235. $\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{\sqrt{x}}\,dx$

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{17cx^{3/2}} - \frac{(7bB-17Ac)\left(\frac{6}{13}b\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}} - \frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\cdots}{\cdots}\right)}{5c\sqrt{bx^2+cx^4}}\cdots\right)\cdots\right)\cdots\right)}{17c}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/Sqrt[x],x]
```

output
```
(2*B*(b*x^2 + c*x^4)^(5/2))/(17*c*x^(3/2)) - ((7*b*B - 17*A*c)*((2*Sqrt[x]
*(b*x^2 + c*x^4)^(3/2))/13 + (6*b*((2*x^(5/2)*Sqrt[b*x^2 + c*x^4])/9 + (2*
b*((2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*c) - (6*b*x*Sqrt[b + c*x^2]*(-((-((S
qrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4
)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[
2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(
5*c*Sqrt[b*x^2 + c*x^4])))/9))/13))/(17*c)
```

3.235.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.235. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx$

3.235.4 Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.71

method	result
risch	$\frac{2\sqrt{x}\,(195B\,c^3x^6+255A\,c^3x^4+285Bb\,c^2x^4+425Ab\,c^2x^2+20B\,b^2c\,x^2+68b^2Ac-28B\,b^3)\sqrt{x^2(c\,x^2+b)}}{3315c^2} - \frac{4b^3(17Ac-7Bb)\sqrt{-bc}\sqrt{\frac{(x}{}}}{}$
default	$-\frac{2(x^4c+b\,x^2)^{\frac{3}{2}}\left(-195B\,c^5x^{10}-255A\,c^5x^8-480Bb\,c^4x^8-680Ab\,c^4x^6-305B\,b^2c^3x^6+204A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\right.}{}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3315/c^2*x^(1/2)*(195*B*c^3*x^6+255*A*c^3*x^4+285*B*b*c^2*x^4+425*A*b*c^
2*x^2+20*B*b^2*c*x^2+68*A*b^2*c-28*B*b^3)*(x^2*(c*x^2+b))^(1/2)-4/1105*b^3
/c^3*(17*A*c-7*B*b)*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/
2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2
)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(
-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^
(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(
c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

3.235.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.31

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx =$$
$$-\frac{2\left(12\left(7\,Bb^4-17\,Ab^3c\right)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\left(195\,Bc^4x^6-28\,Bb\right.\right.}{3315\,c^3}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

output
```
-2/3315*(12*(7*B*b^4 - 17*A*b^3*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weie
rstrassPInverse(-4*b/c, 0, x)) - (195*B*c^4*x^6 - 28*B*b^3*c + 68*A*b^2*c^
2 + 15*(19*B*b*c^3 + 17*A*c^4)*x^4 + 5*(4*B*b^2*c^2 + 85*A*b*c^3)*x^2)*sqr
t(c*x^4 + b*x^2)*sqrt(x))/c^3
```

3.235.6 Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)}{\sqrt{x}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(1/2),x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/sqrt(x), x)
```

3.235.7 Maxima [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{\sqrt{x}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/sqrt(x), x)
```

3.235.8 Giac [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{\sqrt{x}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/sqrt(x), x)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{\sqrt{x}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(1/2),x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(1/2), x)
```

3.236 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx$

3.236.1 Optimal result

Integrand size = 28, antiderivative size = 239

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = -\frac{8b^2(bB-3Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB-3Ac)(bx^2+cx^4)^{3/2}}{33c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{5/2}}{15cx^{5/2}} + \frac{4b^{11/4}(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}$$

output
```
2/15*B*(c*x^4+b*x^2)^(5/2)/c/x^(5/2)-2/33*(-3*A*c+B*b)*(c*x^4+b*x^2)^(3/2)
/c/x^(1/2)-4/77*b*(-3*A*c+B*b)*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c-8/231*b^2*(-3
*A*c+B*b)*(c*x^4+b*x^2)^(1/2)/c^2/x^(1/2)+4/231*b^(11/4)*(-3*A*c+B*b)*x*(c
os(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*
(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(9/4)/(c*x^4
+b*x^2)^(1/2)
```

3.236.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.48

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = \frac{2\sqrt{x^2(b+cx^2)}\left(-(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}(5bB-15Ac-11Bcx^2)+5b^2(bB\right.}{165c^2\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(3/2),x]
```

output
```
(2*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(5*b*B - 15*
A*c - 11*B*c*x^2)) + 5*b^2*(b*B - 3*A*c)*Hypergeometric2F1[-3/2, 1/4, 5/4,
 -((c*x^2)/b)]))/(165*c^2*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

3.236.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1945, 1426, 1426, 1429, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{2B(bx^2+cx^4)^{5/2}}{15cx^{5/2}} - \frac{(bB-3Ac)\int \frac{(cx^4+bx^2)^{3/2}}{x^{3/2}}dx}{3c}$$

$$\Big\downarrow \text{1426}$$

$$\frac{2B(bx^2+cx^4)^{5/2}}{15cx^{5/2}} - \frac{(bB-3Ac)\left(\frac{6}{11}b\int \sqrt{x}\sqrt{cx^4+bx^2}dx+\frac{2(bx^2+cx^4)^{3/2}}{11\sqrt{x}}\right)}{3c}$$

$$\Big\downarrow \text{1426}$$

$$\frac{2B(bx^2+cx^4)^{5/2}}{15cx^{5/2}} - \frac{(bB-3Ac)\left(\frac{6}{11}b\left(\frac{2}{7}b\int \frac{x^{5/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}\right)+\frac{2(bx^2+cx^4)^{3/2}}{11\sqrt{x}}\right)}{3c}$$

$$\Big\downarrow \text{1429}$$

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{15cx^{5/2}}-\frac{(bB-3Ac)\left(\frac{6}{11}b\left(\frac{2}{7}b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int\frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}\right)+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}\right)+\frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}}\right)}{3c}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{15cx^{5/2}}-\frac{(bB-3Ac)\left(\frac{6}{11}b\left(\frac{2}{7}b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}\right)+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}\right)+\frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}}\right)}{3c}$$

$$\Big\downarrow \text{266}$$

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{15cx^{5/2}}-\frac{(bB-3Ac)\left(\frac{6}{11}b\left(\frac{2}{7}b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{2bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}\right)+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}\right)+\frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}}\right)}{3c}$$

$$\Big\downarrow \text{761}$$

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{15cx^{5/2}}-\frac{(bB-3Ac)\left(\frac{6}{11}b\left(\frac{2}{7}b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\right)+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}\right)\right)}{3c}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(3/2),x]
```

output
```
(2*B*(b*x^2 + c*x^4)^(5/2))/(15*c*x^(5/2)) - ((b*B - 3*A*c)*((2*(b*x^2 + c
*x^4)^(3/2))/(11*Sqrt[x]) + (6*b*((2*x^(3/2)*Sqrt[b*x^2 + c*x^4])/7 + (2*b
*((2*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - (b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqr
t[x])/b^(1/4)], 1/2])/(3*c^(5/4)*Sqrt[b*x^2 + c*x^4])))/7))/11))/(3*c)
```

3.236. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx$

3.236.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.236. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx$

3.236.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.01

method	result
risch	$\frac{2(77B\,c^3x^6+105A\,c^3x^4+119Bb\,c^2x^4+195Ab\,c^2x^2+12B\,b^2c\,x^2+60b^2Ac-20B\,b^3)\sqrt{x^2(c\,x^2+b)}}{1155c^2\sqrt{x}} - \frac{4b^3(3Ac-Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)}{\sqrt{-bc}}}}{}$
default	$-\frac{2(x^4c+b\,x^2)^{\frac{3}{2}}\left(-77B\,c^5x^9-105A\,c^5x^7-196Bb\,c^4x^7+30A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b^3c\right.}{}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/1155/c^2*(77*B*c^3*x^6+105*A*c^3*x^4+119*B*b*c^2*x^4+195*A*b*c^2*x^2+12*
B*b^2*c*x^2+60*A*b^2*c-20*B*b^3)/x^(1/2)*(x^2*(c*x^2+b))^(1/2)-4/231*b^3/c
^3*(3*A*c-B*b)*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-
2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*
x^3+b*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2
^(1/2))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

3.236.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.51

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = \frac{2\left(20\,(Bb^4-3\,Ab^3c)\sqrt{c}x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+(77\,Bc^4x^6-20\right.}{}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

output
```
2/1155*(20*(B*b^4 - 3*A*b^3*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x)
 + (77*B*c^4*x^6 - 20*B*b^3*c + 60*A*b^2*c^2 + 7*(17*B*b*c^3 + 15*A*c^4)*x
^4 + 3*(4*B*b^2*c^2 + 65*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^3*x
)
```

3.236.6 Sympy [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}\,(A+Bx^2)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(3/2),x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(3/2), x)
```

3.236.7 Maxima [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(3/2), x)
```

3.236.8 Giac [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(3/2), x)
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{3/2}}\,dx=\int \frac{\left(B\,x^2+A\right)\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(3/2),x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(3/2), x)
```

3.237 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx$

3.237.1 Optimal result

Integrand size = 28, antiderivative size = 369

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx = -\frac{8b^2(3bB-13Ac)x^{3/2}(b+cx^2)}{195c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB-13Ac)(bx^2+cx^4)^{3/2}}{117cx^{3/2}} + \frac{2B(bx^2+cx^4)^{5/2}}{13cx^{7/2}} + \frac{8b^{9/4}(3bB-13Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{7/4}\sqrt{bx^2+cx^4}} - \frac{4b^{9/4}(3bB-13Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{195c^{7/4}\sqrt{bx^2+cx^4}}$$

output
```
-2/117*(-13*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/c/x^(3/2)+2/13*B*(c*x^4+b*x^2)^
(5/2)/c/x^(7/2)-8/195*b^2*(-13*A*c+3*B*b)*x^(3/2)*(c*x^2+b)/c^(3/2)/(b^(1/
2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-4/195*b*(-13*A*c+3*B*b)*x^(1/2)*(c*x^4+b
*x^2)^(1/2)/c+8/195*b^(9/4)*(-13*A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^
2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)-4/195*b^(9/4
)*(-13*A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2
*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2
)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)
```

3.237.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.27

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx = \frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left(3B(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}+b(-3bB+13Ac)\,\text{Hypergeo}\right.}{39c\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(5/2),x]
```

output
```
(2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(3*B*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] +
b*(-3*b*B + 13*A*c)*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)]))/(39*
c*Sqrt[1 + (c*x^2)/b])
```

3.237.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1945, 1426, 1426, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx$$

$$\Big\downarrow \; 1945$$

$$\frac{2B(bx^2+cx^4)^{5/2}}{13cx^{7/2}} - \frac{(3bB-13Ac)\int \frac{(cx^4+bx^2)^{3/2}}{x^{5/2}}dx}{13c}$$

$$\Big\downarrow \; 1426$$

$$\frac{2B(bx^2+cx^4)^{5/2}}{13cx^{7/2}} - \frac{(3bB-13Ac)\left(\frac{2}{3}b\int \frac{\sqrt{cx^4+bx^2}}{\sqrt{x}}dx + \frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}}\right)}{13c}$$

$$\Big\downarrow \; 1426$$

$$\frac{2B(bx^2+cx^4)^{5/2}}{13cx^{7/2}} - \frac{(3bB-13Ac)\left(\frac{2}{3}b\left(\frac{2}{5}b\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx + \frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right) + \frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}}\right)}{13c}$$

$$\Big\downarrow \; 1431$$

$$\frac{2B(bx^2+cx^4)^{5/2}}{13cx^{7/2}} - \frac{(3bB-13Ac)\left(\frac{2}{3}b\left(\frac{2bx\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5\sqrt{bx^2+cx^4}} + \frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right) + \frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}}\right)}{13c}$$

$$\Big\downarrow \; 266$$

$$\frac{2B(bx^2+cx^4)^{5/2}}{13cx^{7/2}} - \frac{(3bB-13Ac)\left(\frac{2}{3}b\left(\frac{4bx\sqrt{b+cx^2}\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5\sqrt{bx^2+cx^4}} + \frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right) + \frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}}\right)}{13c}$$

$$\Big\downarrow \; 834$$

$$\frac{2B(bx^2+cx^4)^{5/2}}{13cx^{7/2}} - \frac{(3bB-13Ac)\left(\frac{2}{3}b\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}} - \frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}} + \frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right) + \frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}}\right)}{13c}$$

$$\Big\downarrow \; 27$$

3.237. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx$

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{13cx^{7/2}}-\frac{(3bB-13Ac)\left(\frac{2}{3}b\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)+\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{3/2}}\right)}{13c}$$

$\downarrow$ 761

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{13cx^{7/2}}-\frac{(3bB-13Ac)\left(\frac{2}{3}b\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right.}{13c}$$

$\downarrow$ 1510

$$\frac{2B\left(bx^2+cx^4\right)^{5/2}}{13cx^{7/2}}-\frac{(3bB-13Ac)\left(\frac{2}{3}b\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\right.\right.}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right.}{5\sqrt{bx^2+cx^4}}\right.\right.}{13c}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(5/2),x]
```

3.237. $\int\frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{5/2}}\,dx$

output
```
(2*B*(b*x^2 + c*x^4)^(5/2))/(13*c*x^(7/2)) - ((3*b*B - 13*A*c)*((2*(b*x^2
+ c*x^4)^(3/2))/(9*x^(3/2)) + (2*b*((2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/5 + (4
*b*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)
) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^
2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b +
c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[
b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2
*c^(3/4)*Sqrt[b + c*x^2])))/(5*Sqrt[b*x^2 + c*x^4])))/3))/(13*c)
```

3.237.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

3.237. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx$

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.237.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.72

method	result
risch	$\frac{2\sqrt{x}\left(45Bc^2x^4+65Ac^2x^2+75Bbc\,x^2+143Abc+12Bb^2\right)\sqrt{x^2(c\,x^2+b)}}{585c}+\frac{4b^2(13Ac-3Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\cdots}{\cdots}$
default	$\frac{2(x^4c+b\,x^2)^{\frac{3}{2}}\left(45B\,x^8c^4+65A\,x^6c^4+120B\,x^6b\,c^3+156A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b^3c-78A\sqrt{\cdots}\cdots\right.}{\cdots}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/585/c*x^(1/2)*(45*B*c^2*x^4+65*A*c^2*x^2+75*B*b*c*x^2+143*A*b*c+12*B*b^2
)*(x^2*(c*x^2+b))^(1/2)+4/195*b^2/c^2*(13*A*c-3*B*b)*(-b*c)^(1/2)*((x+1/c*
(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2
))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/2)*El
lipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1/c*(-b*c
)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
)*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

3.237.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.28

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx = \frac{2\left(12\left(3\,Bb^3-13\,Ab^2c\right)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\right.\right.}{}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

output
```
2/585*(12*(3*B*b^3 - 13*A*b^2*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weiers
trassPInverse(-4*b/c, 0, x)) + (45*B*c^3*x^4 + 12*B*b^2*c + 143*A*b*c^2 +
5*(15*B*b*c^2 + 13*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/c^2
```

3.237.6 Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(5/2),x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(5/2), x)
```

3.237.7 Maxima [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(5/2), x)
```

3.237.8 Giac [F]

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^{5/2}}\,dx = \int \frac{(cx^4 + bx^2)^{\frac{3}{2}}(Bx^2 + A)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(5/2), x)
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^{5/2}}\,dx = \int \frac{(B\,x^2 + A)\,(c\,x^4 + b\,x^2)^{3/2}}{x^{5/2}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(5/2),x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(5/2), x)
```

3.238 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx$

3.238.1 Optimal result

Integrand size = 28, antiderivative size = 201

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx = -\frac{4b(bB-11Ac)\sqrt{bx^2+cx^4}}{77c\sqrt{x}} - \frac{2(bB-11Ac)(bx^2+cx^4)^{3/2}}{77cx^{5/2}} + \frac{2B(bx^2+cx^4)^{5/2}}{11cx^{9/2}} - \frac{4b^{7/4}(bB-11Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77c^{5/4}\sqrt{bx^2+cx^4}}$$

output
```
-2/77*(-11*A*c+B*b)*(c*x^4+b*x^2)^(3/2)/c/x^(5/2)+2/11*B*(c*x^4+b*x^2)^(5/
2)/c/x^(9/2)-4/77*b*(-11*A*c+B*b)*(c*x^4+b*x^2)^(1/2)/c/x^(1/2)-4/77*b^(7/
4)*(-11*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*
arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)
^(1/2)/c^(5/4)/(c*x^4+b*x^2)^(1/2)
```

3.238.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.48

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx = \frac{2\sqrt{x^2(b+cx^2)}\left(B(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}+b(-bB+11Ac)\,\text{Hypergeometri}\right.}{11c\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(7/2),x]
```

output
```
(2*Sqrt[x^2*(b + c*x^2)]*(B*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b*(-(b*B)
+ 11*A*c)*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(11*c*Sqrt[x]*
Sqrt[1 + (c*x^2)/b])
```

3.238.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1945, 1426, 1426, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{2B(bx^2+cx^4)^{5/2}}{11cx^{9/2}}-\frac{(bB-11Ac)\int \frac{(cx^4+bx^2)^{3/2}}{x^{7/2}}dx}{11c}$$

$$\Big\downarrow \text{1426}$$

$$\frac{2B(bx^2+cx^4)^{5/2}}{11cx^{9/2}}-\frac{(bB-11Ac)\left(\frac{6}{7}b\int \frac{\sqrt{cx^4+bx^2}}{x^{3/2}}dx+\frac{2(bx^2+cx^4)^{3/2}}{7x^{5/2}}\right)}{11c}$$

$$\Big\downarrow \text{1426}$$

$$\frac{2B(bx^2+cx^4)^{5/2}}{11cx^{9/2}}-\frac{(bB-11Ac)\left(\frac{6}{7}b\left(\frac{2}{3}b\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)+\frac{2(bx^2+cx^4)^{3/2}}{7x^{5/2}}\right)}{11c}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2B(bx^2+cx^4)^{5/2}}{11cx^{9/2}}-\frac{(bB-11Ac)\left(\frac{6}{7}b\left(\frac{2bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)+\frac{2(bx^2+cx^4)^{3/2}}{7x^{5/2}}\right)}{11c}$$

$$\Big\downarrow \text{266}$$

$$\frac{2B(bx^2+cx^4)^{5/2}}{11cx^{9/2}}-\frac{(bB-11Ac)\left(\frac{6}{7}b\left(\frac{4bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)+\frac{2(bx^2+cx^4)^{3/2}}{7x^{5/2}}\right)}{11c}$$

$$\Big\downarrow \text{761}$$

$$\frac{2B(bx^2+cx^4)^{5/2}}{11cx^{9/2}}-\frac{(bB-11Ac)\left(\frac{6}{7}b\left(\frac{2b^{3/4}x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)+\frac{2(bx^2+cx^4)^{3/2}}{7x^{5/2}}\right)}{11c}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(7/2),x]
```

output
```
(2*B*(b*x^2 + c*x^4)^(5/2))/(11*c*x^(9/2)) - ((b*B - 11*A*c)*((2*(b*x^2 +
c*x^4)^(3/2))/(7*x^(5/2)) + (6*b*((2*Sqrt[b*x^2 + c*x^4])/(3*Sqrt[x]) + (2
*b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]
*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(1/4)*Sqrt[b*x^
2 + c*x^4])))/7))/(11*c)
```

3.238.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.238.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.08

method	result
risch	$\frac{2(7Bc^2x^4+11Ac^2x^2+13Bbcx^2+33Abc+4Bb^2)\sqrt{x^2(cx^2+b)}}{77c\sqrt{x}}+\frac{4b^2(11Ac-Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}}{77c^2\sqrt{cx^3+bx}\,x^{\frac{3}{2}}(c}$
default	$\frac{2(x^4c+bx^2)^{\frac{3}{2}}\left(7Bc^4x^7+22A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b^2c+11Ac^4x^5-2B\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-}}{\sqrt{-bc}}}\right.}{77x^{\frac{7}{2}}(cx^2+b)^2c^2}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/77/c*(7*B*c^2*x^4+11*A*c^2*x^2+13*B*b*c*x^2+33*A*b*c+4*B*b^2)/x^(1/2)*(x
^2*(c*x^2+b))^(1/2)+4/77*b^2/c^2*(11*A*c-B*b)*(-b*c)^(1/2)*((x+1/c*(-b*c)^
(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2
)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/
2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^
2+b)*(x*(c*x^2+b))^(1/2)
```

3.238.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.48

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx =$$

$$-\frac{2\left(4\left(Bb^3-11\,Ab^2c\right)\sqrt{c}x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\left(7\,Bc^3x^4+4\,Bb^2c+33\,Abc^2+\left(13\,Bbc^2+11\,A\right.\right.\right.}{77\,c^2x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="fricas")
```

output
```
-2/77*(4*(B*b^3 - 11*A*b^2*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x)
- (7*B*c^3*x^4 + 4*B*b^2*c + 33*A*b*c^2 + (13*B*b*c^2 + 11*A*c^3)*x^2)*sqr
t(c*x^4 + b*x^2)*sqrt(x))/(c^2*x)
```

3.238.6 Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(7/2),x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(7/2), x)
```

3.238.7 Maxima [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(7/2), x)
```

3.238.8 Giac [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(7/2), x)
```

3.238.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx = \int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2)^{3/2}}{x^{7/2}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(7/2),x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(7/2), x)
```

3.239 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx$

3.239.1 Optimal result

Integrand size = 28, antiderivative size = 356

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx = \frac{8b(bB+9Ac)x^{3/2}(b+cx^2)}{15\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}$$

$$+\frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4}+\frac{2(bB+9Ac)(bx^2+cx^4)^{3/2}}{9bx^{3/2}}-\frac{2A(bx^2+cx^4)^{5/2}}{bx^{11/2}}-\frac{8b^{5/4}(bB+9Ac)x\left(\sqrt{b}+\right.}{}$$

output
```
2/9*(9*A*c+B*b)*(c*x^4+b*x^2)^(3/2)/b/x^(3/2)-2*A*(c*x^4+b*x^2)^(5/2)/b/x^
(11/2)+8/15*b*(9*A*c+B*b)*x^(3/2)*(c*x^2+b)/c^(1/2)/(b^(1/2)+x*c^(1/2))/(c
*x^4+b*x^2)^(1/2)+4/15*(9*A*c+B*b)*x^(1/2)*(c*x^4+b*x^2)^(1/2)-8/15*b^(5/4
)*(9*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/
4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1
/2)/c^(3/4)/(c*x^4+b*x^2)^(1/2)+4/15*b^(5/4)*(9*A*c+B*b)*x*(cos(2*arctan(c
^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*E
llipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^
(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2)^(1/2)
```

3.239.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.24

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx = \frac{2\sqrt{x^2(b+cx^2)}\left(-\frac{3A(b+cx^2)^2}{b}+\frac{(bB+9Ac)x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^2}{b}\right)}{\sqrt{1+\frac{cx^2}{b}}}\right)}{3x^{3/2}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(9/2),x]
```

output
```
(2*Sqrt[x^2*(b + c*x^2)]*((-3*A*(b + c*x^2)^2)/b + ((b*B + 9*A*c)*x^2*Hype
rgeometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)])/Sqrt[1 + (c*x^2)/b]))/(3*x^(3
/2))
```

3.239.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1944, 1426, 1426, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{9/2}}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(9Ac+bB)\int \frac{(cx^4+bx^2)^{3/2}}{x^{5/2}}dx}{b}-\frac{2A(bx^2+cx^4)^{5/2}}{bx^{11/2}}$$

$$\Big\downarrow \text{1426}$$

$$\frac{(9Ac+bB)\left(\frac{2}{3}b\int \frac{\sqrt{cx^4+bx^2}}{\sqrt{x}}dx+\frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}}\right)}{b}-\frac{2A(bx^2+cx^4)^{5/2}}{bx^{11/2}}$$

$$\Big\downarrow \text{1426}$$

$$\frac{(9Ac+bB)\left(\frac{2}{3}b\left(\frac{2}{5}b\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)+\frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}}\right)}{b}-\frac{2A(bx^2+cx^4)^{5/2}}{bx^{11/2}}$$

$$\Big\downarrow 1431$$

$$\frac{(9Ac+bB)\left(\frac{2}{3}b\left(\frac{2bx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)+\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{3/2}}\right)}{b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{bx^{11/2}}$$

$$\Big\downarrow 266$$

$$\frac{(9Ac+bB)\left(\frac{2}{3}b\left(\frac{4bx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)+\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{3/2}}\right)}{b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{bx^{11/2}}$$

$$\Big\downarrow 834$$

$$\frac{(9Ac+bB)\left(\frac{2}{3}b\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)+\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{3/2}}\right)}{b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{bx^{11/2}}$$

$$\Big\downarrow 27$$

$$\frac{(9Ac+bB)\left(\frac{2}{3}b\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)+\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{3/2}}\right)}{b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{bx^{11/2}}$$

$$\Big\downarrow 761$$

$$\frac{(9Ac+bB)\left(\frac{2}{3}b\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)\right)}{b}$$

$$\frac{2A\left(bx^2+cx^4\right)^{5/2}}{bx^{11/2}}$$

$$\Big\downarrow 1510$$

3.239. $\int\frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{9/2}}dx$

$$\frac{(9Ac+bB)\left(\frac{2}{3}b\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}\cdots}{\sqrt[4]{\cdots}}\right.\right.}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\cdots\right.}{5\sqrt{bx^2+cx^4}}\right.\right.}{b}$$

$$\frac{2A\left(bx^2+cx^4\right)^{5/2}}{bx^{11/2}}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(9/2),x]
```

output
```
(-2*A*(b*x^2 + c*x^4)^(5/2))/(b*x^(11/2)) + ((b*B + 9*A*c)*((2*(b*x^2 + c*
x^4)^(3/2))/(9*x^(3/2)) + (2*b*((2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/5 + (4*b*x
*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) +
(b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*E
llipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^
2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(
3/4)*Sqrt[b + c*x^2])))/(5*Sqrt[b*x^2 + c*x^4])))/3))/b
```

3.239.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.239. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx$

3.239.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{2(-5Bc\,x^4-9Ac\,x^2-11bB\,x^2+45Ab)\sqrt{x^2(c\,x^2+b)}}{45x^{\frac{3}{2}}}+\frac{4b(9Ac+Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}}{}\right.}{15c\sqrt{c}}$
default	$\frac{2(x^4c+b\,x^2)^{\frac{3}{2}}\left(5B\,c^3x^6+108A\,b^2c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-54A\,b^2c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-}}{\sqrt{-bc}}}\right.}{}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*(-5*B*c*x^4-9*A*c*x^2-11*B*b*x^2+45*A*b)*(x^2*(c*x^2+b))^(1/2)/x^(3/
2)+4/15*b*(9*A*c+B*b)/c*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))
^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^
(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))
*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b
*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))*(x^2*(c*x^2+b))^(1/2)/x^(3/
2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

3.239.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.26

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx =$$

$$-\frac{2\left(12\left(Bb^2+9\,Abc\right)\sqrt{c}x^2\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\left(5\,Bc^2x^4-45\,Abc+\right.\right.}{45\,cx^2}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="fricas")
```

output
```
-2/45*(12*(B*b^2 + 9*A*b*c)*sqrt(c)*x^2*weierstrassZeta(-4*b/c, 0, weierst
rassPInverse(-4*b/c, 0, x)) - (5*B*c^2*x^4 - 45*A*b*c + (11*B*b*c + 9*A*c^
2)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c*x^2)
```

3.239.6 Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(9/2),x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(9/2), x)
```

3.239.7 Maxima [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(9/2), x)
```

3.239.8 Giac [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(9/2), x)
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{9/2}}\,dx = \int \frac{\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{9/2}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(9/2),x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(9/2), x)
```

3.240 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx$

3.240.1 Optimal result

Integrand size = 28, antiderivative size = 200

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx = \frac{4(3bB+7Ac)\sqrt{bx^2+cx^4}}{21\sqrt{x}} + \frac{2(3bB+7Ac)(bx^2+cx^4)^{3/2}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}} + \frac{4b^{3/4}(3bB+7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

output
```
2/21*(7*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^(5/2)-2/3*A*(c*x^4+b*x^2)^(5/2)
/b/x^(13/2)+4/21*(7*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/x^(1/2)+4/21*b^(3/4)*(7
*A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)
)),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2
)/c^(1/4)/(c*x^4+b*x^2)^(1/2)
```

3.240.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.50

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx = -\frac{2\sqrt{x^2(b+cx^2)}\left(A(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}-b(3bB+7Ac)x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^2}{b}\right)\right)}{3bx^{5/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(11/2),x]
```

output
```
(-2*Sqrt[x^2*(b + c*x^2)]*(A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] - b*(3*b*B
+ 7*A*c)*x^2*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(3*b*x^(5/2
)*Sqrt[1 + (c*x^2)/b])
```

3.240.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1944, 1426, 1426, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(7Ac+3bB)\int \frac{(cx^4+bx^2)^{3/2}}{x^{7/2}}dx}{3b}-\frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}}$$

$$\Big\downarrow \text{1426}$$

$$\frac{(7Ac+3bB)\left(\frac{6}{7}b\int \frac{\sqrt{cx^4+bx^2}}{x^{3/2}}dx+\frac{2(bx^2+cx^4)^{3/2}}{7x^{5/2}}\right)}{3b}-\frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}}$$

$$\Big\downarrow \text{1426}$$

$$\frac{(7Ac+3bB)\left(\frac{6}{7}b\left(\frac{2}{3}b\int\frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)+\frac{2(bx^2+cx^4)^{3/2}}{7x^{5/2}}\right)}{3b}-\frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}}$$

$\downarrow$ 1431

$$\frac{(7Ac+3bB)\left(\frac{6}{7}b\left(\frac{2bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)+\frac{2(bx^2+cx^4)^{3/2}}{7x^{5/2}}\right)}{3b}-\frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}}$$

$\downarrow$ 266

$$\frac{(7Ac+3bB)\left(\frac{6}{7}b\left(\frac{4bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)+\frac{2(bx^2+cx^4)^{3/2}}{7x^{5/2}}\right)}{3b}-\frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}}$$

$\downarrow$ 761

$$\frac{(7Ac+3bB)\left(\frac{6}{7}b\left(\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)+\frac{2(bx^2+cx^4)^{3/2}}{7x^{5/2}}\right)}{3b}-\frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(11/2),x]
```

output
```
(-2*A*(b*x^2 + c*x^4)^(5/2))/(3*b*x^(13/2)) + ((3*b*B + 7*A*c)*((2*(b*x^2
+ c*x^4)^(3/2))/(7*x^(5/2)) + (6*b*((2*Sqrt[b*x^2 + c*x^4])/(3*Sqrt[x]) +
(2*b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^
2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(1/4)*Sqrt[b*
x^2 + c*x^4])))/7))/(3*b)
```

3.240.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

3.240. $\int\frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}}dx$

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.240.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{2\left(-3Bc\,x^{4}-7Ac\,x^{2}-9bB\,x^{2}+7Ab\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{21x^{\frac{5}{2}}}+\frac{4b\left(7Ac+3Bb\right)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{\left(x+\cdots\right.}{\cdots}}\cdots\right.}{21c\sqrt{c\,x^{3}+bx}\,x^{\frac{3}{2}}\left(c\,x^{2}+b\right)}\cdots$
default	$\frac{2\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}\left(14A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-bc}\,bcx+6B\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{x\cdots}{\sqrt{-\cdots}}}\cdots\right.}{21x^{\frac{9}{2}}\left(c\,x^{2}+b\right)^{2}c}\cdots$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(-3*B*c*x^4-7*A*c*x^2-9*B*b*x^2+7*A*b)/x^(5/2)*(x^2*(c*x^2+b))^(1/2)
+4/21*b*(7*A*c+3*B*b)/c*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))
^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^
(1/2)/(c*x^3+b*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/
2)
```

3.240.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.43

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{11/2}}\,dx = \frac{2\left(4\left(3\,Bb^2+7\,Abc\right)\sqrt{c}x^3\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(3\,Bc^2x^4-7\,A\right.\right.}{21\,cx^3}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="fricas")
```

output
```
2/21*(4*(3*B*b^2 + 7*A*b*c)*sqrt(c)*x^3*weierstrassPInverse(-4*b/c, 0, x)
+ (3*B*c^2*x^4 - 7*A*b*c + (9*B*b*c + 7*A*c^2)*x^2)*sqrt(c*x^4 + b*x^2)*sq
rt(x))/(c*x^3)
```

3.240.6 Sympy [F]

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{11/2}}\,dx = \int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}(A+Bx^2)}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(11/2),x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(11/2), x)
```

3.240.7 Maxima [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(11/2), x)
```

3.240.8 Giac [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(11/2), x)
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx = \int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2)^{3/2}}{x^{11/2}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(11/2),x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(11/2), x)
```

3.241 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx$

3.241.1 Optimal result

Integrand size = 28, antiderivative size = 354

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx = \frac{24\sqrt{c}(bB+Ac)x^{3/2}(b+cx^2)}{5\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)(bx^2+cx^4)^{3/2}}{bx^{7/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{5bx^{15/2}} - \frac{24\sqrt[4]{b}\sqrt[4]{c}(bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}} + \frac{12\sqrt[4]{b}\sqrt[4]{c}(bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}$$

output
```
-2*(A*c+B*b)*(c*x^4+b*x^2)^(3/2)/b/x^(7/2)-2/5*A*(c*x^4+b*x^2)^(5/2)/b/x^(
15/2)+24/5*(A*c+B*b)*x^(3/2)*(c*x^2+b)*c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+
b*x^2)^(1/2)+12/5*c*(A*c+B*b)*x^(1/2)*(c*x^4+b*x^2)^(1/2)/b-24/5*b^(1/4)*c
^(1/4)*(A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*
arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)
^(1/2)/(c*x^4+b*x^2)^(1/2)+12/5*b^(1/4)*c^(1/4)*(A*c+B*b)*x*(cos(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*
EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c
^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/(c*x^4+b*x^2)^(1/2)
```

3.241.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.28

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{13/2}}\,dx = -\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(A(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}+5b(bB+Ac)x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},-\frac{cx^2}{b}\right)\right)}{5bx^{7/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(13/2),x]
```

output
```
(-2*Sqrt[x^2*(b + c*x^2)]*(A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + 5*b*(b*B
+ A*c)*x^2*Hypergeometric2F1[-3/2, -1/4, 3/4, -((c*x^2)/b)]))/(5*b*x^(7/2)
*Sqrt[1 + (c*x^2)/b])
```

3.241.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1944, 1425, 1426, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{13/2}}\,dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(Ac+bB)\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^{9/2}}dx}{b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{15/2}}$$

$$\Big\downarrow \text{1425}$$

$$\frac{(Ac+bB)\left(6c\int \frac{\sqrt{cx^4+bx^2}}{\sqrt{x}}dx-\frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}\right)}{b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{15/2}}$$

$$\Big\downarrow \text{1426}$$

$$\frac{(Ac+bB)\left(6c\left(\frac{2}{5}b\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}\right)}{b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{15/2}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{(Ac+bB)\left(6c\left(\frac{2bx\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}\right)}{b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{15/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(Ac+bB)\left(6c\left(\frac{4bx\sqrt{b+cx^2}\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}\right)}{b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{15/2}}$$

$$\Big\downarrow \text{834}$$

$$\frac{(Ac+bB)\left(6c\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}\right)}{b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{15/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(Ac+bB)\left(6c\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}\right)}{b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{15/2}}$$

$$\Big\downarrow \text{761}$$

3.241. $\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{13/2}}dx$

$$\frac{(Ac+bB)\left(6c\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)-\right.}{b}$$

$$\frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{15/2}}$$

$\big\downarrow$ 1510

$$\frac{(Ac+bB)\left(6c\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{\right.}}{\sqrt[4]{b}}\right.\right.}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right.}{5\sqrt{bx^2+cx^4}}\right.\right.}{b}$$

$$\frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{15/2}}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(13/2),x]
```

output
```
(-2*A*(b*x^2 + c*x^4)^(5/2))/(5*b*x^(15/2)) + ((b*B + A*c)*((-2*(b*x^2 + c
*x^4)^(3/2))/x^(7/2) + 6*c*((2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/5 + (4*b*x*Sqr
t[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(
1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellip
ticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))
/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqr
t[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)
*Sqrt[b + c*x^2])))/(5*Sqrt[b*x^2 + c*x^4]))))/b
```

3.241. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx$

3.241.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

3.241. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx$

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.241.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{2\left(-Bc\,x^{4}+7Ac\,x^{2}+5bB\,x^{2}+Ab\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{5x^{\frac{7}{2}}}+\frac{12(Ac+Bb)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\right.}\right.}{}\right.}{5\sqrt{c\,x^{3}+bx}}$
default	$\frac{2\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}\left(12A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc\,x^{2}-6A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\right.}{}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-B*c*x^4+7*A*c*x^2+5*B*b*x^2+A*b)/x^(7/2)*(x^2*(c*x^2+b))^(1/2)+12/5
*(A*c+B*b)*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x
-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+
b*x)^(1/2)*(-2/c*(-b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/
2))^(1/2),1/2*2^(1/2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/
(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*
(x*(c*x^2+b))^(1/2)
```

3.241.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.23

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx =$$
$$-\frac{2\left(12\left(Bb+Ac\right)\sqrt{c}x^4\text{weierstrassZeta}\left(-\frac{4b}{c},0,\text{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)-\left(Bcx^4-\left(5Bb+7Ac\right)x^2\right.}{5x^4}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="fricas")
```

output
```
-2/5*(12*(B*b + A*c)*sqrt(c)*x^4*weierstrassZeta(-4*b/c, 0, weierstrassPIn
verse(-4*b/c, 0, x)) - (B*c*x^4 - (5*B*b + 7*A*c)*x^2 - A*b)*sqrt(c*x^4 +
b*x^2)*sqrt(x))/x^4
```

3.241.6 Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx = \int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}(A+Bx^2)}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(13/2),x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(13/2), x)
```

3.241.7 Maxima [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(13/2), x)
```

3.241.8 Giac [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(13/2), x)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{13/2}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(13/2),x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(13/2), x)
```

3.242 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx$

3.242.1 Optimal result

Integrand size = 28, antiderivative size = 204

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx = \frac{4c(7bB+3Ac)\sqrt{bx^2+cx^4}}{21b\sqrt{x}} - \frac{2(7bB+3Ac)(bx^2+cx^4)^{3/2}}{21bx^{9/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}} + \frac{4c^{3/4}(7bB+3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21\sqrt[4]{b}\sqrt{bx^2+cx^4}}$$

output
```
-2/21*(3*A*c+7*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^(9/2)-2/7*A*(c*x^4+b*x^2)^(5/2
)/b/x^(17/2)+4/21*c*(3*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^(1/2)+4/21*c^(3/
4)*(3*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*
arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)
^(1/2)/b^(1/4)/(c*x^4+b*x^2)^(1/2)
```

3.242.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.50

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx = -\frac{2\sqrt{x^2(b+cx^2)}\left(3A(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}+b(7bB+3Ac)x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},-\frac{cx^2}{b}\right)\right)}{21bx^{9/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(15/2),x]
```

output
```
(-2*Sqrt[x^2*(b + c*x^2)]*(3*A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b*(7*b*
B + 3*A*c)*x^2*Hypergeometric2F1[-3/2, -3/4, 1/4, -((c*x^2)/b)]))/(21*b*x^
(9/2)*Sqrt[1 + (c*x^2)/b])
```

3.242.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1944, 1425, 1426, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{15/2}}\,dx$$

$$\Big\downarrow \; 1944$$

$$\frac{(3Ac+7bB)\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^{11/2}}dx}{7b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{7bx^{17/2}}$$

$$\Big\downarrow \; 1425$$

$$\frac{(3Ac+7bB)\left(2c\int \frac{\sqrt{cx^4+bx^2}}{x^{3/2}}dx-\frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^{9/2}}\right)}{7b}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{7bx^{17/2}}$$

$$\Big\downarrow \; 1426$$

$$\frac{(3Ac+7bB)\left(2c\left(\frac{2}{3}b\int\frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)-\frac{2(bx^2+cx^4)^{3/2}}{3x^{9/2}}\right)}{7b}-\frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{(3Ac+7bB)\left(2c\left(\frac{2bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)-\frac{2(bx^2+cx^4)^{3/2}}{3x^{9/2}}\right)}{7b}-\frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(3Ac+7bB)\left(2c\left(\frac{4bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)-\frac{2(bx^2+cx^4)^{3/2}}{3x^{9/2}}\right)}{7b}-\frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(3Ac+7bB)\left(2c\left(\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)-\frac{2(bx^2+cx^4)^{3/2}}{3x^{9/2}}\right)}{7b}-\frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}}$$

input
```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(15/2),x]
```

output
```
(-2*A*(b*x^2 + c*x^4)^(5/2))/(7*b*x^(17/2)) + ((7*b*B + 3*A*c)*((-2*(b*x^2
 + c*x^4)^(3/2))/(3*x^(9/2)) + 2*c*((2*Sqrt[b*x^2 + c*x^4])/(3*Sqrt[x]) +
(2*b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^
2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(1/4)*Sqrt[b*
x^2 + c*x^4]))))/(7*b)
```

3.242.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

3.242. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx$

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.242. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx$

3.242.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{2(-7Bc\,x^4+9Ac\,x^2+7bB\,x^2+3Ab)\sqrt{x^2(c\,x^2+b)}}{21x^{\frac{9}{2}}}+\frac{4(3Ac+7Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{\left(x+\right.}{\sqrt{}}}\right.}{21\sqrt{c\,x^3+bx}\,x^{\frac{3}{2}}(c\,x^2+b)}$
default	$\frac{2(x^4c+b\,x^2)^{\frac{3}{2}}\left(6A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c\,x^3+14B\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.}{21x^{\frac{13}{2}}(c\,x^2+b)^2}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(-7*B*c*x^4+9*A*c*x^2+7*B*b*x^2+3*A*b)/x^(9/2)*(x^2*(c*x^2+b))^(1/2)
+4/21*(3*A*c+7*B*b)*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/
2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2
)/(c*x^3+b*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),
1/2*2^(1/2))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

3.242.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.37

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx = \frac{2\left(4\,(7\,Bb+3\,Ac)\sqrt{c}x^5\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+(7\,Bcx^4-(7\,Bb\right.}{21\,x^5}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="fricas")
```

output
```
2/21*(4*(7*B*b + 3*A*c)*sqrt(c)*x^5*weierstrassPInverse(-4*b/c, 0, x) + (7
*B*c*x^4 - (7*B*b + 9*A*c)*x^2 - 3*A*b)*sqrt(c*x^4 + b*x^2)*sqrt(x))/x^5
```

3.242.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(15/2),x)
```

output
```
Timed out
```

3.242.7 Maxima [F]

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{15/2}}\,dx = \int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{15}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(15/2), x)
```

3.242.8 Giac [F]

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{15/2}}\,dx = \int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{15}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(15/2), x)
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{15/2}}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{15/2}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(15/2),x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(15/2), x)
```

3.243 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx$

3.243.1 Optimal result

Integrand size = 28, antiderivative size = 364

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx = \frac{8c^{3/2}(9bB+Ac)x^{3/2}(b+cx^2)}{15b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)(bx^2+cx^4)^{3/2}}{45bx^{11/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}} - \frac{8c^{5/4}(9bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}} + \frac{4c^{5/4}(9bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}$$

output
```
-2/45*(A*c+9*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^(11/2)-2/9*A*(c*x^4+b*x^2)^(5/2)
/b/x^(19/2)+8/15*c^(3/2)*(A*c+9*B*b)*x^(3/2)*(c*x^2+b)/b/(b^(1/2)+x*c^(1/2
))/(c*x^4+b*x^2)^(1/2)-4/15*c*(A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^(3/2)-8/
15*c^(5/4)*(A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/
cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/
2))^2)^(1/2)/b^(3/4)/(c*x^4+b*x^2)^(1/2)+4/15*c^(5/4)*(A*c+9*B*b)*x*(cos(2
*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(
1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/(c*x^4+b*x
^2)^(1/2)
```

3.243.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.27

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx = -\frac{2\sqrt{x^2(b+cx^2)}\left(5A(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}+b(9bB+Ac)x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{4},-\frac{1}{4},-\frac{cx^2}{b}\right)\right)}{45bx^{11/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(17/2),x]
```

output
```
(-2*Sqrt[x^2*(b + c*x^2)]*(5*A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b*(9*b*
B + A*c)*x^2*Hypergeometric2F1[-3/2, -5/4, -1/4, -((c*x^2)/b)]))/(45*b*x^(
11/2)*Sqrt[1 + (c*x^2)/b])
```

3.243.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1944, 1425, 1425, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(Ac+9bB)\int \frac{(cx^4+bx^2)^{3/2}}{x^{13/2}}dx}{9b}-\frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}}$$

$$\Big\downarrow \text{1425}$$

$$\frac{(Ac+9bB)\left(\frac{6}{5}c\int \frac{\sqrt{cx^4+bx^2}}{x^{5/2}}dx-\frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}}\right)}{9b}-\frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}}$$

$$\Big\downarrow \text{1425}$$

$$\frac{(Ac+9bB)\left(\frac{6}{5}c\left(2c\int\frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}\right)-\frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}}\right)}{9b}-\frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}}$$

$\downarrow$ 1431

$$\frac{(Ac+9bB)\left(\frac{6}{5}c\left(\frac{2cx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}\right)-\frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}}\right)}{9b}-\frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}}$$

$\downarrow$ 266

$$\frac{(Ac+9bB)\left(\frac{6}{5}c\left(\frac{4cx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}\right)-\frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}}\right)}{9b}-\frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}}$$

$\downarrow$ 834

$$\frac{(Ac+9bB)\left(\frac{6}{5}c\left(\frac{4cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}\right)-\frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}}\right)}{9b}-\frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}}$$

$\downarrow$ 27

$$\frac{(Ac+9bB)\left(\frac{6}{5}c\left(\frac{4cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}\right)-\frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}}\right)}{9b}-\frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}}$$

$\downarrow$ 761

3.243. $\int\frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}}dx$

$$\frac{(Ac+9bB)\left(\frac{6}{5}c\left(\frac{4cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}\right)-\frac{2(bx^2 \cdots}{5 \cdots}\right)}{9b}$$

$$\frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}}$$

$\big\downarrow$ 1510

$$\frac{(Ac+9bB)\left(\frac{6}{5}c\left(\frac{4cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c} \cdots}{\sqrt[4]{\cdots}}\right.\right.}{\sqrt[4]{c}\sqrt{b+cx^2}} \cdots}{\sqrt{c}}\right)}{\sqrt{bx^2+cx^4}} \cdots\right)\right)}{9b}$$

$$\frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}}$$

input

```
Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(17/2),x]
```

output

```
(-2*A*(b*x^2 + c*x^4)^(5/2))/(9*b*x^(19/2)) + ((9*b*B + A*c)*((-2*(b*x^2 +
 c*x^4)^(3/2))/(5*x^(11/2)) + (6*c*((-2*Sqrt[b*x^2 + c*x^4])/x^(3/2) + (4*
c*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x))
 + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c
*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b
] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*
c^(3/4)*Sqrt[b + c*x^2])))/Sqrt[b*x^2 + c*x^4]))/5))/(9*b)
```

3.243. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx$

3.243.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.243. $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx$

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.243.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{2(12A\,c^2x^4+63x^4Bbc+11Abc\,x^2+9b^2B\,x^2+5b^2A)\sqrt{x^2(c\,x^2+b)}}{45x^{\frac{11}{2}}b}+\frac{4c(Ac+9Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{x}{\sqrt{-}}}\cdots}{\cdots}$
default	$\frac{2(x^4c+b\,x^2)^{\frac{3}{2}}\left(12A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b\,c^2x^4-6A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\cdots}{\cdots}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*(12*A*c^2*x^4+63*B*b*c*x^4+11*A*b*c*x^2+9*B*b^2*x^2+5*A*b^2)/x^(11/2
)/b*(x^2*(c*x^2+b))^(1/2)+4/15*c*(A*c+9*B*b)/b*(-b*c)^(1/2)*((x+1/c*(-b*c)
^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/
2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/2)*Elliptic
E(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1/c*(-b*c)^(1/2
)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))*(x^2
*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

3.243.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.28

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx = -\frac{2\left(12\left(9\,Bbc+Ac^2\right)\sqrt{c}x^6\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\left(3\left(21\,Bbc+4\,Ac^2\right)x^4\right.\right.}{45\,bx^6}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="fricas")
```

output
```
-2/45*(12*(9*B*b*c + A*c^2)*sqrt(c)*x^6*weierstrassZeta(-4*b/c, 0, weierst
rassPInverse(-4*b/c, 0, x)) + (3*(21*B*b*c + 4*A*c^2)*x^4 + 5*A*b^2 + (9*B
*b^2 + 11*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b*x^6)
```

3.243.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(17/2),x)
```

output
```
Timed out
```

3.243.7 Maxima [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{17}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(17/2), x)
```

3.243.8 Giac [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{17}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(17/2), x)
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{17/2}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(17/2),x)
```

output
```
int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(17/2), x)
```

3.244 $\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

3.244.1 Optimal result

Integrand size = 28, antiderivative size = 243

$$\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{2b^2(13bB-15Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} + \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} - \frac{2(13bB-15Ac)x^{7/2}\sqrt{bx^2+cx^4}}{165c^2} + \frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c} + \frac{b^{11/4}(13bB-15Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77c^{17/4}\sqrt{bx^2+cx^4}}$$

output

```
6/385*b*(-15*A*c+13*B*b)*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c^3-2/165*(-15*A*c+13
*B*b)*x^(7/2)*(c*x^4+b*x^2)^(1/2)/c^2+2/15*B*x^(11/2)*(c*x^4+b*x^2)^(1/2)/
c-2/77*b^2*(-15*A*c+13*B*b)*(c*x^4+b*x^2)^(1/2)/c^4/x^(1/2)+1/77*b^(11/4)*
(-15*A*c+13*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*
arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)
^(1/2)/c^(17/4)/(c*x^4+b*x^2)^(1/2)
```

3.244.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.59

$$\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{3/2}\Big(-((b+cx^2)\,(195b^3B-7c^3x^4(15A+11Bx^2)-9b^2c(25A+13Bx^2)+bc^2x^2(}{1155c^4\sqrt{}}$$

input
```
Integrate[(x^(13/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*x^(3/2)*(-((b + c*x^2)*(195*b^3*B - 7*c^3*x^4*(15*A + 11*B*x^2) - 9*b^2
*c*(25*A + 13*B*x^2) + b*c^2*x^2*(135*A + 91*B*x^2))) + 15*b^3*(13*b*B - 1
5*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)])
)/(1155*c^4*Sqrt[x^2*(b + c*x^2)])
```

3.244.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1945, 1429, 1429, 1429, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c}-\frac{(13bB-15Ac)\int \frac{x^{13/2}}{\sqrt{cx^4+bx^2}}dx}{15c}$$

$$\Big\downarrow \text{1429}$$

$$\frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c}-\frac{(13bB-15Ac)\left(\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\int \frac{x^{9/2}}{\sqrt{cx^4+bx^2}}dx}{11c}\right)}{15c}$$

$$\Big\downarrow \text{1429}$$

$$\frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c}-\frac{(13bB-15Ac)\left(\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\int\frac{x^{5/2}}{\sqrt{cx^4+bx^2}}dx}{7c}\right)}{11c}\right)}{15c}$$

$\downarrow$ 1429

$$\frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c}-\frac{(13bB-15Ac)\left(\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int\frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{7c}\right)}{11c}\right)}{15c}$$

$\downarrow$ 1431

$$\frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c}-\frac{(13bB-15Ac)\left(\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)}{11c}\right)}{15c}$$

$\downarrow$ 266

$$\frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c}-\frac{(13bB-15Ac)\left(\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{2bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)}{11c}\right)}{15c}$$

3.244. $\int\frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

$$\Big\downarrow \text{761}$$

$$\frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c}-\frac{(13bB-15Ac)\left(\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\right)}{7c}\right)}{11c}\right)}{15c}$$

input
```
Int[(x^(13/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*B*x^(11/2)*Sqrt[b*x^2 + c*x^4])/(15*c) - ((13*b*B - 15*A*c)*((2*x^(7/2)
*Sqrt[b*x^2 + c*x^4])/(11*c) - (9*b*((2*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(7*c)
 - (5*b*((2*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - (b^(3/4)*x*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1
/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(5/4)*Sqrt[b*x^2 + c*x^4])))/(7*c)))/(11
*c)))/(15*c)
```

3.244.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.244.4 Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.98

method	result
risch	$\frac{2\left(77B\,c^{3}x^{6}+105A\,c^{3}x^{4}-91Bb\,c^{2}x^{4}-135Ab\,c^{2}x^{2}+117B\,b^{2}c\,x^{2}+225b^{2}Ac-195B\,b^{3}\right)x^{\frac{3}{2}}\left(c\,x^{2}+b\right)}{1155c^{4}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{b^{3}(15Ac-13Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\sqrt{\ldots}\right.}{\sqrt{-\ldots}}}}{\ldots}$
default	$-\frac{\sqrt{x}\left(-154B\,c^{5}x^{9}-210A\,c^{5}x^{7}+28Bb\,c^{4}x^{7}+225A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b^{3}c+60Ab\,c^{4}\ldots\right.}{\ldots}$

input
```
int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/1155*(77*B*c^3*x^6+105*A*c^3*x^4-91*B*b*c^2*x^4-135*A*b*c^2*x^2+117*B*b^
2*c*x^2+225*A*b^2*c-195*B*b^3)/c^4*x^(3/2)*(c*x^2+b)/(x^2*(c*x^2+b))^(1/2)
-1/77*b^3*(15*A*c-13*B*b)/c^5*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^
(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(
1/2))^(1/2)/(c*x^3+b*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/
2))^(1/2),1/2*2^(1/2))*x^(1/2)/(x^2*(c*x^2+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

3.244.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.50

$$\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2\left(15\left(13\,Bb^4-15\,Ab^3c\right)\sqrt{c}x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(77\,Bc^4x^6-195\,Bb^3\right.\right.}{115}$$

input
```
integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/1155*(15*(13*B*b^4 - 15*A*b^3*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0
, x) + (77*B*c^4*x^6 - 195*B*b^3*c + 225*A*b^2*c^2 - 7*(13*B*b*c^3 - 15*A*
c^4)*x^4 + 9*(13*B*b^2*c^2 - 15*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))
/(c^5*x)
```

3.244.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Timed out
```

3.244.7 Maxima [F]

$$\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{13}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

3.244.8 Giac [F]

$$\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{13}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{13/2}\,(B\,x^2+A)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.245 $\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

3.245.1 Optimal result

Integrand size = 28, antiderivative size = 369

$$\begin{aligned}\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx &= -\frac{14b^2(11bB-13Ac)x^{3/2}(b+cx^2)}{195c^{7/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} \\ &+ \frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3} \\ &- \frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2}+\frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c} \\ &+ \frac{14b^{9/4}(11bB-13Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{15/4}\sqrt{bx^2+cx^4}} \\ &- \frac{7b^{9/4}(11bB-13Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{195c^{15/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

output
```
-14/195*b^2*(-13*A*c+11*B*b)*x^(3/2)*(c*x^2+b)/c^(7/2)/(b^(1/2)+x*c^(1/2))
/(c*x^4+b*x^2)^(1/2)-2/117*(-13*A*c+11*B*b)*x^(5/2)*(c*x^4+b*x^2)^(1/2)/c^
2+2/13*B*x^(9/2)*(c*x^4+b*x^2)^(1/2)/c+14/585*b*(-13*A*c+11*B*b)*x^(1/2)*(
c*x^4+b*x^2)^(1/2)/c^3+14/195*b^(9/4)*(-13*A*c+11*B*b)*x*(cos(2*arctan(c^(
1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ell
ipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1
/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(15/4)/(c*x^4+b*x^2)^(1/2)-
7/195*b^(9/4)*(-13*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2
)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1
/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)
+x*c^(1/2))^2)^(1/2)/c^(15/4)/(c*x^4+b*x^2)^(1/2)
```

3.245.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.33

$$\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{5/2}\Big((b+cx^2)\left(77b^2B+5c^2x^2(13A+9Bx^2)-bc(91A+55Bx^2)\right)+7b^2(-11bB\ \cdots}{585c^3\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[(x^(11/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*x^(5/2)*((b + c*x^2)*(77*b^2*B + 5*c^2*x^2*(13*A + 9*B*x^2) - b*c*(91*A
 + 55*B*x^2)) + 7*b^2*(-11*b*B + 13*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometri
c2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(585*c^3*Sqrt[x^2*(b + c*x^2)])
```

3.245.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1945, 1429, 1429, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$

$\big\downarrow$ 1945

$$\frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c}-\frac{(11bB-13Ac)\int\frac{x^{11/2}}{\sqrt{cx^4+bx^2}}dx}{13c}$$

$\big\downarrow$ 1429

$$\frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c}-\frac{(11bB-13Ac)\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\int\frac{x^{7/2}}{\sqrt{cx^4+bx^2}}dx}{9c}\right)}{13c}$$

$\big\downarrow$ 1429

$$\frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c}-\frac{(11bB-13Ac)\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3b\int\frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{5c}\right)}{9c}\right)}{13c}$$

$\big\downarrow$ 1431

$$\frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c}-\frac{(11bB-13Ac)\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3bx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)}{13c}$$

$\big\downarrow$ 266

$$\frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c}-\frac{(11bB-13Ac)\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)}{13c}$$

$\big\downarrow$ 834

$$\frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c}-\frac{(11bB-13Ac)\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)}{13c}$$

3.245. $\quad \int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

$\downarrow$ 27

$$\frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c}-\frac{(11bB-13Ac)\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)}{13c}$$

$\downarrow$ 761

$$\frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c}-\frac{(11bB-13Ac)\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)}{13c}$$

$\downarrow$ 1510

3.245. $\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

$$\frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c}-\frac{(11bB-13Ac)\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\sqrt[4]{b}(\cdots}{\cdots}\right.}{5c\sqrt{bx^2+cx^4}}\right.}{9c}\right.}{13c}$$

input
```
Int[(x^(11/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*B*x^(9/2)*Sqrt[b*x^2 + c*x^4])/(13*c) - ((11*b*B - 13*A*c)*((2*x^(5/2)*
Sqrt[b*x^2 + c*x^4])/(9*c) - (7*b*((2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*c) -
 (6*b*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]
*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*
x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b
 + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sq
rt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(2*c^(3/4)*Sqrt[b + c*x^2])))/(5*c*Sqrt[b*x^2 + c*x^4])))/(9*c)))/(13*c)
```

3.245. $\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

3.245.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.245. $\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.245.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{2x^{\frac{5}{2}}\left(-45B\,c^{2}x^{4}-65A\,c^{2}x^{2}+55Bbc\,x^{2}+91Abc-77B\,b^{2}\right)\left(c\,x^{2}+b\right)}{585c^{3}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{7b^{2}(13Ac-11Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{\cdots}}{\cdots}$
default	$\frac{\sqrt{x}\left(90B\,x^{8}c^{4}+130A\,x^{6}c^{4}-20B\,x^{6}b\,c^{3}+546A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b^{3}c-273A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{\cdots}\right.}{\cdots}$

input
```
int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/585*x^(5/2)*(-45*B*c^2*x^4-65*A*c^2*x^2+55*B*b*c*x^2+91*A*b*c-77*B*b^2)
/c^3*(c*x^2+b)/(x^2*(c*x^2+b))^(1/2)+7/195*b^2*(13*A*c-11*B*b)/c^4*(-b*c)^
(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))
*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(
-b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2
),1/2*2^(1/2)))*x^(1/2)/(x^2*(c*x^2+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

3.245.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.28

$$\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2\left(21\left(11\,Bb^3-13\,Ab^2c\right)\sqrt{c}\,\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)\right.}{585\,c^4}$$

input
```
integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/585*(21*(11*B*b^3 - 13*A*b^2*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weier
strassPInverse(-4*b/c, 0, x)) + (45*B*c^3*x^4 + 77*B*b^2*c - 91*A*b*c^2 -
5*(11*B*b*c^2 - 13*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/c^4
```

3.245.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Timed out
```

3.245.7 Maxima [F]

$$\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{11}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

3.245.8 Giac [F]

$$\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{11}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{11/2}\,(B\,x^2+A)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.246 $\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

3.246.1 Optimal result

Integrand size = 28, antiderivative size = 204

$$\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\frac{10b(9bB-11Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}}-\frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2}+\frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{5b^{7/4}(9bB-11Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231c^{13/4}\sqrt{bx^2+cx^4}}$$

output
```
-2/77*(-11*A*c+9*B*b)*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c^2+2/11*B*x^(7/2)*(c*x^
4+b*x^2)^(1/2)/c+10/231*b*(-11*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)-
5/231*b^(7/4)*(-11*A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)
^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/
4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+
x*c^(1/2))^2)^(1/2)/c^(13/4)/(c*x^4+b*x^2)^(1/2)
```

3.246.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.60

$$\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{3/2}\left((b+cx^2)\,(45b^2B+3c^2x^2(11A+7Bx^2)-bc(55A+27Bx^2))+5b^2(-9bB+\right.}{231c^3\sqrt{x^2\,(b+cx^2)}}$$

input
```
Integrate[(x^(9/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*x^(3/2)*((b + c*x^2)*(45*b^2*B + 3*c^2*x^2*(11*A + 7*B*x^2) - b*c*(55*A
 + 27*B*x^2)) + 5*b^2*(-9*b*B + 11*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric
2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(231*c^3*Sqrt[x^2*(b + c*x^2)])
```

3.246.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1945, 1429, 1429, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$

$$\Big\downarrow \; 1945$$

$$\frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{(9bB-11Ac)\int \frac{x^{9/2}}{\sqrt{cx^4+bx^2}}dx}{11c}$$

$$\Big\downarrow \; 1429$$

$$\frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{(9bB-11Ac)\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\int \frac{x^{5/2}}{\sqrt{cx^4+bx^2}}dx}{7c}\right)}{11c}$$

$$\Big\downarrow \; 1429$$

$$\frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{(9bB-11Ac)\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int\frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{7c}\right)}{11c}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{(9bB-11Ac)\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)}{11c}$$

$$\Big\downarrow \text{266}$$

$$\frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{(9bB-11Ac)\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{2bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)}{11c}$$

$$\Big\downarrow \text{761}$$

$$\frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{(9bB-11Ac)\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\right)}{7c}\right)}{11c}$$

input
```
Int[(x^(9/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*B*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(11*c) - ((9*b*B - 11*A*c)*((2*x^(3/2)*S
qrt[b*x^2 + c*x^4])/(7*c) - (5*b*((2*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) -
(b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]
*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(5/4)*Sqrt[b*x^
2 + c*x^4])))/(7*c)))/(11*c)
```

3.246. $\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

3.246.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.246. $\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

3.246.4 Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{2(-21B\,c^2x^4-33A\,c^2x^2+27Bbc\,x^2+55Abc-45B\,b^2)x^{\frac{3}{2}}(c\,x^2+b)}{231c^3\sqrt{x^2(c\,x^2+b)}}+\frac{5b^2(11Ac-9Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{}}{231c^4\sqrt{c\,x^3+bx}\,\sqrt{x^2}}$
default	$\frac{\sqrt{x}\left(42B\,c^4x^7+55A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b^2c+66A\,c^4x^5-45B\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{}\right.}{231\sqrt{x^4c+b\,x^2}\,c^4}$

input
```
int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/231*(-21*B*c^2*x^4-33*A*c^2*x^2+27*B*b*c*x^2+55*A*b*c-45*B*b^2)/c^3*x^(
3/2)*(c*x^2+b)/(x^2*(c*x^2+b))^(1/2)+5/231*b^2*(11*A*c-9*B*b)/c^4*(-b*c)^(
1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*
c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*Elliptic
F(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^(1/2)/(x^2*(c
*x^2+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

3.246.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.49

$$\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx =$$
$$-\frac{2\left(5\left(9\,Bb^3-11\,Ab^2c\right)\sqrt{c}x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\left(21\,Bc^3x^4+45\,Bb^2c-55\,Abc^2-3\left(9\,Bbc^2-\right.\right.\right.}{231\,c^4x}$$

input
```
integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/231*(5*(9*B*b^3 - 11*A*b^2*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0,
x) - (21*B*c^3*x^4 + 45*B*b^2*c - 55*A*b*c^2 - 3*(9*B*b*c^2 - 11*A*c^3)*x^
2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^4*x)
```

3.246.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Timed out
```

3.246.7 Maxima [F]

$$\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{9}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^(9/2)/sqrt(c*x^4 + b*x^2), x)
```

3.246.8 Giac [F]

$$\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{9}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^(9/2)/sqrt(c*x^4 + b*x^2), x)
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{9/2}\,(B\,x^2+A)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.247 $\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

3.247.1 Optimal result

Integrand size = 28, antiderivative size = 330

$$\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2b(7bB-9Ac)x^{3/2}(b+cx^2)}{15c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2(7bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c} - \frac{2b^{5/4}(7bB-9Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}} + \frac{b^{5/4}(7bB-9Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}}$$

output
```
2/15*b*(-9*A*c+7*B*b)*x^(3/2)*(c*x^2+b)/c^(5/2)/(b^(1/2)+x*c^(1/2))/(c*x^4
+b*x^2)^(1/2)+2/9*B*x^(5/2)*(c*x^4+b*x^2)^(1/2)/c-2/45*(-9*A*c+7*B*b)*x^(1
/2)*(c*x^4+b*x^2)^(1/2)/c^2-2/15*b^(5/4)*(-9*A*c+7*B*b)*x*(cos(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*El
lipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(
1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)
+1/15*b^(5/4)*(-9*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^
(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4
)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x
*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

3.247.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.29

$$\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{5/2}\left(-\left(\left(b+cx^2\right)\left(7bB-9Ac-5Bcx^2\right)\right)+b(7bB-9Ac)\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometr}\right.}{45c^2\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[(x^(7/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*x^(5/2)*(-((b + c*x^2)*(7*b*B - 9*A*c - 5*B*c*x^2)) + b*(7*b*B - 9*A*c)
*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(45*
c^2*Sqrt[x^2*(b + c*x^2)])
```

3.247.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1945, 1429, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{(7bB-9Ac)\int \frac{x^{7/2}}{\sqrt{cx^4+bx^2}}dx}{9c}$$

$$\Big\downarrow \text{1429}$$

$$\frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{(7bB-9Ac)\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3b\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{5c}\right)}{9c}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{(7bB-9Ac)\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3bx\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5c\sqrt{bx^2+cx^4}}\right)}{9c}$$

$\downarrow$ 266

$$\frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{(7bB-9Ac)\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5c\sqrt{bx^2+cx^4}}\right)}{9c}$$

$\downarrow$ 834

$$\frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{(7bB-9Ac)\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}$$

$\downarrow$ 27

$$\frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{(7bB-9Ac)\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}$$

$\downarrow$ 761

$$\frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{(7bB-9Ac)\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}$$

$\downarrow$ 1510

3.247. $\int\frac{x^{7/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx$

$$\frac{\frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c}-(7bB-9Ac)\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\mathrm{E}}{\sqrt[4]{c}\sqrt{b+}}\right.}{5c\sqrt{bx^2+cx^4}}\right.}{9c}$$

input
```
Int[(x^(7/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*B*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(9*c) - ((7*b*B - 9*A*c)*((2*Sqrt[x]*Sqr
t[b*x^2 + c*x^4])/(5*c) - (6*b*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c
*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c
]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)
*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(5*c*Sqrt[b*x^2 +
c*x^4])))/(9*c)
```

3.247.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.247. $\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

3.247.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.73

method	result
risch	$\frac{2x^{\frac{5}{2}}(5Bc\,x^2+9Ac-7Bb)(c\,x^2+b)}{45c^2\sqrt{x^2(c\,x^2+b)}}-\frac{b(9Ac-7Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\right.}{c}\right.}{15c^3\sqrt{c\,x^3+bx}\,\sqrt{x^2(c\,x^2+b)}}$
default	$-\frac{\sqrt{x}\left(-10B\,c^3x^6+54A\,b^2c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-27A\,b^2c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\right.}{}$

input
```
int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*x^(5/2)*(5*B*c*x^2+9*A*c-7*B*b)/c^2*(c*x^2+b)/(x^2*(c*x^2+b))^(1/2)-1
/15*b*(9*A*c-7*B*b)/c^3*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))
^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^
(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))
*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b
*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))*x^(1/2)/(x^2*(c*x^2+b))^(1/
2)*(x*(c*x^2+b))^(1/2)
```

3.247.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.24

$$\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx =$$
$$-\frac{2\left(3\left(7\,Bb^2-9\,Abc\right)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\left(5\,Bc^2x^2-7\,Bbc+9\,A\right.\right.}{45\,c^3}$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(3*(7*B*b^2 - 9*A*b*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstras
sPInverse(-4*b/c, 0, x)) - (5*B*c^2*x^2 - 7*B*b*c + 9*A*c^2)*sqrt(c*x^4 +
b*x^2)*sqrt(x))/c^3
```

3.247.6 Sympy [F]

$$\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{\frac{7}{2}}(A+Bx^2)}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**(7/2)*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

3.247.7 Maxima [F]

$$\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{7}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^(7/2)/sqrt(c*x^4 + b*x^2), x)
```

3.247.8 Giac [F]

$$\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{7}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^(7/2)/sqrt(c*x^4 + b*x^2), x)
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{x^{7/2}\,(B\,x^2+A)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.248 $\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

3.248.1 Optimal result

Integrand size = 28, antiderivative size = 167

$$\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{2(5bB-7Ac)\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}}+\frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{b^{3/4}(5bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21c^{9/4}\sqrt{bx^2+cx^4}}$$

output
```
2/7*B*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c-2/21*(-7*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2
)/c^2/x^(1/2)+1/21*b^(3/4)*(-7*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/
b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*
arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+
b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(9/4)/(c*x^4+b*x^2)^(1/2)
```

3.248.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.58

$$\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{3/2}\left(-\left(\left(b+cx^2\right)\left(5bB-7Ac-3Bcx^2\right)\right)+b(5bB-7Ac)\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometr}\right.}{21c^2\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[(x^(5/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*x^(3/2)*(-((b + c*x^2)*(5*b*B - 7*A*c - 3*B*c*x^2)) + b*(5*b*B - 7*A*c)
*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(21*
c^2*Sqrt[x^2*(b + c*x^2)])
```

3.248.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1945, 1429, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{(5bB-7Ac)\int \frac{x^{5/2}}{\sqrt{cx^4+bx^2}}dx}{7c}$$

$$\Big\downarrow \text{1429}$$

$$\frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{(5bB-7Ac)\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{7c}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{(5bB-7Ac)\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}\right)}{7c}$$

$$\Big\downarrow \text{266}$$

$$\frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{(5bB-7Ac)\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{2bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}\right)}{7c}$$

$$\Big\downarrow \text{761}$$

$$\frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} - \frac{(5bB-7Ac)\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\right)}{7c}$$

input
```
Int[(x^(5/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*B*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(7*c) - ((5*b*B - 7*A*c)*((2*Sqrt[b*x^2
+ c*x^4])/(3*c*Sqrt[x]) - (b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2
)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
1/2])/(3*c^(5/4)*Sqrt[b*x^2 + c*x^4])))/(7*c)
```

3.248.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.248.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.14

method	result
risch	$\frac{2(3Bc\,x^2+7Ac-5Bb)x^{\frac{3}{2}}(c\,x^2+b)}{21c^2\sqrt{x^2(c\,x^2+b)}}-\frac{b(7Ac-5Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x}\,\sqrt{}}{21c^3\sqrt{c\,x^3+bx}\,\sqrt{x^2(c\,x^2+b)}}$
default	$-\frac{\sqrt{x}\left(7A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc-5B\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\right.}{21\sqrt{x^4c+b\,x^2}\,c^3}$

input
```
int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*(3*B*c*x^2+7*A*c-5*B*b)/c^2*x^(3/2)*(c*x^2+b)/(x^2*(c*x^2+b))^(1/2)-1
/21*b*(7*A*c-5*B*b)/c^3*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))
^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^
(1/2)/(c*x^3+b*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))*x^(1/2)/(x^2*(c*x^2+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

3.248.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.44

$$\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\frac{2\left((5\,Bb^2-7\,Abc)\sqrt{c}x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+(3\,Bc^2x^2-5\,Bbc+7\,Ac^2)\sqrt{}\right.}{21\,c^3x}$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/21*((5*B*b^2 - 7*A*b*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) + (3
*B*c^2*x^2 - 5*B*b*c + 7*A*c^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^3*x)
```

3.248.6 Sympy [F]

$$\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{x^{\frac{5}{2}}(A+Bx^2)}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**(5/2)*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

3.248.7 Maxima [F]

$$\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{(Bx^2+A)x^{\frac{5}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^(5/2)/sqrt(c*x^4 + b*x^2), x)
```

3.248.8 Giac [F]

$$\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{(Bx^2+A)x^{\frac{5}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^(5/2)/sqrt(c*x^4 + b*x^2), x)
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{x^{5/2}\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.249 $\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

3.249.1 Optimal result

Integrand size = 28, antiderivative size = 293

$$\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{2(3bB-5Ac)x^{3/2}(b+cx^2)}{5c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c}$$
$$+\frac{2\sqrt[4]{b}(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}$$
$$-\frac{\sqrt[4]{b}(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}$$

output
```
-2/5*(-5*A*c+3*B*b)*x^(3/2)*(c*x^2+b)/c^(3/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b
*x^2)^(1/2)+2/5*B*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c+2/5*b^(1/4)*(-5*A*c+3*B*b)
*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1
/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(
c*x^4+b*x^2)^(1/2)-1/5*b^(1/4)*(-5*A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(si
n(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*
x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)
```

3.249.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.28

$$\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\frac{2x^{5/2}\left(3B(b+cx^2)+(-3bB+5Ac)\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^2}{b}\right)\right)}{15c\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[(x^(3/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*x^(5/2)*(3*B*(b + c*x^2) + (-3*b*B + 5*A*c)*Sqrt[1 + (c*x^2)/b]*Hyperge
ometric2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(15*c*Sqrt[x^2*(b + c*x^2)])
```

3.249.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1945, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{(3bB-5Ac)\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{5c}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{x\sqrt{b+cx^2}(3bB-5Ac)\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{2x\sqrt{b+cx^2}(3bB-5Ac)\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{834}$$

$$\frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{2x\sqrt{b+cx^2}(3bB-5Ac)\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{2x\sqrt{b+cx^2}(3bB-5Ac)\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{2x\sqrt{b+cx^2}(3bB-5Ac)\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{2x\sqrt{b+cx^2}(3bB-5Ac)\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\cdots\right.\right.}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right.}{5c\sqrt{bx^2+cx^4}}$$

input
```
Int[(x^(3/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*B*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*c) - (2*(3*b*B - 5*A*c)*x*Sqrt[b + c*
x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sq
rt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*A
rcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]
) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^
2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b
+ c*x^2])))/(5*c*Sqrt[b*x^2 + c*x^4])
```

3.249. $\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

3.249.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.249. $\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

3.249.4 Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.77

method	result
risch	$\frac{2B\,x^{\frac{5}{2}}\left(c\,x^{2}+b\right)}{5c\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{\left(5Ac-3Bb\right)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}+\frac{\sqrt{-bc}\,F\left(\sqrt{\frac{\left(\ldots\right.}{\ldots}}\right.}{\ldots}\right.}{5c^{2}\sqrt{c\,x^{3}+bx}\,\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$
default	$\frac{\sqrt{x}\left(10Abc\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-5Abc\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-\ldots}}{\sqrt{-bc}}}\right.\right.}{\ldots}$

input
```
int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*B/c*x^(5/2)*(c*x^2+b)/(x^2*(c*x^2+b))^(1/2)+1/5*(5*A*c-3*B*b)/c^2*(-b*
c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/
2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/
c*(-b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2
^(1/2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(
1/2),1/2*2^(1/2)))*x^(1/2)/(x^2*(c*x^2+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

3.249.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.19

$$\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\frac{2\left(\sqrt{cx^4+bx^2}Bc\sqrt{x}+(3\,Bb-5\,Ac)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\ldots\right.\right.}{5\,c^2}$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(sqrt(c*x^4 + b*x^2)*B*c*sqrt(x) + (3*B*b - 5*A*c)*sqrt(c)*weierstrass
Zeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)))/c^2
```

3.249.6 Sympy [F]

$$\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{\frac{3}{2}}(A+Bx^2)}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**(3/2)*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

3.249.7 Maxima [F]

$$\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{3}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

3.249.8 Giac [F]

$$\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{3}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{x^{3/2}\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.250 $\int \frac{\sqrt{x}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

3.250.1 Optimal result

Integrand size = 28, antiderivative size = 130

$$\int \frac{\sqrt{x}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}c^{5/4}\sqrt{bx^2+cx^4}}$$

output
```
2/3*B*(c*x^4+b*x^2)^(1/2)/c/x^(1/2)-1/3*(-3*A*c+B*b)*x*(cos(2*arctan(c^(1/
4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ellip
ticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2
))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(1/4)/c^(5/4)/(c*x^4+b*x^2)^(
1/2)
```

3.250.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{x}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{3/2}\left(B(b+cx^2)+(-bB+3Ac)\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^2}{b}\right)\right)}{3c\sqrt{x^2(b+cx^2)}}$$

input

```
Integrate[(Sqrt[x]*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output

```
(2*x^(3/2)*(B*(b + c*x^2) + (-(b*B) + 3*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeom
etric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(3*c*Sqrt[x^2*(b + c*x^2)])
```

3.250.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1945, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{(bB-3Ac)\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{x\sqrt{b+cx^2}(bB-3Ac)\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{2x\sqrt{b+cx^2}(bB-3Ac)\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-3Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}c^{5/4}\sqrt{bx^2+cx^4}}$$

input
```
Int[(Sqrt[x]*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*B*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - ((b*B - 3*A*c)*x*(Sqrt[b] + Sqrt
[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/
4)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(1/4)*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

3.250.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.250.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.35

method	result
risch	$\frac{2B\,x^{\frac{3}{2}}\left(c\,x^{2}+b\right)}{3c\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{\left(3Ac-Bb\right)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x}\,\sqrt{x\left(c\,x^{2}+b\right)}}{3c^{2}\sqrt{c\,x^{3}+bx}\,\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$
default	$\frac{\sqrt{x}\left(3A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c-B\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\right.}{3\sqrt{x^{4}c+b\,x^{2}}\,c^{2}}$

input
```
int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*B/c*x^(3/2)*(c*x^2+b)/(x^2*(c*x^2+b))^(1/2)+1/3*(3*A*c-B*b)/c^2*(-b*c)
^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2)
)*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*Ellipt
icF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^(1/2)/(x^2*
(c*x^2+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

3.250.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{x}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left((Bb-3\,Ac)\sqrt{c}x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\sqrt{cx^4+bx^2}Bc\sqrt{x}\right)}{3\,c^2x}$$

input
```
integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*((B*b - 3*A*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) - sqrt(c*x
^4 + b*x^2)*B*c*sqrt(x))/(c^2*x)
```

3.250.6 Sympy [F]

$$\int \frac{\sqrt{x}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{\sqrt{x}\,(A+Bx^2)}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(sqrt(x)*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

3.250.7 Maxima [F]

$$\int \frac{\sqrt{x}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{(Bx^2+A)\sqrt{x}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

3.250.8 Giac [F]

$$\int \frac{\sqrt{x}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{(Bx^2+A)\sqrt{x}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{\sqrt{x}\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.251 $\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx$

3.251.1 Optimal result

Integrand size = 28, antiderivative size = 281

$$\begin{aligned}\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx &= \frac{2(bB+Ac)x^{3/2}(b+cx^2)}{b\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}}\\&-\frac{2(bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}\\&+\frac{(bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

output
```
2*(A*c+B*b)*x^(3/2)*(c*x^2+b)/b/c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^
(1/2)-2*A*(c*x^4+b*x^2)^(1/2)/b/x^(3/2)-2*(A*c+B*b)*x*(cos(2*arctan(c^(1/4
)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ellipt
icE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2)
)*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/c^(3/4)/(c*x^4+b*x^2)^(1
/2)+(A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/
4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1
/2)/b^(3/4)/c^(3/4)/(c*x^4+b*x^2)^(1/2)
```

3.251.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.29

$$\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx = \frac{2\sqrt{x}\left(-3A(b+cx^2)+(bB+Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^2}{b}\right)\right)}{3b\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[x]*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(2*Sqrt[x]*(-3*A*(b + c*x^2) + (b*B + A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hyperge
ometric2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(3*b*Sqrt[x^2*(b + c*x^2)])
```

3.251.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1944, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 1944}$$

$$\frac{(Ac+bB)\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{b}-\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}}$$

$$\Big\downarrow \text{ 1431}$$

$$\frac{x\sqrt{b+cx^2}(Ac+bB)\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{b\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2x\sqrt{b+cx^2}(Ac+bB)\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{b\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}}$$

$$\Big\downarrow \text{834}$$

$$\frac{2x\sqrt{b+cx^2}(Ac+bB)\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x\sqrt{b+cx^2}(Ac+bB)\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2x\sqrt{b+cx^2}(Ac+bB)\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{2x\sqrt{b+cx^2}(Ac+bB)\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\ldots}{\sqrt[4]{\ldots}}\right.\right.}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\ldots\right)}{b\sqrt{bx^2+cx^4}}$$

$$\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}}$$

input
```
Int[(A + B*x^2)/(Sqrt[x]*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*A*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) + (2*(b*B + A*c)*x*Sqrt[b + c*x^2]*
(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (
b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*El
lipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x
^2])))/(b*Sqrt[b*x^2 + c*x^4])
```

3.251. $\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx$

3.251.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.251. $\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx$

3.251.4 Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{2A(c\,x^2+b)\sqrt{x}}{b\sqrt{x^2(c\,x^2+b)}}+\frac{(Ac+Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}+\frac{\sqrt{-bc}\,F\left(\sqrt{\left(\right.}\right.}{}\right.}{bc\sqrt{c\,x^3+bx}\,\sqrt{x^2(c\,x^2+b)}}$
default	$\frac{\sqrt{x}\left(2Abc\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-Abc\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.\right.}{}$

input
```
int((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/b*A*(c*x^2+b)*x^(1/2)/(x^2*(c*x^2+b))^(1/2)+(A*c+B*b)/b/c*(-b*c)^(1/2)*
((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b
*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^
(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1
/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*
2^(1/2)))*x^(1/2)/(x^2*(c*x^2+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

3.251.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.22

$$\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left((Bb+Ac)\sqrt{c}x^2\text{weierstrassZeta}\left(-\frac{4b}{c},0,\text{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}Ac\sqrt{x}\right)}{bcx^2}$$

input
```
integrate((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*((B*b + A*c)*sqrt(c)*x^2*weierstrassZeta(-4*b/c, 0, weierstrassPInverse
(-4*b/c, 0, x)) + sqrt(c*x^4 + b*x^2)*A*c*sqrt(x))/(b*c*x^2)
```

3.251.6 Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx=\int \frac{A+Bx^2}{\sqrt{x}\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate((B*x**2+A)/x**(1/2)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(x)*sqrt(x**2*(b + c*x**2))), x)
```

3.251.7 Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}\sqrt{x}}\,dx$$

input
```
integrate((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

3.251.8 Giac [F]

$$\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}\sqrt{x}}\,dx$$

input
```
integrate((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

3.251.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx=\int \frac{B\,x^2+A}{\sqrt{x}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.252 $\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx$

3.252.1 Optimal result

Integrand size = 28, antiderivative size = 131

$$\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}} + \frac{(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

output
```
-2/3*A*(c*x^4+b*x^2)^(1/2)/b/x^(5/2)+1/3*(-A*c+3*B*b)*x*(cos(2*arctan(c^(1
/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Elli
pticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/
2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(5/4)/c^(1/4)/(c*x^4+b*x^2)^
(1/2)
```

3.252.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left(A(b+cx^2)+(-3bB+Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^2}{b}\right)\right)}{3b\sqrt{x}\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(A + B*x^2)/(x^(3/2)*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*(A*(b + c*x^2) + (-3*b*B + A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric
2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(3*b*Sqrt[x]*Sqrt[x^2*(b + c*x^2)])
```

3.252.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1944, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(3bB-Ac)\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3b}-\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{x\sqrt{b+cx^2}(3bB-Ac)\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3b\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2x\sqrt{b+cx^2}(3bB-Ac)\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3b\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

input
```
Int[(A + B*x^2)/(x^(3/2)*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*A*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5/2)) + ((3*b*B - A*c)*x*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1
/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(5/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

3.252.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.252.4 Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.35

method	result
risch	$-\frac{2A\left(c\,x^{2}+b\right)}{3b\sqrt{x}\,\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{\left(Ac-3Bb\right)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x}\,\sqrt{x\left(c\,x^{2}+b\right)}}{3bc\sqrt{c\,x^{3}+bx}\,\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$
default	$-\frac{A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)cx-3B\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{c\cdots\right.}{3\sqrt{x^{4}c+b\,x^{2}}\,\sqrt{x}\,cb}$

input
```
int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/b*A*(c*x^2+b)/x^(1/2)/(x^2*(c*x^2+b))^(1/2)-1/3*(A*c-3*B*b)/b/c*(-b*c
)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2
))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*Ellip
ticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^(1/2)/(x^2
*(c*x^2+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

3.252.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.44

$$\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx = \frac{2\left((3\,Bb-Ac)\sqrt{c}x^3\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\sqrt{cx^4+bx^2}Ac\sqrt{x}\right)}{3\,bcx^3}$$

input
```
integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/3*((3*B*b - A*c)*sqrt(c)*x^3*weierstrassPInverse(-4*b/c, 0, x) - sqrt(c*
x^4 + b*x^2)*A*c*sqrt(x))/(b*c*x^3)
```

3.252.6 Sympy [F]

$$\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^{\frac{3}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate((B*x**2+A)/x**(3/2)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x**(3/2)*sqrt(x**2*(b + c*x**2))), x)
```

3.252.7 Maxima [F]

$$\int \frac{A + Bx^2}{x^{3/2}\sqrt{bx^2 + cx^4}}\,dx = \int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

3.252.8 Giac [F]

$$\int \frac{A + Bx^2}{x^{3/2}\sqrt{bx^2 + cx^4}}\,dx = \int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

3.252.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2}{x^{3/2}\sqrt{bx^2 + cx^4}}\,dx = \int \frac{B\,x^2 + A}{x^{3/2}\,\sqrt{c\,x^4 + b\,x^2}}\,dx$$

input
```
int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.253 $\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx$

3.253.1 Optimal result

Integrand size = 28, antiderivative size = 332

$$\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx = \frac{2\sqrt{c}(5bB-3Ac)x^{3/2}(b+cx^2)}{5b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} - \frac{2\sqrt[4]{c}(5bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}} + \frac{\sqrt[4]{c}(5bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}$$

output
```
2/5*(-3*A*c+5*B*b)*x^(3/2)*(c*x^2+b)*c^(1/2)/b^2/(b^(1/2)+x*c^(1/2))/(c*x^
4+b*x^2)^(1/2)-2/5*A*(c*x^4+b*x^2)^(1/2)/b/x^(7/2)-2/5*(-3*A*c+5*B*b)*(c*x
^4+b*x^2)^(1/2)/b^2/x^(3/2)-2/5*c^(1/4)*(-3*A*c+5*B*b)*x*(cos(2*arctan(c^(
1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ell
ipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1
/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)+1
/5*c^(1/4)*(-3*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/
2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^
(1/2))^2)^(1/2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)
```

3.253.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.25

$$\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left(A(b+cx^2)+(5bB-3Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{cx^2}{b}\right)\right)}{5bx^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[(A + B*x^2)/(x^(5/2)*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*(A*(b + c*x^2) + (5*b*B - 3*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -((c*x^2)/b)]))/(5*b*x^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

3.253.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1944, 1430, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(5bB-3Ac)\int \frac{1}{\sqrt{x}\sqrt{cx^4+bx^2}}dx}{5b}-\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{(5bB-3Ac)\left(\frac{c\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{b}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{(5bB-3Ac)\left(\frac{cx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{(5bB-3Ac)\left(\frac{2cx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{(5bB-3Ac)\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(5bB-3Ac)\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{(5bB-3Ac)\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

$$\Big\downarrow \text{ 1510}$$

3.253. $\int\frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx$

$$\frac{(5bB-3Ac)\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.\right.}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right.}{b\sqrt{bx^2+cx^4}}\right.}{5b}$$

$$\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

input
```
Int[(A + B*x^2)/(x^(5/2)*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*A*Sqrt[b*x^2 + c*x^4])/(5*b*x^(7/2)) + ((5*b*B - 3*A*c)*((-2*Sqrt[b*x^
2 + c*x^4])/(b*x^(3/2)) + (2*c*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c
*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c
]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)
*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(b*Sqrt[b*x^2 + c*
x^4])))/(5*b)
```

3.253.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.253. $\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx$

3.253.4 Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{2(c\,x^2+b)(-3Ac\,x^2+5bB\,x^2+Ab)}{5b^2x^{\frac{3}{2}}\sqrt{x^2(c\,x^2+b)}}-\frac{(3Ac-5Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\right.}{c}\right.}{5b^2\sqrt{c\,x^3+bx}\,\sqrt{x^2(c\,x^2+b)}}$
default	$-\frac{6A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc\,x^2-3A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\right.}{}$

input
```
int((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(c*x^2+b)*(-3*A*c*x^2+5*B*b*x^2+A*b)/b^2/x^(3/2)/(x^2*(c*x^2+b))^(1/2
)-1/5*(3*A*c-5*B*b)/b^2*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))
^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^
(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))
*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b
*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))*x^(1/2)/(x^2*(c*x^2+b))^(1/
2)*(x*(c*x^2+b))^(1/2)
```

3.253.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.23

$$\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx =$$
$$-\frac{2\left((5\,Bb-3\,Ac)\sqrt{c}x^4\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}((5\,Bb-3\,A\right.}{5\,b^2x^4}$$

input
```
integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/5*((5*B*b - 3*A*c)*sqrt(c)*x^4*weierstrassZeta(-4*b/c, 0, weierstrassPI
nverse(-4*b/c, 0, x)) + sqrt(c*x^4 + b*x^2)*((5*B*b - 3*A*c)*x^2 + A*b)*sq
rt(x))/(b^2*x^4)
```

3.253.6 Sympy [F]

$$\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^{\frac{5}{2}}\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate((B*x**2+A)/x**(5/2)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x**(5/2)*sqrt(x**2*(b + c*x**2))), x)
```

3.253.7 Maxima [F]

$$\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

3.253.8 Giac [F]

$$\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx=\int \frac{B\,x^2+A}{x^{5/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int((A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.254 $\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx$

3.254.1 Optimal result

Integrand size = 28, antiderivative size = 167

$$\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}}-\frac{2(7bB-5Ac)\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} -\frac{c^{3/4}(7bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21b^{9/4}\sqrt{bx^2+cx^4}}$$

output
```
-2/7*A*(c*x^4+b*x^2)^(1/2)/b/x^(9/2)-2/21*(-5*A*c+7*B*b)*(c*x^4+b*x^2)^(1/
2)/b^2/x^(5/2)-1/21*c^(3/4)*(-5*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)
/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2
*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2
+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(9/4)/(c*x^4+b*x^2)^(1/2)
```

3.254.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.51

$$\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = \frac{-6A(b+cx^2)+2(-7bB+5Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-\frac{cx^2}{b}\right)}{21bx^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[(A + B*x^2)/(x^(7/2)*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-6*A*(b + c*x^2) + 2*(-7*b*B + 5*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeomet
ric2F1[-3/4, 1/2, 1/4, -((c*x^2)/b)])/(21*b*x^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

3.254.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1944, 1430, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(7bB-5Ac)\int \frac{1}{x^{3/2}\sqrt{cx^4+bx^2}}dx}{7b}-\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{(7bB-5Ac)\left(-\frac{c\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3b}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{(7bB-5Ac)\left(-\frac{cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(7bB-5Ac)\left(-\frac{2cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(7bB-5Ac)\left(-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

input
```
Int[(A + B*x^2)/(x^(7/2)*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*A*Sqrt[b*x^2 + c*x^4])/(7*b*x^(9/2)) + ((7*b*B - 5*A*c)*((-2*Sqrt[b*x^
2 + c*x^4])/(3*b*x^(5/2)) - (c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)]
, 1/2])/(3*b^(5/4)*Sqrt[b*x^2 + c*x^4])))/(7*b)
```

3.254.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.254.4 Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{2\left(c\,x^{2}+b\right)\left(-5Ac\,x^{2}+7bB\,x^{2}+3Ab\right)}{21b^{2}x^{\frac{5}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{\left(5Ac-7Bb\right)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{\cdots}}{21b^{2}\sqrt{c\,x^{3}+bx}\,\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$
default	$\frac{5A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c\,x^{3}-7B\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\cdots}\right.}{21\sqrt{x^{4}c+b\,x^{2}}\,x^{\frac{5}{2}}b^{2}}$

input
```
int((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(c*x^2+b)*(-5*A*c*x^2+7*B*b*x^2+3*A*b)/b^2/x^(5/2)/(x^2*(c*x^2+b))^(
1/2)+1/21*(5*A*c-7*B*b)/b^2*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1
/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/
2))^(1/2)/(c*x^3+b*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2)
)^(1/2),1/2*2^(1/2))*x^(1/2)/(x^2*(c*x^2+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

3.254.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.41

$$\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left((7\,Bb-5\,Ac)\sqrt{c}x^5\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^4+bx^2}\left((7\,Bb-5\,Ac)x^2+3\,Ab\right)\sqrt{x}\right)}{21\,b^2x^5}$$

input
```
integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/21*((7*B*b - 5*A*c)*sqrt(c)*x^5*weierstrassPInverse(-4*b/c, 0, x) + sqr
t(c*x^4 + b*x^2)*((7*B*b - 5*A*c)*x^2 + 3*A*b)*sqrt(x))/(b^2*x^5)
```

3.254.6 Sympy [F]

$$\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^{\frac{7}{2}}\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate((B*x**2+A)/x**(7/2)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x**(7/2)*sqrt(x**2*(b + c*x**2))), x)
```

3.254.7 Maxima [F]

$$\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

3.254.8 Giac [F]

$$\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

3.254.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{B\,x^2+A}{x^{7/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int((A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.255 $\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx$

3.255.1 Optimal result

Integrand size = 28, antiderivative size = 369

$$\begin{aligned}\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx &= -\frac{2c^{3/2}(9bB-7Ac)x^{3/2}(b+cx^2)}{15b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}}\\
&-\frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}}+\frac{2c(9bB-7Ac)\sqrt{bx^2+cx^4}}{15b^3x^{3/2}}\\
&+\frac{2c^{5/4}(9bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}}\\
&-\frac{c^{5/4}(9bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

output
```
-2/15*c^(3/2)*(-7*A*c+9*B*b)*x^(3/2)*(c*x^2+b)/b^3/(b^(1/2)+x*c^(1/2))/(c*
x^4+b*x^2)^(1/2)-2/9*A*(c*x^4+b*x^2)^(1/2)/b/x^(11/2)-2/45*(-7*A*c+9*B*b)*
(c*x^4+b*x^2)^(1/2)/b^2/x^(7/2)+2/15*c*(-7*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/
b^3/x^(3/2)+2/15*c^(5/4)*(-7*A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*ar
ctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)
/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(11/4)/(c*x^4+b*x^2)^(1/2)-1/15*c^(5/4)*(-
7*A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/
2)/b^(11/4)/(c*x^4+b*x^2)^(1/2)
```

3.255.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.23

$$\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left(5A(b+cx^2)+(9bB-7Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},-\frac{cx^2}{b}\right)\right)}{45bx^{7/2}\sqrt{x^2\,(b+cx^2)}}$$

input
```
Integrate[(A + B*x^2)/(x^(9/2)*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*(5*A*(b + c*x^2) + (9*b*B - 7*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeomet
ric2F1[-5/4, 1/2, -1/4, -((c*x^2)/b)]))/(45*b*x^(7/2)*Sqrt[x^2*(b + c*x^2)
])
```

3.255.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1944, 1430, 1430, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx$$

$$\Big\downarrow \; 1944$$

$$\frac{(9bB-7Ac)\int \frac{1}{x^{5/2}\sqrt{cx^4+bx^2}}dx}{9b}-\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

$$\Big\downarrow \; 1430$$

$$\frac{(9bB-7Ac)\left(-\frac{3c\int \frac{1}{\sqrt{x}\sqrt{cx^4+bx^2}}dx}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

$$\Big\downarrow \; 1430$$

$$\frac{(9bB-7Ac)\left(-\frac{3c\left(\frac{c\int\frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{b}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{(9bB-7Ac)\left(-\frac{3c\left(\frac{cx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(9bB-7Ac)\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

$$\Big\downarrow \text{834}$$

$$\frac{(9bB-7Ac)\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

$$\Big\downarrow \text{27}$$

3.255. $\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx$

$$\frac{(9bB-7Ac)\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

$\downarrow$ 761

$$\frac{(9bB-7Ac)\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

$\downarrow$ 1510

3.255. $\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx$

$$\frac{(9bB-7Ac)\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{\cdots}}{\cdots}\right.\right.}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\cdots\right)}{b\sqrt{bx^2+cx^4}}\cdots\right)}{5b}\cdots\right)}{9b}$$

$$\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

input
```
Int[(A + B*x^2)/(x^(9/2)*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*A*Sqrt[b*x^2 + c*x^4])/(9*b*x^(11/2)) + ((9*b*B - 7*A*c)*((-2*Sqrt[b*x
^2 + c*x^4])/(5*b*x^(7/2)) - (3*c*((-2*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) +
(2*c*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*
x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x
)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b
+ c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqr
t[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/
(2*c^(3/4)*Sqrt[b + c*x^2])))/(b*Sqrt[b*x^2 + c*x^4])))/(5*b)))/(9*b)
```

3.255. $\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx$

3.255.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.255.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{2(cx^2+b)(21Ac^2x^4-27x^4Bbc-7Abcx^2+9b^2Bx^2+5b^2A)}{45b^3x^{\frac{7}{2}}\sqrt{x^2(cx^2+b)}}+\frac{c(7Ac-9Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(\cdots\right.}{15b^3\sqrt{\cdots}}$
default	$\frac{42A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc^2x^4-21A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\cdots\right.}{\cdots}$

input
```
int((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*(c*x^2+b)*(21*A*c^2*x^4-27*B*b*c*x^4-7*A*b*c*x^2+9*B*b^2*x^2+5*A*b^2
)/b^3/x^(7/2)/(x^2*(c*x^2+b))^(1/2)+1/15*c*(7*A*c-9*B*b)/b^3*(-b*c)^(1/2)*
((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b
*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^
(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1
/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*
2^(1/2)))*x^(1/2)/(x^2*(c*x^2+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

3.255.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.28

$$\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = \frac{2\left(3\left(9\,Bbc-7\,Ac^2\right)\sqrt{c}x^6\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)\right.}{45\,b^3x^6}$$

input
```
integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(3*(9*B*b*c - 7*A*c^2)*sqrt(c)*x^6*weierstrassZeta(-4*b/c, 0, weierst
rassPInverse(-4*b/c, 0, x)) + (3*(9*B*b*c - 7*A*c^2)*x^4 - 5*A*b^2 - (9*B*
b^2 - 7*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^3*x^6)
```

3.255.6 Sympy [F]

$$\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^{\frac{9}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate((B*x**2+A)/x**(9/2)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x**(9/2)*sqrt(x**2*(b + c*x**2))), x)
```

3.255.7 Maxima [F]

$$\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)
```

3.255.8 Giac [F]

$$\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)
```

3.255.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{B\,x^2+A}{x^{9/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int((A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.256 $\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx$

3.256.1 Optimal result

Integrand size = 28, antiderivative size = 204

$$\begin{aligned}\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx &= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}} \\ &\quad -\frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}}+\frac{10c(11bB-9Ac)\sqrt{bx^2+cx^4}}{231b^3x^{5/2}} \\ &\quad +\frac{5c^{7/4}(11bB-9Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231b^{13/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

output
```
-2/11*A*(c*x^4+b*x^2)^(1/2)/b/x^(13/2)-2/77*(-9*A*c+11*B*b)*(c*x^4+b*x^2)^
(1/2)/b^2/x^(9/2)+10/231*c*(-9*A*c+11*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^(5/2)
+5/231*c^(7/4)*(-9*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2
)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1
/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)
+x*c^(1/2))^2)^(1/2)/b^(13/4)/(c*x^4+b*x^2)^(1/2)
```

3.256.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.41

$$\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left(7A(b+cx^2)+(11bB-9Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{2},-\frac{3}{4},-\frac{cx^2}{b}\right)\right)}{77bx^{9/2}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[(A + B*x^2)/(x^(11/2)*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*(7*A*(b + c*x^2) + (11*b*B - 9*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeome
tric2F1[-7/4, 1/2, -3/4, -((c*x^2)/b)]))/(77*b*x^(9/2)*Sqrt[x^2*(b + c*x^2
)])
```

3.256.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1944, 1430, 1430, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(11bB-9Ac)\int \frac{1}{x^{7/2}\sqrt{cx^4+bx^2}}dx}{11b}-\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{(11bB-9Ac)\left(-\frac{5c\int \frac{1}{x^{3/2}\sqrt{cx^4+bx^2}}dx}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)}{11b}-\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{(11bB-9Ac)\left(-\frac{5c\left(-\frac{c\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3b}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)}{11b}-\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{(11bB-9Ac)\left(-\frac{5c\left(-\frac{cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)}{11b}-\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(11bB-9Ac)\left(-\frac{5c\left(-\frac{2cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)}{11b}-\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(11bB-9Ac)\left(-\frac{5c\left(-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)}{11b}-\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

input
```
Int[(A + B*x^2)/(x^(11/2)*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*A*Sqrt[b*x^2 + c*x^4])/(11*b*x^(13/2)) + ((11*b*B - 9*A*c)*((-2*Sqrt[b
*x^2 + c*x^4])/(7*b*x^(9/2)) - (5*c*((-2*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5/2)
) - (c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x
)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[
b*x^2 + c*x^4])))/(7*b)))/(11*b)
```

3.256. $\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx$

3.256.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.256. $\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx$

3.256.4 Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{2(cx^2+b)(45Ac^2x^4-55x^4Bbc-27Abcx^2+33b^2Bx^2+21b^2A)}{231b^3x^{\frac{9}{2}}\sqrt{x^2(cx^2+b)}} - \frac{5c(9Ac-11Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{x}{\sqrt{-}}}}{231b^3\sqrt{cx^3+bx}\sqrt{x^2(c}}$
default	$-\frac{45A\sqrt{-bc}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}c^2x^5-55B\sqrt{-bc}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\right.}{231\sqrt{x^4c+bx^2}\,x^{\frac{9}{2}}b^3}$

input
```
int((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/231*(c*x^2+b)*(45*A*c^2*x^4-55*B*b*c*x^4-27*A*b*c*x^2+33*B*b^2*x^2+21*A
*b^2)/b^3/x^(9/2)/(x^2*(c*x^2+b))^(1/2)-5/231*c*(9*A*c-11*B*b)/b^3*(-b*c)^
(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))
*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*Ellipti
cF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^(1/2)/(x^2*(
c*x^2+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

3.256.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.47

$$\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = \frac{2\left(5\left(11\,Bbc-9\,Ac^2\right)\sqrt{c}x^7\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(5\left(11\,Bbc-9\,Ac^2\right)x^4\right.}{231\,b^3x^7}$$

input
```
integrate((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/231*(5*(11*B*b*c - 9*A*c^2)*sqrt(c)*x^7*weierstrassPInverse(-4*b/c, 0, x
) + (5*(11*B*b*c - 9*A*c^2)*x^4 - 21*A*b^2 - 3*(11*B*b^2 - 9*A*b*c)*x^2)*s
qrt(c*x^4 + b*x^2)*sqrt(x))/(b^3*x^7)
```

3.256.6 Sympy [F]

$$\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^{\frac{11}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate((B*x**2+A)/x**(11/2)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x**(11/2)*sqrt(x**2*(b + c*x**2))), x)
```

3.256.7 Maxima [F]

$$\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

3.256.8 Giac [F]

$$\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

3.256.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{B\,x^2+A}{x^{11/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int((A + B*x^2)/(x^(11/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2)/(x^(11/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.257 $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.257.1 Optimal result

Integrand size = 28, antiderivative size = 251

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=-\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}}+\frac{15b(13bB-11Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}}-\frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^3}+\frac{(13bB-11Ac)x^{7/2}\sqrt{bx^2+cx^4}}{11bc^2}-\frac{15b^{7/4}(13bB-11Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{154c^{17/4}\sqrt{bx^2+cx^4}}$$

output
```
-(-A*c+B*b)*x^(15/2)/b/c/(c*x^4+b*x^2)^(1/2)-9/77*(-11*A*c+13*B*b)*x^(3/2)
*(c*x^4+b*x^2)^(1/2)/c^3+1/11*(-11*A*c+13*B*b)*x^(7/2)*(c*x^4+b*x^2)^(1/2)
/b/c^2+15/77*b*(-11*A*c+13*B*b)*(c*x^4+b*x^2)^(1/2)/c^4/x^(1/2)-15/154*b^(
7/4)*(-11*A*c+13*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/c
os(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2
))^2)^(1/2)/c^(17/4)/(c*x^4+b*x^2)^(1/2)
```

3.257.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.53

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^{3/2}\left(195b^3B+2c^3x^4(11A+7Bx^2)-2bc^2x^2(33A+13Bx^2)+b^2(-165Ac+78Bc\right.}{77c^4\sqrt{x^2\,(b+}$$

input
```
Integrate[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x^(3/2)*(195*b^3*B + 2*c^3*x^4*(11*A + 7*B*x^2) - 2*b*c^2*x^2*(33*A + 13*
B*x^2) + b^2*(-165*A*c + 78*B*c*x^2) + 15*b^2*(-13*b*B + 11*A*c)*Sqrt[1 +
(c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(77*c^4*Sqrt[x
^2*(b + c*x^2)])
```

3.257.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1943, 1429, 1429, 1429, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{(13bB-11Ac)\int \frac{x^{13/2}}{\sqrt{cx^4+bx^2}}dx}{2bc}-\frac{x^{15/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1429}$$

$$\frac{(13bB-11Ac)\left(\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\int \frac{x^{9/2}}{\sqrt{cx^4+bx^2}}dx}{11c}\right)}{2bc}-\frac{x^{15/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1429}$$

$$\frac{(13bB-11Ac)\left(\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\int\frac{x^{5/2}}{\sqrt{cx^4+bx^2}}dx}{7c}\right)}{11c}\right)}{2bc}-\frac{x^{15/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\downarrow$ 1429

$$\frac{(13bB-11Ac)\left(\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int\frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{7c}\right)}{11c}\right)}{2bc}-\frac{x^{15/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\downarrow$ 1431

$$\frac{(13bB-11Ac)\left(\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)}{11c}\right)}{2bc}-\frac{x^{15/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\downarrow$ 266

$$\frac{(13bB-11Ac)\left(\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{2bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)}{11c}\right)}{2bc}-\frac{x^{15/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

3.257. $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

$$\downarrow \text{761}$$

$$\frac{(13bB-11Ac)\left(\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\right)}{7c}\right)}{11c}\right)}{2bc}$$

$$\frac{x^{15/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

input
```
Int[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-(((b*B - A*c)*x^(15/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((13*b*B - 11*A*c)*(
(2*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(11*c) - (9*b*((2*x^(3/2)*Sqrt[b*x^2 + c*x
^4])/(7*c) - (5*b*((2*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - (b^(3/4)*x*(Sqr
t[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*Ar
cTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(5/4)*Sqrt[b*x^2 + c*x^4])))/(
7*c)))/(11*c)))/(2*b*c)
```

3.257.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.257. $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

3.257.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.12

method	result
default	$\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(28B\,c^{4}x^{7}+165A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b^{2}c+44A\,c^{4}x^{5}-195B\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{\cdots}}{\sqrt{\cdots}}}\cdots\right.}{154(x^{4}c+b\,x^{2}\cdots}$
risch	$-\frac{2\left(-7B\,c^{2}x^{4}-11A\,c^{2}x^{2}+20Bbc\,x^{2}+44Abc-59B\,b^{2}\right)x^{\frac{3}{2}}\left(c\,x^{2}+b\right)}{77c^{4}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{b^{2}\left(\frac{121A\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\cdots\right.}{\sqrt{c\,x^{3}+bx}}\cdots\right.}{\cdots}$

input
```
int(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/154/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(28*B*c^4*x^7+165*A*(-b*c)^(1/
2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c+44*A*c^4*x^5-195*B*(-b*c)^(1/2)*((c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2),1/2*2^(1/2))*b^3-52*B*b*c^3*x^5-132*A*b*c^3*x^3+156*B*b^2*c^
2*x^3-330*A*b^2*c^2*x+390*B*b^3*c*x)/c^5
```

3.257.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.62

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{15\left((13\,Bb^3c-11\,Ab^2c^2)x^3+(13\,Bb^4-11\,Ab^3c)x\right)\sqrt{c}\,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-(14\,Bc^4x^6+195\,\ldots}{77\,(c^6x^3+bc^5x)}$$

input
```
integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/77*(15*((13*B*b^3*c - 11*A*b^2*c^2)*x^3 + (13*B*b^4 - 11*A*b^3*c)*x)*sq
rt(c)*weierstrassPInverse(-4*b/c, 0, x) - (14*B*c^4*x^6 + 195*B*b^3*c - 16
5*A*b^2*c^2 - 2*(13*B*b*c^3 - 11*A*c^4)*x^4 + 6*(13*B*b^2*c^2 - 11*A*b*c^3
)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^6*x^3 + b*c^5*x)
```

3.257.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(17/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Timed out
```

3.257.7 Maxima [F]

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{17}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^(17/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.257.8 Giac [F]

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{17}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^(17/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{17/2}\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int((x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.258 $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.258.1 Optimal result

Integrand size = 28, antiderivative size = 377

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} + \frac{7b(11bB-9Ac)x^{3/2}(b+cx^2)}{15c^{7/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3} + \frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+cx^4}}{9bc^2} - \frac{7b^{5/4}(11bB-9Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{15/4}\sqrt{bx^2+cx^4}} + \frac{7b^{5/4}(11bB-9Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{30c^{15/4}\sqrt{bx^2+cx^4}}$$

output
```
-(-A*c+B*b)*x^(13/2)/b/c/(c*x^4+b*x^2)^(1/2)+7/15*b*(-9*A*c+11*B*b)*x^(3/2
)*(c*x^2+b)/c^(7/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+1/9*(-9*A*c+11
*B*b)*x^(5/2)*(c*x^4+b*x^2)^(1/2)/b/c^2-7/45*(-9*A*c+11*B*b)*x^(1/2)*(c*x^
4+b*x^2)^(1/2)/c^3-7/15*b^(5/4)*(-9*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^
(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(
sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((
c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(15/4)/(c*x^4+b*x^2)^(1/2)+7/30*b^
(5/4)*(-9*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/c
os(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2
))^2)^(1/2)/c^(15/4)/(c*x^4+b*x^2)^(1/2)
```

3.258.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.29

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{2x^{5/2}\left(77b^2B+c^2x^2(9A+5Bx^2)-bc(63A+11Bx^2)+7b(-11bB+9Ac)\sqrt{1+\frac{c}{}}\right.}{45c^3\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*x^(5/2)*(77*b^2*B + c^2*x^2*(9*A + 5*B*x^2) - b*c*(63*A + 11*B*x^2) + 7
*b*(-11*b*B + 9*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3/4, 3/2, 7/4,
-((c*x^2)/b)]))/(45*c^3*Sqrt[x^2*(b + c*x^2)])
```

3.258.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1943, 1429, 1429, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{(11bB-9Ac)\int \frac{x^{11/2}}{\sqrt{cx^4+bx^2}}dx}{2bc}-\frac{x^{13/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1429}$$

$$\frac{(11bB-9Ac)\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\int \frac{x^{7/2}}{\sqrt{cx^4+bx^2}}dx}{9c}\right)}{2bc}-\frac{x^{13/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1429}$$

$$\frac{(11bB-9Ac)\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3b\int\frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{5c}\right)}{9c}\right)}{2bc}-\frac{x^{13/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 1431

$$\frac{(11bB-9Ac)\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3bx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)}{2bc}-\frac{x^{13/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 266

$$\frac{(11bB-9Ac)\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)}{2bc}-\frac{x^{13/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 834

$$\frac{(11bB-9Ac)\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)}{2bc}-\frac{x^{13/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 27

3.258. $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

$$\frac{(11bB-9Ac)\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)}{2bc}-\frac{x^{13/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\downarrow$ 761

$$\frac{(11bB-9Ac)\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)}{2bc}-\frac{x^{13/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\downarrow$ 1510

3.258. $\int\frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

$$\frac{(11bB-9Ac)\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\sqrt[4]{b}(\sqrt{\cdots}}{\cdots}\right.}{5c\sqrt{bx^2+cx^4}}\right.}{9c}\right.}{2bc}$$

$$\frac{x^{13/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

input
```
Int[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-(((b*B - A*c)*x^(13/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((11*b*B - 9*A*c)*((
2*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(9*c) - (7*b*((2*Sqrt[x]*Sqrt[b*x^2 + c*x^4
])/(5*c) - (6*b*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b]
 + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
+ Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1
/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/
4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(5*c*Sqrt[b*x^2 + c*x^4])))/(9*c)
))/(2*b*c)
```

3.258. $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.258.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.258. $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

3.258.4 Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.11

method	result
default	$-\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(-20B\,c^{3}x^{6}+378A\,b^{2}c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-189A\,b^{2}c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx}{\ldots}}\right.}{\ldots}$
risch	$\frac{2x^{\frac{5}{2}}\left(5Bc\,x^{2}+9Ac-16Bb\right)\left(c\,x^{2}+b\right)}{45c^{3}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{b\left(\frac{\left(24Ac-31Bb\right)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\ldots\right)}{c}\ldots\right)}{c\sqrt{c\,x^{3}+bx}}\ldots\right)}{\ldots}$

input
```
int(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/90/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(-20*B*c^3*x^6+378*A*b^2*c*((c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2),1/2*2^(1/2))-189*A*b^2*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^
(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-462*B
*b^3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+231*B*b^3*((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(
1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
-36*A*c^3*x^4+44*B*b*c^2*x^4-126*A*b*c^2*x^2+154*B*b^2*c*x^2)/c^4
```

3.258.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.36

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx =$$
$$-\frac{21\,(11\,Bb^3-9\,Ab^2c+(11\,Bb^2c-9\,Abc^2)x^2)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-}{45\,(c^5x^2+bc^4)}$$

input
```
integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/45*(21*(11*B*b^3 - 9*A*b^2*c + (11*B*b^2*c - 9*A*b*c^2)*x^2)*sqrt(c)*we
ierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)) - (10*B*c^3*x^
4 - 77*B*b^2*c + 63*A*b*c^2 - 2*(11*B*b*c^2 - 9*A*c^3)*x^2)*sqrt(c*x^4 + b
*x^2)*sqrt(x))/(c^5*x^2 + b*c^4)
```

3.258.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(15/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Timed out
```

3.258.7 Maxima [F]

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{15}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^(15/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.258.8 Giac [F]

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{15}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^(15/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.258.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\int \frac{x^{15/2}\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}} \,d x$$

input
```
int((x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.259 $\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.259.1 Optimal result

Integrand size = 28, antiderivative size = 214

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=-\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}}-\frac{5(9bB-7Ac)\sqrt{bx^2+cx^4}}{21c^3\sqrt{x}}+\frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2}+\frac{5b^{3/4}(9bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{42c^{13/4}\sqrt{bx^2+cx^4}}$$

output
```
-(-A*c+B*b)*x^(11/2)/b/c/(c*x^4+b*x^2)^(1/2)+1/7*(-7*A*c+9*B*b)*x^(3/2)*(c
*x^4+b*x^2)^(1/2)/b/c^2-5/21*(-7*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2
)+5/42*b^(3/4)*(-7*A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)
^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/
4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+
x*c^(1/2))^2)^(1/2)/c^(13/4)/(c*x^4+b*x^2)^(1/2)
```

3.259.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.51

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^{3/2}\left(-45b^2B+bc(35A-18Bx^2)+2c^2x^2(7A+3Bx^2)+5b(9bB-7Ac)\sqrt{1+\frac{cx^2}{b}}\right.}{21c^3\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x^(3/2)*(-45*b^2*B + b*c*(35*A - 18*B*x^2) + 2*c^2*x^2*(7*A + 3*B*x^2) +
5*b*(9*b*B - 7*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -
((c*x^2)/b)]))/(21*c^3*Sqrt[x^2*(b + c*x^2)])
```

3.259.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1943, 1429, 1429, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{(9bB-7Ac)\int \frac{x^{9/2}}{\sqrt{cx^4+bx^2}}dx}{2bc}-\frac{x^{11/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1429}$$

$$\frac{(9bB-7Ac)\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\int \frac{x^{5/2}}{\sqrt{cx^4+bx^2}}dx}{7c}\right)}{2bc}-\frac{x^{11/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1429}$$

$$\frac{(9bB-7Ac)\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int\frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{7c}\right)}{2bc}-\frac{x^{11/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\downarrow$ 1431

$$\frac{(9bB-7Ac)\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)}{2bc}-\frac{x^{11/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\downarrow$ 266

$$\frac{(9bB-7Ac)\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{2bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)}{2bc}-\frac{x^{11/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\downarrow$ 761

$$\frac{(9bB-7Ac)\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\right)}{7c}\right)}{2bc}-\frac{x^{11/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

input
```
Int[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-(((b*B - A*c)*x^(11/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((9*b*B - 7*A*c)*((2
*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(7*c) - (5*b*((2*Sqrt[b*x^2 + c*x^4])/(3*c*S
qrt[x]) - (b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqr
t[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(5/4)
*Sqrt[b*x^2 + c*x^4])))/(7*c)))/(2*b*c)
```

3.259. $\int\frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.259.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

3.259. $\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.259.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.19

method	result
default	$-\frac{x^{\frac{5}{2}}(cx^2+b)\left(35A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc-45B\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.}{42(x^4c+bx^2)^{\frac{3}{2}}c^4}$
risch	$\frac{2(3Bcx^2+7Ac-12Bb)x^{\frac{3}{2}}(cx^2+b)}{21c^3\sqrt{x^2(cx^2+b)}}-\frac{b\left(\frac{28A\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{cx^3+bx}}-\frac{33Bb\sqrt{-bc}}{}\right.}{}$

input
```
int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/42/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(35*A*(-b*c)^(1/2)*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2),1/2*2^(1/2))*b*c-45*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2)
)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2
-12*B*c^3*x^5-28*A*c^3*x^3+36*B*b*c^2*x^3-70*A*b*c^2*x+90*B*b^2*c*x)/c^4
```

3.259.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.60

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{5\left((9\,Bb^2c-7\,Abc^2)x^3+(9\,Bb^3-7\,Ab^2c)x\right)\sqrt{c}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+(6}{21\,(c^5x^3+bc^4x)}$$

input
```
integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/21*(5*((9*B*b^2*c - 7*A*b*c^2)*x^3 + (9*B*b^3 - 7*A*b^2*c)*x)*sqrt(c)*we
ierstrassPInverse(-4*b/c, 0, x) + (6*B*c^3*x^4 - 45*B*b^2*c + 35*A*b*c^2 -
 2*(9*B*b*c^2 - 7*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^5*x^3 + b*c^
4*x)
```

3.259.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Timed out
```

3.259.7 Maxima [F]

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{13}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^(13/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.259.8 Giac [F]

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{13}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^(13/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{13/2}\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.260 $\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.260.1 Optimal result

Integrand size = 28, antiderivative size = 340

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}} - \frac{3(7bB-5Ac)x^{3/2}(b+cx^2)}{5c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2} + \frac{3\sqrt[4]{b}(7bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{11/4}\sqrt{bx^2+cx^4}} - \frac{3\sqrt[4]{b}(7bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10c^{11/4}\sqrt{bx^2+cx^4}}$$

output
```
-(-A*c+B*b)*x^(9/2)/b/c/(c*x^4+b*x^2)^(1/2)-3/5*(-5*A*c+7*B*b)*x^(3/2)*(c*
x^2+b)/c^(5/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+1/5*(-5*A*c+7*B*b)*
x^(1/2)*(c*x^4+b*x^2)^(1/2)/b/c^2+3/5*b^(1/4)*(-5*A*c+7*B*b)*x*(cos(2*arct
an(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)
))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+
x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^
(1/2)-3/10*b^(1/4)*(-5*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))
)^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c
^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1
/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

3.260.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.25

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{2x^{5/2}\left(-7bB+5Ac+Bcx^2+(7bB-5Ac)\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},\right.\right.}{5c^2\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*x^(5/2)*(-7*b*B + 5*A*c + B*c*x^2 + (7*b*B - 5*A*c)*Sqrt[1 + (c*x^2)/b]
*Hypergeometric2F1[3/4, 3/2, 7/4, -((c*x^2)/b)]))/(5*c^2*Sqrt[x^2*(b + c*x
^2)])
```

3.260.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1943, 1429, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{(7bB-5Ac)\int \frac{x^{7/2}}{\sqrt{cx^4+bx^2}}dx}{2bc}-\frac{x^{9/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1429}$$

$$\frac{(7bB-5Ac)\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3b\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{5c}\right)}{2bc}-\frac{x^{9/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{(7bB-5Ac)\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3bx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5c\sqrt{bx^2+cx^4}}\right)}{2bc}-\frac{x^{9/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\downarrow$ 266

$$\frac{(7bB-5Ac)\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5c\sqrt{bx^2+cx^4}}\right)}{2bc}-\frac{x^{9/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\downarrow$ 834

$$\frac{(7bB-5Ac)\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{2bc}-\frac{x^{9/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\downarrow$ 27

$$\frac{(7bB-5Ac)\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{2bc}-\frac{x^{9/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\downarrow$ 761

$$\frac{(7bB-5Ac)\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{2bc}-\frac{x^{9/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\downarrow$ 1510

3.260. $\int\frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

$$\frac{(7bB-5Ac)\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,E}{\sqrt[4]{c}\sqrt{b+}}\right.}{5c\sqrt{bx^2+cx^4}}\right.}{2bc}$$

$$\frac{x^{9/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

input
```
Int[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-(((b*B - A*c)*x^(9/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((7*b*B - 5*A*c)*((2*
Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*c) - (6*b*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]
*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*
Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[
b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcT
an[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(5*c*Sq
rt[b*x^2 + c*x^4])))/(2*b*c)
```

3.260.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

3.260. $\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.260.4 Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.16

method	result
default	$\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(30Abc\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-15Abc\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\cdots\right.}{}$
risch	$\frac{2B\,x^{\frac{5}{2}}\left(c\,x^{2}+b\right)}{5c^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{\left(\frac{(5Ac-8Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}+\frac{\sqrt{-bc}\,F\left(\sqrt{\frac{\left(x\cdots\right.}{}}\right.}{}\right.}{c\sqrt{c\,x^{3}+bx}}\cdots}{}$

input
```
int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/10/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(30*A*b*c*((c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/
(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*
2^(1/2))-15*A*b*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x
+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-42*B*b^2*((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-
x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),
1/2*2^(1/2))+21*B*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF((
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+4*B*c^2*x^4-10*A*c^2*x
^2+14*B*b*c*x^2)/c^3
```

3.260.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.31

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{3\,(7\,Bb^2-5\,Abc+(7\,Bbc-5\,Ac^2)x^2)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInve}\right.}{5\,(c^4x^2+bc^3)}$$

input
```
integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/5*(3*(7*B*b^2 - 5*A*b*c + (7*B*b*c - 5*A*c^2)*x^2)*sqrt(c)*weierstrassZe
ta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)) + (2*B*c^2*x^2 + 7*B*b*c
- 5*A*c^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^4*x^2 + b*c^3)
```

3.260.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Timed out
```

3.260.7 Maxima [F]

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{11}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^(11/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.260.8 Giac [F]

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{11}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^(11/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.260.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{11/2}\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.261 $\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.261.1 Optimal result

Integrand size = 28, antiderivative size = 178

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(5bB-3Ac)\sqrt{bx^2+cx^4}}{3bc^2\sqrt{x}} - \frac{(5bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{6\sqrt[4]{b}c^{9/4}\sqrt{bx^2+cx^4}}$$

output
```
-(-A*c+B*b)*x^(7/2)/b/c/(c*x^4+b*x^2)^(1/2)+1/3*(-3*A*c+5*B*b)*(c*x^4+b*x^
2)^(1/2)/b/c^2/x^(1/2)-1/6*(-3*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/
b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*
arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+
b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(1/4)/c^(9/4)/(c*x^4+b*x^2)^(1/2)
```

3.261.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.48

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^{3/2}\left(5bB-3Ac+2Bcx^2+(-5bB+3Ac)\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\right.\right.}{3c^2\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x^(3/2)*(5*b*B - 3*A*c + 2*B*c*x^2 + (-5*b*B + 3*A*c)*Sqrt[1 + (c*x^2)/b]
*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(3*c^2*Sqrt[x^2*(b + c*x
^2)])
```

3.261.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1943, 1429, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{(5bB-3Ac)\int \frac{x^{5/2}}{\sqrt{cx^4+bx^2}}dx}{2bc}-\frac{x^{7/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1429}$$

$$\frac{(5bB-3Ac)\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{2bc}-\frac{x^{7/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{(5bB-3Ac)\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}\right)}{2bc}-\frac{x^{7/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(5bB-3Ac)\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{2bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}\right)}{2bc}-\frac{x^{7/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(5bB-3Ac)\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\right)}{2bc}-\frac{x^{7/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

input
```
Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-(((b*B - A*c)*x^(7/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((5*b*B - 3*A*c)*((2*
Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - (b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(3*c^(5/4)*Sqrt[b*x^2 + c*x^4])))/(2*b*c)
```

3.261.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

3.261. $\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

3.261.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.29

method	result
default	$\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(3A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c-5B\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\cdots}\right.}{6\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}c^{3}}$
risch	$\frac{2B\,x^{\frac{3}{2}}\left(c\,x^{2}+b\right)}{3c^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{\left(\frac{3A\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{c\,x^{3}+bx}}-\frac{4Bb\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(\cdots\right.}{\cdots}}}{\cdots}\right.}{\cdots}$

input
```
int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(3*A*(-b*c)^(1/2)*((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2),1/2*2^(1/2))*c-5*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2
)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b+4*B*c^2
*x^3-6*A*c^2*x+10*B*b*c*x)/c^3
```

3.261.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.57

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{((5\,Bbc-3\,Ac^2)x^3+(5\,Bb^2-3\,Abc)x)\sqrt{c}\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-(2\,Bc^2x^2+5\,Bbc-3\,Ac^2)\sqrt{c}}{3\,(c^4x^3+bc^3x)}$$

input
```
integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(((5*B*b*c - 3*A*c^2)*x^3 + (5*B*b^2 - 3*A*b*c)*x)*sqrt(c)*weierstras
sPInverse(-4*b/c, 0, x) - (2*B*c^2*x^2 + 5*B*b*c - 3*A*c^2)*sqrt(c*x^4 + b
*x^2)*sqrt(x))/(c^4*x^3 + b*c^3*x)
```

3.261.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Timed out
```

3.261.7 Maxima [F]

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{9}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^(9/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.261.8 Giac [F]

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{9}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^(9/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.261.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{9/2}\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.262 $\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.262.1 Optimal result

Integrand size = 28, antiderivative size = 299

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(3bB-Ac)x^{3/2}(b+cx^2)}{bc^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{7/4}\sqrt{bx^2+cx^4}} + \frac{(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{3/4}c^{7/4}\sqrt{bx^2+cx^4}}$$

output
```
-(-A*c+B*b)*x^(5/2)/b/c/(c*x^4+b*x^2)^(1/2)+(-A*c+3*B*b)*x^(3/2)*(c*x^2+b)
/b/c^(3/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-(-A*c+3*B*b)*x*(cos(2*a
rctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1
/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/
2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/c^(7/4)/(c*x
^4+b*x^2)^(1/2)+1/2*(-A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))
^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/
2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/c^(7/4)/(c*x^4+b*x^2)^(1/2)
```

3.262.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.26

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{2x^{5/2}\left(-3bB+(3bB-Ac)\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{cx^2}{b}\right)\right)}{3bc\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-2*x^(5/2)*(-3*b*B + (3*b*B - A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[
3/4, 3/2, 7/4, -((c*x^2)/b)]))/(3*b*c*Sqrt[x^2*(b + c*x^2)])
```

3.262.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1943, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 1943}$$

$$\frac{(3bB-Ac)\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{2bc}-\frac{x^{5/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1431}$$

$$\frac{x\sqrt{b+cx^2}(3bB-Ac)\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{2bc\sqrt{bx^2+cx^4}}-\frac{x^{5/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{x\sqrt{b+cx^2}(3bB-Ac)\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{bc\sqrt{bx^2+cx^4}}-\frac{x^{5/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 834

$$\frac{x\sqrt{b+cx^2}(3bB-Ac)\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{bc\sqrt{bx^2+cx^4}}-\frac{x^{5/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 27

$$\frac{x\sqrt{b+cx^2}(3bB-Ac)\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{bc\sqrt{bx^2+cx^4}}-\frac{x^{5/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 761

$$\frac{x\sqrt{b+cx^2}(3bB-Ac)\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{bc\sqrt{bx^2+cx^4}}-\frac{x^{5/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 1510

$$\frac{x\sqrt{b+cx^2}(3bB-Ac)\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\right.}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right.}{bc\sqrt{bx^2+cx^4}}-\frac{x^{5/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

input
```
Int[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-(((b*B - A*c)*x^(5/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((3*b*B - A*c)*x*Sqrt
[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1
/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellipt
icE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/
Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt
[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*
Sqrt[b + c*x^2])))/(b*c*Sqrt[b*x^2 + c*x^4])
```

3.262. $\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.262.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

3.262. $\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.262.4 Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.30

method	result
default	$-\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(2Abc\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-Abc\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\right.\right.}{}$

input
```
int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(2*A*b*c*((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(
-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2
^(1/2))-A*b*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-6*B*b^2*((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(
-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2
^(1/2))+3*B*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-2*A*c^2*x^2+2*B*b*c*x^2)/c^2
/b
```

3.262.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.33

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx =$$
$$-\frac{(3\,Bb^2-Abc+(3\,Bbc-Ac^2)x^2)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}}{bc^3x^2+b^2c^2}$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-((3*B*b^2 - A*b*c + (3*B*b*c - A*c^2)*x^2)*sqrt(c)*weierstrassZeta(-4*b/c
, 0, weierstrassPInverse(-4*b/c, 0, x)) + sqrt(c*x^4 + b*x^2)*(B*b*c - A*c
^2)*sqrt(x))/(b*c^3*x^2 + b^2*c^2)
```

3.262.6 Sympy [F]

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{7}{2}}(A+Bx^2)}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**(7/2)*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.262.7 Maxima [F]

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{7}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^(7/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.262.8 Giac [F]

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{7}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^(7/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.262.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\int \frac{x^{7/2}\,(B\,x^2+A)}{{(c\,x^4+b\,x^2)}^{3/2}} \,d x$$

input
```
int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.263 $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.263.1 Optimal result

Integrand size = 28, antiderivative size = 137

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^{3/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{5/4}c^{5/4}\sqrt{bx^2+cx^4}}$$

output
```
-(-A*c+B*b)*x^(3/2)/b/c/(c*x^4+b*x^2)^(1/2)+1/2*(A*c+B*b)*x*(cos(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*
EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c
^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(5/4)/c^(5/4)/(c*x^4+b*x
^2)^(1/2)
```

3.263.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.55

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^{3/2}\left(-bB+Ac+(bB+Ac)\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^2}{b}\right)\right)}{bc\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x^(3/2)*(-(b*B) + A*c + (b*B + A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1
[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(b*c*Sqrt[x^2*(b + c*x^2)])
```

3.263.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1943, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{(Ac+bB)\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{2bc}-\frac{x^{3/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{x\sqrt{b+cx^2}(Ac+bB)\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{2bc\sqrt{bx^2+cx^4}}-\frac{x^{3/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{x\sqrt{b+cx^2}(Ac+bB)\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{bc\sqrt{bx^2+cx^4}}-\frac{x^{3/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+bB)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{5/4}c^{5/4}\sqrt{bx^2+cx^4}}-\frac{x^{3/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

input
```
Int[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-(((b*B - A*c)*x^(3/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((b*B + A*c)*x*(Sqrt[
b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcT
an[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*b^(5/4)*c^(5/4)*Sqrt[b*x^2 + c*x^4
])
```

3.263.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

3.263.4 Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.62

method	result
default	$\frac{x^{\frac{5}{2}} \left(c\,x^{2}+b\right) \left(A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c+B\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\right.}{2\left(x^{4} c+b\,x^{2}\right)^{\frac{3}{2}} b\,c^{2}}$

input
```
int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(A*(-b*c)^(1/2)*((c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(
-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
,1/2*2^(1/2))*c+B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(
1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*El
lipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b+2*A*c^2*x-2
*B*b*c*x)/b/c^2
```

3.263.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.66

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{((Bbc+Ac^2)x^3+(Bb^2+Abc)x)\sqrt{c}\text{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)-\sqrt{cx^4+bx^2}(B}{bc^3x^3+b^2c^2x}$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
(((B*b*c + A*c^2)*x^3 + (B*b^2 + A*b*c)*x)*sqrt(c)*weierstrassPInverse(-4*
b/c, 0, x) - sqrt(c*x^4 + b*x^2)*(B*b*c - A*c^2)*sqrt(x))/(b*c^3*x^3 + b^2
*c^2*x)
```

3.263.6 Sympy [F]

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{5}{2}}(A+Bx^2)}{(x^2(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**(5/2)*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.263.7 Maxima [F]

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{5}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^(5/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.263.8 Giac [F]

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{5}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^(5/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.263.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{5/2}\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.264 $\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.264.1 Optimal result

Integrand size = 28, antiderivative size = 318

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{(bB-3Ac)x^{3/2}(b+cx^2)}{b^2\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{7/4}c^{3/4}\sqrt{bx^2+cx^4}} - \frac{(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{7/4}c^{3/4}\sqrt{bx^2+cx^4}}$$

output
```
(-3*A*c+B*b)*x^(5/2)/b^2/(c*x^4+b*x^2)^(1/2)-(-3*A*c+B*b)*x^(3/2)*(c*x^2+b
)/b^2/c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2*A*x^(1/2)/b/(c*x^4
+b*x^2)^(1/2)+(-3*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*
x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c
^(1/2))^2)^(1/2)/b^(7/4)/c^(3/4)/(c*x^4+b*x^2)^(1/2)-1/2*(-3*A*c+B*b)*x*(c
os(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*
(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/c^(3/4
)/(c*x^4+b*x^2)^(1/2)
```

3.264.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.24

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{2\sqrt{x}\left(-3Ab+(bB-3Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{cx^2}{b}\right)\right)}{3b^2\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*Sqrt[x]*(-3*A*b + (b*B - 3*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2
F1[3/4, 3/2, 7/4, -((c*x^2)/b)]))/(3*b^2*Sqrt[x^2*(b + c*x^2)])
```

3.264.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1944, 1428, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(bB-3Ac)\int \frac{x^{7/2}}{(cx^4+bx^2)^{3/2}}dx}{b} - \frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1428}$$

$$\frac{(bB-3Ac)\left(\frac{x^{5/2}}{b\sqrt{bx^2+cx^4}} - \frac{\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{2b}\right)}{b} - \frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{(bB-3Ac)\left(\frac{x^{5/2}}{b\sqrt{bx^2+cx^4}} - \frac{x\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{2b\sqrt{bx^2+cx^4}}\right)}{b} - \frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \; 266$$

$$\frac{(bB-3Ac)\left(\frac{x^{5/2}}{b\sqrt{bx^2+cx^4}}-\frac{x\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{b\sqrt{bx^2+cx^4}}\right)}{b}-\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \; 834$$

$$\frac{(bB-3Ac)\left(\frac{x^{5/2}}{b\sqrt{bx^2+cx^4}}-\frac{x\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}\right)}{b}-\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{(bB-3Ac)\left(\frac{x^{5/2}}{b\sqrt{bx^2+cx^4}}-\frac{x\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}\right)}{b}-\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \; 761$$

$$\frac{(bB-3Ac)\left(\frac{x^{5/2}}{b\sqrt{bx^2+cx^4}}-\frac{x\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}\right)}{b}-\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \; 1510$$

3.264. $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

$$\frac{(bB-3Ac)\left(\frac{x^{5/2}}{b\sqrt{bx^2+cx^4}}-\frac{x\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\,\text{arct}\cdots\right.}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\cdots\right)}{b\sqrt{bx^2+cx^4}}\right)}{b}$$

$$\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

input
```
Int[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-2*A*Sqrt[x])/(b*Sqrt[b*x^2 + c*x^4]) + ((b*B - 3*A*c)*(x^(5/2)/(b*Sqrt[b
*x^2 + c*x^4]) - (x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[
b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b
] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^
(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
+ c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(
1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(b*Sqrt[b*x^2 + c*x^4])))/b
```

3.264.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1428
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] + Simp[d^2*
((m + 4*p + 3)/(2*b*(p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[p, -1]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.264. $\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.264.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.23

method	result
default	$\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(6Abc\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-3Abc\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\cdots}\right.\right.}{\cdots}$
risch	$-\frac{2A\left(c\,x^{2}+b\right)\sqrt{x}}{b^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{\left(\frac{A\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}+\frac{\sqrt{-bc}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)\cdots}{\sqrt{-bc}}}\right.}{c}\cdots\right.}{\sqrt{c\,x^{3}+bx}}\cdots\right.}{\cdots}$

input

```
int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(6*A*b*c*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-
b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^
(1/2))-3*A*b*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-2*B*b^2*((c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/
(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*
2^(1/2))+B*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-6*A*c^2*x^2+2*B*b*c*x^2-4*A*b
*c)/c/b^2
```

3.264.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.36

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{((Bbc-3\,Ac^2)x^4+(Bb^2-3\,Abc)x^2)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInvers}\right.}{b^2c^2x^4+b^3cx^2}$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
(((B*b*c - 3*A*c^2)*x^4 + (B*b^2 - 3*A*b*c)*x^2)*sqrt(c)*weierstrassZeta(-
4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)) - sqrt(c*x^4 + b*x^2)*(2*A*b*
c - (B*b*c - 3*A*c^2)*x^2)*sqrt(x))/(b^2*c^2*x^4 + b^3*c*x^2)
```

3.264.6 Sympy [F]

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}}(A+Bx^2)}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**(3/2)*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.264.7 Maxima [F]

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{3}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^(3/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.264.8 Giac [F]

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{3}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^(3/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{3/2}\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.265 $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

3.265.1 Optimal result

Integrand size = 28, antiderivative size = 167

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}}+\frac{(3bB-5Ac)x^{3/2}}{3b^2\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{6b^{9/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

output
```
1/3*(-5*A*c+3*B*b)*x^(3/2)/b^2/(c*x^4+b*x^2)^(1/2)-2/3*A/b/x^(1/2)/(c*x^4+
b*x^2)^(1/2)+1/6*(-5*A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^
2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(
1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2
)+x*c^(1/2))^2)^(1/2)/b^(9/4)/c^(1/4)/(c*x^4+b*x^2)^(1/2)
```

3.265.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{-2Ab+3bBx^2-5Acx^2+(3bB-5Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{c}{...}\right.}{3b^2\sqrt{x}\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-2*A*b + 3*b*B*x^2 - 5*A*c*x^2 + (3*b*B - 5*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*
Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)])/(3*b^2*Sqrt[x]*Sqrt[x^2*(b
 + c*x^2)])
```

3.265.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1944, 1428, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(3bB-5Ac)\int \frac{x^{5/2}}{(cx^4+bx^2)^{3/2}}dx}{3b}-\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1428}$$

$$\frac{(3bB-5Ac)\left(\frac{\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{2b}+\frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}\right)}{3b}-\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{(3bB-5Ac)\left(\frac{x\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{2b\sqrt{bx^2+cx^4}}+\frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}\right)}{3b}-\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(3bB-5Ac)\left(\frac{x\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{b\sqrt{bx^2+cx^4}}+\frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}\right)}{3b}-\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(3bB-5Ac)\left(\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{5/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}\right)}{3b}-\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}}$$

input
```
Int[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-2*A)/(3*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4]) + ((3*b*B - 5*A*c)*(x^(3/2)/(b*Sq
rt[b*x^2 + c*x^4]) + (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*b^(5
/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])))/(3*b)
```

3.265.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1428
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] + Simp[d^2*
((m + 4*p + 3)/(2*b*(p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[p, -1]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.265.4 Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.41

method	result
default	$-\frac{x^{\frac{3}{2}}\left(c\,x^{2}+b\right)\left(5A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)cx-3B\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{\cdots}\right.}{6\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}c\,b^{2}}$
risch	$-\frac{2A\left(c\,x^{2}+b\right)}{3b^{2}\sqrt{x}\,\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{\left(\frac{A\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{c\,x^{3}+bx}}+3b(Ac-Bb)\left(\frac{x}{b\sqrt{\left(x^{2}+\frac{b}{c}\right)cx}}\cdots\right.\right.}{3b^{2}\sqrt{x^{2}\left(c\,x^{2}\cdots\right.}}$

input
```
int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(c*x^4+b*x^2)^(3/2)*x^(3/2)*(c*x^2+b)*(5*A*(-b*c)^(1/2)*((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))*c*x-3*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(
1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*x+10
*A*c^2*x^2-6*B*b*c*x^2+4*A*b*c)/c/b^2
```

3.265.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{((3\,Bbc-5\,Ac^2)x^5+(3\,Bb^2-5\,Abc)x^3)\sqrt{c}\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\sqrt{cx^4}}{3\,(b^2c^2x^5+b^3cx^3)}$$

input
```
integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(((3*B*b*c - 5*A*c^2)*x^5 + (3*B*b^2 - 5*A*b*c)*x^3)*sqrt(c)*weierstra
ssPInverse(-4*b/c, 0, x) - sqrt(c*x^4 + b*x^2)*(2*A*b*c - (3*B*b*c - 5*A*c
^2)*x^2)*sqrt(x))/(b^2*c^2*x^5 + b^3*c*x^3)
```

3.265.6 Sympy [F]

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{x}(A+Bx^2)}{(x^2(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(sqrt(x)*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.265.7 Maxima [F]

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{x}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

3.265.8 Giac [F]

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{x}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{x}\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.266 $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^{3/2}}\,dx$

3.266.1 Optimal result

Integrand size = 28, antiderivative size = 368

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^{3/2}}\,dx = -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}}+\frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}}+\frac{3\sqrt{c}(5bB-7Ac)x^{3/2}(b+cx^2)}{5b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}}-\frac{3\sqrt[4]{c}(5bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{bx^2+cx^4}}+\frac{3\sqrt[4]{c}(5bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10b^{11/4}\sqrt{bx^2+cx^4}}$$

output
```
-2/5*A/b/x^(3/2)/(c*x^4+b*x^2)^(1/2)+3/5*(-7*A*c+5*B*b)*x^(3/2)*(c*x^2+b)*
c^(1/2)/b^3/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+1/5*(-7*A*c+5*B*b)*x^(
1/2)/b^2/(c*x^4+b*x^2)^(1/2)-3/5*(-7*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^
(3/2)-3/5*c^(1/4)*(-7*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))
^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/
2)+x*c^(1/2))^2)^(1/2)/b^(11/4)/(c*x^4+b*x^2)^(1/2)+3/10*c^(1/4)*(-7*A*c+5
*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1
/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2
*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(1
1/4)/(c*x^4+b*x^2)^(1/2)
```

3.266.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.21

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^{3/2}}\,dx = \frac{-2Ab+2(-5bB+7Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{4},-\frac{cx^2}{b}\right)}{5b^2x^{3/2}\sqrt{x^2\,(b+cx^2)}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-2*A*b + 2*(-5*b*B + 7*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-1/
4, 3/2, 3/4, -((c*x^2)/b)])/(5*b^2*x^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

3.266.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1944, 1428, 1430, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(5bB-7Ac)\int \frac{x^{3/2}}{(cx^4+bx^2)^{3/2}}dx}{5b}-\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1428}$$

$$\frac{(5bB-7Ac)\left(\frac{3\int \frac{1}{\sqrt{x}\sqrt{cx^4+bx^2}}dx}{2b}+\frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}\right)}{5b}-\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{(5bB-7Ac)\left(\frac{3\left(\frac{c\int\frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{b}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{2b}+\frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}\right)}{5b}-\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}}$$

$\downarrow$ 1431

$$\frac{(5bB-7Ac)\left(\frac{3\left(\frac{cx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{2b}+\frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}\right)}{5b}-\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}}$$

$\downarrow$ 266

$$\frac{(5bB-7Ac)\left(\frac{3\left(\frac{2cx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{2b}+\frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}\right)}{5b}-\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}}$$

$\downarrow$ 834

$$\frac{(5bB-7Ac)\left(\frac{3\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{2b}+\frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}\right)}{5b}-\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}}$$

$\downarrow$ 27

3.266. $\int\frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^{3/2}}\,dx$

$$\frac{(5bB-7Ac)\left(\frac{3\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{2b}+\frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}\right)}{5b}-\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{(5bB-7Ac)\left(\frac{3\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{2b}+\frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}\right)}{5b}$$

$$\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1510}$$

3.266. $\int\frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^{3/2}}\,dx$

$$\frac{(5bB-7Ac)\left(\frac{3\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.\right.}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right.}{b\sqrt{bx^2+cx^4}}\right.}{2b}\right.}{5b}$$

$$\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}}$$

input
```
Int[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-2*A)/(5*b*x^(3/2)*Sqrt[b*x^2 + c*x^4]) + ((5*b*B - 7*A*c)*(Sqrt[x]/(b*Sq
rt[b*x^2 + c*x^4]) + (3*((-2*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) + (2*c*x*Sqr
t[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(
1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellip
ticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))
/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqr
t[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)
*Sqrt[b + c*x^2])))/(b*Sqrt[b*x^2 + c*x^4])))/(2*b)))/(5*b)
```

3.266. $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^{3/2}}\,dx$

3.266.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1428
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] + Simp[d^2*
((m + 4*p + 3)/(2*b*(p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[p, -1]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.266.4 Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.14

method	result
default	$-\frac{\sqrt{x}\,(c\,x^2+b)\left(42A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc\,x^2-21A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\right.}{}$
risch	$-\frac{2(c\,x^2+b)\,(-8Ac\,x^2+5bB\,x^2+Ab)}{5b^3x^{\frac{3}{2}}\sqrt{x^2(c\,x^2+b)}}-\frac{c^2\left(\frac{(8Ac-5Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\right.}{c}\right.}{c^2\sqrt{c\,x^3+bx}}\right.}{}$

input
```
int((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10/(c*x^4+b*x^2)^(3/2)*x^(1/2)*(c*x^2+b)*(42*A*((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b
*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(
1/2))*b*c*x^2-21*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c*x^2-30*B*((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2),1/2*2^(1/2))*b^2*x^2+15*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(
1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*El
lipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*x^2-42*A*
c^2*x^4+30*x^4*B*b*c-28*A*b*c*x^2+20*b^2*B*x^2+4*b^2*A)/b^3
```

3.266. $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^{3/2}}\,dx$

3.266.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{3\left((5\,Bbc-7\,Ac^2)x^6+(5\,Bb^2-7\,Abc)x^4\right)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+(}{5\left(b^3cx^6+b^4x^4\right)}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

output
```
-1/5*(3*((5*B*b*c - 7*A*c^2)*x^6 + (5*B*b^2 - 7*A*b*c)*x^4)*sqrt(c)*weiers
trassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)) + (3*(5*B*b*c - 7*
A*c^2)*x^4 + 2*A*b^2 + 2*(5*B*b^2 - 7*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt
(x))/(b^3*c*x^6 + b^4*x^4)
```

3.266.6 Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{A+Bx^2}{\sqrt{x}\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)/(c*x**4+b*x**2)**(3/2)/x**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(x)*(x**2*(b + c*x**2))**(3/2)), x)
```

3.266.7 Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{Bx^2+A}{\left(cx^4+bx^2\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

3.266.8 Giac [F]

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^{3/2}}\,dx=\int \frac{Bx^2+A}{(cx^4+bx^2)^{\frac{3}{2}}\sqrt{x}}\,dx$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

3.266.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^{3/2}}\,dx=\int \frac{B\,x^2+A}{\sqrt{x}\,(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

3.267 $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^{3/2}}\,dx$

3.267.1 Optimal result

Integrand size = 28, antiderivative size = 203

$$\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^{3/2}}\,dx = -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5(7bB-9Ac)\sqrt{bx^2+cx^4}}{21b^3x^{5/2}} - \frac{5c^{3/4}(7bB-9Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{42b^{13/4}\sqrt{bx^2+cx^4}}$$

output
```
-2/7*A/b/x^(5/2)/(c*x^4+b*x^2)^(1/2)+1/7*(-9*A*c+7*B*b)/b^2/x^(1/2)/(c*x^4
+b*x^2)^(1/2)-5/21*(-9*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^(5/2)-5/42*c^(
3/4)*(-9*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos
(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)
/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))
^2)^(1/2)/b^(13/4)/(c*x^4+b*x^2)^(1/2)
```

3.267.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx=\frac{-6Ab+2(-7bB+9Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},-\frac{cx^2}{b}\right)}{21b^2x^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-6*A*b + 2*(-7*b*B + 9*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-3/
4, 3/2, 1/4, -((c*x^2)/b)])/(21*b^2*x^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

3.267.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1944, 1428, 1430, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(7bB-9Ac)\int \frac{\sqrt{x}}{\left(cx^4+bx^2\right)^{3/2}}dx}{7b}-\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1428}$$

$$\frac{(7bB-9Ac)\left(\frac{5\int \frac{1}{x^{3/2}\sqrt{cx^4+bx^2}}dx}{2b}+\frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}}\right)}{7b}-\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{(7bB-9Ac)\left(\frac{5\left(-\frac{c\int\frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3b}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{2b}+\frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}}\right)}{7b}-\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{(7bB-9Ac)\left(\frac{5\left(-\frac{cx\sqrt{b+cx^2}\int\frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{2b}+\frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}}\right)}{7b}-\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(7bB-9Ac)\left(\frac{5\left(-\frac{2cx\sqrt{b+cx^2}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{2b}+\frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}}\right)}{7b}-\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(7bB-9Ac)\left(\frac{5\left(-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{2b}+\frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}}\right)}{7b}-\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}}$$

input
```
Int[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-2*A)/(7*b*x^(5/2)*Sqrt[b*x^2 + c*x^4]) + ((7*b*B - 9*A*c)*(1/(b*Sqrt[x]*
Sqrt[b*x^2 + c*x^4]) + (5*((-2*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5/2)) - (c^(3/
4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellip
ticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^2 + c*
x^4])))/(2*b)))/(7*b)
```

3.267. $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^{3/2}}\,dx$

3.267.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1428
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] + Simp[d^2*
((m + 4*p + 3)/(2*b*(p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[p, -1]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.267. $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^{3/2}} \, dx$

3.267.4 Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.25

method	result
default	$\frac{\left(c\,x^{2}+b\right)\left(45A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c\,x^{3}-35B\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\right.}{42\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}\sqrt{x}\,b^{3}}$
risch	$-\frac{2\left(c\,x^{2}+b\right)\left(-12Ac\,x^{2}+7bB\,x^{2}+3Ab\right)}{21b^{3}x^{\frac{5}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{c\left(-\frac{7Bb\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c\sqrt{c\,x^{3}+bx}}+\frac{12A\sqrt{\ }}{}\right.}{}$

input
```
int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/42/(c*x^4+b*x^2)^(3/2)/x^(1/2)*(c*x^2+b)*(45*A*(-b*c)^(1/2)*((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2),1/2*2^(1/2))*c*x^3-35*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2
))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*
x^3+90*A*c^2*x^4-70*x^4*B*b*c+36*A*b*c*x^2-28*b^2*B*x^2-12*b^2*A)/b^3
```

3.267.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.62

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx =$$
$$-\frac{5\left((7\,Bbc-9\,Ac^2)x^7+(7\,Bb^2-9\,Abc)x^5\right)\sqrt{c}\,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(5\,(7\,Bbc-9\,Ac^2)x^4+6\,A\right.}{21\left(b^3cx^7+b^4x^5\right)}$$

input
```
integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/21*(5*((7*B*b*c - 9*A*c^2)*x^7 + (7*B*b^2 - 9*A*b*c)*x^5)*sqrt(c)*weier
strassPInverse(-4*b/c, 0, x) + (5*(7*B*b*c - 9*A*c^2)*x^4 + 6*A*b^2 + 2*(7
*B*b^2 - 9*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^3*c*x^7 + b^4*x^5)
```

3.267.6 Sympy [F]

$$\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^{3/2}}\,dx = \int \frac{A+Bx^2}{x^{\frac{3}{2}}\left(x^2(b+cx^2)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)/x**(3/2)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral((A + B*x**2)/(x**(3/2)*(x**2*(b + c*x**2))**(3/2)), x)
```

3.267.7 Maxima [F]

$$\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+bx^2)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^(3/2)), x)
```

3.267.8 Giac [F]

$$\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+bx^2)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^(3/2)), x)
```

3.267.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{B\,x^2+A}{x^{3/2}\,\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

input
```
int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

3.268 $\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)^{3/2}}\,dx$

3.268.1 Optimal result

Integrand size = 28, antiderivative size = 405

$$\begin{aligned}
\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx &= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} \\
&+\frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}}-\frac{7c^{3/2}(9bB-11Ac)x^{3/2}(b+cx^2)}{15b^4\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} \\
&-\frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}}+\frac{7c(9bB-11Ac)\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} \\
&+\frac{7c^{5/4}(9bB-11Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{15/4}\sqrt{bx^2+cx^4}} \\
&-\frac{7c^{5/4}(9bB-11Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{30b^{15/4}\sqrt{bx^2+cx^4}}
\end{aligned}$$

output
```
-2/9*A/b/x^(7/2)/(c*x^4+b*x^2)^(1/2)+1/9*(-11*A*c+9*B*b)/b^2/x^(3/2)/(c*x^
4+b*x^2)^(1/2)-7/15*c^(3/2)*(-11*A*c+9*B*b)*x^(3/2)*(c*x^2+b)/b^4/(b^(1/2)
+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-7/45*(-11*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b
^3/x^(7/2)+7/15*c*(-11*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b^4/x^(3/2)+7/15*c^(
5/4)*(-11*A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/co
s(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2)
)^2)^(1/2)/b^(15/4)/(c*x^4+b*x^2)^(1/2)-7/30*c^(5/4)*(-11*A*c+9*B*b)*x*(co
s(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)
/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(
b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(15/4)/(c*x^4
+b*x^2)^(1/2)
```

3.268.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.20

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx=\frac{-10Ab+2(-9bB+11Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{2},-\frac{1}{4},-\frac{cx^2}{b}\right)}{45b^2x^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[(A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-10*A*b + 2*(-9*b*B + 11*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-
5/4, 3/2, -1/4, -((c*x^2)/b)])/(45*b^2*x^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

3.268.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1944, 1428, 1430, 1430, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1944}$$

$$\frac{(9bB-11Ac)\int \frac{1}{\sqrt{x}(cx^4+bx^2)^{3/2}}dx}{9b}-\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1428}$$

$$\frac{(9bB-11Ac)\left(\frac{7\int \frac{1}{x^{5/2}\sqrt{cx^4+bx^2}}dx}{2b}+\frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}\right)}{9b}-\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1430}$$

$$\frac{(9bB-11Ac)\left(\frac{7\left(-\frac{3c\int \frac{1}{\sqrt{x}\sqrt{cx^4+bx^2}}dx}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{2b}+\frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}\right)}{9b}-\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1430}$$

$$\frac{(9bB-11Ac)\left(\frac{7\left(-\frac{3c\left(\frac{c\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{b}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{2b}+\frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}\right)}{9b}-\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1431}$$

$$\frac{(9bB-11Ac)\left(\frac{7\left(-\frac{3c\left(\frac{cx\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{2b}+\frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}\right)}{9b}-\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 266}$$

3.268. $\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)^{3/2}}\,dx$

$$\frac{(9bB-11Ac)\left(\frac{7\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{2b}+\frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}\right)}{9b}-\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \; 834$$

$$\frac{(9bB-11Ac)\left(\frac{7\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{2b}+\frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}\right)}{9b}-\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \; 27$$

3.268. $\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)^{3/2}}\,dx$

$$\frac{(9bB-11Ac)\left(\frac{7\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{2b}+\frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}\right)}{9b}-\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \; 761$$

3.268. $\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)^{3/2}}\,dx$

$$\frac{(9bB-11Ac)\left(\frac{7\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2\cdots}}{5bx^7\cdots}\right.\cdots}{2b}\cdots\right.}{9b}$$

$$\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}}$$

$\downarrow$ 1510

3.268. $\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)^{3/2}}\,dx$

$$\frac{(9bB-11Ac)\left(\frac{7\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,E\left(2\arctan\left(\cdots\right.\right.}{\sqrt[4]{c}\sqrt{b+cx^2}\cdots}}{\sqrt{c}\cdots}\cdots\right)}{b\sqrt{bx^2+cx^4}\cdots}\cdots\right)}{5b\cdots}\cdots\right)}{2b\cdots}\cdots\right)}{9b\cdots}$$

$$\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}}$$

3.268. $\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)^{3/2}}\,dx$

input
```
Int[(A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-2*A)/(9*b*x^(7/2)*Sqrt[b*x^2 + c*x^4]) + ((9*b*B - 11*A*c)*(1/(b*x^(3/2)
*Sqrt[b*x^2 + c*x^4]) + (7*((-2*Sqrt[b*x^2 + c*x^4])/(5*b*x^(7/2)) - (3*c*
((-2*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) + (2*c*x*Sqrt[b + c*x^2]*(-((-((Sqrt
[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*
x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*S
qrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sq
rt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*A
rcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(b*S
qrt[b*x^2 + c*x^4])))/(5*b)))/(2*b)))/(9*b)
```

3.268.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1428
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] + Simp[d^2*
((m + 4*p + 3)/(2*b*(p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[p, -1]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

3.268.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.11

method	result
default	$\frac{(c\,x^2+b)\left(462A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b\,c^2x^4-231A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F}{}$
risch	$-\frac{2(c\,x^2+b)\,(93A\,c^2x^4-72x^4Bbc-16Abc\,x^2+9b^2B\,x^2+5b^2A)}{45b^4x^{\frac{7}{2}}\sqrt{x^2(c\,x^2+b)}}+\frac{c^2\left(\frac{(31Ac-24Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}}{c\sqrt{}}\right.}{}$

input
```
int((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/90/(c*x^4+b*x^2)^(3/2)/x^(3/2)*(c*x^2+b)*(462*A*((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b
*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(
1/2))*b*c^2*x^4-231*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF((
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2*x^4-378*B*((c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2),1/2*2^(1/2))*b^2*c*x^4+189*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))
^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*
c*x^4-462*A*c^3*x^6+378*x^6*B*b*c^2-308*A*b*c^2*x^4+252*x^4*B*b^2*c+44*A*b
^2*c*x^2-36*b^3*B*x^2-20*b^3*A)/b^4
```

3.268.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.40

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{21\left((9\,Bbc^2-11\,Ac^3)x^8+(9\,Bb^2c-11\,Abc^2)x^6\right)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weie}\right.}{}$$

input
```
integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/45*(21*((9*B*b*c^2 - 11*A*c^3)*x^8 + (9*B*b^2*c - 11*A*b*c^2)*x^6)*sqrt(
c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)) + (21*(9*
B*b*c^2 - 11*A*c^3)*x^6 + 14*(9*B*b^2*c - 11*A*b*c^2)*x^4 - 10*A*b^3 - 2*(
9*B*b^3 - 11*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^4*c*x^8 + b^5*x
^6)
```

3.268.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)/x**(5/2)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Timed out
```

3.268.7 Maxima [F]

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{Bx^2+A}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^(5/2)), x)
```

3.268.8 Giac [F]

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{Bx^2+A}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^(5/2)), x)
```

3.268.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{B\,x^2+A}{x^{5/2}\,\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

input
```
int((A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int((A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

3.269 $\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx$

3.269.1 Optimal result

Integrand size = 24, antiderivative size = 96

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx=\frac{Ab^3x^{7+m}}{7+m}+\frac{b^2(bB+3Ac)x^{9+m}}{9+m}+\frac{3bc(bB+Ac)x^{11+m}}{11+m}+\frac{c^2(3bB+Ac)x^{13+m}}{13+m}+\frac{Bc^3x^{15+m}}{15+m}$$

output
```
A*b^3*x^(7+m)/(7+m)+b^2*(3*A*c+B*b)*x^(9+m)/(9+m)+3*b*c*(A*c+B*b)*x^(11+m)
/(11+m)+c^2*(A*c+3*B*b)*x^(13+m)/(13+m)+B*c^3*x^(15+m)/(15+m)
```

3.269.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.93

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx=x^{7+m}\left(\frac{Ab^3}{7+m}+\frac{b^2(bB+3Ac)x^2}{9+m}+\frac{3bc(bB+Ac)x^4}{11+m}+\frac{c^2(3bB+Ac)x^6}{13+m}+\frac{Bc^3x^8}{15+m}\right)$$

input
```
Integrate[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

output
```
x^(7 + m)*((A*b^3)/(7 + m) + (b^2*(b*B + 3*A*c)*x^2)/(9 + m) + (3*b*c*(b*B
 + A*c)*x^4)/(11 + m) + (c^2*(3*b*B + A*c)*x^6)/(13 + m) + (B*c^3*x^8)/(15
 + m))
```

3.269.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(A+Bx^2\right) \left(bx^2+cx^4\right)^3 \, dx$$

$$\Big\downarrow \; 9$$

$$\int x^{m+6}\left(A+Bx^2\right) \left(b+cx^2\right)^3 dx$$

$$\Big\downarrow \; 355$$

$$\int \left(Ab^3x^{m+6}+b^2x^{m+8}(3Ac+bB)+c^2x^{m+12}(Ac+3bB)+3bcx^{m+10}(Ac+bB)+Bc^3x^{m+14}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{Ab^3x^{m+7}}{m+7}+\frac{b^2x^{m+9}(3Ac+bB)}{m+9}+\frac{c^2x^{m+13}(Ac+3bB)}{m+13}+\frac{3bcx^{m+11}(Ac+bB)}{m+11}+\frac{Bc^3x^{m+15}}{m+15}$$

input
```
Int[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

output
```
(A*b^3*x^(7 + m))/(7 + m) + (b^2*(b*B + 3*A*c)*x^(9 + m))/(9 + m) + (3*b*c
*(b*B + A*c)*x^(11 + m))/(11 + m) + (c^2*(3*b*B + A*c)*x^(13 + m))/(13 + m
) + (B*c^3*x^(15 + m))/(15 + m)
```

3.269.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.269.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(96) = 192.

Time = 2.05 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.94

method	result
gosper	$\frac{x^{7+m}\left(B\,c^{3}m^{4}x^{8}+40B\,c^{3}m^{3}x^{8}+A\,c^{3}m^{4}x^{6}+3Bb\,c^{2}m^{4}x^{6}+590B\,c^{3}m^{2}x^{8}+42A\,c^{3}m^{3}x^{6}+126Bb\,c^{2}m^{3}x^{6}+3800m\,x^{8}B\,c^{3}+3Ab\right.}{}$
risch	$\frac{x^{m}\left(B\,c^{3}m^{4}x^{8}+40B\,c^{3}m^{3}x^{8}+A\,c^{3}m^{4}x^{6}+3Bb\,c^{2}m^{4}x^{6}+590B\,c^{3}m^{2}x^{8}+42A\,c^{3}m^{3}x^{6}+126Bb\,c^{2}m^{3}x^{6}+3800m\,x^{8}B\,c^{3}+3Ab\,c^{2}\right.}{}$
parallelrisch	$\frac{3B\,x^{13}x^{m}b\,c^{2}m^{4}+126B\,x^{13}x^{m}b\,c^{2}m^{3}+3A\,x^{11}x^{m}b\,c^{2}m^{4}+1932B\,x^{13}x^{m}b\,c^{2}m^{2}+3B\,x^{11}x^{m}b^{2}c\,m^{4}+132A\,x^{11}x^{m}b\,c^{2}m^{3}+1283}{}$

input
```
int(x^m*(B*x^2+A)*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
x^(7+m)/(7+m)/(9+m)/(11+m)/(13+m)/(15+m)*(B*c^3*m^4*x^8+40*B*c^3*m^3*x^8+A
*c^3*m^4*x^6+3*B*b*c^2*m^4*x^6+590*B*c^3*m^2*x^8+42*A*c^3*m^3*x^6+126*B*b*
c^2*m^3*x^6+3800*B*c^3*m*x^8+3*A*b*c^2*m^4*x^4+644*A*c^3*m^2*x^6+3*B*b^2*c
*m^4*x^4+1932*B*b*c^2*m^2*x^6+9009*B*c^3*x^8+132*A*b*c^2*m^3*x^4+4278*A*c^
3*m*x^6+132*B*b^2*c*m^3*x^4+12834*B*b*c^2*m*x^6+3*A*b^2*c*m^4*x^2+2118*A*b
*c^2*m^2*x^4+10395*A*c^3*x^6+B*b^3*m^4*x^2+2118*B*b^2*c*m^2*x^4+31185*B*b*
c^2*x^6+138*A*b^2*c*m^3*x^2+14652*A*b*c^2*m*x^4+46*B*b^3*m^3*x^2+14652*B*b
^2*c*m*x^4+A*b^3*m^4+2328*A*b^2*c*m^2*x^2+36855*A*b*c^2*x^4+776*B*b^3*m^2*
x^2+36855*B*b^2*c*x^4+48*A*b^3*m^3+16998*A*b^2*c*m*x^2+5666*B*b^3*m*x^2+85
4*A*b^3*m^2+45045*A*b^2*c*x^2+15015*B*b^3*x^2+6672*A*b^3*m+19305*A*b^3)
```

3.269.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(96) = 192.

Time = 0.42 (sec) , antiderivative size = 381, normalized size of antiderivative = 3.97

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx$$
$$=\frac{((Bc^3m^4+40\,Bc^3m^3+590\,Bc^3m^2+3800\,Bc^3m+9009\,Bc^3)x^{15}+((3\,Bbc^2+Ac^3)m^4+31185\,Bbc^2+1}{}$$

input
```
integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
((B*c^3*m^4 + 40*B*c^3*m^3 + 590*B*c^3*m^2 + 3800*B*c^3*m + 9009*B*c^3)*x^
15 + ((3*B*b*c^2 + A*c^3)*m^4 + 31185*B*b*c^2 + 10395*A*c^3 + 42*(3*B*b*c^
2 + A*c^3)*m^3 + 644*(3*B*b*c^2 + A*c^3)*m^2 + 4278*(3*B*b*c^2 + A*c^3)*m)
*x^13 + 3*((B*b^2*c + A*b*c^2)*m^4 + 12285*B*b^2*c + 12285*A*b*c^2 + 44*(B
*b^2*c + A*b*c^2)*m^3 + 706*(B*b^2*c + A*b*c^2)*m^2 + 4884*(B*b^2*c + A*b*
c^2)*m)*x^11 + ((B*b^3 + 3*A*b^2*c)*m^4 + 15015*B*b^3 + 45045*A*b^2*c + 46
*(B*b^3 + 3*A*b^2*c)*m^3 + 776*(B*b^3 + 3*A*b^2*c)*m^2 + 5666*(B*b^3 + 3*A
*b^2*c)*m)*x^9 + (A*b^3*m^4 + 48*A*b^3*m^3 + 854*A*b^3*m^2 + 6672*A*b^3*m
+ 19305*A*b^3)*x^7)*x^m/(m^5 + 55*m^4 + 1190*m^3 + 12650*m^2 + 66009*m + 1
35135)
```

3.269.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2077 vs. $2(87) = 174$.

Time = 1.06 (sec) , antiderivative size = 2077, normalized size of antiderivative = 21.64

$$\int x^m (A + Bx^2) (bx^2 + cx^4)^3 \, dx = \text{Too large to display}$$

input
```
integrate(x**m*(B*x**2+A)*(c*x**4+b*x**2)**3,x)
```

output
```
Piecewise((-A*b**3/(8*x**8) - A*b**2*c/(2*x**6) - 3*A*b*c**2/(4*x**4) - A*
c**3/(2*x**2) - B*b**3/(6*x**6) - 3*B*b**2*c/(4*x**4) - 3*B*b*c**2/(2*x**2
) + B*c**3*log(x), Eq(m, -15)), (-A*b**3/(6*x**6) - 3*A*b**2*c/(4*x**4) -
3*A*b*c**2/(2*x**2) + A*c**3*log(x) - B*b**3/(4*x**4) - 3*B*b**2*c/(2*x**2
) + 3*B*b*c**2*log(x) + B*c**3*x**2/2, Eq(m, -13)), (-A*b**3/(4*x**4) - 3*
A*b**2*c/(2*x**2) + 3*A*b*c**2*log(x) + A*c**3*x**2/2 - B*b**3/(2*x**2) +
3*B*b**2*c*log(x) + 3*B*b*c**2*x**2/2 + B*c**3*x**4/4, Eq(m, -11)), (-A*b*
*3/(2*x**2) + 3*A*b**2*c*log(x) + 3*A*b*c**2*x**2/2 + A*c**3*x**4/4 + B*b*
*3*log(x) + 3*B*b**2*c*x**2/2 + 3*B*b*c**2*x**4/4 + B*c**3*x**6/6, Eq(m, -
9)), (A*b**3*log(x) + 3*A*b**2*c*x**2/2 + 3*A*b*c**2*x**4/4 + A*c**3*x**6/
6 + B*b**3*x**2/2 + 3*B*b**2*c*x**4/4 + B*b*c**2*x**6/2 + B*c**3*x**8/8, E
q(m, -7)), (A*b**3*m**4*x**7*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2
 + 66009*m + 135135) + 48*A*b**3*m**3*x**7*x**m/(m**5 + 55*m**4 + 1190*m**
3 + 12650*m**2 + 66009*m + 135135) + 854*A*b**3*m**2*x**7*x**m/(m**5 + 55*
m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 6672*A*b**3*m*x**7*x**
m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 19305*A*b
**3*x**7*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135)
 + 3*A*b**2*c*m**4*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66
009*m + 135135) + 138*A*b**2*c*m**3*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3
+ 12650*m**2 + 66009*m + 135135) + 2328*A*b**2*c*m**2*x**9*x**m/(m**5 +...
```

3.269.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.34

$$\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{Bc^3x^{m+15}}{m+15}+\frac{3\,Bbc^2x^{m+13}}{m+13}+\frac{Ac^3x^{m+13}}{m+13}+\frac{3\,Bb^2cx^{m+11}}{m+11}+\frac{3\,Abc^2x^{m+11}}{m+11}+\frac{Bb^3x^{m+9}}{m+9}+\frac{3\,Ab^2cx^{m+9}}{m+9}+\frac{Ab^3x^{m+7}}{m+7}$$

input
```
integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
B*c^3*x^(m + 15)/(m + 15) + 3*B*b*c^2*x^(m + 13)/(m + 13) + A*c^3*x^(m + 1
3)/(m + 13) + 3*B*b^2*c*x^(m + 11)/(m + 11) + 3*A*b*c^2*x^(m + 11)/(m + 11
) + B*b^3*x^(m + 9)/(m + 9) + 3*A*b^2*c*x^(m + 9)/(m + 9) + A*b^3*x^(m + 7
)/(m + 7)
```

3.269.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(96) = 192.

Time = 0.29 (sec) , antiderivative size = 603, normalized size of antiderivative = 6.28

$$\int x^m (A+Bx^2) (bx^2+cx^4)^3 \, dx = \frac{Bc^3m^4x^{15}x^m + 40\,Bc^3m^3x^{15}x^m + 3\,Bbc^2m^4x^{13}x^m + Ac^3m^4x^{13}x^m + 590\,Bc^3m^2x^{15}x^m + 126\,Bbc^2m^3x^{13}x}{}$$

input
```
integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
(B*c^3*m^4*x^15*x^m + 40*B*c^3*m^3*x^15*x^m + 3*B*b*c^2*m^4*x^13*x^m + A*c
^3*m^4*x^13*x^m + 590*B*c^3*m^2*x^15*x^m + 126*B*b*c^2*m^3*x^13*x^m + 42*A
*c^3*m^3*x^13*x^m + 3800*B*c^3*m*x^15*x^m + 3*B*b^2*c*m^4*x^11*x^m + 3*A*b
*c^2*m^4*x^11*x^m + 1932*B*b*c^2*m^2*x^13*x^m + 644*A*c^3*m^2*x^13*x^m + 9
009*B*c^3*x^15*x^m + 132*B*b^2*c*m^3*x^11*x^m + 132*A*b*c^2*m^3*x^11*x^m +
 12834*B*b*c^2*m*x^13*x^m + 4278*A*c^3*m*x^13*x^m + B*b^3*m^4*x^9*x^m + 3*
A*b^2*c*m^4*x^9*x^m + 2118*B*b^2*c*m^2*x^11*x^m + 2118*A*b*c^2*m^2*x^11*x^
m + 31185*B*b*c^2*x^13*x^m + 10395*A*c^3*x^13*x^m + 46*B*b^3*m^3*x^9*x^m +
 138*A*b^2*c*m^3*x^9*x^m + 14652*B*b^2*c*m*x^11*x^m + 14652*A*b*c^2*m*x^11
*x^m + A*b^3*m^4*x^7*x^m + 776*B*b^3*m^2*x^9*x^m + 2328*A*b^2*c*m^2*x^9*x^
m + 36855*B*b^2*c*x^11*x^m + 36855*A*b*c^2*x^11*x^m + 48*A*b^3*m^3*x^7*x^m
 + 5666*B*b^3*m*x^9*x^m + 16998*A*b^2*c*m*x^9*x^m + 854*A*b^3*m^2*x^7*x^m
+ 15015*B*b^3*x^9*x^m + 45045*A*b^2*c*x^9*x^m + 6672*A*b^3*m*x^7*x^m + 193
05*A*b^3*x^7*x^m)/(m^5 + 55*m^4 + 1190*m^3 + 12650*m^2 + 66009*m + 135135)
```

3.269.9 Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.03

$$\begin{aligned}\int x^m (A+Bx^2) (bx^2+cx^4)^3 \, dx &= \frac{A\,b^3\,x^m\,x^7\,(m^4+48\,m^3+854\,m^2+6672\,m+19305)}{m^5+55\,m^4+1190\,m^3+12650\,m^2+66009\,m+135135}\\ &\quad+\frac{B\,c^3\,x^m\,x^{15}\,(m^4+40\,m^3+590\,m^2+3800\,m+9009)}{m^5+55\,m^4+1190\,m^3+12650\,m^2+66009\,m+135135}\\ &\quad+\frac{b^2\,x^m\,x^9\,(3\,A\,c+B\,b)\,(m^4+46\,m^3+776\,m^2+5666\,m+15015)}{m^5+55\,m^4+1190\,m^3+12650\,m^2+66009\,m+135135}\\ &\quad+\frac{c^2\,x^m\,x^{13}\,(A\,c+3\,B\,b)\,(m^4+42\,m^3+644\,m^2+4278\,m+10395)}{m^5+55\,m^4+1190\,m^3+12650\,m^2+66009\,m+135135}\\ &\quad+\frac{3\,b\,c\,x^m\,x^{11}\,(A\,c+B\,b)\,(m^4+44\,m^3+706\,m^2+4884\,m+12285)}{m^5+55\,m^4+1190\,m^3+12650\,m^2+66009\,m+135135}\end{aligned}$$

input
```
int(x^m*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)
```

output
```
(A*b^3*x^m*x^7*(6672*m + 854*m^2 + 48*m^3 + m^4 + 19305))/(66009*m + 12650
*m^2 + 1190*m^3 + 55*m^4 + m^5 + 135135) + (B*c^3*x^m*x^15*(3800*m + 590*m
^2 + 40*m^3 + m^4 + 9009))/(66009*m + 12650*m^2 + 1190*m^3 + 55*m^4 + m^5
+ 135135) + (b^2*x^m*x^9*(3*A*c + B*b)*(5666*m + 776*m^2 + 46*m^3 + m^4 +
15015))/(66009*m + 12650*m^2 + 1190*m^3 + 55*m^4 + m^5 + 135135) + (c^2*x^
m*x^13*(A*c + 3*B*b)*(4278*m + 644*m^2 + 42*m^3 + m^4 + 10395))/(66009*m +
 12650*m^2 + 1190*m^3 + 55*m^4 + m^5 + 135135) + (3*b*c*x^m*x^11*(A*c + B*
b)*(4884*m + 706*m^2 + 44*m^3 + m^4 + 12285))/(66009*m + 12650*m^2 + 1190*
m^3 + 55*m^4 + m^5 + 135135)
```

3.269. $\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx$

3.270 $\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$

3.270.1 Optimal result

Integrand size = 24, antiderivative size = 71

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx=\frac{Ab^2x^{5+m}}{5+m}+\frac{b(bB+2Ac)x^{7+m}}{7+m}+\frac{c(2bB+Ac)x^{9+m}}{9+m}+\frac{Bc^2x^{11+m}}{11+m}$$

output
```
A*b^2*x^(5+m)/(5+m)+b*(2*A*c+B*b)*x^(7+m)/(7+m)+c*(A*c+2*B*b)*x^(9+m)/(9+m
)+B*c^2*x^(11+m)/(11+m)
```

3.270.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx=x^{5+m}\left(\frac{Ab^2}{5+m}+\frac{b(bB+2Ac)x^2}{7+m}+\frac{c(2bB+Ac)x^4}{9+m}+\frac{Bc^2x^6}{11+m}\right)$$

input
```
Integrate[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

output
```
x^(5 + m)*((A*b^2)/(5 + m) + (b*(b*B + 2*A*c)*x^2)/(7 + m) + (c*(2*b*B + A
*c)*x^4)/(9 + m) + (B*c^2*x^6)/(11 + m))
```

3.270.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (A+Bx^2) \left(bx^2+cx^4\right)^2 \, dx$$

$$\Big\downarrow \; 9$$

$$\int x^{m+4} (A+Bx^2) \left(b+cx^2\right)^2 dx$$

$$\Big\downarrow \; 355$$

$$\int \left(Ab^2x^{m+4}+bx^{m+6}(2Ac+bB)+cx^{m+8}(Ac+2bB)+Bc^2x^{m+10}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{Ab^2x^{m+5}}{m+5}+\frac{bx^{m+7}(2Ac+bB)}{m+7}+\frac{cx^{m+9}(Ac+2bB)}{m+9}+\frac{Bc^2x^{m+11}}{m+11}$$

input
```
Int[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

output
```
(A*b^2*x^(5 + m))/(5 + m) + (b*(b*B + 2*A*c)*x^(7 + m))/(7 + m) + (c*(2*b*
B + A*c)*x^(9 + m))/(9 + m) + (B*c^2*x^(11 + m))/(11 + m)
```

3.270.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.270.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(71) = 142$.

Time = 2.11 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.69

method	result
gosper	$\frac{x^{5+m}(B\,c^2m^3x^6+21B\,c^2m^2x^6+A\,c^2m^3x^4+2Bbc\,m^3x^4+143m\,x^6B\,c^2+23A\,c^2m^2x^4+46Bbc\,m^2x^4+315B\,c^2x^6+2Abc\,m^3x^2}{}$
risch	$\frac{x^m(B\,c^2m^3x^6+21B\,c^2m^2x^6+A\,c^2m^3x^4+2Bbc\,m^3x^4+143m\,x^6B\,c^2+23A\,c^2m^2x^4+46Bbc\,m^2x^4+315B\,c^2x^6+2Abc\,m^3x^2+1}{}$
parallelrisch	$\frac{2B\,x^9x^mbc\,m^3+46B\,x^9x^mbc\,m^2+25B\,x^7x^mb^2m^2+A\,x^5x^mb^2m^3+199B\,x^7x^mb^2m+990A\,x^7x^mbc+27A\,x^5x^mb^2m^2+239A\,x^5}{}$

input
```
int(x^m*(B*x^2+A)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
x^(5+m)/(5+m)/(7+m)/(9+m)/(11+m)*(B*c^2*m^3*x^6+21*B*c^2*m^2*x^6+A*c^2*m^3
*x^4+2*B*b*c*m^3*x^4+143*B*c^2*m*x^6+23*A*c^2*m^2*x^4+46*B*b*c*m^2*x^4+315
*B*c^2*x^6+2*A*b*c*m^3*x^2+167*A*c^2*m*x^4+B*b^2*m^3*x^2+334*B*b*c*m*x^4+5
0*A*b*c*m^2*x^2+385*A*c^2*x^4+25*B*b^2*m^2*x^2+770*B*b*c*x^4+A*b^2*m^3+398
*A*b*c*m*x^2+199*B*b^2*m*x^2+27*A*b^2*m^2+990*A*b*c*x^2+495*B*b^2*x^2+239*
A*b^2*m+693*A*b^2)
```

3.270.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(71) = 142$.

Time = 0.31 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.06

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx$$
$$=\frac{((Bc^2m^3+21\,Bc^2m^2+143\,Bc^2m+315\,Bc^2)x^{11}+((2\,Bbc+Ac^2)m^3+770\,Bbc+385\,Ac^2+23\,(2\,Bbc+}{}$$

input
```
integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
((B*c^2*m^3 + 21*B*c^2*m^2 + 143*B*c^2*m + 315*B*c^2)*x^11 + ((2*B*b*c + A
*c^2)*m^3 + 770*B*b*c + 385*A*c^2 + 23*(2*B*b*c + A*c^2)*m^2 + 167*(2*B*b*
c + A*c^2)*m)*x^9 + ((B*b^2 + 2*A*b*c)*m^3 + 495*B*b^2 + 990*A*b*c + 25*(B
*b^2 + 2*A*b*c)*m^2 + 199*(B*b^2 + 2*A*b*c)*m)*x^7 + (A*b^2*m^3 + 27*A*b^2
*m^2 + 239*A*b^2*m + 693*A*b^2)*x^5)*x^m/(m^4 + 32*m^3 + 374*m^2 + 1888*m
+ 3465)
```

3.270.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1051 vs. $2(63) = 126$.

Time = 0.67 (sec) , antiderivative size = 1051, normalized size of antiderivative = 14.80

$$\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$$
$$= \begin{cases} -\frac{Ab^2}{6x^6}-\frac{Abc}{2x^4}-\frac{Ac^2}{2x^2}-\frac{Bb^2}{4x^4}-\frac{Bbc}{x^2}+Bc^2\log(x) \\ -\frac{Ab^2}{4x^4}-\frac{Abc}{x^2}+Ac^2\log(x)-\frac{Bb^2}{2x^2}+2Bbc\log(x)+\frac{Bc^2x^2}{2} \\ -\frac{Ab^2}{2x^2}+2Abc\log(x)+\frac{Ac^2x^2}{2}+Bb^2\log(x)+Bbcx^2+\frac{Bc^2x^4}{4} \\ Ab^2\log(x)+Abcx^2+\frac{Ac^2x^4}{4}+\frac{Bb^2x^2}{2}+\frac{Bbcx^4}{2}+\frac{Bc^2x^6}{6} \\ \frac{Ab^2m^3x^5x^m}{m^4+32m^3+374m^2+1888m+3465}+\frac{27Ab^2m^2x^5x^m}{m^4+32m^3+374m^2+1888m+3465}+\frac{239Ab^2mx^5x^m}{m^4+32m^3+374m^2+1888m+3465}+\frac{693Ab^2x^5x^m}{m^4+32m^3+374m^2+188} \end{cases}$$

input
```
integrate(x**m*(B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

output
```
Piecewise((-A*b**2/(6*x**6) - A*b*c/(2*x**4) - A*c**2/(2*x**2) - B*b**2/(4
*x**4) - B*b*c/x**2 + B*c**2*log(x), Eq(m, -11)), (-A*b**2/(4*x**4) - A*b*
c/x**2 + A*c**2*log(x) - B*b**2/(2*x**2) + 2*B*b*c*log(x) + B*c**2*x**2/2,
 Eq(m, -9)), (-A*b**2/(2*x**2) + 2*A*b*c*log(x) + A*c**2*x**2/2 + B*b**2*l
og(x) + B*b*c*x**2 + B*c**2*x**4/4, Eq(m, -7)), (A*b**2*log(x) + A*b*c*x**
2 + A*c**2*x**4/4 + B*b**2*x**2/2 + B*b*c*x**4/2 + B*c**2*x**6/6, Eq(m, -5
)), (A*b**2*m**3*x**5*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 2
7*A*b**2*m**2*x**5*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 239*
A*b**2*m*x**5*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 693*A*b**
2*x**5*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 2*A*b*c*m**3*x**
7*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 50*A*b*c*m**2*x**7*x*
*m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 398*A*b*c*m*x**7*x**m/(m*
*4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 990*A*b*c*x**7*x**m/(m**4 + 32*
m**3 + 374*m**2 + 1888*m + 3465) + A*c**2*m**3*x**9*x**m/(m**4 + 32*m**3 +
 374*m**2 + 1888*m + 3465) + 23*A*c**2*m**2*x**9*x**m/(m**4 + 32*m**3 + 37
4*m**2 + 1888*m + 3465) + 167*A*c**2*m*x**9*x**m/(m**4 + 32*m**3 + 374*m**
2 + 1888*m + 3465) + 385*A*c**2*x**9*x**m/(m**4 + 32*m**3 + 374*m**2 + 188
8*m + 3465) + B*b**2*m**3*x**7*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m +
3465) + 25*B*b**2*m**2*x**7*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 346
5) + 199*B*b**2*m*x**7*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465)...
```

3.270.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.28

$$\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx = \frac{Bc^2x^{m+11}}{m+11}+\frac{2\,Bbcx^{m+9}}{m+9}+\frac{Ac^2x^{m+9}}{m+9}+\frac{Bb^2x^{m+7}}{m+7}+\frac{2\,Abcx^{m+7}}{m+7}+\frac{Ab^2x^{m+5}}{m+5}$$

input
```
integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
B*c^2*x^(m + 11)/(m + 11) + 2*B*b*c*x^(m + 9)/(m + 9) + A*c^2*x^(m + 9)/(m
 + 9) + B*b^2*x^(m + 7)/(m + 7) + 2*A*b*c*x^(m + 7)/(m + 7) + A*b^2*x^(m +
 5)/(m + 5)
```

3.270.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(71) = 142$.

Time = 0.27 (sec) , antiderivative size = 340, normalized size of antiderivative = 4.79

$$\int x^m (A + Bx^2) \left(bx^2 + cx^4\right)^2 \, dx = \frac{Bc^2m^3x^{11}x^m + 21\,Bc^2m^2x^{11}x^m + 2\,Bbcm^3x^9x^m + Ac^2m^3x^9x^m + 143\,Bc^2mx^{11}x^m + 46\,Bbcm^2x^9x^m + 23}{}$$

input
```
integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
(B*c^2*m^3*x^11*x^m + 21*B*c^2*m^2*x^11*x^m + 2*B*b*c*m^3*x^9*x^m + A*c^2*
m^3*x^9*x^m + 143*B*c^2*m*x^11*x^m + 46*B*b*c*m^2*x^9*x^m + 23*A*c^2*m^2*x
^9*x^m + 315*B*c^2*x^11*x^m + B*b^2*m^3*x^7*x^m + 2*A*b*c*m^3*x^7*x^m + 33
4*B*b*c*m*x^9*x^m + 167*A*c^2*m*x^9*x^m + 25*B*b^2*m^2*x^7*x^m + 50*A*b*c*
m^2*x^7*x^m + 770*B*b*c*x^9*x^m + 385*A*c^2*x^9*x^m + A*b^2*m^3*x^5*x^m +
199*B*b^2*m*x^7*x^m + 398*A*b*c*m*x^7*x^m + 27*A*b^2*m^2*x^5*x^m + 495*B*b
^2*x^7*x^m + 990*A*b*c*x^7*x^m + 239*A*b^2*m*x^5*x^m + 693*A*b^2*x^5*x^m)/
(m^4 + 32*m^3 + 374*m^2 + 1888*m + 3465)
```

3.270.9 Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.52

$$\int x^m (A + Bx^2) \left(bx^2 + cx^4\right)^2 \, dx = x^m \left(\frac{A\,b^2\,x^5\,(m^3 + 27\,m^2 + 239\,m + 693)}{m^4 + 32\,m^3 + 374\,m^2 + 1888\,m + 3465} + \frac{B\,c^2\,x^{11}\,(m^3 + 21\,m^2 + 143\,m + 315)}{m^4 + 32\,m^3 + 374\,m^2 + 1888\,m + 3465} + \frac{b\,x^7\,(2\,A\,c + B\,b)\,(m^3 + 25\,m^2 + 199\,m + 495)}{m^4 + 32\,m^3 + 374\,m^2 + 1888\,m + 3465} + \frac{c\,x^9\,(A\,c + 2\,B\,b)\,(m^3 + 23\,m^2 + 167\,m + 385)}{m^4 + 32\,m^3 + 374\,m^2 + 1888\,m + 3465}\right)$$

input
```
int(x^m*(A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

output
```
x^m*((A*b^2*x^5*(239*m + 27*m^2 + m^3 + 693))/(1888*m + 374*m^2 + 32*m^3 +
 m^4 + 3465) + (B*c^2*x^11*(143*m + 21*m^2 + m^3 + 315))/(1888*m + 374*m^2
 + 32*m^3 + m^4 + 3465) + (b*x^7*(2*A*c + B*b)*(199*m + 25*m^2 + m^3 + 495
))/(1888*m + 374*m^2 + 32*m^3 + m^4 + 3465) + (c*x^9*(A*c + 2*B*b)*(167*m
+ 23*m^2 + m^3 + 385))/(1888*m + 374*m^2 + 32*m^3 + m^4 + 3465))
```

3.270. $\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$

3.271 $\int x^m (A + Bx^2) (bx^2 + cx^4) \, dx$

3.271.1 Optimal result

Integrand size = 22, antiderivative size = 45

$$\int x^m (A + Bx^2) (bx^2 + cx^4) \, dx = \frac{Abx^{3+m}}{3+m} + \frac{(bB + Ac)x^{5+m}}{5+m} + \frac{Bcx^{7+m}}{7+m}$$

output
```
A*b*x^(3+m)/(3+m)+(A*c+B*b)*x^(5+m)/(5+m)+B*c*x^(7+m)/(7+m)
```

3.271.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int x^m (A + Bx^2) (bx^2 + cx^4) \, dx = x^{3+m} \left(\frac{Ab}{3+m} + \frac{(bB + Ac)x^2}{5+m} + \frac{Bcx^4}{7+m} \right)$$

input
```
Integrate[x^m*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

output
```
x^(3 + m)*((A*b)/(3 + m) + ((b*B + A*c)*x^2)/(5 + m) + (B*c*x^4)/(7 + m))
```

3.271.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {9, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (A+Bx^2) (bx^2+cx^4) \, dx$$

$$\Big\downarrow \; 9$$

$$\int x^{m+2} (A+Bx^2) (b+cx^2) \, dx$$

$$\Big\downarrow \; 355$$

$$\int \left(x^{m+4}(Ac+bB)+Abx^{m+2}+Bcx^{m+6}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^{m+5}(Ac+bB)}{m+5}+\frac{Abx^{m+3}}{m+3}+\frac{Bcx^{m+7}}{m+7}$$

input
```
Int[x^m*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

output
```
(A*b*x^(3 + m))/(3 + m) + ((b*B + A*c)*x^(5 + m))/(5 + m) + (B*c*x^(7 + m)
)/(7 + m)
```

3.271.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.271.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

method	result
norman	$\frac{(Ac+Bb)x^5 e^{m\ln(x)}}{5+m}+\frac{Ab\,x^3 e^{m\ln(x)}}{3+m}+\frac{Bc\,x^7 e^{m\ln(x)}}{7+m}$
gosper	$\frac{x^{3+m}\left(Bc\,m^2x^4+8Bcm\,x^4+Ac\,m^2x^2+Bb\,m^2x^2+15Bc\,x^4+10Acm\,x^2+10Bbm\,x^2+Ab\,m^2+21Ac\,x^2+21bB\,x^2+12Abm+35A\right.}{(3+m)(5+m)(7+m)}$
risch	$\frac{x^m\left(Bc\,m^2x^4+8Bcm\,x^4+Ac\,m^2x^2+Bb\,m^2x^2+15Bc\,x^4+10Acm\,x^2+10Bbm\,x^2+Ab\,m^2+21Ac\,x^2+21bB\,x^2+12Abm+35Ab\right)x}{(7+m)(5+m)(3+m)}$
parallelrisch	$\frac{B\,x^7x^mc\,m^2+8B\,x^7x^mcm+A\,x^5x^mc\,m^2+15B\,x^7x^mc+B\,x^5x^mb\,m^2+10A\,x^5x^mcm+10B\,x^5x^mbm+21A\,x^5x^mc+A\,x^3x^mb\,m}{(7+m)(5+m)(3+m)}$

input
```
int(x^m*(B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
(A*c+B*b)/(5+m)*x^5*exp(m*ln(x))+A*b/(3+m)*x^3*exp(m*ln(x))+B*c/(7+m)*x^7*
exp(m*ln(x))
```

3.271.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(45) = 90.

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.09

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx$$
$$=\frac{((Bcm^2+8\,Bcm+15\,Bc)x^7+((Bb+Ac)m^2+21\,Bb+21\,Ac+10\,(Bb+Ac)m)x^5+(Abm^2+12\,Abm}{m^3+15\,m^2+71\,m+105}$$

input
```
integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
((B*c*m^2 + 8*B*c*m + 15*B*c)*x^7 + ((B*b + A*c)*m^2 + 21*B*b + 21*A*c + 1
0*(B*b + A*c)*m)*x^5 + (A*b*m^2 + 12*A*b*m + 35*A*b)*x^3)*x^m/(m^3 + 15*m^
2 + 71*m + 105)
```

3.271.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(37) = 74.

Time = 0.40 (sec) , antiderivative size = 415, normalized size of antiderivative = 9.22

$$\int x^m (A+Bx^2)(bx^2+cx^4)\,dx$$
$$= \begin{cases} -\frac{Ab}{4x^4}-\frac{Ac}{2x^2}-\frac{Bb}{2x^2}+Bc\log(x) \\ -\frac{Ab}{2x^2}+Ac\log(x)+Bb\log(x)+\frac{Bcx^2}{2} \\ Ab\log(x)+\frac{Acx^2}{2}+\frac{Bbx^2}{2}+\frac{Bcx^4}{4} \\ \frac{Abm^2x^3x^m}{m^3+15m^2+71m+105}+\frac{12Abmx^3x^m}{m^3+15m^2+71m+105}+\frac{35Abx^3x^m}{m^3+15m^2+71m+105}+\frac{Acm^2x^5x^m}{m^3+15m^2+71m+105}+\frac{10Acmx^5x^m}{m^3+15m^2+71m+105}+\frac{2}{m^3+15} \end{cases}$$

input
```
integrate(x**m*(B*x**2+A)*(c*x**4+b*x**2),x)
```

output
```
Piecewise((-A*b/(4*x**4) - A*c/(2*x**2) - B*b/(2*x**2) + B*c*log(x), Eq(m,
 -7)), (-A*b/(2*x**2) + A*c*log(x) + B*b*log(x) + B*c*x**2/2, Eq(m, -5)),
(A*b*log(x) + A*c*x**2/2 + B*b*x**2/2 + B*c*x**4/4, Eq(m, -3)), (A*b*m**2*
x**3*x**m/(m**3 + 15*m**2 + 71*m + 105) + 12*A*b*m*x**3*x**m/(m**3 + 15*m*
*2 + 71*m + 105) + 35*A*b*x**3*x**m/(m**3 + 15*m**2 + 71*m + 105) + A*c*m*
*2*x**5*x**m/(m**3 + 15*m**2 + 71*m + 105) + 10*A*c*m*x**5*x**m/(m**3 + 15
*m**2 + 71*m + 105) + 21*A*c*x**5*x**m/(m**3 + 15*m**2 + 71*m + 105) + B*b
*m**2*x**5*x**m/(m**3 + 15*m**2 + 71*m + 105) + 10*B*b*m*x**5*x**m/(m**3 +
 15*m**2 + 71*m + 105) + 21*B*b*x**5*x**m/(m**3 + 15*m**2 + 71*m + 105) +
B*c*m**2*x**7*x**m/(m**3 + 15*m**2 + 71*m + 105) + 8*B*c*m*x**7*x**m/(m**3
 + 15*m**2 + 71*m + 105) + 15*B*c*x**7*x**m/(m**3 + 15*m**2 + 71*m + 105),
 True))
```

3.271.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int x^m (A+Bx^2)(bx^2+cx^4)\,dx = \frac{Bcx^{m+7}}{m+7}+\frac{Bbx^{m+5}}{m+5}+\frac{Acx^{m+5}}{m+5}+\frac{Abx^{m+3}}{m+3}$$

input
```
integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
B*c*x^(m + 7)/(m + 7) + B*b*x^(m + 5)/(m + 5) + A*c*x^(m + 5)/(m + 5) + A*
b*x^(m + 3)/(m + 3)
```

3.271.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(45) = 90$.

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.31

$$\int x^m(A+Bx^2)(bx^2+cx^4)\,dx = \frac{Bcm^2x^7x^m+8\,Bcmx^7x^m+Bbm^2x^5x^m+Acm^2x^5x^m+15\,Bcx^7x^m+10\,Bbmx^5x^m+10\,Acmx^5x^m+Ab}{m^3+15\,m^2+71\,m+105}$$

input
```
integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
(B*c*m^2*x^7*x^m + 8*B*c*m*x^7*x^m + B*b*m^2*x^5*x^m + A*c*m^2*x^5*x^m + 1
5*B*c*x^7*x^m + 10*B*b*m*x^5*x^m + 10*A*c*m*x^5*x^m + A*b*m^2*x^3*x^m + 21
*B*b*x^5*x^m + 21*A*c*x^5*x^m + 12*A*b*m*x^3*x^m + 35*A*b*x^3*x^m)/(m^3 +
15*m^2 + 71*m + 105)
```

3.271.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.16

$$\int x^m(A+Bx^2)(bx^2+cx^4)\,dx = x^m\left(\frac{x^5\,(A\,c+B\,b)\,(m^2+10\,m+21)}{m^3+15\,m^2+71\,m+105} + \frac{A\,b\,x^3\,(m^2+12\,m+35)}{m^3+15\,m^2+71\,m+105} + \frac{B\,c\,x^7\,(m^2+8\,m+15)}{m^3+15\,m^2+71\,m+105}\right)$$

input
```
int(x^m*(A + B*x^2)*(b*x^2 + c*x^4),x)
```

output
```
x^m*((x^5*(A*c + B*b)*(10*m + m^2 + 21))/(71*m + 15*m^2 + m^3 + 105) + (A*
b*x^3*(12*m + m^2 + 35))/(71*m + 15*m^2 + m^3 + 105) + (B*c*x^7*(8*m + m^2
 + 15))/(71*m + 15*m^2 + m^3 + 105))
```

3.272 $\int \frac{x^m(A+Bx^2)}{bx^2+cx^4}\,dx$

3.272.1 Optimal result

Integrand size = 24, antiderivative size = 71

$$\int \frac{x^m(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{Bx^{-1+m}}{c(1-m)} + \frac{(bB-Ac)x^{-1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1+m),\frac{1+m}{2},-\frac{cx^2}{b}\right)}{bc(1-m)}$$

output
```
-B*x^(-1+m)/c/(1-m)+(-A*c+B*b)*x^(-1+m)*hypergeom([1, -1/2+1/2*m],[1/2+1/2
*m],-c*x^2/b)/b/c/(1-m)
```

3.272.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{x^m(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{x^{-1+m}\left(bB+(-bB+Ac)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1+m),\frac{1+m}{2},-\frac{cx^2}{b}\right)\right)}{bc(-1+m)}$$

input
```
Integrate[(x^m*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(x^(-1 + m)*(b*B + (-(b*B) + A*c)*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)
/2, -((c*x^2)/b)]))/(b*c*(-1 + m))
```

3.272.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(A+Bx^2)}{bx^2+cx^4}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{x^{m-2}(A+Bx^2)}{b+cx^2}dx$$
$$\Big\downarrow \text{ 363}$$
$$-\frac{(bB-Ac)\int \frac{x^{m-2}}{cx^2+b}dx}{c}-\frac{Bx^{m-1}}{c(1-m)}$$
$$\Big\downarrow \text{ 278}$$
$$\frac{x^{m-1}(bB-Ac)\,\text{Hypergeometric2F1}\left(1,\frac{m-1}{2},\frac{m+1}{2},-\frac{cx^2}{b}\right)}{bc(1-m)}-\frac{Bx^{m-1}}{c(1-m)}$$

input
```
Int[(x^m*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

output
```
-((B*x^(-1 + m))/(c*(1 - m))) + ((b*B - A*c)*x^(-1 + m)*Hypergeometric2F1[
1, (-1 + m)/2, (1 + m)/2, -((c*x^2)/b)])/(b*c*(1 - m))
```

3.272.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.272.4 Maple [F]

$$\int \frac{x^m(x^2B+A)}{x^4c+b\,x^2}dx$$

input
```
int(x^m*(B*x^2+A)/(c*x^4+b*x^2),x)
```

output
```
int(x^m*(B*x^2+A)/(c*x^4+b*x^2),x)
```

3.272.5 Fricas [F]

$$\int \frac{x^m(A+Bx^2)}{bx^2+cx^4}\,dx = \int \frac{(Bx^2+A)x^m}{cx^4+bx^2}\,dx$$

input
```
integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*x^m/(c*x^4 + b*x^2), x)
```

3.272.6 Sympy [F]

$$\int \frac{x^m(A+Bx^2)}{bx^2+cx^4}\,dx = \int \frac{x^m(A+Bx^2)}{x^2\,(b+cx^2)}\,dx$$

input
```
integrate(x**m*(B*x**2+A)/(c*x**4+b*x**2),x)
```

output
```
Integral(x**m*(A + B*x**2)/(x**2*(b + c*x**2)), x)
```

3.272.7 Maxima [F]

$$\int \frac{x^m(A+Bx^2)}{bx^2+cx^4}\,dx = \int \frac{(Bx^2+A)x^m}{cx^4+bx^2}\,dx$$

input
```
integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^m/(c*x^4 + b*x^2), x)
```

3.272.8 Giac [F]

$$\int \frac{x^m(A+Bx^2)}{bx^2+cx^4}\,dx = \int \frac{(Bx^2+A)x^m}{cx^4+bx^2}\,dx$$

input
```
integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^m/(c*x^4 + b*x^2), x)
```

3.272.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(A+Bx^2)}{bx^2+cx^4}\,dx=\int \frac{x^m\,\left(B\,x^2+A\right)}{c\,x^4+b\,x^2}\,dx$$

input
```
int((x^m*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

output
```
int((x^m*(A + B*x^2))/(b*x^2 + c*x^4), x)
```

3.273 $\int \frac{x^m(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

3.273.1 Optimal result

Integrand size = 24, antiderivative size = 98

$$\int \frac{x^m(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(bB-Ac)x^{-3+m}}{2bc\,(b+cx^2)} + \frac{(bB(3-m)-Ac(5-m))x^{-3+m}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),-\frac{cx^2}{b}\right)}{2b^2c(3-m)}$$

output
```
-1/2*(-A*c+B*b)*x^(-3+m)/b/c/(c*x^2+b)+1/2*(b*B*(3-m)-A*c*(5-m))*x^(-3+m)*
hypergeom([1, -3/2+1/2*m],[-1/2+1/2*m],-c*x^2/b)/b^2/c/(3-m)
```

3.273.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{x^m(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{x^{-3+m}\left(bB\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),-\frac{cx^2}{b}\right)+(-bB+Ac)\,\text{Hypergeometric2F1}\left(2\right.\right.}{b^2c(-3+m)}$$

input
```
Integrate[(x^m*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
(x^(-3 + m)*(b*B*Hypergeometric2F1[1, (-3 + m)/2, (-1 + m)/2, -((c*x^2)/b)
] + (-(b*B) + A*c)*Hypergeometric2F1[2, (-3 + m)/2, (-1 + m)/2, -((c*x^2)/
b)]))/(b^2*c*(-3 + m))
```

3.273.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 362, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (A+Bx^2)}{(bx^2+cx^4)^2}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^{m-4}(A+Bx^2)}{(b+cx^2)^2}dx$$

$$\Big\downarrow \; 362$$

$$-\frac{(bB(3-m)-Ac(5-m))\int \frac{x^{m-4}}{cx^2+b}dx}{2bc}-\frac{x^{m-3}(bB-Ac)}{2bc(b+cx^2)}$$

$$\Big\downarrow \; 278$$

$$\frac{x^{m-3}(bB(3-m)-Ac(5-m))\,\text{Hypergeometric2F1}\left(1,\frac{m-3}{2},\frac{m-1}{2},-\frac{cx^2}{b}\right)}{2b^2c(3-m)}-\frac{x^{m-3}(bB-Ac)}{2bc(b+cx^2)}$$

input
```
Int[(x^m*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*((b*B - A*c)*x^(-3 + m))/(b*c*(b + c*x^2)) + ((b*B*(3 - m) - A*c*(5 -
 m))*x^(-3 + m)*Hypergeometric2F1[1, (-3 + m)/2, (-1 + m)/2, -((c*x^2)/b)]
)/(2*b^2*c*(3 - m))
```

3.273.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

3.273.4 Maple [F]

$$\int \frac{x^m(x^2B+A)}{(x^4c+b\,x^2)^2}dx$$

input
```
int(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

output
```
int(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

3.273.5 Fricas [F]

$$\int \frac{x^m(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{(Bx^2+A)x^m}{(cx^4+bx^2)^2}\,dx$$

input
```
integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*x^m/(c^2*x^8 + 2*b*c*x^6 + b^2*x^4), x)
```

3.273.6 Sympy [F]

$$\int \frac{x^m(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{x^m(A+Bx^2)}{x^4\,(b+cx^2)^2}\,dx$$

input
```
integrate(x**m*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

output
```
Integral(x**m*(A + B*x**2)/(x**4*(b + c*x**2)**2), x)
```

3.273.7 Maxima [F]

$$\int \frac{x^m(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{(Bx^2+A)x^m}{(cx^4+bx^2)^2}\,dx$$

input
```
integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^m/(c*x^4 + b*x^2)^2, x)
```

3.273.8 Giac [F]

$$\int \frac{x^m(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{(Bx^2+A)x^m}{(cx^4+bx^2)^2}\,dx$$

input
```
integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^m/(c*x^4 + b*x^2)^2, x)
```

3.273.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{x^m\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^2}\,dx$$

input
```
int((x^m*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

output
```
int((x^m*(A + B*x^2))/(b*x^2 + c*x^4)^2, x)
```

3.274 $\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^p\,dx$

3.274.1 Optimal result

Integrand size = 24, antiderivative size = 140

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^p\,dx=\frac{Bx^{-1+m}\left(bx^2+cx^4\right)^{1+p}}{c(3+m+4p)}-\frac{(bB(1+m+2p)-Ac(3+m+4p))x^{1+m}\left(1+\frac{cx^2}{b}\right)^{-p}\left(bx^2+cx^4\right)^p\,\text{Hypergeometric2F1}\left(-p,\frac{1}{2}(1+m\right.}{c(1+m+2p)(3+m+4p)}$$

output
```
B*x^(-1+m)*(c*x^4+b*x^2)^(p+1)/c/(3+m+4*p)-(b*B*(1+m+2*p)-A*c*(3+m+4*p))*x
^(1+m)*(c*x^4+b*x^2)^p*hypergeom([-p, 1/2+1/2*m+p],[3/2+1/2*m+p],-c*x^2/b)
/c/(1+m+2*p)/(3+m+4*p)/((1+c*x^2/b)^p)
```

3.274.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^p\,dx=\frac{x^{1+m}\left(x^2\left(b+cx^2\right)\right)^p\left(1+\frac{cx^2}{b}\right)^{-p}\left(A(3+m+2p)\,\text{Hypergeometric2F1}\left(-p,\frac{1}{2}(1+m+2p),\frac{1}{2}(3+m+2p\right.\right.}{(1+m+2p)(3+}$$

input
```
Integrate[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^p,x]
```

output
```
(x^(1 + m)*(x^2*(b + c*x^2))^p*(A*(3 + m + 2*p)*Hypergeometric2F1[-p, (1 +
 m + 2*p)/2, (3 + m + 2*p)/2, -((c*x^2)/b)] + B*(1 + m + 2*p)*x^2*Hypergeo
metric2F1[-p, (3 + m + 2*p)/2, (5 + m + 2*p)/2, -((c*x^2)/b)]))/((1 + m +
2*p)*(3 + m + 2*p)*(1 + (c*x^2)/b)^p)
```

3.274.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1945, 1431, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (A+Bx^2) \left(bx^2+cx^4\right)^p \, dx$$

$$\Big\downarrow \text{1945}$$

$$\left(A-\frac{bB(m+2p+1)}{c(m+4p+3)}\right)\int x^m \left(cx^4+bx^2\right)^p dx+\frac{Bx^{m-1}\left(bx^2+cx^4\right)^{p+1}}{c(m+4p+3)}$$

$$\Big\downarrow \text{1431}$$

$$x^{-2p}\left(b+cx^2\right)^{-p}\left(bx^2+cx^4\right)^p\left(A-\frac{bB(m+2p+1)}{c(m+4p+3)}\right)\int x^{m+2p}\left(cx^2+b\right)^p dx+\frac{Bx^{m-1}\left(bx^2+cx^4\right)^{p+1}}{c(m+4p+3)}$$

$$\Big\downarrow \text{279}$$

$$x^{-2p}\left(\frac{cx^2}{b}+1\right)^{-p}\left(bx^2+cx^4\right)^p\left(A-\frac{bB(m+2p+1)}{c(m+4p+3)}\right)\int x^{m+2p}\left(\frac{cx^2}{b}+1\right)^p dx+\frac{Bx^{m-1}\left(bx^2+cx^4\right)^{p+1}}{c(m+4p+3)}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^{m+1}\left(\frac{cx^2}{b}+1\right)^{-p}\left(bx^2+cx^4\right)^p\left(A-\frac{bB(m+2p+1)}{c(m+4p+3)}\right)\text{Hypergeometric2F1}\left(-p,\frac{1}{2}(m+2p+1),\frac{1}{2}(m+2p+3),-\frac{cx^2}{b}\right)}{m+2p+1}$$
$$\frac{Bx^{m-1}\left(bx^2+cx^4\right)^{p+1}}{c(m+4p+3)}$$

input
```
Int[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^p,x]
```

output
```
(B*x^(-1 + m)*(b*x^2 + c*x^4)^(1 + p))/(c*(3 + m + 4*p)) + ((A - (b*B*(1 +
 m + 2*p))/(c*(3 + m + 4*p)))*x^(1 + m)*(b*x^2 + c*x^4)^p*Hypergeometric2F
1[-p, (1 + m + 2*p)/2, (3 + m + 2*p)/2, -((c*x^2)/b)])/((1 + m + 2*p)*(1 +
 (c*x^2)/b)^p)
```

3.274.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.274.4 Maple [F]

$$\int x^m (x^2B + A) (x^4c + b\,x^2)^p\,dx$$

input
```
int(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x)
```

output
```
int(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x)
```

3.274.5 Fricas [F]

$$\int x^m (A + Bx^2) (bx^2 + cx^4)^p\,dx = \int (Bx^2 + A)(cx^4 + bx^2)^p x^m\,dx$$

input
```
integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(c*x^4 + b*x^2)^p*x^m, x)
```

3.274.6 Sympy [F]

$$\int x^m (A + Bx^2) (bx^2 + cx^4)^p\,dx = \int x^m (x^2(b + cx^2))^p (A + Bx^2)\,dx$$

input
```
integrate(x**m*(B*x**2+A)*(c*x**4+b*x**2)**p,x)
```

output
```
Integral(x**m*(x**2*(b + c*x**2))**p*(A + B*x**2), x)
```

3.274.7 Maxima [F]

$$\int x^m (A + Bx^2) (bx^2 + cx^4)^p \, dx = \int (Bx^2 + A) (cx^4 + bx^2)^p x^m \, dx$$

input
```
integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(c*x^4 + b*x^2)^p*x^m, x)
```

3.274.8 Giac [F]

$$\int x^m (A + Bx^2) (bx^2 + cx^4)^p \, dx = \int (Bx^2 + A) (cx^4 + bx^2)^p x^m \, dx$$

input
```
integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(c*x^4 + b*x^2)^p*x^m, x)
```

3.274.9 Mupad [F(-1)]

Timed out.

$$\int x^m (A + Bx^2) (bx^2 + cx^4)^p \, dx = \int x^m \left(B\,x^2 + A\right) \left(c\,x^4 + b\,x^2\right)^p \, dx$$

input
```
int(x^m*(A + B*x^2)*(b*x^2 + c*x^4)^p,x)
```

output
```
int(x^m*(A + B*x^2)*(b*x^2 + c*x^4)^p, x)
```

3.275 $\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx$

3.275.1 Optimal result

Integrand size = 32, antiderivative size = 95

$$\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx=-\frac{(ad-bc(2+p))x^{-j(1+p)}\left(ax^j+bx^{j+n}\right)^{1+p}}{b^2n(1+p)(2+p)}+\frac{dx^{n-j(1+p)}\left(ax^j+bx^{j+n}\right)^{1+p}}{bn(2+p)}$$

output
```
-(a*d-b*c*(2+p))*(a*x^j+b*x^(j+n))^(p+1)/b^2/n/(p^2+3*p+2)/(x^(j*(p+1)))+d
*x^(n-j*(p+1))*(a*x^j+b*x^(j+n))^(p+1)/b/n/(2+p)
```

3.275.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx=\frac{x^{-jp}(a+bx^n)\left(x^j(a+bx^n)\right)^p(-ad+bc(2+p)+bd(1+p)x^n)}{b^2n(1+p)(2+p)}$$

input
```
Integrate[x^(-1 + n - j*p)*(c + d*x^n)*(a*x^j + b*x^(j + n))^p,x]
```

output
```
((a + b*x^n)*(x^j*(a + b*x^n))^p*(-(a*d) + b*c*(2 + p) + b*d*(1 + p)*x^n))
/(b^2*n*(1 + p)*(2 + p)*x^(j*p))
```

3.275.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1945, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^n)\, x^{-jp+n-1} \left(ax^j+bx^{j+n}\right)^p \, dx$$

$$\Big\downarrow \text{1945}$$

$$\left(c-\frac{ad}{b(p+2)}\right)\int x^{n-jp-1}\left(ax^j+bx^{j+n}\right)^p dx+\frac{dx^{n-j(p+1)}\left(ax^j+bx^{j+n}\right)^{p+1}}{bn(p+2)}$$

$$\Big\downarrow \text{1920}$$

$$\frac{x^{-j(p+1)}\left(c-\frac{ad}{b(p+2)}\right)\left(ax^j+bx^{j+n}\right)^{p+1}}{bn(p+1)}+\frac{dx^{n-j(p+1)}\left(ax^j+bx^{j+n}\right)^{p+1}}{bn(p+2)}$$

input
```
Int[x^(-1 + n - j*p)*(c + d*x^n)*(a*x^j + b*x^(j + n))^p,x]
```

output
```
((c - (a*d)/(b*(2 + p)))*(a*x^j + b*x^(j + n))^(1 + p))/(b*n*(1 + p)*x^(j*
(1 + p))) + (d*x^(n - j*(1 + p))*(a*x^j + b*x^(j + n))^(1 + p))/(b*n*(2 +
p))
```

3.275.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

3.275.4 Maple [F]

$$\int x^{-jp+n-1}(c+d\,x^{n})\left(a\,x^{j}+b\,x^{j+n}\right)^{p}dx$$

input
```
int(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x)
```

output
```
int(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x)
```

3.275.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.47

$$\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx$$
$$=\frac{((b^2dp+b^2d)xx^{-jp+n-1}x^{2\,n}+(2\,b^2c+(b^2c+abd)p)xx^{-jp+n-1}x^n+(abcp+2\,abc-a^2d)xx^{-jp+n-1})\left(\frac{(bx^n+}{}\right.}{(b^2np^2+3\,b^2np+2\,b^2n)x^n}$$

input
```
integrate(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x, algorithm="fricas"
)
```

output
```
((b^2*d*p + b^2*d)*x*x^(-j*p + n - 1)*x^(2*n) + (2*b^2*c + (b^2*c + a*b*d)
*p)*x*x^(-j*p + n - 1)*x^n + (a*b*c*p + 2*a*b*c - a^2*d)*x*x^(-j*p + n - 1
))*((b*x^n + a)*x^(j + n)/x^n)^p/((b^2*n*p^2 + 3*b^2*n*p + 2*b^2*n)*x^n)
```

3.275.6 Sympy [F(-1)]

Timed out.

$$\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**(-j*p+n-1)*(c+d*x**n)*(a*x**j+b*x**(j+n))**p,x)
```

output
```
Timed out
```

3.275.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.18

$$\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx=\frac{(bx^n+a)ce^{\left(-jp\log(x)+p\log(bx^n+a)+p\log(x^j)\right)}}{bn(p+1)}+\frac{(b^2(p+1)x^{2n}+abpx^n-a^2)de^{\left(-jp\log(x)+p\log(bx^n+a)+p\log(x^j)\right)}}{(p^2+3p+2)b^2n}$$

input
```
integrate(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x, algorithm="maxima"
)
```

output
```
(b*x^n + a)*c*e^(-j*p*log(x) + p*log(b*x^n + a) + p*log(x^j))/(b*n*(p + 1)
) + (b^2*(p + 1)*x^(2*n) + a*b*p*x^n - a^2)*d*e^(-j*p*log(x) + p*log(b*x^n
 + a) + p*log(x^j))/((p^2 + 3*p + 2)*b^2*n)
```

3.275.8 Giac [F]

$$\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx=\int(dx^n+c)\left(bx^{j+n}+ax^j\right)^p x^{-jp+n-1}\,dx$$

input
```
integrate(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)*(b*x^(j + n) + a*x^j)^p*x^(-j*p + n - 1), x)
```

3.275.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx=\int x^{n-j\,p-1}\,\left(a\,x^j+b\,x^{j+n}\right)^p\,(c+d\,x^n)\,dx$$

input
```
int(x^(n - j*p - 1)*(a*x^j + b*x^(j + n))^p*(c + d*x^n),x)
```

output
```
int(x^(n - j*p - 1)*(a*x^j + b*x^(j + n))^p*(c + d*x^n), x)
```

3.276 $\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p \, dx$

3.276.1 Optimal result

Integrand size = 30, antiderivative size = 113

$$\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p \, dx = \frac{x(ex)^m \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \left(ax^j+bx^{j+n}\right)^p \operatorname{AppellF1}\left(\frac{1+m+jp}{n},-p,-q,\frac{1+m+n+jp}{n},-\frac{bx^n}{a},-\frac{d}{}\right.}{1+m+jp}$$

output
```
x*(e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p*AppellF1((j*p+m+1)/n,-p,-q,(j*p+
m+n+1)/n,-b*x^n/a,-d*x^n/c)/(j*p+m+1)/((1+b*x^n/a)^p)/((1+d*x^n/c)^q)
```

3.276.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98

$$\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p \, dx = \frac{x(ex)^m \left(x^j(a+bx^n)\right)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \operatorname{AppellF1}\left(\frac{1+m+jp}{n},-p,-q,\frac{1+m+n+jp}{n},-\frac{bx^n}{a},-\frac{d}{}\right.}{1+m+jp}$$

input
```
Integrate[(e*x)^m*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^p,x]
```

output
```
(x*(e*x)^m*(x^j*(a + b*x^n))^p*(c + d*x^n)^q*AppellF1[(1 + m + j*p)/n, -p,
 -q, (1 + m + n + j*p)/n, -((b*x^n)/a), -((d*x^n)/c)])/((1 + m + j*p)*(1 +
 (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

3.276.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1948, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p dx$$

$$\Big\downarrow \text{1948}$$

$$(ex)^m x^{-jp-m}(a+bx^n)^{-p} \left(ax^j+bx^{j+n}\right)^p \int x^{m+jp}(bx^n+a)^p (dx^n+c)^q dx$$

$$\Big\downarrow \text{1013}$$

$$(ex)^m x^{-jp-m}\left(\frac{bx^n}{a}+1\right)^{-p} \left(ax^j+bx^{j+n}\right)^p \int x^{m+jp}\left(\frac{bx^n}{a}+1\right)^p (dx^n+c)^q dx$$

$$\Big\downarrow \text{1013}$$

$$(ex)^m x^{-jp-m}\left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} \left(ax^j+bx^{j+n}\right)^p \int x^{m+jp}\left(\frac{bx^n}{a}+1\right)^p \left(\frac{dx^n}{c}+1\right)^q dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{x(ex)^m \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} \left(ax^j+bx^{j+n}\right)^p \text{AppellF1}\left(\frac{m+jp+1}{n},-p,-q,\frac{m+n+jp+1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{jp+m+1}$$

input
```
Int[(e*x)^m*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^p,x]
```

output
```
(x*(e*x)^m*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^p*AppellF1[(1 + m + j*p)/n,
 -p, -q, (1 + m + n + j*p)/n, -((b*x^n)/a), -((d*x^n)/c)])/((1 + m + j*p)*
(1 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

3.276.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

3.276.4 Maple [F]

$$\int (ex)^m (c + d\,x^n)^q \left(a\,x^j + b\,x^{j+n}\right)^p dx$$

input
```
int((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x)
```

output
```
int((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x)
```

3.276.5 Fricas [F]

$$\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p \,dx = \int (bx^{j+n}+ax^j)^p (dx^n+c)^q (ex)^m \,dx$$

input
```
integrate((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x, algorithm="fricas")
```

output
```
integral((b*x^(j + n) + a*x^j)^p*(d*x^n + c)^q*(e*x)^m, x)
```

3.276.6 Sympy [F(-1)]

Timed out.

$$\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p \,dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(c+d*x**n)**q*(a*x**j+b*x**(j+n))**p,x)
```

output
```
Timed out
```

3.276.7 Maxima [F]

$$\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p \,dx = \int (bx^{j+n}+ax^j)^p (dx^n+c)^q (ex)^m \,dx$$

input
```
integrate((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x, algorithm="maxima")
```

output
```
integrate((b*x^(j + n) + a*x^j)^p*(d*x^n + c)^q*(e*x)^m, x)
```

3.276.8 Giac [F]

$$\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p \, dx = \int \left(bx^{j+n}+ax^j\right)^p (dx^n+c)^q (ex)^m \, dx$$

input
```
integrate((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x, algorithm="giac")
```

output
```
integrate((b*x^(j + n) + a*x^j)^p*(d*x^n + c)^q*(e*x)^m, x)
```

3.276.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p \, dx = \int \left(a\,x^j+b\,x^{j+n}\right)^p (e\,x)^m (c+d\,x^n)^q \, dx$$

input
```
int((a*x^j + b*x^(j + n))^p*(e*x)^m*(c + d*x^n)^q,x)
```

output
```
int((a*x^j + b*x^(j + n))^p*(e*x)^m*(c + d*x^n)^q, x)
```

3.277 $\int (ex)^{7/4} (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^{5/3} dx$

3.277.1 Optimal result

Integrand size = 34, antiderivative size = 129

$$\int (ex)^{7/4} (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^{5/3} dx = \frac{12aex^{2+j}(ex)^{3/4}(c+dx^n)^q\left(1+\frac{dx^n}{c}\right)^{-q}\left(ax^j+bx^{j+n}\right)^{2/3}\text{AppellF1}\left(\frac{33+20j}{12n},-\frac{5}{3},-q,\frac{33+20j+1}{12n}\right.}{(33+20j)\left(1+\frac{bx^n}{a}\right)^{2/3}}$$

output
```
12*a*e*x^(2+j)*(e*x)^(3/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(2/3)*AppellF1(1/
12*(33+20*j)/n,-5/3,-q,1+(11/4+5/3*j)/n,-b*x^n/a,-d*x^n/c)/(33+20*j)/(1+b*
x^n/a)^(2/3)/((1+d*x^n/c)^q)
```

3.277.2 Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.63

$$\int (ex)^{7/4} (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^{5/3} dx = \frac{12x^{1+j}(ex)^{7/4}\left(x^j(a+bx^n)\right)^{2/3}(c+dx^n)^q\left(1+\frac{dx^n}{c}\right)^{-q}\left(a(33+20j+12n)\text{AppellF1}\left(\frac{33+20}{12n}\right.\right.}{(33+20j)(3}$$

input
```
Integrate[(e*x)^(7/4)*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^(5/3),x]
```

output
```
(12*x^(1 + j)*(e*x)^(7/4)*(x^j*(a + b*x^n))^(2/3)*(c + d*x^n)^q*(a*(33 + 2
0*j + 12*n)*AppellF1[(33 + 20*j)/(12*n), -2/3, -q, (11/4 + (5*j)/3 + n)/n,
 -((b*x^n)/a), -((d*x^n)/c)] + b*(33 + 20*j)*x^n*AppellF1[(33 + 20*j + 12*
n)/(12*n), -2/3, -q, (33 + 20*j + 24*n)/(12*n), -((b*x^n)/a), -((d*x^n)/c)
]))/((33 + 20*j)*(33 + 20*j + 12*n)*(1 + (b*x^n)/a)^(2/3)*(1 + (d*x^n)/c)^
q)
```

3.277.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1948, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{7/4} \left(ax^j + bx^{j+n}\right)^{5/3} (c + dx^n)^q \, dx$$

$$\Big\downarrow \text{1948}$$

$$\frac{e(ex)^{3/4} x^{\frac{1}{12}(-8j-9)} \left(ax^j + bx^{j+n}\right)^{2/3} \int x^{\frac{1}{12}(20j+21)} (bx^n + a)^{5/3} (dx^n + c)^q dx}{(a + bx^n)^{2/3}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{ae(ex)^{3/4} x^{\frac{1}{12}(-8j-9)} \left(ax^j + bx^{j+n}\right)^{2/3} \int x^{\frac{1}{12}(20j+21)} \left(\frac{bx^n}{a} + 1\right)^{5/3} (dx^n + c)^q dx}{\left(\frac{bx^n}{a} + 1\right)^{2/3}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{ae(ex)^{3/4} x^{\frac{1}{12}(-8j-9)} \left(ax^j + bx^{j+n}\right)^{2/3} (c + dx^n)^q \left(\frac{dx^n}{c} + 1\right)^{-q} \int x^{\frac{1}{12}(20j+21)} \left(\frac{bx^n}{a} + 1\right)^{5/3} \left(\frac{dx^n}{c} + 1\right)^q dx}{\left(\frac{bx^n}{a} + 1\right)^{2/3}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{12ae(ex)^{3/4} x^{\frac{1}{12}(-8j-9)+\frac{1}{12}(20j+33)} \left(ax^j + bx^{j+n}\right)^{2/3} (c + dx^n)^q \left(\frac{dx^n}{c} + 1\right)^{-q} \text{AppellF1}\left(\frac{20j+33}{12n}, -\frac{5}{3}, -q, \frac{20j+12n+33}{12n},\right.}{(20j + 33) \left(\frac{bx^n}{a} + 1\right)^{2/3}}$$

input
```
Int[(e*x)^(7/4)*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^(5/3),x]
```

output
```
(12*a*e*x^((-9 - 8*j)/12 + (33 + 20*j)/12)*(e*x)^(3/4)*(c + d*x^n)^q*(a*x^
j + b*x^(j + n))^(2/3)*AppellF1[(33 + 20*j)/(12*n), -5/3, -q, (33 + 20*j +
 12*n)/(12*n), -((b*x^n)/a), -((d*x^n)/c)])/((33 + 20*j)*(1 + (b*x^n)/a)^(
2/3)*(1 + (d*x^n)/c)^q)
```

3.277.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

3.277.4 Maple [F]

$$\int (ex)^{\frac{7}{4}} (c + d\,x^n)^q \left(a\,x^j + b\,x^{j+n}\right)^{\frac{5}{3}} dx$$

input
```
int((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x)
```

output
```
int((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x)
```

3.277.5 Fricas [F]

$$\int (ex)^{7/4} (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^{5/3} dx = \int \left(bx^{j+n}+ax^j\right)^{\frac{5}{3}} (ex)^{\frac{7}{4}} (dx^n+c)^q \, dx$$

input
```
integrate((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x, algorithm="fr
icas")
```

output
```
integral((b*e*x*x^(j + n) + a*e*x*x^j)*(b*x^(j + n) + a*x^j)^(2/3)*(e*x)^(
3/4)*(d*x^n + c)^q, x)
```

3.277.6 Sympy [F(-1)]

Timed out.

$$\int (ex)^{7/4} (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^{5/3} dx = \text{Timed out}$$

input
```
integrate((e*x)**(7/4)*(c+d*x**n)**q*(a*x**j+b*x**(j+n))**(5/3),x)
```

output
```
Timed out
```

3.277.7 Maxima [F]

$$\int (ex)^{7/4} (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^{5/3} dx = \int \left(bx^{j+n}+ax^j\right)^{\frac{5}{3}} (ex)^{\frac{7}{4}} (dx^n+c)^q \, dx$$

input
```
integrate((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x, algorithm="ma
xima")
```

output
```
integrate((b*x^(j + n) + a*x^j)^(5/3)*(e*x)^(7/4)*(d*x^n + c)^q, x)
```

3.277.8 Giac [F]

$$\int (ex)^{7/4}(c+dx^n)^q\left(ax^j+bx^{j+n}\right)^{5/3}\,dx=\int \left(bx^{j+n}+ax^j\right)^{\frac{5}{3}}(ex)^{\frac{7}{4}}(dx^n+c)^q\,dx$$

input
```
integrate((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x, algorithm="gi
ac")
```

output
```
integrate((b*x^(j + n) + a*x^j)^(5/3)*(e*x)^(7/4)*(d*x^n + c)^q, x)
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^{7/4}(c+dx^n)^q\left(ax^j+bx^{j+n}\right)^{5/3}\,dx=\int \left(a\,x^j+b\,x^{j+n}\right)^{5/3}(e\,x)^{7/4}\,(c+d\,x^n)^q\,dx$$

input
```
int((a*x^j + b*x^(j + n))^(5/3)*(e*x)^(7/4)*(c + d*x^n)^q,x)
```

output
```
int((a*x^j + b*x^(j + n))^(5/3)*(e*x)^(7/4)*(c + d*x^n)^q, x)
```

3.278 $\int \frac{4+3x^4}{5x+2x^5} dx$

3.278.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{4+3x^4}{5x+2x^5}\,dx=\frac{4\log(x)}{5}+\frac{7}{40}\log\left(5+2x^4\right)$$

output
```
4/5*ln(x)+7/40*ln(2*x^4+5)
```

3.278.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{4+3x^4}{5x+2x^5}\,dx=\frac{4\log(x)}{5}+\frac{7}{40}\log\left(5+2x^4\right)$$

input
```
Integrate[(4 + 3*x^4)/(5*x + 2*x^5),x]
```

output
```
(4*Log[x])/5 + (7*Log[5 + 2*x^4])/40
```

3.278.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2026, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^4+4}{2x^5+5x}\,dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{3x^4+4}{x\left(2x^4+5\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{3x^4+4}{x^4\left(2x^4+5\right)}dx^4$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{4}\int \left(\frac{4}{5x^4}+\frac{7}{5\left(2x^4+5\right)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{4\log\left(x^4\right)}{5}+\frac{7}{10}\log\left(2x^4+5\right)\right)$$

input
```
Int[(4 + 3*x^4)/(5*x + 2*x^5),x]
```

output
```
((4*Log[x^4])/5 + (7*Log[5 + 2*x^4])/10)/4
```

3.278.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.278.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$\frac{4\ln(x)}{5}+\frac{7\ln\left(x^4+\frac{5}{2}\right)}{40}$	14
default	$\frac{4\ln(x)}{5}+\frac{7\ln(2x^4+5)}{40}$	16
norman	$\frac{4\ln(x)}{5}+\frac{7\ln(2x^4+5)}{40}$	16
risch	$\frac{4\ln(x)}{5}+\frac{7\ln(2x^4+5)}{40}$	16
meijerg	$\frac{4\ln(x)}{5}+\frac{\ln(2)}{5}-\frac{\ln(5)}{5}+\frac{7\ln\left(1+\frac{2x^4}{5}\right)}{40}$	24

input
```
int((3*x^4+4)/(2*x^5+5*x),x,method=_RETURNVERBOSE)
```

output
```
4/5*ln(x)+7/40*ln(x^4+5/2)
```

3.278.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{4+3x^4}{5x+2x^5} \, dx = \frac{7}{40} \log \left(2 x^4 + 5\right) + \frac{4}{5} \log \left(x\right)$$

input
```
integrate((3*x^4+4)/(2*x^5+5*x),x, algorithm="fricas")
```

output
```
7/40*log(2*x^4 + 5) + 4/5*log(x)
```

3.278.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{4+3x^4}{5x+2x^5} \, dx = \frac{4\log \left(x\right)}{5} + \frac{7\log \left(2x^4 + 5\right)}{40}$$

input
```
integrate((3*x**4+4)/(2*x**5+5*x),x)
```

output
```
4*log(x)/5 + 7*log(2*x**4 + 5)/40
```

3.278.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{4+3x^4}{5x+2x^5} \, dx = \frac{7}{40} \log \left(2 x^4 + 5\right) + \frac{4}{5} \log \left(x\right)$$

input
```
integrate((3*x^4+4)/(2*x^5+5*x),x, algorithm="maxima")
```

output
```
7/40*log(2*x^4 + 5) + 4/5*log(x)
```

3.278.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{4+3x^4}{5x+2x^5}\,dx = \frac{7}{40}\,\log\left(2\,x^4+5\right)+\frac{1}{5}\,\log\left(x^4\right)$$

input
```
integrate((3*x^4+4)/(2*x^5+5*x),x, algorithm="giac")
```

output
```
7/40*log(2*x^4 + 5) + 1/5*log(x^4)
```

3.278.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{4+3x^4}{5x+2x^5}\,dx = \frac{7\,\ln\left(x^4+\frac{5}{2}\right)}{40}+\frac{4\,\ln\left(x\right)}{5}$$

input
```
int((3*x^4 + 4)/(5*x + 2*x^5),x)
```

output
```
(7*log(x^4 + 5/2))/40 + (4*log(x))/5
```

3.279 $\int \frac{1+x^6}{x-x^7} dx$

3.279.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{1+x^6}{x-x^7}\,dx=\log(x)-\frac{1}{3}\log\left(1-x^6\right)$$

output
```
ln(x)-1/3*ln(-x^6+1)
```

3.279.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1+x^6}{x-x^7}\,dx=\log(x)-\frac{1}{3}\log\left(1-x^6\right)$$

input
```
Integrate[(1 + x^6)/(x - x^7),x]
```

output
```
Log[x] - Log[1 - x^6]/3
```

3.279.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2026, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6+1}{x-x^7} \, dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{x^6+1}{x\left(1-x^6\right)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{6}\int \frac{x^6+1}{x^6\left(1-x^6\right)}dx^6$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{6}\int \left(\frac{1}{x^6}-\frac{2}{x^6-1}\right)dx^6$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{6}\left(\log\left(x^6\right)-2\log\left(1-x^6\right)\right)$$

input
```
Int[(1 + x^6)/(x - x^7),x]
```

output
```
(Log[x^6] - 2*Log[1 - x^6])/6
```

3.279.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.279.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln\left(x\right)-\frac{\ln\left(x^{6}-1\right)}{3}$	12
meijerg	$-\frac{\ln\left(-x^{6}+1\right)}{3}+\ln\left(x\right)+\frac{i\pi}{6}$	18
default	$\ln\left(x\right)-\frac{\ln\left(-1+x\right)}{3}-\frac{\ln\left(1+x\right)}{3}-\frac{\ln\left(x^{2}+x+1\right)}{3}-\frac{\ln\left(x^{2}-x+1\right)}{3}$	36
norman	$\ln\left(x\right)-\frac{\ln\left(-1+x\right)}{3}-\frac{\ln\left(1+x\right)}{3}-\frac{\ln\left(x^{2}+x+1\right)}{3}-\frac{\ln\left(x^{2}-x+1\right)}{3}$	36
parallelrisch	$\ln\left(x\right)-\frac{\ln\left(-1+x\right)}{3}-\frac{\ln\left(1+x\right)}{3}-\frac{\ln\left(x^{2}+x+1\right)}{3}-\frac{\ln\left(x^{2}-x+1\right)}{3}$	36

input
```
int((x^6+1)/(-x^7+x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/3*ln(x^6-1)
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1+x^6}{x-x^7}\,dx = -\frac{1}{3}\log\left(x^6-1\right)+\log\left(x\right)$$

input
```
integrate((x^6+1)/(-x^7+x),x, algorithm="fricas")
```

output
```
-1/3*log(x^6 - 1) + log(x)
```

3.279.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1+x^6}{x-x^7}\,dx = \log\left(x\right)-\frac{\log\left(x^6-1\right)}{3}$$

input
```
integrate((x**6+1)/(-x**7+x),x)
```

output
```
log(x) - log(x**6 - 1)/3
```

3.279.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(13) = 26.

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \frac{1+x^6}{x-x^7}\,dx = -\frac{1}{3}\log\left(x^2+x+1\right)-\frac{1}{3}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)-\frac{1}{3}\log\left(x-1\right)+\log\left(x\right)$$

input
```
integrate((x^6+1)/(-x^7+x),x, algorithm="maxima")
```

output
```
-1/3*log(x^2 + x + 1) - 1/3*log(x^2 - x + 1) - 1/3*log(x + 1) - 1/3*log(x
- 1) + log(x)
```

3.279.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{1+x^6}{x-x^7}\,dx = \frac{1}{6}\log\left(x^6\right) - \frac{1}{3}\log\left(\left|x^6-1\right|\right)$$

input
```
integrate((x^6+1)/(-x^7+x),x, algorithm="giac")
```

output
```
1/6*log(x^6) - 1/3*log(abs(x^6 - 1))
```

3.279.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1+x^6}{x-x^7}\,dx = \ln(x) - \frac{\ln\left(x^6-1\right)}{3}$$

input
```
int((x^6 + 1)/(x - x^7),x)
```

output
```
log(x) - log(x^6 - 1)/3
```

3.280 $\int \frac{8+5x^{10}}{2x-x^{11}}\,dx$

3.280.1 Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \frac{8+5x^{10}}{2x-x^{11}}\,dx=4\log(x)-\frac{9}{10}\log\left(2-x^{10}\right)$$

output
```
4*ln(x)-9/10*ln(-x^10+2)
```

3.280.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{8+5x^{10}}{2x-x^{11}}\,dx=4\log(x)-\frac{9}{10}\log\left(2-x^{10}\right)$$

input
```
Integrate[(8 + 5*x^10)/(2*x - x^11),x]
```

output
```
4*Log[x] - (9*Log[2 - x^10])/10
```

3.280.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2026, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^{10}+8}{2x-x^{11}}\,dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{5x^{10}+8}{x\left(2-x^{10}\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{10}\int \frac{5x^{10}+8}{x^{10}\left(2-x^{10}\right)}dx^{10}$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{10}\int \left(\frac{4}{x^{10}}-\frac{9}{x^{10}-2}\right)dx^{10}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{10}\left(4\log\left(x^{10}\right)-9\log\left(2-x^{10}\right)\right)$$

input
```
Int[(8 + 5*x^10)/(2*x - x^11),x]
```

output
```
(4*Log[x^10] - 9*Log[2 - x^10])/10
```

3.280.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.280.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$4\ln\left(x\right)-\frac{9\ln\left(x^{10}-2\right)}{10}$	14
norman	$4\ln\left(x\right)-\frac{9\ln\left(x^{10}-2\right)}{10}$	14
risch	$4\ln\left(x\right)-\frac{9\ln\left(x^{10}-2\right)}{10}$	14
parallelrisch	$4\ln\left(x\right)-\frac{9\ln\left(x^{10}-2\right)}{10}$	14
meijerg	$4\ln\left(x\right)-\frac{2\ln\left(2\right)}{5}+\frac{2i\pi}{5}-\frac{9\ln\left(1-\frac{x^{10}}{2}\right)}{10}$	24

input
```
int((5*x^10+8)/(-x^11+2*x),x,method=_RETURNVERBOSE)
```

output
```
4*ln(x)-9/10*ln(x^10-2)
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{8+5x^{10}}{2x-x^{11}}\,dx = -\frac{9}{10}\,\log\left(x^{10}-2\right)+4\,\log\left(x\right)$$

input
```
integrate((5*x^10+8)/(-x^11+2*x),x, algorithm="fricas")
```

output
```
-9/10*log(x^10 - 2) + 4*log(x)
```

3.280.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{8+5x^{10}}{2x-x^{11}}\,dx = 4\log\left(x\right)-\frac{9\log\left(x^{10}-2\right)}{10}$$

input
```
integrate((5*x**10+8)/(-x**11+2*x),x)
```

output
```
4*log(x) - 9*log(x**10 - 2)/10
```

3.280.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{8+5x^{10}}{2x-x^{11}}\,dx = -\frac{9}{10}\,\log\left(x^{10}-2\right)+4\,\log\left(x\right)$$

input
```
integrate((5*x^10+8)/(-x^11+2*x),x, algorithm="maxima")
```

output
```
-9/10*log(x^10 - 2) + 4*log(x)
```

3.280.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{8+5x^{10}}{2x-x^{11}}\,dx = \frac{2}{5}\log\left(x^{10}\right) - \frac{9}{10}\log\left(\left|x^{10}-2\right|\right)$$

input
```
integrate((5*x^10+8)/(-x^11+2*x),x, algorithm="giac")
```

output
```
2/5*log(x^10) - 9/10*log(abs(x^10 - 2))
```

3.280.9 Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{8+5x^{10}}{2x-x^{11}}\,dx = 4\ln(x) - \frac{9\ln\left(x^{10}-2\right)}{10}$$

input
```
int((5*x^10 + 8)/(2*x - x^11),x)
```

output
```
4*log(x) - (9*log(x^10 - 2))/10
```

3.281 $\int \frac{-3+2x}{-x^2+x^3} dx$

3.281.1 Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \frac{-3+2x}{-x^2+x^3} dx = -\frac{3}{x} - \log(1-x) + \log(x)$$

output
```
-3/x-ln(1-x)+ln(x)
```

3.281.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{-3+2x}{-x^2+x^3} dx = -\frac{3}{x} - \log(1-x) + \log(x)$$

input
```
Integrate[(-3 + 2*x)/(-x^2 + x^3),x]
```

output
```
-3/x - Log[1 - x] + Log[x]
```

3.281.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2026, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x-3}{x^3-x^2}\,dx$$

$$\Big\downarrow \ 2026$$

$$\int \frac{2x-3}{(x-1)x^2}dx$$

$$\Big\downarrow \ 86$$

$$\int \left(\frac{3}{x^2}+\frac{1}{x}+\frac{1}{1-x}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{3}{x}-\log(1-x)+\log(x)$$

input
```
Int[(-3 + 2*x)/(-x^2 + x^3),x]
```

output
```
-3/x - Log[1 - x] + Log[x]
```

3.281.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.281.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\ln(x)-\frac{3}{x}-\ln(-1+x)$	15
norman	$\ln(x)-\frac{3}{x}-\ln(-1+x)$	15
risch	$\ln(x)-\frac{3}{x}-\ln(-1+x)$	15
parallelrisch	$\frac{\ln(x)x-\ln(-1+x)x-3}{x}$	18
meijerg	$-\frac{3}{x}+\ln(x)+i\pi-\ln(1-x)$	21

input
```
int((-3+2*x)/(x^3-x^2),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-3/x-ln(-1+x)
```

3.281.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{-3+2x}{-x^2+x^3}\,dx=-\frac{x\log(x-1)-x\log(x)+3}{x}$$

input
```
integrate((-3+2*x)/(x^3-x^2),x, algorithm="fricas")
```

output
```
-(x*log(x - 1) - x*log(x) + 3)/x
```

3.281.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{-3+2x}{-x^2+x^3}\,dx = \log(x) - \log(x-1) - \frac{3}{x}$$

input
```
integrate((-3+2*x)/(x**3-x**2),x)
```

output
```
log(x) - log(x - 1) - 3/x
```

3.281.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-3+2x}{-x^2+x^3}\,dx = -\frac{3}{x} - \log(x-1) + \log(x)$$

input
```
integrate((-3+2*x)/(x^3-x^2),x, algorithm="maxima")
```

output
```
-3/x - log(x - 1) + log(x)
```

3.281.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{-3+2x}{-x^2+x^3}\,dx = -\frac{3}{x} - \log(|x-1|) + \log(|x|)$$

input
```
integrate((-3+2*x)/(x^3-x^2),x, algorithm="giac")
```

output
```
-3/x - log(abs(x - 1)) + log(abs(x))
```

3.281.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-3+2x}{-x^2+x^3}\,dx = 2\,\mathrm{atanh}(2\,x-1)-\frac{3}{x}$$

input
```
int(-(2*x - 3)/(x^2 - x^3),x)
```

output
```
2*atanh(2*x - 1) - 3/x
```

3.282 $\int \frac{ax^m+bx^n}{cx^m+dx^n}\,dx$

3.282.1 Optimal result

Integrand size = 25, antiderivative size = 54

$$\int \frac{ax^m+bx^n}{cx^m+dx^n}\,dx = \frac{ax}{c}+\frac{(bc-ad)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{m-n},1+\frac{1}{m-n},-\frac{cx^{m-n}}{d}\right)}{cd}$$

output
```
a*x/c+(-a*d+b*c)*x*hypergeom([1, 1/(m-n)],[1+1/(m-n)],-c*x^(m-n)/d)/c/d
```

3.282.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{ax^m+bx^n}{cx^m+dx^n}\,dx = \frac{x\left(ad+(bc-ad)\,\text{Hypergeometric2F1}\left(1,\frac{1}{m-n},1+\frac{1}{m-n},-\frac{cx^{m-n}}{d}\right)\right)}{cd}$$

input
```
Integrate[(a*x^m + b*x^n)/(c*x^m + d*x^n),x]
```

output
```
(x*(a*d + (b*c - a*d)*Hypergeometric2F1[1, (m - n)^(-1), 1 + (m - n)^(-1),
 -((c*x^(m - n))/d)]))/(c*d)
```

3.282.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2027, 10, 913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^m+bx^n}{cx^m+dx^n}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^n(ax^{m-n}+b)}{cx^m+dx^n}dx$$

$$\Big\downarrow \text{10}$$

$$\int \frac{ax^{m-n}+b}{cx^{m-n}+d}dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(bc-ad)\int \frac{1}{cx^{m-n}+d}dx}{c}+\frac{ax}{c}$$

$$\Big\downarrow \text{778}$$

$$\frac{x(bc-ad)\,\text{Hypergeometric2F1}\left(1,\frac{1}{m-n},1+\frac{1}{m-n},-\frac{cx^{m-n}}{d}\right)}{cd}+\frac{ax}{c}$$

input
```
Int[(a*x^m + b*x^n)/(c*x^m + d*x^n),x]
```

output
```
(a*x)/c + ((b*c - a*d)*x*Hypergeometric2F1[1, (m - n)^(-1), 1 + (m - n)^(-
1), -((c*x^(m - n))/d)])/(c*d)
```

3.282.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

3.282.4 Maple [F]

$$\int \frac{x^m a + b\,x^n}{c\,x^m + d\,x^n}dx$$

input
```
int((x^m*a+b*x^n)/(c*x^m+d*x^n),x)
```

output
```
int((x^m*a+b*x^n)/(c*x^m+d*x^n),x)
```

3.282.5 Fricas [F]

$$\int \frac{ax^m + bx^n}{cx^m + dx^n}\,dx = \int \frac{ax^m + bx^n}{cx^m + dx^n}\,dx$$

input
```
integrate((a*x^m+b*x^n)/(c*x^m+d*x^n),x, algorithm="fricas")
```

output
```
integral((a*x^m + b*x^n)/(c*x^m + d*x^n), x)
```

3.282.6 Sympy [F]

$$\int \frac{ax^m + bx^n}{cx^m + dx^n} dx = \int \frac{ax^m + bx^n}{cx^m + dx^n} dx$$

input
```
integrate((a*x**m+b*x**n)/(c*x**m+d*x**n),x)
```

output
```
Integral((a*x**m + b*x**n)/(c*x**m + d*x**n), x)
```

3.282.7 Maxima [F]

$$\int \frac{ax^m + bx^n}{cx^m + dx^n} dx = \int \frac{ax^m + bx^n}{cx^m + dx^n} dx$$

input
```
integrate((a*x^m+b*x^n)/(c*x^m+d*x^n),x, algorithm="maxima")
```

output
```
-(b*c - a*d)*integrate(x^m/(c*d*x^m + d^2*x^n), x) + b*x/d
```

3.282.8 Giac [F]

$$\int \frac{ax^m + bx^n}{cx^m + dx^n} dx = \int \frac{ax^m + bx^n}{cx^m + dx^n} dx$$

input
```
integrate((a*x^m+b*x^n)/(c*x^m+d*x^n),x, algorithm="giac")
```

output
```
integrate((a*x^m + b*x^n)/(c*x^m + d*x^n), x)
```

3.282.9 Mupad [F(-1)]

Timed out.

$$\int \frac{ax^m + bx^n}{cx^m + dx^n}\,dx = \int \frac{a\,x^m + b\,x^n}{c\,x^m + d\,x^n}\,dx$$

input
```
int((a*x^m + b*x^n)/(c*x^m + d*x^n),x)
```

output
```
int((a*x^m + b*x^n)/(c*x^m + d*x^n), x)
```

3.283 $\int x^m(a+bx^n)^p\left(a(1+m+q)x^q+b(1+m+n(1+$

3.283.1 Optimal result

Integrand size = 39, antiderivative size = 18

$$\int x^m(a+bx^n)^p\left(a(1+m+q)x^q+b(1+m+n(1+p)+q)x^{n+q}\right)\,dx=x^{1+m+q}(a+bx^n)^{1+p}$$

output
```
x^(1+m+q)*(a+b*x^n)^(p+1)
```

3.283.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 6.44

$$\int x^m(a+bx^n)^p\left(a(1+m+q)x^q+b(1+m+n(1+p)+q)x^{n+q}\right)\,dx$$
$$=x^{1+m+q}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(a\,\text{Hypergeometric2F1}\left(-p,\frac{1+m+q}{n},\frac{1+m+n+q}{n},-\frac{bx^n}{a}\right)+\frac{b(1+m+n+np+q)x^n\,\text{Hypergeometric2F1}\left(-p,\frac{1+m+n+q}{n},\frac{1+m+2n+q}{n},-\frac{bx^n}{a}\right)}{1+m+n+q}\right)$$

input
```
Integrate[x^m*(a + b*x^n)^p*(a*(1 + m + q)*x^q + b*(1 + m + n*(1 + p) + q)
*x^(n + q)),x]
```

output
```
(x^(1 + m + q)*(a + b*x^n)^p*(a*Hypergeometric2F1[-p, (1 + m + q)/n, (1 +
m + n + q)/n, -((b*x^n)/a)] + (b*(1 + m + n + n*p + q)*x^n*Hypergeometric2
F1[-p, (1 + m + n + q)/n, (1 + m + 2*n + q)/n, -((b*x^n)/a)])/(1 + m + n +
 q)))/(1 + (b*x^n)/a)^p
```

3.283.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {10, 951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (a+bx^n)^p \left(a(m+q+1)x^q+bx^{n+q}(m+n(p+1)+q+1)\right) dx$$

$$\Big\downarrow \; 10$$

$$\int x^{m+q}(a+bx^n)^p \left(a(m+q+1)+bx^n(m+np+n+q+1)\right) dx$$

$$\Big\downarrow \; 951$$

$$x^{m+q+1}(a+bx^n)^{p+1}$$

input
```
Int[x^m*(a + b*x^n)^p*(a*(1 + m + q)*x^q + b*(1 + m + n*(1 + p) + q)*x^(n
+ q)),x]
```

output
```
x^(1 + m + q)*(a + b*x^n)^(1 + p)
```

3.283.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 951
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

3.283.4 Maple [F]

$$\int x^m(a+b\,x^n)^p\left(a(1+m+q)\,x^q+b(1+m+n(1+p)+q)\,x^{n+q}\right)dx$$

input
```
int(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x)
```

output
```
int(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x)
```

3.283.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.39

$$\int x^m(a+bx^n)^p\left(a(1+m+q)x^q+b(1+m+n(1+p)+q)x^{n+q}\right)\,dx$$
$$=\left(bxx^mx^{n+q}+axx^mx^q\right)\left(\frac{bx^{n+q}+ax^q}{x^q}\right)^p$$

input
```
integrate(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x, alg
orithm="fricas")
```

output
```
(b*x*x^m*x^(n + q) + a*x*x^m*x^q)*((b*x^(n + q) + a*x^q)/x^q)^p
```

3.283.6 Sympy [F(-1)]

Timed out.

$$\int x^m(a+bx^n)^p\left(a(1+m+q)x^q+b(1+m+n(1+p)+q)x^{n+q}\right)\,dx=\text{Timed out}$$

input
```
integrate(x**m*(a+b*x**n)**p*(a*(1+m+q)*x**q+b*(1+m+n*(1+p)+q)*x**(n+q)),x
)
```

output
```
Timed out
```

3.283.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int x^m(a+bx^n)^p\left(a(1+m+q)x^q+b(1+m+n(1+p)+q)x^{n+q}\right)\,dx$$
$$=\left(axx^m+bxe^{(m\log(x)+n\log(x))}\right)e^{(p\log(bx^n+a)+q\log(x))}$$

input
```
integrate(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x, alg
orithm="maxima")
```

output
```
(a*x*x^m + b*x*e^(m*log(x) + n*log(x)))*e^(p*log(b*x^n + a) + q*log(x))
```

3.283.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(18) = 36.

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.67

$$\int x^m(a+bx^n)^p\left(a(1+m+q)x^q+b(1+m+n(1+p)+q)x^{n+q}\right)\,dx$$
$$=(bx^n+a)^pbxx^ne^{(m\log(x)+q\log(x))}+(bx^n+a)^paxe^{(m\log(x)+q\log(x))}$$

input
```
integrate(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x, alg
orithm="giac")
```

output
```
(b*x^n + a)^p*b*x*x^n*e^(m*log(x) + q*log(x)) + (b*x^n + a)^p*a*x*e^(m*log
(x) + q*log(x))
```

3.283.9 Mupad [F(-1)]

Timed out.

$$\int x^m(a+bx^n)^p\left(a(1+m+q)x^q+b(1+m+n(1+p)+q)x^{n+q}\right)\,dx$$
$$=\int x^m\left(a\,x^q\,(m+q+1)+b\,x^{n+q}\,(m+q+n\,(p+1)+1)\right)\,(a+b\,x^n)^p\,dx$$

input
```
int(x^m*(a*x^q*(m + q + 1) + b*x^(n + q)*(m + q + n*(p + 1) + 1))*(a + b*x
^n)^p,x)
```

output
```
int(x^m*(a*x^q*(m + q + 1) + b*x^(n + q)*(m + q + n*(p + 1) + 1))*(a + b*x
^n)^p, x)
```

3.284 $\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx} dx$

3.284.1 Optimal result

Integrand size = 20, antiderivative size = 64

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx} dx = \frac{\left(a+\frac{b}{x}\right)^n \left(1+\frac{b}{ax}\right)^{-n} x^m \operatorname{AppellF1}\left(-m,-n,1,1-m,-\frac{b}{ax},-\frac{c}{dx}\right)}{dm}$$

output
```
(a+b/x)^n*x^m*AppellF1(-m,-n,1,1-m,-b/a/x,-c/d/x)/d/m/((1+b/a/x)^n)
```

3.284.2 Mathematica [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx} dx = \int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx} dx$$

input
```
Integrate[((a + b/x)^n*x^m)/(c + d*x),x]
```

output
```
Integrate[((a + b/x)^n*x^m)/(c + d*x), x]
```

3.284.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1016, 999, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a+\frac{b}{x}\right)^n}{c+dx} \, dx$$

$$\Big\downarrow \text{ 1016}$$

$$\int \frac{x^{m-1} \left(a+\frac{b}{x}\right)^n}{\frac{c}{x}+d}dx$$

$$\Big\downarrow \text{ 999}$$

$$-\left(\frac{1}{x}\right)^m x^m \int \frac{\left(a+\frac{b}{x}\right)^n \left(\frac{1}{x}\right)^{-m-1}}{\frac{c}{x}+d}d\frac{1}{x}$$

$$\Big\downarrow \text{ 152}$$

$$\left(\frac{1}{x}\right)^m x^m \left(-\left(a+\frac{b}{x}\right)^n\right) \left(\frac{b}{ax}+1\right)^{-n} \int \frac{\left(\frac{b}{ax}+1\right)^n \left(\frac{1}{x}\right)^{-m-1}}{\frac{c}{x}+d}d\frac{1}{x}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{x^m \left(a+\frac{b}{x}\right)^n \left(\frac{b}{ax}+1\right)^{-n} \text{AppellF1}\left(-m,-n,1,1-m,-\frac{b}{ax},-\frac{c}{dx}\right)}{dm}$$

input
```
Int[((a + b/x)^n*x^m)/(c + d*x),x]
```

output
```
((a + b/x)^n*x^m*AppellF1[-m, -n, 1, 1 - m, -(b/(a*x)), -(c/(d*x))])/(d*m*
(1 + b/(a*x))^n)
```

3.284.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

rule 999
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(-(e*x)^m)*(x^(-1))^m   Subst[Int[(a + b/x^n)^p*(
(c + d/x^n)^q/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q}
, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && !RationalQ[m]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] || !I
ntegerQ[p])
```

3.284.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{n} x^{m}}{dx+c}dx$$

input
```
int((a+b/x)^n*x^m/(d*x+c),x)
```

output
```
int((a+b/x)^n*x^m/(d*x+c),x)
```

3.284.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{dx+c}\,dx$$

input
```
integrate((a+b/x)^n*x^m/(d*x+c),x, algorithm="fricas")
```

output
```
integral(x^m*((a*x + b)/x)^n/(d*x + c), x)
```

3.284.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx=\int \frac{x^m\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx$$

input
```
integrate((a+b/x)**n*x**m/(d*x+c),x)
```

output
```
Integral(x**m*(a + b/x)**n/(c + d*x), x)
```

3.284.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{dx+c}\,dx$$

input
```
integrate((a+b/x)^n*x^m/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((a + b/x)^n*x^m/(d*x + c), x)
```

3.284.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x^m}{dx+c}\,dx$$

input
```
integrate((a+b/x)^n*x^m/(d*x+c),x, algorithm="giac")
```

output
```
integrate((a + b/x)^n*x^m/(d*x + c), x)
```

3.284.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx = \int \frac{x^m\left(a+\frac{b}{x}\right)^n}{c+d\,x}\,dx$$

input
```
int((x^m*(a + b/x)^n)/(c + d*x),x)
```

output
```
int((x^m*(a + b/x)^n)/(c + d*x), x)
```

3.285 $\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx} \, dx$

3.285.1 Optimal result

Integrand size = 20, antiderivative size = 195

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx} \, dx = -\frac{(2ac+bd(1-n))\left(a+\frac{b}{x}\right)^{1+n} x}{2a^2d^2} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x^2}{2ad} - \frac{c^3\left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^3(ac-bd)(1+n)} + \frac{(2a^2c^2-2abcdn-b^2d^2(1-n)n)\left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b}{ax}\right)}{2a^3d^3(1+n)}$$

output
```
-1/2*(2*a*c+b*d*(1-n))*(a+b/x)^(1+n)*x/a^2/d^2+1/2*(a+b/x)^(1+n)*x^2/a/d-c
^3*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/d^3/(a*c-b*
d)/(1+n)+1/2*(2*a^2*c^2-2*a*b*c*d*n-b^2*d^2*(1-n)*n)*(a+b/x)^(1+n)*hyperge
om([1, 1+n],[2+n],1+b/a/x)/a^3/d^3/(1+n)
```

3.285.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx}\,dx$$
$$=\frac{\left(a+\frac{b}{x}\right)^n(b+ax)\left(-2a^3c^3\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(ac-bd)\left(ad(1+n)x(bd(-\right.\right.}{2a^3d^3(ac-}$$

input
```
Integrate[((a + b/x)^n*x^2)/(c + d*x),x]
```

output
```
((a + b/x)^n*(b + a*x)*(-2*a^3*c^3*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(
a + b/x))/(a*c - b*d)] + (a*c - b*d)*(a*d*(1 + n)*x*(b*d*(-1 + n) + a*(-2*
c + d*x)) + (2*a^2*c^2 - 2*a*b*c*d*n + b^2*d^2*(-1 + n)*n)*Hypergeometric2
F1[1, 1 + n, 2 + n, 1 + b/(a*x)])))/(2*a^3*d^3*(a*c - b*d)*(1 + n)*x)
```

3.285.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1016, 948, 114, 168, 174, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx$$

$$\Big\downarrow \; 1016$$

$$\int \frac{x\left(a+\frac{b}{x}\right)^n}{\frac{c}{x}+d}dx$$

$$\Big\downarrow \; 948$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^n x^3}{\frac{c}{x}+d}d\frac{1}{x}$$

$$\Big\downarrow \; 114$$

$$\frac{\int \frac{\left(a+\frac{b}{x}\right)^n\left(2ac+\frac{b(1-n)c}{x}+bd(1-n)\right)x^2}{\frac{c}{x}+d}d\frac{1}{x}}{2ad}+\frac{x^2\left(a+\frac{b}{x}\right)^{n+1}}{2ad}$$

$$\Big\downarrow \ 168$$

$$-\frac{-\frac{\int \frac{\left(a+\frac{b}{x}\right)^n\left(2a^2c^2-2abdnc-\frac{b(2ac+bd(1-n))nc}{x}-b^2d^2(1-n)n\right)x}{\frac{c}{x}+d}d\frac{1}{x}}{ad}-\frac{x\left(a+\frac{b}{x}\right)^{n+1}(2ac+bd(1-n))}{ad}}{2ad}+\frac{x^2\left(a+\frac{b}{x}\right)^{n+1}}{2ad}$$

$$\Big\downarrow \ 174$$

$$-\frac{-\frac{\frac{\left(2a^2c^2-2abcdn-b^2d^2(1-n)n\right)\int\left(a+\frac{b}{x}\right)^nxd\frac{1}{x}}{d}-\frac{2a^2c^3\int\frac{\left(a+\frac{b}{x}\right)^n}{\frac{c}{x}+d}d\frac{1}{x}}{d}}{ad}-\frac{x\left(a+\frac{b}{x}\right)^{n+1}(2ac+bd(1-n))}{ad}}{2ad}+\frac{x^2\left(a+\frac{b}{x}\right)^{n+1}}{2ad}$$

$$\Big\downarrow \ 75$$

$$-\frac{-\frac{-\frac{2a^2c^3\int\frac{\left(a+\frac{b}{x}\right)^n}{\frac{c}{x}+d}d\frac{1}{x}}{d}-\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(2a^2c^2-2abcdn-b^2d^2(1-n)n\right)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b}{ax}+1\right)}{ad(n+1)}}{ad}-\frac{x\left(a+\frac{b}{x}\right)^{n+1}(2ac+bd(1-n))}{ad}}{2ad}+\frac{x^2\left(a+\frac{b}{x}\right)^{n+1}}{2ad}$$

$$\Big\downarrow \ 78$$

$$-\frac{\frac{-\frac{2a^2c^3\left(a+\frac{b}{x}\right)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(n+1)(ac-bd)}-\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(2a^2c^2-2abcdn-b^2d^2(1-n)n\right)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b}{ax}+1\right)}{ad(n+1)}}{ad}-\frac{x\left(a-\right.}{}}{2ad}$$

$$\frac{x^2\left(a+\frac{b}{x}\right)^{n+1}}{2ad}$$

input
```
Int[((a + b/x)^n*x^2)/(c + d*x),x]
```

output
```
((a + b/x)^(1 + n)*x^2)/(2*a*d) + (-((((2*a*c + b*d*(1 - n))*(a + b/x)^(1 +
 n)*x)/(a*d)) - ((2*a^2*c^3*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, (c*(a + b/x))/(a*c - b*d)])/(d*(a*c - b*d)*(1 + n)) - ((2*a^2*c^2 -
 2*a*b*c*d*n - b^2*d^2*(1 - n)*n)*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1
 + n, 2 + n, 1 + b/(a*x)])/(a*d*(1 + n)))/(a*d))/(2*a*d)
```

3.285. $\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx}\,dx$

3.285.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.285. $\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx} \, dx$

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

3.285.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{dx+c}dx$$

input
```
int((a+b/x)^n*x^2/(d*x+c),x)
```

output
```
int((a+b/x)^n*x^2/(d*x+c),x)
```

3.285.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{dx+c}\,dx$$

input
```
integrate((a+b/x)^n*x^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(x^2*((a*x + b)/x)^n/(d*x + c), x)
```

3.285.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx}\,dx=\int \frac{x^2\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx$$

input
```
integrate((a+b/x)**n*x**2/(d*x+c),x)
```

output
```
Integral(x**2*(a + b/x)**n/(c + d*x), x)
```

3.285.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x^2}{dx+c}\,dx$$

input
```
integrate((a+b/x)^n*x^2/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((a + b/x)^n*x^2/(d*x + c), x)
```

3.285.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x^2}{dx+c}\,dx$$

input
```
integrate((a+b/x)^n*x^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate((a + b/x)^n*x^2/(d*x + c), x)
```

3.285.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx}\,dx = \int \frac{x^2\left(a+\frac{b}{x}\right)^n}{c+d\,x}\,dx$$

input
```
int((x^2*(a + b/x)^n)/(c + d*x),x)
```

output
```
int((x^2*(a + b/x)^n)/(c + d*x), x)
```

3.286 $\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx}\,dx$

3.286.1 Optimal result

Integrand size = 18, antiderivative size = 131

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx}\,dx=\frac{\left(a+\frac{b}{x}\right)^{1+n}x}{ad}+\frac{c^2\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(ac-bd)(1+n)}-\frac{(ac-bdn)\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b}{ax}\right)}{a^2d^2(1+n)}$$

output
```
(a+b/x)^(1+n)*x/a/d+c^2*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(
a*c-b*d))/d^2/(a*c-b*d)/(1+n)-(-b*d*n+a*c)*(a+b/x)^(1+n)*hypergeom([1, 1+n
],[2+n],1+b/a/x)/a^2/d^2/(1+n)
```

3.286.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx}\,dx=\frac{\left(a+\frac{b}{x}\right)^n(b+ax)\left(a^2c^2\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(ac-bd)\left(ad(1+n)x+(-ac+\right.\right.}{a^2d^2(ac-bd)(1+n)x}$$

input
```
Integrate[((a + b/x)^n*x)/(c + d*x),x]
```

output
```
((a + b/x)^n*(b + a*x)*(a^2*c^2*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a +
 b/x))/(a*c - b*d)] + (a*c - b*d)*(a*d*(1 + n)*x + (-(a*c) + b*d*n)*Hyperg
eometric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)])))/(a^2*d^2*(a*c - b*d)*(1 + n)*
x)
```

3.286.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1016, 899, 114, 174, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+\frac{b}{x}\right)^n}{c+dx}dx$$

$$\Big\downarrow \text{ 1016}$$

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{\frac{c}{x}+d}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{\frac{c}{x}+d}d\frac{1}{x}$$

$$\Big\downarrow \text{ 114}$$

$$\frac{\int \frac{\left(a+\frac{b}{x}\right)^n\left(ac-\frac{bnc}{x}-bdn\right)x}{\frac{c}{x}+d}d\frac{1}{x}}{ad}+\frac{x\left(a+\frac{b}{x}\right)^{n+1}}{ad}$$

$$\Big\downarrow \text{ 174}$$

$$\frac{\frac{(ac-bdn)\int \left(a+\frac{b}{x}\right)^n xd\frac{1}{x}}{d}-\frac{ac^2\int \frac{\left(a+\frac{b}{x}\right)^n}{\frac{c}{x}+d}d\frac{1}{x}}{d}}{ad}+\frac{x\left(a+\frac{b}{x}\right)^{n+1}}{ad}$$

$$\Big\downarrow \text{ 75}$$

$$\frac{-\frac{ac^2\int \frac{\left(a+\frac{b}{x}\right)^n}{\frac{c}{x}+d}d\frac{1}{x}}{d}-\frac{\left(a+\frac{b}{x}\right)^{n+1}(ac-bdn)\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b}{ax}+1\right)}{ad(n+1)}}{ad}+\frac{x\left(a+\frac{b}{x}\right)^{n+1}}{ad}$$

3.286. $\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx}\,dx$

$$\Big\downarrow 78$$

$$\frac{\frac{ac^2\left(a+\frac{b}{x}\right)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(n+1)(ac-bd)}-\frac{\left(a+\frac{b}{x}\right)^{n+1}(ac-bdn)\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b}{ax}+1\right)}{ad(n+1)}}{ad}+\frac{x\left(a+\frac{b}{x}\right)^{n+1}}{ad}$$

input
```
Int[((a + b/x)^n*x)/(c + d*x),x]
```

output
```
((a + b/x)^(1 + n)*x)/(a*d) + ((a*c^2*(a + b/x)^(1 + n)*Hypergeometric2F1[
1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(d*(a*c - b*d)*(1 + n)) - ((a
*c - b*d*n)*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/(a*
x)])/(a*d*(1 + n)))/(a*d)
```

3.286.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

3.286. $\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx}\,dx$

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

3.286.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{n} x}{dx+c}dx$$

input
```
int((a+b/x)^n*x/(d*x+c),x)
```

output
```
int((a+b/x)^n*x/(d*x+c),x)
```

3.286.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{n} x}{dx+c}\,dx$$

input
```
integrate((a+b/x)^n*x/(d*x+c),x, algorithm="fricas")
```

output
```
integral(x*((a*x + b)/x)^n/(d*x + c), x)
```

3.286.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx}\,dx=\int \frac{x\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx$$

input
```
integrate((a+b/x)**n*x/(d*x+c),x)
```

output
```
Integral(x*(a + b/x)**n/(c + d*x), x)
```

3.286.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n x}{dx+c}\,dx$$

input
```
integrate((a+b/x)^n*x/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((a + b/x)^n*x/(d*x + c), x)
```

3.286.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n x}{dx+c}\,dx$$

input
```
integrate((a+b/x)^n*x/(d*x+c),x, algorithm="giac")
```

output
```
integrate((a + b/x)^n*x/(d*x + c), x)
```

3.286.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx}\,dx = \int \frac{x\left(a+\frac{b}{x}\right)^n}{c+d\,x}\,dx$$

input
```
int((x*(a + b/x)^n)/(c + d*x),x)
```

output
```
int((x*(a + b/x)^n)/(c + d*x), x)
```

3.287 $\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx$

3.287.1 Optimal result

Integrand size = 17, antiderivative size = 101

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx = -\frac{c\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(ac-bd)(1+n)} + \frac{\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b}{ax}\right)}{ad(1+n)}$$

output
```
-c*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/d/(a*c-b*d)
/(1+n)+(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],1+b/a/x)/a/d/(1+n)
```

3.287.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx = \frac{\left(a+\frac{b}{x}\right)^n(b+ax)\left(ac\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(-ac+bd)\,\text{Hypergeometric2F1}\left(\right.\right.}{ad(-ac+bd)(1+n)x}$$

input
```
Integrate[(a + b/x)^n/(c + d*x),x]
```

output
```
((a + b/x)^n*(b + a*x)*(a*c*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x
))/(a*c - b*d)] + (-(a*c) + b*d)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/
(a*x)]))/(a*d*(-(a*c) + b*d)*(1 + n)*x)
```

3.287.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {941, 948, 97, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx} \, dx$$

$$\Big\downarrow \ 941$$

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x\left(\frac{c}{x}+d\right)}dx$$

$$\Big\downarrow \ 948$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^n x}{\frac{c}{x}+d}d\frac{1}{x}$$

$$\Big\downarrow \ 97$$

$$\frac{c\int \frac{\left(a+\frac{b}{x}\right)^n}{\frac{c}{x}+d}d\frac{1}{x}}{d}-\frac{\int \left(a+\frac{b}{x}\right)^n xd\frac{1}{x}}{d}$$

$$\Big\downarrow \ 75$$

$$\frac{c\int \frac{\left(a+\frac{b}{x}\right)^n}{\frac{c}{x}+d}d\frac{1}{x}}{d}+\frac{\left(a+\frac{b}{x}\right)^{n+1} \text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b}{ax}+1\right)}{ad(n+1)}$$

$$\Big\downarrow \ 78$$

$$\frac{\left(a+\frac{b}{x}\right)^{n+1} \text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b}{ax}+1\right)}{ad(n+1)}-\frac{c\left(a+\frac{b}{x}\right)^{n+1} \text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(n+1)(ac-bd)}$$

input
```
Int[(a + b/x)^n/(c + d*x),x]
```

output
```
-((c*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a
*c - b*d)])/(d*(a*c - b*d)*(1 + n))) + ((a + b/x)^(1 + n)*Hypergeometric2F
1[1, 1 + n, 2 + n, 1 + b/(a*x)])/(a*d*(1 + n))
```

3.287.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.287. $\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx$

3.287.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{dx+c}dx$$

input
```
int((a+b/x)^n/(d*x+c),x)
```

output
```
int((a+b/x)^n/(d*x+c),x)
```

3.287.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{dx+c}\,dx$$

input
```
integrate((a+b/x)^n/(d*x+c),x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^n/(d*x + c), x)
```

3.287.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx$$

input
```
integrate((a+b/x)**n/(d*x+c),x)
```

output
```
Integral((a + b/x)**n/(c + d*x), x)
```

3.287.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{dx+c}\,dx$$

input
```
integrate((a+b/x)^n/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((a + b/x)^n/(d*x + c), x)
```

3.287.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{dx+c}\,dx$$

input
```
integrate((a+b/x)^n/(d*x+c),x, algorithm="giac")
```

output
```
integrate((a + b/x)^n/(d*x + c), x)
```

3.287.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{c+d\,x}\,dx$$

input
```
int((a + b/x)^n/(c + d*x),x)
```

output
```
int((a + b/x)^n/(c + d*x), x)
```

3.288 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx$

3.288.1 Optimal result

Integrand size = 20, antiderivative size = 54

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx=\frac{\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)(1+n)}$$

output
```
(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/(a*c-b*d)/(1+n
)
```

3.288.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx=\frac{\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)(1+n)}$$

input
```
Integrate[(a + b/x)^n/(x*(c + d*x)),x]
```

output
```
((a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c -
 b*d)])/((a*c - b*d)*(1 + n))
```

3.288.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1016, 946, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx$$

$$\Big\downarrow \text{1016}$$

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2\left(\frac{c}{x}+d\right)}dx$$

$$\Big\downarrow \text{946}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^n}{\frac{c}{x}+d}d\frac{1}{x}$$

$$\Big\downarrow \text{78}$$

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)}$$

input
```
Int[(a + b/x)^n/(x*(c + d*x)),x]
```

output
```
((a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c -
 b*d)])/((a*c - b*d)*(1 + n))
```

3.288.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

3.288.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{x \left(d x +c \right)}d x$$

input
```
int((a+b/x)^n/x/(d*x+c),x)
```

output
```
int((a+b/x)^n/x/(d*x+c),x)
```

3.288.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x}\,dx$$

input
```
integrate((a+b/x)^n/x/(d*x+c),x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^n/(d*x^2 + c*x), x)
```

3.288.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{x\,(c+dx)}\,dx$$

input
```
integrate((a+b/x)**n/x/(d*x+c),x)
```

output
```
Integral((a + b/x)**n/(x*(c + d*x)), x)
```

3.288.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x}\,dx$$

input
```
integrate((a+b/x)^n/x/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((a + b/x)^n/((d*x + c)*x), x)
```

3.288.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x}\,dx$$

input
```
integrate((a+b/x)^n/x/(d*x+c),x, algorithm="giac")
```

output
```
integrate((a + b/x)^n/((d*x + c)*x), x)
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{x\,\left(c+d\,x\right)}\,dx$$

input
```
int((a + b/x)^n/(x*(c + d*x)),x)
```

output
```
int((a + b/x)^n/(x*(c + d*x)), x)
```

3.289 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx$

3.289.1 Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx = -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc(1+n)} - \frac{d\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(ac-bd)(1+n)}$$

output
```
-(a+b/x)^(1+n)/b/c/(1+n)-d*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x
)/(a*c-b*d))/c/(a*c-b*d)/(1+n)
```

3.289.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx = \frac{\left(a+\frac{b}{x}\right)^n(b+ax)\left(ac-bd+bd\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)\right)}{bc(-ac+bd)(1+n)x}$$

input
```
Integrate[(a + b/x)^n/(x^2*(c + d*x)),x]
```

output
```
((a + b/x)^n*(b + a*x)*(a*c - b*d + b*d*Hypergeometric2F1[1, 1 + n, 2 + n,
 (c*(a + b/x))/(a*c - b*d)]))/(b*c*(-(a*c) + b*d)*(1 + n)*x)
```

3.289.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1016, 948, 90, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}dx$$

$$\Big\downarrow \text{1016}$$

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3\left(\frac{c}{x}+d\right)}dx$$

$$\Big\downarrow \text{948}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^n}{\left(\frac{c}{x}+d\right)x}d\frac{1}{x}$$

$$\Big\downarrow \text{90}$$

$$\frac{d\int \frac{\left(a+\frac{b}{x}\right)^n}{\frac{c}{x}+d}d\frac{1}{x}}{c}-\frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc(n+1)}$$

$$\Big\downarrow \text{78}$$

$$-\frac{d\left(a+\frac{b}{x}\right)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(n+1)(ac-bd)}-\frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc(n+1)}$$

input
```
Int[(a + b/x)^n/(x^2*(c + d*x)),x]
```

output
```
-((a + b/x)^(1 + n)/(b*c*(1 + n))) - (d*(a + b/x)^(1 + n)*Hypergeometric2F
1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(c*(a*c - b*d)*(1 + n))
```

3.289.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && !IntegerQ[m] && IntegerQ[n]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

3.289.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{x^{2}\left(dx+c\right)}dx$$

input
```
int((a+b/x)^n/x^2/(d*x+c),x)
```

output
```
int((a+b/x)^n/x^2/(d*x+c),x)
```

3.289.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^2}\,dx$$

input
```
integrate((a+b/x)^n/x^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^n/(d*x^3 + c*x^2), x)
```

3.289.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2\,(c+dx)}\,dx$$

input
```
integrate((a+b/x)**n/x**2/(d*x+c),x)
```

output
```
Integral((a + b/x)**n/(x**2*(c + d*x)), x)
```

3.289.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^2}\,dx$$

input
```
integrate((a+b/x)^n/x^2/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((a + b/x)^n/((d*x + c)*x^2), x)
```

3.289.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^2}\,dx$$

input
```
integrate((a+b/x)^n/x^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate((a + b/x)^n/((d*x + c)*x^2), x)
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{x^2\,(c+d\,x)}\,dx$$

input
```
int((a + b/x)^n/(x^2*(c + d*x)),x)
```

output
```
int((a + b/x)^n/(x^2*(c + d*x)), x)
```

3.290 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx$

3.290.1 Optimal result

Integrand size = 20, antiderivative size = 115

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx = \frac{(ac+bd)\left(a+\frac{b}{x}\right)^{1+n}}{b^2c^2(1+n)} - \frac{\left(a+\frac{b}{x}\right)^{2+n}}{b^2c(2+n)} + \frac{d^2\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(ac-bd)(1+n)}$$

output
```
(a*c+b*d)*(a+b/x)^(1+n)/b^2/c^2/(1+n)-(a+b/x)^(2+n)/b^2/c/(2+n)+d^2*(a+b/x
)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/c^2/(a*c-b*d)/(1+n)
```

3.290.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx = -\frac{\left(a+\frac{b}{x}\right)^n(b+ax)\left((ac-bd)(-bc(1+n)+acx+bd(2+n)x)+b^2d^2(2+n)x\,\text{Hypergeometric2F1}\left(1,\right.\right.}{b^2c^2(-ac+bd)(1+n)(2+n)x^2}$$

input
```
Integrate[(a + b/x)^n/(x^3*(c + d*x)),x]
```

output
```
-(((a + b/x)^n*(b + a*x)*((a*c - b*d)*(-(b*c*(1 + n)) + a*c*x + b*d*(2 + n
)*x) + b^2*d^2*(2 + n)*x*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/
(a*c - b*d)]))/(b^2*c^2*(-(a*c) + b*d)*(1 + n)*(2 + n)*x^2))
```

3.290.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1016, 948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}dx$$

$$\Big\downarrow \text{ 1016}$$

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^4\left(\frac{c}{x}+d\right)}dx$$

$$\Big\downarrow \text{ 948}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^n}{\left(\frac{c}{x}+d\right)x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 99}$$

$$-\int \left(\frac{(-ac-bd)\left(a+\frac{b}{x}\right)^n}{bc^2}+\frac{d^2\left(a+\frac{b}{x}\right)^n}{c^2\left(\frac{c}{x}+d\right)}+\frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(ac+bd)\left(a+\frac{b}{x}\right)^{n+1}}{b^2c^2(n+1)}-\frac{\left(a+\frac{b}{x}\right)^{n+2}}{b^2c(n+2)}+\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(n+1)(ac-bd)}$$

input
```
Int[(a + b/x)^n/(x^3*(c + d*x)),x]
```

output
```
((a*c + b*d)*(a + b/x)^(1 + n))/(b^2*c^2*(1 + n)) - (a + b/x)^(2 + n)/(b^2
*c*(2 + n)) + (d^2*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c
*(a + b/x))/(a*c - b*d)])/(c^2*(a*c - b*d)*(1 + n))
```

3.290.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.290.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{x^{3} \left(d x +c \right)}d x$$

input
```
int((a+b/x)^n/x^3/(d*x+c),x)
```

output
```
int((a+b/x)^n/x^3/(d*x+c),x)
```

3.290.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^3}\,dx$$

input
```
integrate((a+b/x)^n/x^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^n/(d*x^4 + c*x^3), x)
```

3.290.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3\,(c+dx)}\,dx$$

input
```
integrate((a+b/x)**n/x**3/(d*x+c),x)
```

output
```
Integral((a + b/x)**n/(x**3*(c + d*x)), x)
```

3.290.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^3}\,dx$$

input
```
integrate((a+b/x)^n/x^3/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((a + b/x)^n/((d*x + c)*x^3), x)
```

3.290.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^3}\,dx$$

input
```
integrate((a+b/x)^n/x^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate((a + b/x)^n/((d*x + c)*x^3), x)
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3\,\left(c+d\,x\right)}\,dx$$

input
```
int((a + b/x)^n/(x^3*(c + d*x)),x)
```

output
```
int((a + b/x)^n/(x^3*(c + d*x)), x)
```

3.291 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx$

3.291.1 Optimal result

Integrand size = 20, antiderivative size = 207

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx = \frac{(ac+bd)\left(a^2c^2+b^2d^2\right)\left(a+\frac{b}{x}\right)^{1+n}}{b^4c^4(1+n)} - \frac{\left(3a^2c^2+2abcd+b^2d^2\right)\left(a+\frac{b}{x}\right)^{2+n}}{b^4c^3(2+n)} + \frac{(3ac+bd)\left(a+\frac{b}{x}\right)^{3+n}}{b^4c^2(3+n)} - \frac{\left(a+\frac{b}{x}\right)^{4+n}}{b^4c(4+n)} + \frac{d^4\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^4(ac-bd)(1+n)}$$

output
```
(a*c+b*d)*(a^2*c^2+b^2*d^2)*(a+b/x)^(1+n)/b^4/c^4/(1+n)-(3*a^2*c^2+2*a*b*c
*d+b^2*d^2)*(a+b/x)^(2+n)/b^4/c^3/(2+n)+(3*a*c+b*d)*(a+b/x)^(3+n)/b^4/c^2/
(3+n)-(a+b/x)^(4+n)/b^4/c/(4+n)+d^4*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n]
,c*(a+b/x)/(a*c-b*d))/c^4/(a*c-b*d)/(1+n)
```

3.291.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx = \frac{\left(a+\frac{b}{x}\right)^{1+n}\left(\frac{(ac+bd)(a^2c^2+b^2d^2)}{b^4(1+n)}-\frac{c(3a^2c^2+2abcd+b^2d^2)\left(a+\frac{b}{x}\right)}{b^4(2+n)}+\frac{c^2(3ac+bd)\left(a+\frac{b}{x}\right)^2}{b^4(3+n)}-\frac{c^3\left(a+\frac{b}{x}\right)^3}{b^4(4+n)}+\frac{d^4\,\text{Hypergeometric2F1}\left(\right.}{(ac-bd}\right.}{c^4}$$

input
```
Integrate[(a + b/x)^n/(x^5*(c + d*x)),x]
```

output
```
((a + b/x)^(1 + n)*(((a*c + b*d)*(a^2*c^2 + b^2*d^2))/(b^4*(1 + n)) - (c*(
3*a^2*c^2 + 2*a*b*c*d + b^2*d^2)*(a + b/x))/(b^4*(2 + n)) + (c^2*(3*a*c +
b*d)*(a + b/x)^2)/(b^4*(3 + n)) - (c^3*(a + b/x)^3)/(b^4*(4 + n)) + (d^4*H
ypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/((a*c - b*d)
*(1 + n))))/c^4
```

3.291.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1016, 948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx$$

$$\Big\downarrow \; 1016$$

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^6\left(\frac{c}{x}+d\right)}dx$$

$$\Big\downarrow \; 948$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^n}{\left(\frac{c}{x}+d\right)x^4}d\frac{1}{x}$$

$$\Big\downarrow \; 99$$

$$-\int \left(\frac{(ac+bd)\left(-a^2c^2-b^2d^2\right)\left(a+\frac{b}{x}\right)^n}{b^3c^4}+\frac{d^4\left(a+\frac{b}{x}\right)^n}{c^4\left(\frac{c}{x}+d\right)}+\frac{\left(3a^2c^2+2abdc+b^2d^2\right)\left(a+\frac{b}{x}\right)^{n+1}}{b^3c^3}+\frac{(-3ac-bd)\left(a\right.}{b^3c^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ac+bd)\left(a^2c^2+b^2d^2\right)\left(a+\frac{b}{x}\right)^{n+1}}{b^4c^4(n+1)}-\frac{\left(3a^2c^2+2abcd+b^2d^2\right)\left(a+\frac{b}{x}\right)^{n+2}}{b^4c^3(n+2)}+\frac{(3ac+bd)\left(a+\frac{b}{x}\right)^{n+3}}{b^4c^2(n+3)}-\frac{\left(a+\frac{b}{x}\right)^{n+4}}{b^4c(n+4)}+\frac{d^4\left(a+\frac{b}{x}\right)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^4(n+1)(ac-bd)}$$

input
```
Int[(a + b/x)^n/(x^5*(c + d*x)),x]
```

output
```
((a*c + b*d)*(a^2*c^2 + b^2*d^2)*(a + b/x)^(1 + n))/(b^4*c^4*(1 + n)) - ((
3*a^2*c^2 + 2*a*b*c*d + b^2*d^2)*(a + b/x)^(2 + n))/(b^4*c^3*(2 + n)) + ((
3*a*c + b*d)*(a + b/x)^(3 + n))/(b^4*c^2*(3 + n)) - (a + b/x)^(4 + n)/(b^4
*c*(4 + n)) + (d^4*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c
*(a + b/x))/(a*c - b*d)])/(c^4*(a*c - b*d)*(1 + n))
```

3.291.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

3.291. $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.291.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5 (dx+c)}dx$$

input
```
int((a+b/x)^n/x^5/(d*x+c),x)
```

output
```
int((a+b/x)^n/x^5/(d*x+c),x)
```

3.291.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^5}\,dx$$

input
```
integrate((a+b/x)^n/x^5/(d*x+c),x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^n/(d*x^6 + c*x^5), x)
```

3.291.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{x^5\,(c+dx)}\,dx$$

input
```
integrate((a+b/x)**n/x**5/(d*x+c),x)
```

output
```
Integral((a + b/x)**n/(x**5*(c + d*x)), x)
```

3.291.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^5}\,dx$$

input
```
integrate((a+b/x)^n/x^5/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((a + b/x)^n/((d*x + c)*x^5), x)
```

3.291.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^5}\,dx$$

input
```
integrate((a+b/x)^n/x^5/(d*x+c),x, algorithm="giac")
```

output
```
integrate((a + b/x)^n/((d*x + c)*x^5), x)
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{x^5\,(c+d\,x)}\,dx$$

input
```
int((a + b/x)^n/(x^5*(c + d*x)),x)
```

output
```
int((a + b/x)^n/(x^5*(c + d*x)), x)
```

3.292 $\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2} \, dx$

3.292.1 Optimal result

Integrand size = 20, antiderivative size = 73

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2} \, dx = -\frac{\left(a+\frac{b}{x}\right)^n \left(1+\frac{b}{ax}\right)^{-n} x^{-1+m} \operatorname{AppellF1}\left(1-m,-n,2,2-m,-\frac{b}{ax},-\frac{c}{dx}\right)}{d^2(1-m)}$$

output
```
-(a+b/x)^n*x^(-1+m)*AppellF1(1-m,-n,2,2-m,-b/a/x,-c/d/x)/d^2/(1-m)/((1+b/a
/x)^n)
```

3.292.2 Mathematica [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2} \, dx = \int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2} \, dx$$

input
```
Integrate[((a + b/x)^n*x^m)/(c + d*x)^2,x]
```

output
```
Integrate[((a + b/x)^n*x^m)/(c + d*x)^2, x]
```

3.292.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1016, 999, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a+\frac{b}{x}\right)^n}{(c+dx)^2} \, dx$$

$$\Big\downarrow \text{1016}$$

$$\int \frac{x^{m-2} \left(a+\frac{b}{x}\right)^n}{\left(\frac{c}{x}+d\right)^2}dx$$

$$\Big\downarrow \text{999}$$

$$-\left(\frac{1}{x}\right)^m x^m \int \frac{\left(a+\frac{b}{x}\right)^n \left(\frac{1}{x}\right)^{-m}}{\left(\frac{c}{x}+d\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{152}$$

$$\left(\frac{1}{x}\right)^m x^m \left(-\left(a+\frac{b}{x}\right)^n\right) \left(\frac{b}{ax}+1\right)^{-n} \int \frac{\left(\frac{b}{ax}+1\right)^n \left(\frac{1}{x}\right)^{-m}}{\left(\frac{c}{x}+d\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{150}$$

$$-\frac{x^{m-1} \left(a+\frac{b}{x}\right)^n \left(\frac{b}{ax}+1\right)^{-n} \text{AppellF1}\left(1-m,-n,2,2-m,-\frac{b}{ax},-\frac{c}{dx}\right)}{d^2 (1-m)}$$

input
```
Int[((a + b/x)^n*x^m)/(c + d*x)^2,x]
```

output
```
-(((a + b/x)^n*x^(-1 + m)*AppellF1[1 - m, -n, 2, 2 - m, -(b/(a*x)), -(c/(d
*x))])/(d^2*(1 - m)*(1 + b/(a*x))^n))
```

3.292.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 999
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(-(e*x)^m)*(x^(-1))^m   Subst[Int[(a + b/x^n)^p*(
(c + d/x^n)^q/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q}
, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

3.292.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{n} x^{m}}{\left(dx+c\right)^{2}}dx$$

input
```
int((a+b/x)^n*x^m/(d*x+c)^2,x)
```

output
```
int((a+b/x)^n*x^m/(d*x+c)^2,x)
```

3.292.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(dx+c)^2}\,dx$$

input
```
integrate((a+b/x)^n*x^m/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(x^m*((a*x + b)/x)^n/(d^2*x^2 + 2*c*d*x + c^2), x)
```

3.292.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2}\,dx=\int \frac{x^m\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx$$

input
```
integrate((a+b/x)**n*x**m/(d*x+c)**2,x)
```

output
```
Integral(x**m*(a + b/x)**n/(c + d*x)**2, x)
```

3.292.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(dx+c)^2}\,dx$$

input
```
integrate((a+b/x)^n*x^m/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^n*x^m/(d*x + c)^2, x)
```

3.292.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{\left(dx+c\right)^2}\,dx$$

input
```
integrate((a+b/x)^n*x^m/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((a + b/x)^n*x^m/(d*x + c)^2, x)
```

3.292.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2}\,dx=\int \frac{x^m\left(a+\frac{b}{x}\right)^n}{\left(c+d\,x\right)^2}\,dx$$

input
```
int((x^m*(a + b/x)^n)/(c + d*x)^2,x)
```

output
```
int((x^m*(a + b/x)^n)/(c + d*x)^2, x)
```

3.293 $\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx$

3.293.1 Optimal result

Integrand size = 20, antiderivative size = 202

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx &= \frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{1+n}}{ad^2(ac-bd)\left(d+\frac{c}{x}\right)}+\frac{\left(a+\frac{b}{x}\right)^{1+n}x}{ad\left(d+\frac{c}{x}\right)}\\ &+\frac{c^2(2ac-bd(2-n))\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^3(ac-bd)^2(1+n)}\\ &-\frac{(2ac-bdn)\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b}{ax}\right)}{a^2d^3(1+n)}\end{aligned}$$

output
```
c*(2*a*c-b*d)*(a+b/x)^(1+n)/a/d^2/(a*c-b*d)/(d+c/x)+(a+b/x)^(1+n)*x/a/d/(d
+c/x)+c^2*(2*a*c-b*d*(2-n))*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/
x)/(a*c-b*d))/d^3/(a*c-b*d)^2/(1+n)-(-b*d*n+2*a*c)*(a+b/x)^(1+n)*hypergeom
([1, 1+n],[2+n],1+b/a/x)/a^2/d^3/(1+n)
```

3.293.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx$$
$$=\frac{\left(a+\frac{b}{x}\right)^{1+n}\left(acd(ac-bd)(2ac-bd)(1+n)x+ad^2(ac-bd)^2(1+n)x^2+(c+dx)\left(a^2c^2(2ac+bd(-2+\right.\right.}{a^2d^3(}$$

input
```
Integrate[((a + b/x)^n*x^2)/(c + d*x)^2,x]
```

output
```
((a + b/x)^(1 + n)*(a*c*d*(a*c - b*d)*(2*a*c - b*d)*(1 + n)*x + a*d^2*(a*c
 - b*d)^2*(1 + n)*x^2 + (c + d*x)*(a^2*c^2*(2*a*c + b*d*(-2 + n))*Hypergeo
metric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)] - (a*c - b*d)^2*(2*a
*c - b*d*n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)])))/(a^2*d^3*(a
*c - b*d)^2*(1 + n)*(c + d*x))
```

3.293.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1016, 899, 114, 168, 174, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx$$
$$\Big\downarrow \; 1016$$
$$\int \frac{\left(a+\frac{b}{x}\right)^n}{\left(\frac{c}{x}+d\right)^2}dx$$
$$\Big\downarrow \; 899$$
$$-\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{\left(\frac{c}{x}+d\right)^2}d\frac{1}{x}$$
$$\Big\downarrow \; 114$$

$$\frac{\int \frac{\left(a+\frac{b}{x}\right)^n \left(2ac+\frac{b(1-n)c}{x}-bdn\right)x}{\left(\frac{c}{x}+d\right)^2}d\frac{1}{x}}{ad}+\frac{x\left(a+\frac{b}{x}\right)^{n+1}}{ad\left(\frac{c}{x}+d\right)}$$

$$\Big\downarrow \text{168}$$

$$\frac{\frac{\int \frac{\left(a+\frac{b}{x}\right)^n \left((ac-bd)(2ac-bdn)-\frac{bc(2ac-bd)n}{x}\right)x}{\frac{c}{x}+d}d\frac{1}{x}}{d(ac-bd)}+\frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{n+1}}{d\left(\frac{c}{x}+d\right)(ac-bd)}}{ad}+\frac{x\left(a+\frac{b}{x}\right)^{n+1}}{ad\left(\frac{c}{x}+d\right)}$$

$$\Big\downarrow \text{174}$$

$$\frac{\frac{\frac{(ac-bd)(2ac-bdn)\int \left(a+\frac{b}{x}\right)^n xd\frac{1}{x}}{d}-\frac{ac^2(2ac-bd(2-n))\int \frac{\left(a+\frac{b}{x}\right)^n}{\frac{c}{x}+d}d\frac{1}{x}}{d}}{d(ac-bd)}+\frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{n+1}}{d\left(\frac{c}{x}+d\right)(ac-bd)}}{ad}+\frac{x\left(a+\frac{b}{x}\right)^{n+1}}{ad\left(\frac{c}{x}+d\right)}$$

$$\Big\downarrow \text{75}$$

$$\frac{\frac{-\frac{ac^2(2ac-bd(2-n))\int \frac{\left(a+\frac{b}{x}\right)^n}{\frac{c}{x}+d}d\frac{1}{x}}{d}-\frac{(ac-bd)\left(a+\frac{b}{x}\right)^{n+1}(2ac-bdn)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b}{ax}+1\right)}{ad(n+1)}}{d(ac-bd)}+\frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{n+1}}{d\left(\frac{c}{x}+d\right)(ac-bd)}}{ad}+\frac{x\left(a+\frac{b}{x}\right)^{n+1}}{ad\left(\frac{c}{x}+d\right)}$$

$$\Big\downarrow \text{78}$$

$$\frac{\frac{\frac{ac^2\left(a+\frac{b}{x}\right)^{n+1}(2ac-bd(2-n))\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(n+1)(ac-bd)}-\frac{(ac-bd)\left(a+\frac{b}{x}\right)^{n+1}(2ac-bdn)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b}{ax}+1\right)}{ad(n+1)}}{d(ac-bd)}+\frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{n+1}}{d\left(\frac{c}{x}+d\right)(ac-bd)}}{ad}+\frac{x\left(a+\frac{b}{x}\right)^{n+1}}{ad\left(\frac{c}{x}+d\right)}$$

input
```
Int[((a + b/x)^n*x^2)/(c + d*x)^2,x]
```

output
```
((a + b/x)^(1 + n)*x)/(a*d*(d + c/x)) + ((c*(2*a*c - b*d)*(a + b/x)^(1 + n
))/(d*(a*c - b*d)*(d + c/x)) + ((a*c^2*(2*a*c - b*d*(2 - n))*(a + b/x)^(1
+ n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(d*(a*
c - b*d)*(1 + n)) - ((a*c - b*d)*(2*a*c - b*d*n)*(a + b/x)^(1 + n)*Hyperge
ometric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)])/(a*d*(1 + n)))/(d*(a*c - b*d)))/
(a*d)
```

3.293. $\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx$

3.293.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && !IntegerQ[m] && IntegerQ[n]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.293. $\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2} \, dx$

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

3.293.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{n} x^{2}}{\left(dx+c\right)^{2}}dx$$

input
```
int((a+b/x)^n*x^2/(d*x+c)^2,x)
```

output
```
int((a+b/x)^n*x^2/(d*x+c)^2,x)
```

3.293.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^{n} x^{2}}{\left(dx+c\right)^{2}}\,dx$$

input
```
integrate((a+b/x)^n*x^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(x^2*((a*x + b)/x)^n/(d^2*x^2 + 2*c*d*x + c^2), x)
```

3.293.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx = \int \frac{x^{2}\left(a+\frac{b}{x}\right)^{n}}{\left(c+dx\right)^{2}}\,dx$$

input
```
integrate((a+b/x)**n*x**2/(d*x+c)**2,x)
```

output
```
Integral(x**2*(a + b/x)**n/(c + d*x)**2, x)
```

3.293.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(dx+c)^2}\,dx$$

input
```
integrate((a+b/x)^n*x^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^n*x^2/(d*x + c)^2, x)
```

3.293.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(dx+c)^2}\,dx$$

input
```
integrate((a+b/x)^n*x^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((a + b/x)^n*x^2/(d*x + c)^2, x)
```

3.293.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx = \int \frac{x^2\left(a+\frac{b}{x}\right)^n}{(c+d\,x)^2}\,dx$$

input
```
int((x^2*(a + b/x)^n)/(c + d*x)^2,x)
```

output
```
int((x^2*(a + b/x)^n)/(c + d*x)^2, x)
```

3.294 $\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2} \, dx$

3.294.1 Optimal result

Integrand size = 18, antiderivative size = 150

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2} \, dx &= -\frac{c\left(a+\frac{b}{x}\right)^{1+n}}{d(ac-bd)\left(d+\frac{c}{x}\right)} \\ &\quad - \frac{c(ac-bd(1-n))\left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(ac-bd)^2(1+n)} \\ &\quad + \frac{\left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b}{ax}\right)}{ad^2(1+n)}\end{aligned}$$

output
```
-c*(a+b/x)^(1+n)/d/(a*c-b*d)/(d+c/x)-c*(a*c-b*d*(1-n))*(a+b/x)^(1+n)*hyper
geom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/d^2/(a*c-b*d)^2/(1+n)+(a+b/x)^(1+
n)*hypergeom([1, 1+n],[2+n],1+b/a/x)/a/d^2/(1+n)
```

3.294.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2}\,dx = \frac{\left(a+\frac{b}{x}\right)^{1+n}\left(-\frac{cdx}{(ac-bd)(c+dx)}-\frac{c(ac+bd(-1+n))\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)^2(1+n)}+\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\right.}{a(1+n)}\right.}{d^2}$$

input
```
Integrate[((a + b/x)^n*x)/(c + d*x)^2,x]
```

output
```
((a + b/x)^(1 + n)*(-((c*d*x)/((a*c - b*d)*(c + d*x))) - (c*(a*c + b*d*(-1
 + n))*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/((a*
c - b*d)^2*(1 + n)) + Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)]/(a*(
1 + n))))/d^2
```

3.294.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1016, 948, 114, 174, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{1016}$$

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x\left(\frac{c}{x}+d\right)^2}dx$$

$$\Big\downarrow \text{948}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^n x}{\left(\frac{c}{x}+d\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{114}$$

$$-\frac{\int \frac{\left(a+\frac{b}{x}\right)^n \left(ac-\frac{bnc}{x}-bd\right)x}{\frac{c}{x}+d}d\frac{1}{x}}{d(ac-bd)}-\frac{c\left(a+\frac{b}{x}\right)^{n+1}}{d\left(\frac{c}{x}+d\right)(ac-bd)}$$

$$\Big\downarrow \text{174}$$

$$-\frac{\frac{(ac-bd)\int \left(a+\frac{b}{x}\right)^n xd\frac{1}{x}}{d}-\frac{c(ac-bd(1-n))\int \frac{\left(a+\frac{b}{x}\right)^n}{\frac{c}{x}+d}d\frac{1}{x}}{d}}{d(ac-bd)}-\frac{c\left(a+\frac{b}{x}\right)^{n+1}}{d\left(\frac{c}{x}+d\right)(ac-bd)}$$

$$\Big\downarrow \text{75}$$

$$-\frac{-\frac{c(ac-bd(1-n))\int \frac{\left(a+\frac{b}{x}\right)^n}{\frac{c}{x}+d}d\frac{1}{x}}{d}-\frac{(ac-bd)\left(a+\frac{b}{x}\right)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b}{ax}+1\right)}{ad(n+1)}}{d(ac-bd)}-\frac{c\left(a+\frac{b}{x}\right)^{n+1}}{d\left(\frac{c}{x}+d\right)(ac-bd)}$$

$$\Big\downarrow \text{78}$$

$$-\frac{\frac{c\left(a+\frac{b}{x}\right)^{n+1}(ac-bd(1-n))\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(n+1)(ac-bd)}-\frac{(ac-bd)\left(a+\frac{b}{x}\right)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b}{ax}+1\right)}{ad(n+1)}}{d(ac-bd)}-\frac{c\left(a+\frac{b}{x}\right)^{n+1}}{d\left(\frac{c}{x}+d\right)(ac-bd)}$$

input
```
Int[((a + b/x)^n*x)/(c + d*x)^2,x]
```

output
```
-((c*(a + b/x)^(1 + n))/(d*(a*c - b*d)*(d + c/x))) - ((c*(a*c - b*d*(1 - n
))*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c
 - b*d)])/(d*(a*c - b*d)*(1 + n)) - ((a*c - b*d)*(a + b/x)^(1 + n)*Hyperge
ometric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)])/(a*d*(1 + n)))/(d*(a*c - b*d))
```

3.294.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

3.294. $\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2}\,dx$

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

3.294.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{n} x}{\left(dx+c\right)^{2}}dx$$

input
```
int((a+b/x)^n*x/(d*x+c)^2,x)
```

output
```
int((a+b/x)^n*x/(d*x+c)^2,x)
```

3.294. $\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2}\,dx$

3.294.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2} \, dx = \int \frac{\left(a+\frac{b}{x}\right)^n x}{\left(dx+c\right)^2} \, dx$$

input
```
integrate((a+b/x)^n*x/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(x*((a*x + b)/x)^n/(d^2*x^2 + 2*c*d*x + c^2), x)
```

3.294.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2} \, dx = \int \frac{x\left(a+\frac{b}{x}\right)^n}{\left(c+dx\right)^2} \, dx$$

input
```
integrate((a+b/x)**n*x/(d*x+c)**2,x)
```

output
```
Integral(x*(a + b/x)**n/(c + d*x)**2, x)
```

3.294.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2} \, dx = \int \frac{\left(a+\frac{b}{x}\right)^n x}{\left(dx+c\right)^2} \, dx$$

input
```
integrate((a+b/x)^n*x/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^n*x/(d*x + c)^2, x)
```

3.294.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n x}{(dx+c)^2}\,dx$$

input
```
integrate((a+b/x)^n*x/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((a + b/x)^n*x/(d*x + c)^2, x)
```

3.294.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2}\,dx=\int \frac{x\left(a+\frac{b}{x}\right)^n}{(c+d\,x)^2}\,dx$$

input
```
int((x*(a + b/x)^n)/(c + d*x)^2,x)
```

output
```
int((x*(a + b/x)^n)/(c + d*x)^2, x)
```

3.295 $\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2} dx$

3.295.1 Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2} dx = -\frac{b\left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)^2(1+n)}$$

output
```
-b*(a+b/x)^(1+n)*hypergeom([2, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/(a*c-b*d)^2
/(1+n)
```

3.295.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2} dx = -\frac{b\left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left(2,1+n,2+n,-\frac{c\left(a+\frac{b}{x}\right)}{-ac+bd}\right)}{(-ac+bd)^2(1+n)}$$

input
```
Integrate[(a + b/x)^n/(c + d*x)^2,x]
```

output
```
-((b*(a + b/x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, -((c*(a + b/x))/
(-(a*c) + b*d))])/((-(a*c) + b*d)^2*(1 + n)))
```

3.295.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {941, 946, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2} \, dx$$

$$\Big\downarrow \text{941}$$

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2\left(\frac{c}{x}+d\right)^2}dx$$

$$\Big\downarrow \text{946}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^n}{\left(\frac{c}{x}+d\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{78}$$

$$-\frac{b\left(a+\frac{b}{x}\right)^{n+1} \text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)^2}$$

input
```
Int[(a + b/x)^n/(c + d*x)^2,x]
```

output
```
-((b*(a + b/x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (c*(a + b/x))/(a
*c - b*d)])/((a*c - b*d)^2*(1 + n)))
```

3.295.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.295.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{\left(dx+c\right)^{2}}dx$$

input
```
int((a+b/x)^n/(d*x+c)^2,x)
```

output
```
int((a+b/x)^n/(d*x+c)^2,x)
```

3.295.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{n}}{\left(dx+c\right)^{2}}\,dx$$

input
```
integrate((a+b/x)^n/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^n/(d^2*x^2 + 2*c*d*x + c^2), x)
```

3.295.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(c+dx\right)^2}\,dx$$

input
```
integrate((a+b/x)**n/(d*x+c)**2,x)
```

output
```
Integral((a + b/x)**n/(c + d*x)**2, x)
```

3.295.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(dx+c\right)^2}\,dx$$

input
```
integrate((a+b/x)^n/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^n/(d*x + c)^2, x)
```

3.295.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(dx+c\right)^2}\,dx$$

input
```
integrate((a+b/x)^n/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((a + b/x)^n/(d*x + c)^2, x)
```

3.295.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{\left(c+d\,x\right)^2}\,dx$$

input
```
int((a + b/x)^n/(c + d*x)^2,x)
```

output
```
int((a + b/x)^n/(c + d*x)^2, x)
```

3.296 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}\,dx$

3.296.1 Optimal result

Integrand size = 20, antiderivative size = 105

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}\,dx = -\frac{d\left(a+\frac{b}{x}\right)^{1+n}}{c(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{(ac-bd(1+n))\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(ac-bd)^2(1+n)}$$

output
```
-d*(a+b/x)^(1+n)/c/(a*c-b*d)/(d+c/x)+(a*c-b*d*(1+n))*(a+b/x)^(1+n)*hyperge
om([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/c/(a*c-b*d)^2/(1+n)
```

3.296.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}\,dx = \frac{\left(a+\frac{b}{x}\right)^{1+n}\left(\frac{d(-ac+bd)x}{c+dx} + \frac{(ac-bd(1+n))\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{1+n}\right)}{c(ac-bd)^2}$$

input
```
Integrate[(a + b/x)^n/(x*(c + d*x)^2),x]
```

output
```
((a + b/x)^(1 + n)*((d*(-(a*c) + b*d)*x)/(c + d*x) + ((a*c - b*d*(1 + n))*
Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(1 + n)))/(
c*(a*c - b*d)^2)
```

3.296.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1016, 948, 87, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}dx$$

$$\Big\downarrow \text{1016}$$

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3\left(\frac{c}{x}+d\right)^2}dx$$

$$\Big\downarrow \text{948}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^n}{\left(\frac{c}{x}+d\right)^2x}d\frac{1}{x}$$

$$\Big\downarrow \text{87}$$

$$-\frac{(ac-bd(n+1))\int \frac{\left(a+\frac{b}{x}\right)^n}{\frac{c}{x}+d}d\frac{1}{x}}{c(ac-bd)}-\frac{d\left(a+\frac{b}{x}\right)^{n+1}}{c\left(\frac{c}{x}+d\right)(ac-bd)}$$

$$\Big\downarrow \text{78}$$

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}(ac-bd(n+1))\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(n+1)(ac-bd)^2}-\frac{d\left(a+\frac{b}{x}\right)^{n+1}}{c\left(\frac{c}{x}+d\right)(ac-bd)}$$

input
```
Int[(a + b/x)^n/(x*(c + d*x)^2),x]
```

output
```
-((d*(a + b/x)^(1 + n))/(c*(a*c - b*d)*(d + c/x))) + ((a*c - b*d*(1 + n))*
(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c -
b*d)])/(c*(a*c - b*d)^2*(1 + n))
```

3.296.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

3.296.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{x \left(d x +c \right)^{2}}d x$$

input
```
int((a+b/x)^n/x/(d*x+c)^2,x)
```

output
```
int((a+b/x)^n/x/(d*x+c)^2,x)
```

3.296. $\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2} dx$

3.296.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{\left(dx+c\right)^2x}\,dx$$

input
```
integrate((a+b/x)^n/x/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^n/(d^2*x^3 + 2*c*d*x^2 + c^2*x), x)
```

3.296.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{x\left(c+dx\right)^2}\,dx$$

input
```
integrate((a+b/x)**n/x/(d*x+c)**2,x)
```

output
```
Integral((a + b/x)**n/(x*(c + d*x)**2), x)
```

3.296.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{\left(dx+c\right)^2x}\,dx$$

input
```
integrate((a+b/x)^n/x/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^n/((d*x + c)^2*x), x)
```

3.296.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)^2x}\,dx$$

input
```
integrate((a+b/x)^n/x/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((a + b/x)^n/((d*x + c)^2*x), x)
```

3.296.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{x\,(c+d\,x)^2}\,dx$$

input
```
int((a + b/x)^n/(x*(c + d*x)^2),x)
```

output
```
int((a + b/x)^n/(x*(c + d*x)^2), x)
```

3.297 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx$

3.297.1 Optimal result

Integrand size = 20, antiderivative size = 133

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx = -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc^2(1+n)}+\frac{d^2\left(a+\frac{b}{x}\right)^{1+n}}{c^2(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{d(2ac-bd(2+n))\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(ac-bd)^2(1+n)}$$

output
```
-(a+b/x)^(1+n)/b/c^2/(1+n)+d^2*(a+b/x)^(1+n)/c^2/(a*c-b*d)/(d+c/x)-d*(2*a*
c-b*d*(2+n))*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/c
^2/(a*c-b*d)^2/(1+n)
```

3.297.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx = -\frac{\left(a+\frac{b}{x}\right)^n(b+ax)\left((ac-bd)(ac(c+dx)-bd(c+d(2+n)x))+bd(2ac-bd(2+n))(c+dx)\,\text{Hyperge}\right.}{bc^2(ac-bd)^2(1+n)x(c+dx)}$$

input
```
Integrate[(a + b/x)^n/(x^2*(c + d*x)^2),x]
```

output
```
-(((a + b/x)^n*(b + a*x)*((a*c - b*d)*(a*c*(c + d*x) - b*d*(c + d*(2 + n)*
x)) + b*d*(2*a*c - b*d*(2 + n))*(c + d*x)*Hypergeometric2F1[1, 1 + n, 2 +
n, (c*(a + b/x))/(a*c - b*d)]))/(b*c^2*(a*c - b*d)^2*(1 + n)*x*(c + d*x)))
```

3.297.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1016, 948, 100, 25, 90, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}dx$$

$$\Big\downarrow \text{ 1016}$$

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^4\left(\frac{c}{x}+d\right)^2}dx$$

$$\Big\downarrow \text{ 948}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^n}{\left(\frac{c}{x}+d\right)^2x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 100}$$

$$\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}}{c^2\left(\frac{c}{x}+d\right)(ac-bd)}-\frac{\int -\frac{\left(a+\frac{b}{x}\right)^n\left(d(ac-bd(n+1))-\frac{c(ac-bd)}{x}\right)}{\frac{c}{x}+d}d\frac{1}{x}}{c^2(ac-bd)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\left(a+\frac{b}{x}\right)^n\left(d(ac-bd(n+1))-\frac{c(ac-bd)}{x}\right)}{\frac{c}{x}+d}d\frac{1}{x}}{c^2(ac-bd)}+\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}}{c^2\left(\frac{c}{x}+d\right)(ac-bd)}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{d(2ac-bd(n+2))\int \frac{\left(a+\frac{b}{x}\right)^n}{\frac{c}{x}+d}d\frac{1}{x}-\frac{(ac-bd)\left(a+\frac{b}{x}\right)^{n+1}}{b(n+1)}}{c^2(ac-bd)}+\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}}{c^2\left(\frac{c}{x}+d\right)(ac-bd)}$$

$$\Big\downarrow 78$$

$$\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}}{c^2\left(\frac{c}{x}+d\right)(ac-bd)}+\frac{-\frac{d\left(a+\frac{b}{x}\right)^{n+1}(2ac-bd(n+2))\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)}-\frac{(ac-bd)\left(a+\frac{b}{x}\right)^{n+1}}{b(n+1)}}{c^2(ac-bd)}$$

input
```
Int[(a + b/x)^n/(x^2*(c + d*x)^2),x]
```

output
```
(d^2*(a + b/x)^(1 + n))/(c^2*(a*c - b*d)*(d + c/x)) + (-(((a*c - b*d)*(a +
 b/x)^(1 + n))/(b*(1 + n))) - (d*(2*a*c - b*d*(2 + n))*(a + b/x)^(1 + n)*H
ypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/((a*c - b*d)
*(1 + n)))/(c^2*(a*c - b*d))
```

3.297.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

3.297. $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx$

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

3.297.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{x^{2} \left(d x+c\right)^{2}}d x$$

input
```
int((a+b/x)^n/x^2/(d*x+c)^2,x)
```

output
```
int((a+b/x)^n/x^2/(d*x+c)^2,x)
```

3.297.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\, dx = \int \frac{\left(a+\frac{b}{x}\right)^{n}}{\left(dx+c\right)^{2} x^{2}}\, dx$$

input
```
integrate((a+b/x)^n/x^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^n/(d^2*x^4 + 2*c*d*x^3 + c^2*x^2), x)
```

3.297.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2\left(c+dx\right)^2}\,dx$$

input
```
integrate((a+b/x)**n/x**2/(d*x+c)**2,x)
```

output
```
Integral((a + b/x)**n/(x**2*(c + d*x)**2), x)
```

3.297.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{\left(dx+c\right)^2x^2}\,dx$$

input
```
integrate((a+b/x)^n/x^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^n/((d*x + c)^2*x^2), x)
```

3.297.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{\left(dx+c\right)^2x^2}\,dx$$

input
```
integrate((a+b/x)^n/x^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((a + b/x)^n/((d*x + c)^2*x^2), x)
```

3.297.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((a + b/x)^n/(x^2*(c + d*x)^2),x)
```

output
```
int((a + b/x)^n/(x^2*(c + d*x)^2), x)
```

3.298 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx$

3.298.1 Optimal result

Integrand size = 20, antiderivative size = 217

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx = -\frac{\left(a+\frac{b}{x}\right)^{1+n}\left(d(bd(2+n)(ac+bd(3+n))-ac(ac+bd(5+3n)))-\frac{c(ac-bd)(ac+bd(3+n))}{x}\right)}{b^2c^3(ac-bd)(1+n)(2+n)\left(d+\frac{c}{x}\right)} - \frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc(2+n)\left(d+\frac{c}{x}\right)x^2} + \frac{d^2(3ac-bd(3+n))\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^3(ac-bd)^2(1+n)}$$

output
```
-(a+b/x)^(1+n)*(d*(b*d*(2+n)*(a*c+b*d*(3+n))-a*c*(a*c+b*d*(5+3*n)))-c*(a*c
-b*d)*(a*c+b*d*(3+n))/x)/b^2/c^3/(a*c-b*d)/(1+n)/(2+n)/(d+c/x)-(a+b/x)^(1+
n)/b/c/(2+n)/(d+c/x)/x^2+d^2*(3*a*c-b*d*(3+n))*(a+b/x)^(1+n)*hypergeom([1,
 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/c^3/(a*c-b*d)^2/(1+n)
```

3.298.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx$$
$$= \frac{\left(a+\frac{b}{x}\right)^{1+n}\left(-\frac{1}{x(c+dx)}+\frac{-a^2c^2(c+dx)+b^2d^2(3+n)(c+d(2+n)x)-abcd(c(2+n)+d(3+2n)x)}{bc^2(-ac+bd)(1+n)(c+dx)}-\frac{bd^2(2+n)(-3ac+bd(3+n))\,\text{Hypergeon}}{c^2(ac-bd)^2}\right.}{bc(2+n)}$$

input
```
Integrate[(a + b/x)^n/(x^3*(c + d*x)^2),x]
```

output
```
((a + b/x)^(1 + n)*(-(1/(x*(c + d*x))) + (-(a^2*c^2*(c + d*x)) + b^2*d^2*(
3 + n)*(c + d*(2 + n)*x) - a*b*c*d*(c*(2 + n) + d*(3 + 2*n)*x))/(b*c^2*(-(
a*c) + b*d)*(1 + n)*(c + d*x)) - (b*d^2*(2 + n)*(-3*a*c + b*d*(3 + n))*Hyp
ergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(c^2*(a*c - b*
d)^2*(1 + n))))/(b*c*(2 + n))
```

3.298.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1016, 948, 111, 25, 163, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx$$
$$\Big\downarrow \text{1016}$$
$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5\left(\frac{c}{x}+d\right)^2}dx$$
$$\Big\downarrow \text{948}$$
$$-\int \frac{\left(a+\frac{b}{x}\right)^n}{\left(\frac{c}{x}+d\right)^2x^3}d\frac{1}{x}$$
$$\Big\downarrow \text{111}$$

$$-\frac{\int -\frac{\left(a+\frac{b}{x}\right)^n\left(2ad+\frac{ac+bd(n+3)}{x}\right)}{\left(\frac{c}{x}+d\right)^2x}d\frac{1}{x}}{bc(n+2)}-\frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc(n+2)x^2\left(\frac{c}{x}+d\right)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{\left(a+\frac{b}{x}\right)^n\left(2ad+\frac{ac+bd(n+3)}{x}\right)}{\left(\frac{c}{x}+d\right)^2x}d\frac{1}{x}}{bc(n+2)}-\frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc(n+2)x^2\left(\frac{c}{x}+d\right)}$$

$\Big\downarrow$ 163

$$\frac{-\frac{bd^2(n+2)(3ac-bd(n+3))\int \frac{\left(a+\frac{b}{x}\right)^n}{\frac{c}{x}+d}d\frac{1}{x}}{c^2(ac-bd)}-\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(d(bd(n+2)(ac+bd(n+3))-ac(ac+bd(3n+5)))-\frac{c(ac-bd)(ac+bd(n+3))}{x}\right)}{bc^2(n+1)\left(\frac{c}{x}+d\right)(ac-bd)}}{bc(n+2)}-\frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc(n+2)x^2\left(\frac{c}{x}+d\right)}$$

$\Big\downarrow$ 78

$$\frac{\frac{bd^2(n+2)\left(a+\frac{b}{x}\right)^{n+1}(3ac-bd(n+3))\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(n+1)(ac-bd)^2}-\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(d(bd(n+2)(ac+bd(n+3))-ac(ac+bd(3n+5)))-\right.}{bc^2(n+1)\left(\frac{c}{x}+d\right)(ac-bd)}}{bc(n+2)}$$

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc(n+2)x^2\left(\frac{c}{x}+d\right)}$$

input
```
Int[(a + b/x)^n/(x^3*(c + d*x)^2),x]
```

output
```
-((a + b/x)^(1 + n)/(b*c*(2 + n)*(d + c/x)*x^2)) + (-(((a + b/x)^(1 + n)*(
d*(b*d*(2 + n)*(a*c + b*d*(3 + n)) - a*c*(a*c + b*d*(5 + 3*n))) - (c*(a*c
- b*d)*(a*c + b*d*(3 + n)))/x))/(b*c^2*(a*c - b*d)*(1 + n)*(d + c/x))) + (
b*d^2*(2 + n)*(3*a*c - b*d*(3 + n))*(a + b/x)^(1 + n)*Hypergeometric2F1[1,
 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(c^2*(a*c - b*d)^2*(1 + n)))/(b
*c*(2 + n))
```

3.298. $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx$

3.298.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

3.298. $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx$

3.298.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{x^{3} \left(d x +c \right)^{2}}d x$$

input
```
int((a+b/x)^n/x^3/(d*x+c)^2,x)
```

output
```
int((a+b/x)^n/x^3/(d*x+c)^2,x)
```

3.298.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\, dx = \int \frac{\left(a+\frac{b}{x}\right)^{n}}{{\left(dx+c\right)}^{2} x^{3}}\, dx$$

input
```
integrate((a+b/x)^n/x^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^n/(d^2*x^5 + 2*c*d*x^4 + c^2*x^3), x)
```

3.298.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\, dx = \int \frac{\left(a+\frac{b}{x}\right)^{n}}{x^{3} \left(c+dx\right)^{2}}\, dx$$

input
```
integrate((a+b/x)**n/x**3/(d*x+c)**2,x)
```

output
```
Integral((a + b/x)**n/(x**3*(c + d*x)**2), x)
```

3.298.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)^2x^3}\,dx$$

input
```
integrate((a+b/x)^n/x^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^n/((d*x + c)^2*x^3), x)
```

3.298.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)^2x^3}\,dx$$

input
```
integrate((a+b/x)^n/x^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((a + b/x)^n/((d*x + c)^2*x^3), x)
```

3.298.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{x^3\,(c+d\,x)^2}\,dx$$

input
```
int((a + b/x)^n/(x^3*(c + d*x)^2),x)
```

output
```
int((a + b/x)^n/(x^3*(c + d*x)^2), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```